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Dynamical Symmetry and Truncation of the Spherical Shell Model

Mike Guidry

Department of Physics, University of Tennessee,
Knoxville, TN 37996-1200 USA

and

Physics Division, Oak Ridge National Laboratory,
Oak Ridge, TN 37831 USA

The Fermion Dynamical Symmetry Model is used to introduce the idea of solving the shell model problem by means of a symmetry-dictated truncation. Several calculations are presented suggesting that such truncations can lead to a model space in which a variety of nuclear structure phenomena are described economically using a simple effective interaction.

1. INTRODUCTION

The shell model is the most fundamental tractible theory of nuclear structure, but it cannot be used for systems with many valence particles outside closed shells. There are two basic reasons for this.

1. The matrix dimensionalities are too large for even the best modern computers and algorithms to handle.
2. There are too many effective interactions parameters.

The first problem is well known; the second is just as important, but seems to be far less appreciated [1, 2]. We may illustrate this second problem by noting that a Wildenthal type approach to the heaviest nuclei, where one views the effective interaction for a major shell of neutrons and of protons to be specified by a set of matrix elements to be determined by fits to existing data, would require a *minimum* of about 2500 parameters (matrix elements of allowed one and two-body interactions) to be determined from some combination of theory and data. By comparison, the same approach in the sd shell requires almost two orders of magnitude fewer parameters.

Thus, in addition to solving the matrix dimensionality problem, one needs a consistent way to select a highly restricted subset of the effective interaction parameters as the ones relevant for low energy structure. That such a restricted set exists is suggested strongly by the relative simplicity of observed low-energy nuclear structure, but the crucial question is how to select, from the thousands of allowed one and two-body scatterings, the relatively small number of favored linear combinations that (one suspects) are responsible for the dominant features of low-energy nuclear structure. Ideally, one would like an approach that accomplishes both of these tasks in a self-consistent manner. A traditional prescription is to use some form of energy-dictated truncation. This restricts the size of the shell model space, thereby helping with the matrix dimensionality problem, and at the same time limiting the number of parameters that must

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Shell Model Truncations

Example: Energy-Dictated Truncation

SU_2 Generators: $\{J_1, J_2, J_3\}$

$$H(SU_2) \approx aJ^2$$

Example: Symmetry-Dictated Truncation

$$H(SU_2 \supset U_1) \approx aJ^2 + bJ_3^2$$

Selection of Directions in Space
Spontaneous Symmetry Breaking
Phase Transitions

Figure 1. An illustration of energy-dictated and symmetry-dictated truncations for a simple symmetry.

be determined. Indeed, the restriction of traditional shell model calculations to a single major shell is an example of an energy-dictated truncation. However, energy-dictated truncations have not been very successful for highly collective nuclei far removed from closed shells. Although the influence of any single high-lying configuration may be negligible, the coherent nature of collective excitations implies that one cannot ensure that the aggregate contribution of many high-lying configurations to the collective subspace may be neglected.

2. THE SHELL MODEL PROBLEM

There are three modern approaches to this problem of trying to extend the shell model to the description of heavy nuclei far removed from closed shells.

1. Improved algorithms and computers for traditional shell models. This approach is exemplified by the new shell model code DUSM developed by Vallieres and coworkers, and is discussed by Michel Vallieres in a separate paper contained in this volume.
2. Path integral solutions of the shell model using Monte Carlo algorithms on fast supercomputers. The talk by Erich Ormand in these proceedings illustrates this approach.
3. Symmetry-dictated truncations of the shell model space.

It is this latter topic that I wish to discuss in this paper. Let us note that approaches (2) and (3) have had some success with the matrix dimensionality problem, either by attacking it more efficiently, or by avoiding it altogether in the path integral approach. However, neither of these

methods offers an intrinsic solution to the proliferation of effective interaction parameters in the heavy nuclei. One must supplement these approaches with a prescription for selecting the components of the interaction to emphasize.

