Intrabeam Scattering in the HEB

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Abstract

A study of Intrabeam Scattering (IBS) in the High Energy Booster (HEB) is presented. Piwinski's formulas for IBS are presented and evaluated for the HEB. A computer code written to evaluate Piwinski's formulas is discussed. The result of the study is that IBS should not be a problem for the HEB, although the safety factor is not enormous.

Key Words: Intrabeam Scattering, IBS, Piwinski, High Energy Booster, HEB.

1.0 PURPOSE OF THIS REPORT

The purpose of this report is to evaluate the expected intrabeam scattering (IBS) rates associated with the High Energy Booster (HEB) design parameters.

2.0 PIWINSKI'S IBS FORMULAS

Piwinski¹ has generated equations for the determination of intrabeam scattering, and he has pointed out to Evans and Zotter² that the original formulas need to be multiplied by a factor of 1/2, leaving:

$$1/\tau_h = (1/2\sigma_h^2)(d\sigma_h^2/dt) = jA(1 - [D^2\sigma_h^2/\sigma_x^2])f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y); \quad (1)$$

$$\begin{aligned} 1/\tau_{x'} &= (1/2\sigma_{x'}^2)(d\sigma_{x'}^2/dt) \\ &= A[f(\sigma_{x'}/\sigma_y, \sigma_{x'}/\sigma_z, q\sigma_{x'}) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)]; \end{aligned} \quad (2)$$

$$1/\tau_{z'} = (1/2\sigma_{z'}^2)(d\sigma_{z'}^2/dt) = Af(\sigma_{z'}/\sigma_y, \sigma_{z'}/\sigma_{x'}, q\sigma_{z'}). \quad (3)$$

In the above equations Piwinski defines

$$A = (cr_0^2 \Lambda N_b) / (32\pi^{3/2} \sigma_h \sigma_x \beta \sigma_{x'} \sigma_z \sigma_{z'} \beta^3 \gamma^4); \quad (4)$$

$$\sigma_y = \sigma_h \sigma_x \beta / \gamma \sigma_{x'}; q = \beta \gamma (2b_0/r_0)^{1/2}; \text{ and } b_0 = 0.5(\gamma/n_p)^{1/3}. \quad (5a)$$

$$\Lambda = N_b/2\pi^{1/2}\sigma_s \text{ and } j = 1 \text{ for bunched beams.} \quad (5b)$$

$$\Lambda = N/C \text{ and } j = 2 \text{ for unbunched beams.} \quad (5c)$$

In the above expressions σ corresponds to the standard deviation of the quantity subscripted, x and x' are the horizontal half beam size and divergence respectively, z and z' are the vertical half beam size and divergence respectively, h is equal to the relative momentum spread, $h = \Delta p/p$, D is the value of the dispersion, r_0 is the classical radius of the stored particle (protons here), N_b is the number of particles in each bunch, N is the total number of particles stored in the ring, n_p is the density of the particles stored in the beam, C is the circumference of the ring, and σ_s is the one standard deviation bunch length.

Piwinski³ defines $f(a,b,c)$ as:

$$f(a,b,c) = 8\pi \int_0^1 dx (1 - 3x^2)(1/pq) [\ln(0.5c^2(1/q+1/p) - 0.577 \dots)] \quad (6)$$

where

$$p = [a^2 + x^2(1 - a^2)]^{1/2} \text{ and } q = [b^2 + x^2(1 - b^2)]^{1/2}. \quad (7)$$

A computer code written for this report calculates the relations used in the IBS equations as well as $f(a,b,c)$. The code is discussed in Appendixes B and C below.

3.0 CALCULATION OF IBS DURING HEB INJECTION FLAT, $\epsilon_L = 0.1$ eV-SECONDS

By noting that $\epsilon_{nx} = \beta\gamma\pi\sigma_x\beta\sigma_x$, and $\epsilon_{nz} = \beta\gamma\pi\sigma_z\sigma_z$, Eq. (4) can be re-expressed for bunched beams as

$$A = (cr_0^2 N_b) / (64\epsilon_{nx}\epsilon_{nz}\sigma_h\sigma_s\beta\gamma^2). \quad (8)$$

With $\epsilon_L = 0.1$ eV-seconds and $N_b = 1 \times 10^{10}$, $\epsilon_{nx} = \epsilon_{nz} = 0.7\pi$ mm-mr, $\sigma_s = 0.3$ m, $\sigma_\eta = 2.7 \times 10^{-5}$ and $\gamma = 200$ at injection,

$$A = 7.00 \times 10^{-8} \text{ s}^{-1} \text{ (at injection)}. \quad (9)$$

The IBS Eqs. (1), (2) and (3) involve the function f , which has arguments that are various combinations of the quantities σ_y , σ_z , σ_x , and q . These quantities are functions of the Twiss beta function of the ring. This report will now determine IBS rates for positions within the HEB where the beam has a maximum horizontal size, maximum vertical size, and is round. All calculations here will assume a value of the dispersion function of $D = 1.7$ meters, which is roughly equal to a weighted average of the HEB dispersion function.

3.1. IBS at HEB Injection, $\epsilon_L = 0.1$ eV-seconds, Maximum Horizontal Beam Size

At injection, and at a part of the trajectory where the beam is largest horizontally $\beta_{Tx} = 120$ m and $\beta_{Tz} = 20$ m leaving

$$\begin{aligned} \sigma_{x\beta} &= (\beta_{Tx}\epsilon_{nx}/\pi\beta\gamma)^{1/2} = 6.48 \times 10^{-4} \text{ m}, \sigma_z = (\beta_{Tz}\epsilon_{nz}/\pi\beta\gamma)^{1/2} = 2.65 \times 10^{-4} \text{ m}, \\ \sigma_{x'} &= (\epsilon_{nx}/\pi\beta\gamma\beta_{Tx})^{1/2} = 5.40 \times 10^{-6}, \sigma_{z'} = (\epsilon_{nz}/\pi\beta\gamma\beta_{Tz})^{1/2} = 1.32 \times 10^{-5}, \\ \sigma_x &= \sigma_{x\beta} + D\sigma_h = 6.94 \times 10^{-4} \text{ m and } \sigma_y = \sigma_h\sigma_{x\beta}/\gamma\sigma_x = 1.26 \times 10^{-7}. \end{aligned} \quad (10)$$

