AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

Imaging Targets Embedded in a Lossy Half Space with Synthetic Aperture Radar

A. W. Doerry, B. C. Brock, B. Boverie, D. Cress

Sandia National Laboratories

MS 0529

P.O. Box 5800

Albuquerque, NM 87185-0529

ABSTRACT

This paper addresses theoretical aspects of forming images from an airborne Synthetic Aperture Radar (SAR) of targets buried below the earth's surface. Soil is generally a lossy, dispersive medium, with wide ranging variability in these attributes depending on soil type, moisture content, and a host of other physical properties. Focussing a SAR subsurface image presents new dimensions of complexity relative to its surface-image counterpart, even when the soil's properties are known. This paper treats the soil as a lossy, dispersive half space, and presents a practical model for the radar echo-delay time to point scatterers within it. This model is then used to illustrate effects of refraction, dispersion, and attenuation on a SAR's phase histories, and the resulting image. Various data collection geometries and processing strategies are examined for both 2-Dimensional and 3-Dimensional SAR images. The conclusions from this work are that 1) focussing a SAR image must generally take into account both refraction and dispersion, 2) resolving targets at different depths in lossy soils requires perhaps unprecedented sidelobe attenuation, that for some soils may only be achievable with specialized window functions, 3) the impulse response of the soil itself places a practical limit on the usable bandwidth of the radar, and 4) dynamic ranges and sensitivities will need to be orders of magnitude greater than typical surface-imaging SARs, leading to significant impact on SAR parameters, for example compressing the usable range of pulse repetition frequencies (PRFs).

INTRODUCTION

There is significant interest in detecting, locating, and identifying 'targets' beneath the earth's surface. Applications span the areas of national defense, environmental monitoring, arms control and nonproliferation, geologic and mining exploration, and archeology. There is a healthy industry today of ground-based vehicle- or hand-towed ground penetrating radars (GPRs). The challenge is to cover more ground, deeper, quicker, and with better resolution. To this end, experimental systems are emerging using airborne GPRs. Existing airborne systems are limited by signal-to-noise ratio (SNR) performance and resolution. Synthetic aperture radar (SAR) techniques offer advantages in maintaining resolution even with increasing range, and large integration gains for good SNR performance. SAR with 2-Dimensional synthetic apertures offer 3-Dimensional imaging to help discriminate buried targets from surface clutter. To date, however, most SAR efforts seem to apply surface-imaging techniques to the subsurface-imaging problem. Very little has been written in the literature, thus far, dealing with problems unique to imaging underground targets with an airborne SAR. The impact of signal refraction, accurate echo delay, propagation losses, dispersion, and volumetric scattering on SAR data seems to have been largely ignored in an analytical manner. This report attempts to fill some of this information void.

THE RADAR ECHO

Consider a dielectric boundary, above which is free space, and below which is a homogeneous, lossy, and dispersive medium. Let a cartesian coordinate system be chosen where the x-y plane is the dielectric boundary, with free space in the +z direction, as in figure 1. Furthermore, the dielectric has complex relative permittivity ϵ , and complex relative permeability μ . The conductivity is included in ϵ . Both ϵ and μ are generally functions of frequency. The origin of this coordinate system is identified as the Target Surface Location (TSL). Beneath the TSL we place a target at location $(0,0,-d)$. The radar sensor is placed with phase center at location (x_0, y_0, z_0) , in free space. A vector from the TSL to this point is defined as r_s . The angle between r_s and the x-y plane is defined as the elevation angle ψ .

Figure 1.
Geometry of Radar Echo

Using a plane-wave assumption, if the radar emits energy that reflects from the target, the echo will follow the ray path identified by vectors r_{s1} and r_{s2} , which make angles with the x-y plane of ψ_1 and ψ_2 respectively.

The echo delay time is then given by

$$\tau_s = \frac{2\|r_{s1}\|}{c_l} + \frac{2\|r_{s2}\|}{c}, \quad (1)$$

where τ_s is the radar echo delay time, c is the velocity of light in free space, and c_l is the velocity of propagation in the lossy half space.

Imaging using (1) is discussed in [7], (Xu, Cao, and Su, 1993). The problem is that for a lossy medium, even if ϵ and μ are known, given only r_s and d , exact values for $\|r_{s1}\|$ and $\|r_{s2}\|$ are generally difficult to compute.

However, by using a plane-wave approximation, where $\psi_2 \approx \psi_1$, (plausible for an airborne SAR) we can find a fairly accurate approximate solution for the echo delay time as

$$\tau_s \approx \frac{2}{c} \left(\|r_s\| + \text{Real} \left[d \eta \sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2} \right] \right), \quad (2)$$

where η is the complex refraction index for the lossy dielectric ($\eta = \eta_R + j\eta_I = \sqrt{\epsilon\mu}$).

If $\left| \frac{\cos \psi}{\eta} \right|^2 \ll 1$, then the further approximation may be made

$$\tau_s \approx \frac{2}{c} \left(\|r_s\| + d\eta_R \left(1 - \frac{\cos^2 \psi}{2\eta_I^2} \right) \right). \quad (3)$$

The coarsest approximation ignores elevation-angle effects, and is given by

$$\tau_s \approx \frac{2}{c} (\|r_s\| + d\eta_R). \quad (4)$$

Which approximation to use depends largely on the accuracy required, which in turn depends on the circumstances of its use.