On the other hand, the symmetry-dictated truncation that I will discuss here offers an integrated solution to the problem: (1) the symmetries dictate a severe truncation of the shell model space, thus taming the matrix dimensionality problem; (2) the requirement that the dominant interactions respect these symmetries provides a methodology for selecting a limited subset of interactions, thereby severely restricting the number of parameters that must be determined. Whether such an integrated approach is correct depends on the ability of the resulting theory to describe a broad range of nuclear structure data. Thus, in this paper I will introduce the basic ideas of symmetry-dictated truncation, illustrate some simple effective interactions resulting from such an approach, and survey some comparisons of such calculations with data.

3. SYMMETRY-DICTATED TRUNCATION

The idea of symmetry-dictated truncation is illustrated for a simple model assuming an SU_2 symmetry in Fig. 1, and for a shell model dynamical symmetry in Fig. 2. Stated somewhat loosely, the essential idea of an energy-dictated truncation is to truncate the space "spherically" in the space of symmetry generators (for example, the three angular momentum components in Fig. 1), but the idea of a symmetry-dictated truncation is to truncate the space by selecting a particular direction or set of directions in the space of symmetry generators to single out for emphasis. These directions are indicated schematically in Fig. 1 and Fig. 2 by the heavy arrows. Since such an approach selects particular directions in the space, it is closely associated with spontaneous symmetry breaking and phase transitions.

4. DETERMINATION OF EFFECTIVE INTERACTIONS PARAMETERS

Let us now examine some fdsm calculations for rare earth nuclei using the code fdu0 developed by Wu and Vallieres [3]. For notational convenience, the Hamiltonian may be expressed as [3, 1]

$$\begin{aligned}
 H = \sum_{\sigma=\pi,\nu} & \left(E^\sigma J^\sigma (J^\sigma + 1) + B_2^\sigma P_\sigma^2 \cdot P_\sigma^2 \right. \\
 & + B_3^\sigma P_3^\sigma \cdot P_3^\sigma + G_0^\sigma S_\sigma^\dagger S_\sigma + G_2^\sigma D_\sigma^\dagger D_\sigma \\
 & + E^{\pi\nu} J^\pi \cdot J^\nu + B_2^{\pi\nu} P_2^\pi \cdot P_2^\nu + B_3^{\pi\nu} P_3^\pi \cdot P_3^\nu \\
 & \left. + B_1^\nu P_1^\nu \cdot P_1^\nu + B_1^\pi P_1^\pi \cdot P_1^\pi + B_1^{\pi\nu} P_1^\nu \cdot P_1^\pi + E_J J(J+1) \right) \quad (1)
 \end{aligned}$$

where generally S denotes monopole pair operators, D denotes D -pair operators, and P_r denotes multipole operators of order r , with all quantities constructed in the $k-i$ truncation scheme illustrated in Fig. 3. However, in this form not all parameters are independent and not all contribute for a particular highest symmetry: only one of (E^σ, B_1^σ) and one of $(E_J, E^{\pi\nu}, B_1^{\pi\nu})$ are independent, while B_3^σ and $B_3^{\pi\nu}$ only contribute for $SO(8)$ shells. Thus, there are at most 13 parameters for $SO_8^\pi \times SO_8^\nu$, 10 for $Sp_6^\pi \times Sp_6^\nu$, and 11 for $SO_8^\pi \times Sp_6^\nu$.

In Fig. 4 we display some parameters of the fdsm effective interaction as determined by a systematic fit to the lowest energy states of isotopes of Gd, Dy, and Er. An example of such fits is shown in Fig. 5 for three Gd isotopes that span the range from near vibrational to very well deformed. Some $B(E2)$ values calculated from the corresponding wavefunctions are shown in Fig. 6.

Selection of Directions in Space Spontaneous Symmetry Breaking Phase Transitions

FDSM Coupling Scheme

$$\vec{j} = \vec{k} + \vec{i}$$

$$a_{jm}^\dagger = \sum \langle km_k im_i | jm \rangle b_{jm_k im_i}^\dagger$$

- The essence of the FDSM method is the selection of a collective subspace by the k - i truncation
- There is no direct reference to the nature of the larger single-particle space; in principle, many spaces could be truncated in this way

Figure 3. The fdsm coupling scheme, which is a generalization of the original Ginocchio [4] coupling scheme. More details may be found in [5] and [1].

Figure 4. fdsm effective interaction parameters as determined by fits to 16 low-lying energy levels of Gd, Dy, and Er isotopes using the code `fdv0`.