The number density enters in the expressions only very weakly, and this can be approximated as

$$n_p = N_b / (\sigma_x\sigma_z\sigma_s) = 1.82 \times 10^{17} \text{ m}^{-3}. \quad (11)$$

Thus,

$$b_0 = 0.5(\gamma/n_p)^{1/3} = 5.17 \times 10^{-6} \text{ m and } q = \beta\gamma(2b_0/r_0)^{1/2} = 5.20 \times 10^8 . \quad (12)$$

So,

$$\begin{aligned} 1/\tau_h &= (1/2\sigma_h^2)(d\sigma_h^2/dt) = A(1 - [D^2\sigma_h^2/\sigma_x^2])f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y) \quad (13) \\ &= 7.00 \times 10^{-8} \text{ s}^{-1}(1 - 0.0044)f(0.023, 0.010, 65.5) = \sim 7.00 \times 10^{-8} \text{ s}^{-1} (3 \times 10^4) = 2 \times 10^{-3} \text{ s}^{-1} \\ &\Rightarrow \tau_h = 500 \text{ seconds} . \end{aligned}$$

$$\begin{aligned} 1/\tau_{x'} &= (1/2\sigma_{x'}^2)(d\sigma_{x'}^2/dt) = A[f(\sigma_{x'}/\sigma_y, \sigma_{x'}/\sigma_z, q\sigma_{x'}) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)] \\ &= 7.00 \times 10^{-8} \text{ s}^{-1}[f(42.8, 0.41, 2810) + (0.0044)f(0.023, 0.010, 65.5)] \\ &= \sim 7.00 \times 10^{-8} \text{ s}^{-1}[-3 + (0.0044)(3 \times 10^4)] = 1 \times 10^{-5} \text{ s}^{-1} \quad (14) \\ &\Rightarrow \tau_{x'} = 100 \text{ 000 seconds} . \end{aligned}$$

$$\begin{aligned} 1/\tau_{z'} &= (1/2\sigma_{z'}^2)(d\sigma_{z'}^2/dt) = A f(\sigma_{z'}/\sigma_y, \sigma_{z'}/\sigma_x, q\sigma_{z'}) \\ &= 7.00 \times 10^{-8} \text{ s}^{-1} f(105, 2.45, 6880) = 7.00 \times 10^{-8} (-3) \text{ s}^{-1} = -2 \times 10^{-7} \text{ s}^{-1} . \quad (15) \\ &\Rightarrow \tau_{z'} = -5 \text{ 000 000 seconds} . \end{aligned}$$

3.2. IBS at HEB Injection, $\varepsilon_L = 0.1$ eV-seconds, Maximum Vertical Beam Size

At injection, and at a part of the trajectory where the beam is largest vertically $\beta_{Tx} = 20$ m and $\beta_{Tz} = 120$ m leaving

$$\begin{aligned} \sigma_{x\beta} &= (\beta_{Tx}\varepsilon_{nx}/\pi\beta\gamma)^{1/2} = 2.65 \times 10^{-4} \text{ m} , \sigma_z = (\beta_{Tz}\varepsilon_{nz}/\pi\beta\gamma)^{1/2} = 6.48 \times 10^{-4} \text{ m} , \\ \sigma_{x'} &= (\varepsilon_{nx}/\pi\beta\gamma\beta_{Tx})^{1/2} = 1.32 \times 10^{-5} , \sigma_{z'} = (\varepsilon_{nz}/\pi\beta\gamma\beta_{Tz})^{1/2} = 5.40 \times 10^{-6} , \quad (16) \\ \sigma_x &= \sigma_{x\beta} + D\sigma_h = 3.10 \times 10^{-4} \text{ m and } \sigma_y = \sigma_h\sigma_{x\beta}/\gamma\sigma_x = 1.15 \times 10^{-7} . \end{aligned}$$

The number density enters in the expressions only very weakly, and this can be approximated as

$$n_p = N_b/(\sigma_x\sigma_z\sigma_y) = 1.66 \times 10^{17} \text{ m}^{-3} . \quad (17)$$

Thus,

$$b_o = 0.5(\gamma/n_p)^{1/3} = 5.33 \times 10^{-6} \text{ m and } q = \beta\gamma(2b_o/r_o)^{1/2} = 5.28 \times 10^8. \quad (18)$$

So,

$$\begin{aligned} 1/\tau_h &= (1/2\sigma_h^2)(d\sigma_h^2/dt) = A(1 - [D^2\sigma_h^2/\sigma_x^2])f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y) \\ &= 7.00 \times 10^{-8} \text{ s}^{-1} (1 - 0.022)f(0.009, 0.021, 60.7) \approx (0.978) 7.00 \times 10^{-8} \text{ s}^{-1} (3 \times 10^4) \\ &= 2 \times 10^{-3} \text{ s}^{-1} = > \tau_h = 500 \text{ seconds}. \end{aligned} \quad (19)$$

$$\begin{aligned} 1/\tau_{x'} &= (1/2\sigma_{x'}^2)(d\sigma_{x'}^2/dt) = A[f(\sigma_{x'}/\sigma_y, \sigma_{x'}/\sigma_z, q\sigma_{x'}) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)] \\ &= 7.00 \times 10^{-8} \text{ s}^{-1} [f(115, 2.45, 6980) + (0.022)f(0.009, 0.021, 60.7)] \\ &\approx 7.00 \times 10^{-8} \text{ s}^{-1} [-3 + (0.022)(3 \times 10^4)] = 5 \times 10^{-5} \text{ s}^{-1} \\ &\Rightarrow \tau_{x'} = 21 \text{ 000 seconds}. \end{aligned} \quad (20)$$

$$\begin{aligned} 1/\tau_{z'} &= (1/2\sigma_{z'}^2)(d\sigma_{z'}^2/dt) = A f(\sigma_{z'}/\sigma_y, \sigma_{z'}/\sigma_x, q\sigma_{z'}) \\ &= 7.00 \times 10^{-8} \text{ s}^{-1} f(46.9, 0.41, 2850) = 7.00 \times 10^{-8} \text{ s}^{-1} (-3) = -2 \times 10^{-7} \text{ s}^{-1} \\ &\Rightarrow \tau_{z'} = -5 \text{ 000 000 seconds}. \end{aligned} \quad (21)$$

3.3. IBS at HEB Injection, $\epsilon_L = 0.1$ eV-seconds, Round Beam

At injection, and at a part of the trajectory where the beam is round $\beta_{Tx} = \beta_{Tz} = 70$ m leaving

$$\begin{aligned} \sigma_{x\beta} &= (\beta_{Tx}\epsilon_{nx}/\pi\beta\gamma)^{1/2} = 4.95 \times 10^{-4} \text{ m}, \sigma_z = (\beta_{Tz}\epsilon_{nz}/\pi\beta\gamma)^{1/2} = 4.95 \times 10^{-4} \text{ m}, \\ \sigma_{x'} &= (\epsilon_{nx}/\pi\beta\gamma\beta_{Tx})^{1/2} = 7.07 \times 10^{-6}, \sigma_{z'} = (\epsilon_{nz}/\pi\beta\gamma\beta_{Tz})^{1/2} = 7.07 \times 10^{-6}, \\ \sigma_x &= \sigma_{x\beta} + D\sigma_h = 5.41 \times 10^{-4} \text{ m and } \sigma_y = \sigma_h\sigma_{x\beta}/\gamma\sigma_x = 1.24 \times 10^{-7}. \end{aligned} \quad (22)$$

The number density enters in the expressions only very weakly, and this can be approximated as

$$n_p = N_b/(\sigma_x\sigma_z\sigma_s) = 1.24 \times 10^{17} \text{ m}^{-3}. \quad (23)$$

Thus,

$$b_o = 0.5(\gamma/n_p)^{1/3} = 5.86 \times 10^{-6} \text{ m and } q = \beta\gamma(2b_o/r_o)^{1/2} = 5.54 \times 10^8 . \quad (24)$$

So,

$$\begin{aligned} 1/\tau_h &= (1/2\sigma_h^2)(d\sigma_h^2/dt) = A(1 - [D^2\sigma_h^2/\sigma_x^2])f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y) \\ &= 7.00 \times 10^{-8} \text{ s}^{-1}(1 - 0.007)f(0.017, 0.017, 68.4) \approx 7.00 \times 10^{-8} \text{ s}^{-1}(3 \times 10^4) = 2 \times 10^{-3} \text{ s}^{-1} \\ &\Rightarrow \tau_h = 500 \text{ seconds} . \end{aligned} \quad (25)$$

$$\begin{aligned} 1/\tau_x &= (1/2\sigma_x^2)(d\sigma_x^2/dt) = A[f(\sigma_x/\sigma_y, \sigma_x/\sigma_z, q\sigma_x) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)] \\ &= 7.00 \times 10^{-8} \text{ s}^{-1}[f(57.2, 1.0, 3910) + (0.007)f(0.017, 0.017, 68.4)] \\ &\approx 7.00 \times 10^{-8} \text{ s}^{-1}[-5 + (0.007)(3 \times 10^4)] = 1 \times 10^{-5} \text{ s}^{-1} \\ &\Rightarrow \tau_x = 100 \text{ 000 seconds} . \end{aligned} \quad (26)$$

$$\begin{aligned} 1/\tau_z &= (1/2\sigma_z^2)(d\sigma_z^2/dt) = A f(\sigma_z/\sigma_y, \sigma_z/\sigma_x, q\sigma_z) \\ &= 7.00 \times 10^{-8} \text{ s}^{-1}f(57.2, 1.0, 3910) = 7.00 \times 10^{-8} \text{ s}^{-1}(-5) = -3.5 \times 10^{-7} \text{ s}^{-1} \\ &\Rightarrow \tau_z = -3 \text{ 000 000 seconds} . \end{aligned} \quad (27)$$

3.4. IBS at Injection Into the HEB, $\epsilon_L = 0.1$ eV-seconds, Summary

The IBS rates for the HEB at injection with $\epsilon_L = 0.1$ eV-seconds for various points within the lattice are summarized in Table 1.

Table 1. IBS Rates in the HEB During Injection.

β_{Tx} (m)	β_{Tz} (m)	τ_h (s)	τ_x (s)	τ_z (s)
120	20	500	100 000	-5 000 000
20	120	500	21 000	-5 000 000
70	70	500	100 000	-3 000 000

A good estimate for the IBS scattering rates in the HEB at injection is:

$$\tau_h = 500 \text{ seconds} ; \tau_x = 50 \text{ 000 seconds} ; \tau_z = -4 \text{ 000 000 seconds} . \quad (28)$$

The above estimate is a guess based on the observation that the values of IBS at the extremes of the betatron oscillations will pull the IBS rate somewhat away from the value IBS has for the round beam case.

4.0 CALCULATION OF IBS DURING HEB INJECTION FLAT, $\epsilon_L = 0.33$ eV-SECONDS

Eq. (8) will now be used.

$$A = (cr_0^2 N_b) / (64 \epsilon_{nx} \epsilon_{nz} \sigma_h \sigma_s \beta \gamma^2) . \quad (8)$$

With $\epsilon_L = 0.33$ eV-seconds and $N_b = 1 \times 10^{10}$, $\epsilon_{nx} = \epsilon_{nz} = 0.7\pi$ mm-mr, $\sigma_s = 0.3$ m, $\sigma_h = 8.8 \times 10^{-5}$ and $\gamma = 200$ at injection, so,

$$A = 2.15 \times 10^{-8} \text{ s}^{-1} \quad (\text{at injection}) . \quad (29)$$

The analysis completed in Section 3 indicates that the longitudinal IBS growth rate is about the same for regions of large horizontal, small horizontal and round beam size. Thus it is only required to calculate the longitudinal IBS rate at one place in the ring, chosen here to be that place in the ring where the beam is round. The transverse heating times will be seen to be too large to be important.