Finally, Fig. 7 shows a detailed spectrum and relative transition rates for the nucleus ^{196}Pt that is taken from reference [6]. In this case only 5 of the effective interaction parameters have been allowed to vary in the fit to the spectrum. Remarkably, the corresponding parameters describe the SO_6 behavior of the platinum region and at the same time are closely related to sets that describe data in the beginning and middle of the rare earth region where the structure is SU_3 or vibrational in nature. Thus, there are strong indications that it will be possible to obtain a satisfactory description of the entire rare-earth region with a single set of effective interaction parameters varying in a simple way with particle number.

5. SUMMARY

The Fermion Dynamical Symmetry Model (fdsm) provides a systematic method for truncating the spherical shell model. In this scheme, a valence space is selected using energy considerations and principles of dynamical symmetry are then used to radically truncate the valence space. We term this a symmetry-dictated truncation. The resulting truncated space presents the possibility of systematic shell model calculations for all heavy nuclei. Since the space has been severely truncated, the corresponding interactions are highly effective with respect to the original shell model. Thus, the first step in systematic calculations with this truncation scheme is to determine the appropriate fdsm effective interaction for each valence space. Although it is of considerable interest in the longer term to relate such effective interactions to standard shell model ones, the most practical initial way to determine the required interaction is to construct it phenomenologically by taking its matrix elements as parameters to be constrained by a carefully chosen data set. I have presented examples of systematic fdsm calculations that have been used to determine an effective interaction appropriate for configurations in heavy nuclei with no broken pairs. This interaction is simple and has a rather weak dependence on particle number within major-shell valence spaces. Calculations using this interaction reproduce low-lying spectra, moments, and

Figure 5. Calculated and observed spectra for some Gd, Dy, and Er isotopes using the parameters in Fig. 4. The energy scales for the three cases are somewhat different.

transition rates for broad ranges of nuclei that exhibit varied collective behavior: axial rotors, anharmonic vibrators, and gamma-soft rotors. These results constitute a practical demonstration that systematic shell model calculations are now feasible for very heavy nuclei far removed from closed shells.

REFERENCES

1. C.-L. Wu, D. H. Feng, and M. W. Guidry, in press, *Advances in Nucl. Phys.*
2. M. W. Guidry in *Proceedings of First Symposium on Nuclear Physics in the Universe*, edited by M. W. Guidry and M. R. Strayer, Elsevier (1993).
3. H. Wu and M. Vallieres, *Phys. Rev. C* 39 (1989) 1066.
4. J. Ginocchio, *Ann. Phys.* 126 (1980) 234.
5. C.-L. Wu, D. H. Feng, X.-G. Chen, J.-Q. Chen, and M. W. Guidry, *Phys. Rev. C* 36 (1987) 1157.

Figure 6. Calculated and observed $B(E2)$ values for some Gd, Dy, and Er isotopes using the parameters in Fig. 4.

6. D. H. Feng, M. W. Guidry, X.-W. Pan, C.-L. Wu, and I. Zlatev, Phys. Rev. C48 (1993) R1488.
7. J. A. Cizewski, R. F. Casten, G. J. Smith, M. L. Stelts, and W. R. Kane, Phys. Rev. Lett. 40 (1978) 167.

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Figure 7. Level scheme for positive-parity states in ^{196}Pt . Experimental levels are taken from [7]. The theoretical levels are from the fdsm calculation using the fdm0 code. The parameters are $G_{0\nu} = -48$, $G_{0\pi} = -65$, $B_{2np} = -300$, $B_{2\nu} = 66$, $B_{2\pi} = 32$, with all units in keV. The upper number on the transition arrows is the measured relative $B(E2)$ value; the middle number is the ibm-1 predicted value; the lowest number on each transition arrow is the fdsm prediction. There are only two transitions where the fdsm calculation does not agree with the ibm-1 prediction: the $2_4^+ \rightarrow 3_1^+$ and $0_3^+ \rightarrow 2_2^+$ transitions are forbidden in the ibm-1, but not in the fdsm. For weak $\Delta\tau = 0, \pm 2$ transitions the fdsm results agree with data quite well. An additional $d^\dagger \tilde{d}$ term is required in the $E2$ operator to reproduce these transitions with ibm-1.

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