At a part of the trajectory where the beam is round $\beta_{Tx} = \beta_{Tz} = 70$ m leaving

$$\begin{aligned} \sigma_{x\beta} &= (\beta_{Tx} \epsilon_{nx} / \pi \beta \gamma)^{1/2} = 4.95 \times 10^{-4} \text{ m} , \quad \sigma_z = (\beta_{Tz} \epsilon_{nz} / \pi \beta \gamma)^{1/2} = 4.95 \times 10^{-4} \text{ m} , \\ \sigma_{x'} &= (\epsilon_{nx} / \pi \beta \gamma \beta_{Tx})^{1/2} = 7.07 \times 10^{-6} , \quad \sigma_{z'} = (\epsilon_{nz} / \pi \beta \gamma \beta_{Tz})^{1/2} = 7.07 \times 10^{-6} , \quad (30) \\ \sigma_x &= \sigma_{x\beta} + D\sigma_h = 6.45 \times 10^{-4} \text{ m and } \sigma_y = \sigma_h \sigma_{x\beta} / \gamma \sigma_x = 3.38 \times 10^{-7} . \end{aligned}$$

The number density enters in the expressions only very weakly, and this can be approximated as

$$n_p = N_b / (\sigma_x \sigma_z \sigma_s) = 1.04 \times 10^{17} \text{ m}^{-3} . \quad (31)$$

Thus,

$$b_0 = 0.5(\gamma/n_p)^{1/3} = 6.22 \times 10^{-6} \text{ m and } q = \beta \gamma (2b_0/r_0)^{1/2} = 5.70 \times 10^8 . \quad (32)$$

So,

$$\begin{aligned}
1/\tau_h &= (1/2\sigma_h^2)(d\sigma_h^2/dt) = A[(-D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)] \\
&= 2.15 \times 10^{-8} \text{ s}^{-1} (1 - 0.054)f(0.048, 0.048, 193) \\
&\sim 2.15 \times 10^{-8} \text{ s}^{-1} (1 \times 10^4) = 2 \times 10^{-4} \text{ s}^{-1} \\
&\Rightarrow \tau_h = 5 \text{ 000 seconds} .
\end{aligned} \tag{33}$$

$$\begin{aligned}
1/\tau_{x'} &= (1/2\sigma_{x'}^2)(d\sigma_{x'}^2/dt) = A[f(\sigma_{x'}/\sigma_y, \sigma_{x'}/\sigma_z, q\sigma_{x'}) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)] \\
&= 2.15 \times 10^{-8} \text{ s}^{-1} [f(20.9, 1.0, 4030) + (0.054)f(0.048, 0.048, 193)] \\
&\sim 2.15 \times 10^{-8} \text{ s}^{-1} [-13 + (0.054)(1 \times 10^4)] = 1 \times 10^{-5} \text{ s}^{-1} \\
&\Rightarrow \tau_{x'} = 100 \text{ 000 seconds} .
\end{aligned} \tag{34}$$

$$\begin{aligned}
1/\tau_{z'} &= (1/2\sigma_{z'}^2)(d\sigma_{z'}^2/dt) = A f(\sigma_{z'}/\sigma_y, \sigma_{z'}/\sigma_{x'}, q\sigma_{z'}) \\
&= 2.15 \times 10^{-8} \text{ s}^{-1} f(20.9, 1.0, 4030) = -3 \times 10^{-7} \text{ s}^{-1} \\
&\Rightarrow \tau_{z'} = -3 \text{ 000 000 s} .
\end{aligned} \tag{35}$$

5.0 CALCULATION OF IBS DURING HEB EJECTION FLAT TOP

The calculation will again be done where the beam is round. Eq. (8) will again be used.

$$A = (cr_0^2 N_b) / (64 \epsilon_{nx} \epsilon_{nz} \sigma_h \sigma_s \beta \gamma^2) . \tag{8}$$

With $\epsilon_L = 0.66$ eV-seconds and $N_b = 1 \times 10^{10}$, $\sigma_s = 0.300$ m, $\sigma_h = 1.8 \times 10^{-5}$, $\epsilon_{nx} = \epsilon_{nz} = 0.8\pi$ mm-mr and $\gamma = 2000$ at ejection,

$$A = 8.04 \times 10^{-10} \text{ s}^{-1} \text{ (at ejection)} . \tag{36}$$

At a part of the trajectory where the beam is round $\beta_{Tx} = \beta_{Tz} = 70$ m leaving

$$\begin{aligned}
\sigma_{x\beta} &= (\beta_{Tx} \epsilon_{nx} / \pi \beta \gamma)^{1/2} = 1.67 \times 10^{-4} \text{ m} , \sigma_z = (\beta_{Tz} \epsilon_{nz} / \pi \beta \gamma)^{1/2} = 1.67 \times 10^{-4} \text{ m} , \\
\sigma_{x'} &= (\epsilon_{nx} / \pi \beta \gamma \beta_{Tx})^{1/2} = 2.39 \times 10^{-6} , \sigma_{z'} = (\epsilon_{nz} / \pi \beta \gamma \beta_{Tz})^{1/2} = 2.39 \times 10^{-6} , \\
\sigma_x &= \sigma_{x\beta} + D\sigma_h = 1.98 \times 10^{-4} \text{ m and } \sigma_y = \sigma_h \sigma_{x\beta} / \gamma \sigma_x = 7.61 \times 10^{-9} .
\end{aligned} \tag{37}$$

The number density enters in the expressions only very weakly, and this can be approximated as

$$n_p = N_b/(\sigma_x \sigma_z \sigma_y) = 1.01 \times 10^{18} \text{ m}^{-3}. \quad (38)$$

Thus,

$$b_o = 0.5(\gamma/n_p)^{1/3} = 6.29 \times 10^{-6} \text{ m and } q = \beta\gamma(2b_o/r_o)^{1/2} = 5.74 \times 10^9. \quad (39)$$

So,

$$\begin{aligned} 1/\tau_h &= (1/2\sigma_h^2)(d\sigma_h^2/dt) = A(1 - [D^2\sigma_h^2/\sigma_x^2])f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y) \\ &= 8.04 \times 10^{-10} \text{ s}^{-1}(1 - 0.024)f(0.0032, 0.0032, 43.6) \\ &\approx 8.04 \times 10^{-10} \text{ s}^{-1}(1.5 \times 10^5) = 1.21 \times 10^{-4} \text{ s}^{-1} \\ &\Rightarrow \tau_h = 8 \text{ 300 seconds}. \end{aligned} \quad (40)$$

$$\begin{aligned} 1/\tau_x &= (1/2\sigma_x^2)(d\sigma_x^2/dt) = A[f(\sigma_x/\sigma_y, \sigma_x/\sigma_z, q\sigma_x) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y)] \\ &= 8.04 \times 10^{-10} \text{ s}^{-1} [f(314, 1.0, 13 \text{ 700}) + (0.024)f(0.0032, 0.0032, 43.6)] \\ &\approx 8.04 \times 10^{-10} \text{ s}^{-1} [-1 + (0.024)(1.5 \times 10^5)] = 3 \times 10^{-6} \text{ s}^{-1} \\ &\Rightarrow \tau_x = 350 \text{ 000 seconds}. \end{aligned} \quad (41)$$

$$\begin{aligned} 1/\tau_z &= (1/2\sigma_z^2)(d\sigma_z^2/dt) = A f(\sigma_z/\sigma_y, \sigma_z/\sigma_x, q\sigma_z) \\ &= 8.04 \times 10^{-10} \text{ s}^{-1} f(314, 1.0, 13 \text{ 700}) = -8.04 \times 10^{-10} \text{ s}^{-1} \\ &\Rightarrow \tau_z = -1 \text{ 000 000 000 seconds}. \end{aligned} \quad (42)$$

6.0 CONCLUSION

The IBS heating rates for the HEB at injection under the assumption that the longitudinal emittance is 0.1 eV-seconds are:

$$\tau_h = 500 \text{ seconds} ; \tau_x = 50 \text{ 000 seconds} ; \tau_z = -4 \text{ 000 000 seconds}. \quad (43)$$

The IBS heating rates for the HEB at injection under the assumption that the longitudinal emittance is 0.33 eV-seconds are:

$$\tau_h = 5000 \text{ seconds} ; \tau_x = 50 \text{ 000 seconds} ; \tau_z = -3 \text{ 000 000 seconds}. \quad (44)$$

The IBS heating rates for the HEB at ejection under the assumption that the longitudinal emittance is 0.66 eV-seconds are:

$$\tau_h = 8\,300 \text{ seconds} ; \tau_x = 350\,000 \text{ seconds} ; \tau_z = -1\,000\,000\,000 \text{ seconds.} \quad (45)$$

The injection flat of the HEB is 20 seconds, and therefore the longitudinal emittance growth due to IBS should be 4% under the assumption that the longitudinal emittance is 0.1 eV-seconds. The IBS heating decreases as the longitudinal emittance is increased.

It is therefore the conclusion of this work that IBS heating should not be a problem in the HEB, although the safety factor is not enormous.

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APPENDIX A VELOCITY SPREAD CHECK

Neglecting dispersion, a simple check to verify the velocity spread ratio in the moving frame can be obtained by noting that the longitudinal velocity spread is given by

$$\beta_h = \beta \Delta p / p = 1.8 \times 10^{-5}, \quad (46)$$

while the transverse velocity spread is given by

$$\beta_z = \epsilon_{nz} / \pi z = 8 \times 10^{-7} / 1.7 \times 10^{-4} = 4.7 \times 10^{-3}. \quad (47)$$

Therefore the ratio of the velocity spreads is $\beta_h / \beta_z = 1.8 \times 10^{-5} / 4.7 \times 10^{-3} = 0.0033$, close to the value of 0.0032 found above in Eq. (40). (The difference is due to the included dispersion.)

APPENDIX B

DOCUMENTATION OF THE CODE IBS

I have written a computer code to calculate intrabeam scattering rates. The source code for the program is listed in Appendix C. This appendix will document how to run the code, and what the lines in the source code mean.

What the code does is to evaluate the IBS rate at a single point within the machine. If one wanted to calculate the IBS rate for the entire machine, one would have to calculate the IBS rate at various points within the machine and weight the rate proportional to the amount of lattice space where that data is appropriate, and then sum over all such points.

The lines prior to the line labeled 0001 are standard variable declarations.

Line 0001 specifies the speed of light, c .

Line 0002 specifies the ratio of the circumference of a circle to its diameter, π , coded as PI.

Line 0003 specifies the square root of PI.

The block of code starting with line 0010 and ending with line 0015 reads input to the code from the file IBS.IN. In order to understand the input it is relevant to cite Piwinski's formulas for the IBS rates:

$$1/\tau_h = (1/2\sigma_h^2)(d\sigma_h^2/dt) = jA(1 - [D^2\sigma_h^2/\sigma_x^2])f(\sigma_y/\sigma_x, \sigma_y/\sigma_z, q\sigma_y); \quad (1)$$

$$\begin{aligned} 1/\tau_{x'} &= (1/2\sigma_{x'}^2)(d\sigma_{x'}^2/dt) \\ &= A[f(\sigma_{x'}/\sigma_y, \sigma_{x'}/\sigma_z, q\sigma_{x'}) + (D^2\sigma_h^2/\sigma_x^2)f(\sigma_y/\sigma_{x'}, \sigma_y/\sigma_z, q\sigma_y)]; \end{aligned} \quad (2)$$

$$1/\tau_{z'} = (1/2\sigma_{z'}^2)(d\sigma_{z'}^2/dt) = Af(\sigma_{z'}/\sigma_y, \sigma_{z'}/\sigma_{x'}, q\sigma_{z'}). \quad (3)$$

In the above equations Piwinski defines

$$A = (cr_0^2 N_b)/(64\pi^2 \sigma_h \sigma_s \sigma_x \beta \sigma_{x'} \sigma_z \sigma_{z'} \beta^3 \gamma^4); j = 1; \text{ for bunched beams and} \quad (4a)$$

$$A = (cr_0^2 N\pi^{1/2})/(C32\pi^2 \sigma_h \sigma_x \beta \sigma_{x'} \sigma_z \sigma_{z'} \beta^3 \gamma^4); j = 2; \text{ for unbunched beams.} \quad (4b)$$

where

$$\sigma_y = \sigma_h \sigma_x \beta / \gamma \sigma_{x'}; q = \beta \gamma (2b_0/r_0)^{1/2}; \text{ and } b_0 = 0.5 (\gamma/n_p)^{1/3}. \quad (5)$$

Piwinski³ defines $f(a,b,c)$ as

$$f(a,b,c) = 8\pi \int_0^1 dx (1-3x^2)(1/pq)[\ln(0.5c^2(1/q+1/p) - 0.577 \dots)] \quad (6)$$

where

$$p = [a^2 + x^2(1 - a^2)]^{1/2} \text{ and } q = [b^2 + x^2(1 - b^2)]^{1/2}. \quad (7)$$

The definition of all of the symbols used in Equations (1) through (7) is done above in Section 2 of this report.

The input to the code is entered in four lines in free input format. Line 1 contains NMODE, XORBETA, and DX. Line 2 contains NBEAM, CIRC, EPNX, and EPNZ. Line 3 contains DPOP, GAMMA, and MASS. Line 4 contains XIN, ZIN, and DISP.

What these input variables are is as follows:

Line 1

NMODE—a flag which tells the code whether the beam is bunched or unbunched. NMODE=1 for bunched beams, and NMODE = 2 for unbunched beams.

XORBETA – a flag which tells the code whether beam size will be entered as physical size or the beta function. If XORBETA = -1, beam size is entered as a beta function. (For XORBETA = -1 XIN is the horizontal beta function at the point of interest and ZIN is the vertical beta function at the point of interest.) If XORBETA = +1, beam size is entered as physical beam size. (For XORBETA = +1 XIN is the horizontal beam size at the point of interest and ZIN is the vertical beam size at the point of interest.)

DX – The step size for the numerical integration. By varying this parameter to smaller values one can check for code convergence.

Line 2

NBEAM – the number of particles in the beam (for unbunched beams, NMODE = 2) or the number of particles in a bunch (for bunched beams, NMODE = 1).

CIRC – the circumference of the ring (for unbunched beams, NMODE = 2) or the bunch length (for bunched beams, NMODE = 1).

EPNX – the normalized one sigma horizontal emittance of the beam.

EPNZ – the normalized one sigma vertical emittance of the beam.

Line 3

DPOP – the relative momentum one sigma momentum spread of the beam.

GAMMA – the beam energy divided by the rest mass energy of the particles.

MASS – the rest mass of beam particles. For ions enter in units of the proton mass (for protons MASS = 1, for deuterons MASS = 2, etc.). For electrons enter 0. (The code treats MASS = 0 as indicating the electron mass).

Line 4

XIN – if XORBETA = +1, the total horizontal one sigma beam size including the dispersion component at the point of interest, if XORBETA = -1, the horizontal beta function at the point of interest.

ZIN – if XORBETA = +1, the total vertical one sigma beam size at the point of interest, if XORBETA = -1, the vertical beta function at the point of interest.

DISP – the value of the dispersion function at the point of interest.

Line 30 opens the output file IBS.OUT.

Line 35 calculates BETA from GAMMA.

Lines 40–44 calculate the classical radius of the beam particles.

Lines 45 to 49 calculate A for the cases of bunched and unbunched beams. Use is made of Eqs. (4a) and (4b) above, along with the definition of normalized emittance, $EPNX = SIGXB * SIGXP * PI * BETA * GAMMA$. Here SIGXB is the beam size sans dispersion, SIGXP is the spread in angles, and each quantity is one sigma.

Lines 50 to 56 calculate the one sigma beam size sans dispersion.

Line 57 calculates the beam size with dispersion.

Lines 58 and 59 calculate SIGXP and SIGZP, respectively.

Line 60 calculates SIGY, which is found in Eq. (5) above.

Lines 61 to 63 calculate q , which is found in Eq. (5) above.

Line 64 calculates $STUFF = (D^2 \sigma_h^2 / \sigma_x^2)$ which is used in several of the expressions for IBS.

Lines 65 to 75 write alphanumerics of some of the variables, =, and the value of the variable to the output file.

Lines 76 to 90 write statements indicating the values of parameters used to evaluate the longitudinal IBS rate.

Line 0092 sets the flag PASS = 1, indicating that $f(a,b,c)$ will now be calculated for the longitudinal IBS rate. This flag is later used to send the program to the correct place for further calculations once f is calculated.

Line 0095 sends the program to line 800, which is the section devoted to calculating $f(a,b,c)$.

Lines 110 to 120 calculate the IBS growth rate for dp/p . FOUT is the value of $f(a,b,c)$ calculated when the program is sent to line 800, A and STUFF are calculated in lines 45–49 and line 64, respectively. Note here that there is an additional factor of 2 in the IBS rate for unbunched beams, as indicated in Reference 1. Reference 3 does not have the factor of 2, but Reference 3 refers to 1 when the equations are stated, so I have coded using the expression of Reference 1.

Lines 130 to 140 print out the dp/p IBS growth rate and its inverse, the characteristic time constant for a $1/e$ emittance growth, as well as some identifying alphanumerics, to the output file.

Line 150 writes some dots to the output file so as to separate sections of the output file.

Lines 160 and 170 write some alphanumerics to the output file to begin the identification of the section on the horizontal emittance growth rate.

Lines 180 through 185 calculate the a , b , and c coefficients of Piwinski's $f(a,b,c)$ relevant to the first term in the horizontal emittance IBS growth rate. (Note that Eq. (2) above contains two terms.)

Line 190 sets the PASS variable so that the return from the $f(a,b,c)$ calculation knows where to go.

Line 195 sends the program to line 800, where the calculation of $f(a,b,c)$ is done.

Line 200 is the line where the program returns to after the calculation of $f(a,b,c)$ is complete.

Line 210 stores the value of $f(a,b,c)$ for the first term for later use in calculation the horizontal IBS rate on line 320.

Lines 260 and 270 write some alphanumerics and data to the output file relevant to the identification of the section on the horizontal emittance growth rate.

Lines 280 through 285 calculate the a , b , and c coefficients of Piwinski's $f(a,b,c)$ relevant to the second term in the horizontal emittance IBS growth rate. (Note that Eq. (2) above contains two terms.)

Line 290 sets the PASS variable so that the return from the $f(a,b,c)$ calculation knows where to go.

Line 295 sends the program to line 800, where the calculation of $f(a,b,c)$ is done.

Line 300 is the line where the program returns to after the calculation of $f(a,b,c)$ is complete.

Line 310 writes some alphanumerics and data to the output file relevant to a , b , and c for the second term of the horizontal emittance growth rate.

Line 320 calculates the horizontal IBS growth rate.

Lines 330 to 340 print out the horizontal emittance IBS growth rate and its inverse, the characteristic time constant for a $1/e$ emittance growth, as well as some identifying alphanumerics, to the output file.

Line 350 writes some dots to the output file so as to separate sections of the output file.

Lines 360 and 370 write some alphanumerics to the output file to begin the identification of the section on the vertical emittance growth rate.

Lines 380 through 385 calculate the a , b , and c coefficients of Piwinski's $f(a,b,c)$ relevant to the vertical emittance IBS growth rate.

Line 390 sets the PASS variable so that the return from the $f(a,b,c)$ calculation knows where to go.

Line 395 sends the program to line 800, where the calculation of $f(a,b,c)$ is done.

Line 400 is the line where the program returns to after the calculation of $f(a,b,c)$ is complete.

Line 420 calculates the vertical IBS growth rate.

Lines 430 to 440 print out the vertical emittance IBS growth rate and its inverse, the characteristic time constant for a $1/e$ emittance growth, as well as some identifying alphanumerics, to the output file.

Line 499 sends the program to the line 9999, after which the program stops.

Line 800 is the line where the calculation of $f(a,b,c)$ begins.

Line 810 sets the dummy integration variable, x , which goes from 0 to 1 in the integrand, to zero.

Line 820 sets the $FLAG = -1$, signifying that the first step in the integration is about to take place.

Lines 830 and 840 set $FOUT = 0$ and $F = 0$, respectively. $FOUT$ is used for $f(a,b,c)$ and F is used for the summed value of the integrand prior to multiplying by 8π .

Line 1100 is the return point for the loop which calculates $f(a,b,c)$.

Line 1120 and 1140 calculate p and q , respectively, as given in Eq. (7)

Line 1160 forms the argument of the logarithm, as given in Eq. (6).

Line 1180 forms the integrand of the integral given in Eq. (6).

Lines 1200 through 1290 are executed only if the evaluation of the integral is in the first pass through the loop. Line 1220 sets $FLAG = 1$, signifying that the integrand is no longer in the first pass through the loop. Line 1240 sets $S1 = S$, which is the present value of the integrand. Line 1260 increments the dummy variable of integration by DX . Line 1280 sends the program back to Line 1100 so as to get the second value of the integrand. The reason two values are for the integrand are needed is that the integral is evaluated by approximating the integral as a series of trapezoids, with height given by the integrand at two points and width given by dx .

Lines 1300 through 1400 evaluate the area of the trapezoid and add it to the previous total area of the trapezoids. Line 1300 sets $S2$ equal to the present value of the integrand. Line 1320 evaluates the area of the trapezoid. Line 1340 adds the present trapezoid area to the total summed area so far. Line 1360 sets $S1$ equal to the present value of the integrand, so that on the next pass $S1$ is the previous value of the integrand in line 1320. Line 1380 increments the dummy variable of integration. Line 1400 checks to see if the dummy variable has reached the upper limit of 1, if not it returns the program to line 1100 to do the next step in the integral, if so it proceeds to line 1500.

Line 1500 multiplies the summed area of all trapezoids by 8π , so as to form the value $f(a,b,c)$.

Line 1510 sends the program to line 100, if this is the first time $f(a,b,c)$ has been calculated.

Line 1520 sends the program to line 200, if this is the second time $f(a,b,c)$ has been calculated.

Line 1530 sends the program to line 300, if this is the third time $f(a,b,c)$ has been calculated.

Line 1540 sends the program to line 400, if this is the fourth time $f(a,b,c)$ has been calculated.

Line 9999 is where the program is sent when all calculations and writes to the output file are complete. The lines following line 9999 stop and end the program.

APPENDIX C

SOURCE FILE FOR IBS

C PROGRAM IBS

```
DOUBLE PRECISION C,PI,PIRT,NMODE,XORBETA,NBEAM,CIRC,EPNX,EPNZ
DOUBLE PRECISION DPOP,GAMMA,MASS,XIN,ZIN,DISP,BETA,R0,A,DX
DOUBLE PRECISION SIGXB,SIGZ,SIGX,SIGXP,SIGZP,SIGY,NP,B0,Q,STUFF
DOUBLE PRECISION FOUT,F1OUT,TDPOPINV,TDPOP,TXINV,TX,TZINV,TZ
DOUBLE PRECISION A1,B1,C1,DX,X,P,Q,R,S,S1,S2,F,DF,PASS,Q1
```

INTEGER FLAG

```
0001    C=2.99792458E8
0002    PI=3.141592653589793238
0003    PIRT=PI**0.5

0010    OPEN (UNIT=10,STATUS='OLD',FILE='IBS.IN')
          READ (10,*) NMODE,XORBETA,DX
          READ (10,*) NBEAM,CIRC,EPNX,EPNZ
          READ (10,*) DPOP,GAMMA,MASS
          READ (10,*) XIN,ZIN,DISP
0015    CLOSE (10)

0030    OPEN (UNIT=30,STATUS='NEW',FILE='IBS.OUT')

0035    BETA=(1-(1/GAMMA**2))**0.5

0040    IF (MASS.LT.0.9) THEN
          R0=2.82E-15
          ELSE
          R0=1.53E-18/MASS
0044    ENDIF

0045    IF (NMODE.LT.1.5) THEN
          A=(C*R0*R0*NBEAM)/(64*EPNX*PI*EPNZ*PI*DPOP*CIRC*BETA*GAMMA*GAMMA)
          ELSE
          A=(C*R0*R0*NBEAM*PIRT)/(32*EPNX*EPNZ*DPOP*CIRC*BETA*GAMMA*GAMMA)
0049    ENDIF

0050    IF (XORBETA.LT.0) THEN
          SIGXB=((XIN*EPNX)/(BETA*GAMMA))**0.5
          SIGZ=((ZIN*EPNZ)/(BETA*GAMMA))**0.5
          ELSE
          SIGXB=XIN-DISP*DPOP
          SIGZ=ZIN
0056    ENDIF

0057    SIGX=SIGXB+DISP*DPOP
0058    SIGXP=EPNX/(BETA*GAMMA*SIGXB)
          SIGZP=EPNZ/(BETA*GAMMA*SIGZ)
0060    SIGY=(DPOP*SIGXB)/(GAMMA*SIGX)
```

```

0061  NP=NBEAM/(SIGX*SIGZ*CIRC)
      B0=0.5*(GAMMA/NP)**0.3333
      Q=BETA*GAMMA*(2*B0/R0)**0.5
0064  STUFF=((DISP*DPOP)/SIGX)**2

0065  WRITE (30,*) 'A = ',A
      WRITE (30,*) 'SIGXB = ',SIGXB
      WRITE (30,*) 'SIGZ = ',SIGZ
      WRITE (30,*) 'SIGXP = ',SIGXP
      WRITE (30,*) 'SIGZP = ',SIGZP
      WRITE (30,*) 'SIGX = ',SIGX
      WRITE (30,*) 'SIGY = ',SIGY
      WRITE (30,*) 'NP = ',NP
      WRITE (30,*) 'B0 = ',B0
0075  WRITE (30,*) 'Q = ',Q

0076  WRITE (30,*) '.....'
      WRITE (30,*) '1/T_DPOP = '
      IF (NMODE.LT.1.5) THEN
      WRITE (30,*) A,'(1 - ',STUFF,')'
      ELSE
      WRITE (30,*) 2*A,'(1 - ',STUFF,')'
      ENDIF
      A1=SIGY/SIGXP
      B1=SIGY/SIGZP
      C1=Q*SIGY
0090  WRITE (30,*) F('A1,',B1,',',C1,')
0092  PASS=1
0095  GOTO 0800

0100  CONTINUE
0110  IF (NMODE.LT.1.5) THEN
      TDPOPINV=A*(1-STUFF)*FOUT
      ELSE
      TDPOPINV=2*A*(1-STUFF)*FOUT
0120  ENDIF
0130  WRITE (30,*) '1/TAU_DPOP = ',TDPOPINV
      TDPOP=1/TDPOPINV
      WRITE (30,*) 'T_DPOP = ',TDPOP
0140  WRITE (30,*) FOUT,DX

0150  WRITE (30,*) '.....'
0160  WRITE (30,*) '1/TAU_X'
0170  WRITE (30,*) A,F(
0180  A1=SIGXP/SIGY
      B1=SIGXP/SIGZP
0185  C1=Q*SIGXP
0190  PASS=2
0195  GOTO 0800

```

```

0200  CONTINUE
0210  F1OUT=FOUT
0260  WRITE (30,*) A1,',B1,',C1,')
0270  WRITE (30,*) 'PLUS',A,',STUFF,F(
0280  A1=SIGY/SIGXP
      B1=SIGY/SIGZP
0285  C1=Q*SIGY
0290  PASS=3
0295  GOTO 0800

0300  CONTINUE
0310  WRITE (30,*) A1,',B1,',C1,')
0320  TXINV=A*F1OUT+A*STUFF*FOUT
0330  WRITE (30,*) '1/TAU_X =',TXINV
      TX=1/TXINV
      WRITE (30,*) 'TAU_X =',TX
0340  WRITE (30,*) FOUT,DX

0350  WRITE (30,*) '.....'
0360  WRITE (30,*) 'TAU_Z'
0370  WRITE (30,*) A,F(
0380  A1=SIGZP/SIGY
      B1=SIGZP/SIGXP
0385  C1=Q*SIGZP
0387  WRITE (30,*) A1,',B1,',C1,')
0390  PASS=4
0395  GOTO 0800

0400  CONTINUE
0420  TZINV=A*FOUT
0430  WRITE (30,*) '1/TAU_Z =',TZINV
      TZ=1/TZINV
      WRITE (30,*) 'TAU_Z =',TZ
0440  WRITE (30,*) FOUT,DX

0499  GOTO 9999

0800  CONTINUE

0810  X=0.0
0820  FLAG=-1
0830  FOUT=0
0840  F=0.

1100  CONTINUE
1120  P=((A1*A1)+(X*X*(1-A1*A1)))**-.5
1140  Q1=((B1*B1)+(X*X*(1-B1*B1)))**-.5
1160  R=((C1*C1)/2)*(P+Q1)
1180  S=(LOG(R)-0.577)*(1-3*X*X)*P*Q1

```



```
1200  IF (FLAG.LT.0) THEN
1220      FLAG=1.
1240      S1=S
1260      X=X+DX
1280  GOTO 1100
1290  ENDIF

1300  S2=S
1320  DF=(S2+S1)*DX/2
1340  F=F+DF
1360  S1=S
1380  X=X+DX
1400  IF (X.LT.1) GOTO 1100

1500  FOUT=F*8*3.141592654

1510  IF (PASS.LT.1.5) GOTO 100
1520  IF (PASS.LT.2.5) GOTO 200
1530  IF (PASS.LT.3.5) GOTO 300
1540  IF (PASS.LT.4.5) GOTO 400

9999  CONTINUE

      STOP
      END
```

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END