Similarly, the echo signal strength relative to a surface target with the same cross section is approximated by

$$G_\eta(\omega) \approx |1 - \Gamma|^2 \exp \left(\frac{2\omega}{c} \text{Imag} \left[d\eta \sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2} \right] \right), \quad (5)$$

where $|1 - \Gamma|^2$ is the two-way transmissivity loss in crossing the dielectric boundary, and the remainder is the two-way propagation loss. The transmissivity loss accounts for the impedance mismatch at the dielectric boundary and represents typically a few dB, making it small when compared to penetration loss for deep targets [3] (Brock and Patitz, 1993). The Fresnel reflection coefficient Γ is defined as

$$\Gamma = \frac{\zeta - \left(\sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2} \right) / \sin \psi}{\zeta + \left(\sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2} \right) / \sin \psi} \text{ for perpendicular polarization,}$$

and

$$\Gamma = \frac{\zeta - \sin \psi / \sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2}}{\zeta + \sin \psi / \sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2}} \text{ for parallel polarization,}$$

where $\zeta = \sqrt{\mu/\epsilon}$, the dielectric impedance normalized to free space.

Putting (2) and (5) together we can write the frequency response of the ground relative to the surface for a plane wave as

$$H_\eta(\omega) = \left(1 - \Gamma^2 \right) \exp \left(-j \frac{2\omega d}{c} \left[\eta \sqrt{1 - \left(\frac{\cos \psi}{\eta} \right)^2} \right] \right). \quad (6)$$

A target's cross section will also generally depend on whether it is embedded in the dielectric, and how deep, but for our purposes we shall presume it constant.

At this point we observe the following:

- Equation (6) is valid for any radar to the extent that the plane-wave approximation is valid for the radar's operating geometry.
- Both loss and delay depend on elevation angle ψ .
- Both loss and delay depend on both the real part and the imaginary part of dielectric properties μ and ϵ , and vary as a function of their frequency dependence. In addition, loss also depends explicitly on frequency ω , even for constant μ and ϵ .
- The change in delay τ_s as a function of frequency is generally referred to as dispersion. When delay is transformed into phase, by multiplying it with ω , this represents a frequency dependent phase error.
- The transmissivity includes a phase rotation that is elevation

angle dependent, and frequency dependent to the extent that η and ζ are frequency dependent. It is expected, however, that for depths of interest this dependence on frequency is dwarfed by the dependence of propagation loss on frequency.

THE SOIL MODEL'S IMPULSE RESPONSE

Equation (6) represents the frequency response of the lossy, dispersive dielectric. The Inverse Fourier Transform of (6) is then the Impulse Response (IPR) of the soil. In practice, however, we deal with band-limited impulse-like signals, whether directly generated or synthesized. If we assume transmitted signals with uniform frequency weighting, the soil's time-domain response (with amplitude normalized to a lossless dielectric) is calculated as

$$h_\eta(t, \omega_1, \omega_2) = \frac{1}{(\omega_2 - \omega_1)} \int_{\omega_1}^{\omega_2} H_\eta(\omega) \exp(j\omega t) d\omega. \quad (7)$$

This represents the dielectric's band-limited IPR.

Example

Consider a soil known as San Antonio Clay Loam with 5% water content by weight, characterized in reference [6] (Hipp, 1974), and modelled in [3] (Brock and Patitz, 1993). We plot the dielectric constant model vs. frequency in figure 2. The two-way loss to a subsurface target relative to a surface target at normal incidence ($\psi=90^\circ$) is plotted in figure 3, for targets at several depths.

Figure 2.
Dielectric Constant vs. frequency for San Antonio Clay Loam with 5% water content by weight.

Figure 3.
Loss relative to the surface to various specified depths below the surface, with the dielectric characterized by figure 2, at normal incidence.

Using equation (7), we plot the band-limited IPR of this soil (band limited from 1 MHz to 1 GHz) to various depths in figure 4.

The IPR broadening as a function of depth for the 1-MHz to 1-GHz signal is plotted in figure 5. The solid lines show IPR broadening due to both loss and dispersion, whereas the dashed lines show IPR broadening due to loss alone (assumes perfect dispersion compensation).

Figure 6 illustrates the effects of bandwidth on the IPR for the soil at a depth of 5 meters below the dielectric surface. For a constant total power signal, as bandwidth increases beyond the soil's cutoff frequency, a lesser portion of the energy is returned in the echo.

Figure 4.

Band-limited IPR to various depths in dielectric of figure 2. Signal is band-limited from 1 MHz to 1 GHz.

Figure 5.

IPR broadening as a function of depth. Solid lines are due to both loss and dispersion. Dashed lines are due to loss alone.

From these plots we make some observations for San Antonio Clay Loam with 5% water content by weight.

- Loss is not always monotonic with frequency. Lower isn't always better. As frequency decreases, or as depths become shallower, the transmissivity loss will tend to dominate propagation losses.
- More bandwidth doesn't always mean better resolution. The soil's frequency response limits the usable bandwidth, and hence achievable resolution.
- For this soil, loss as a function of frequency seems to play a bigger role in resolution than dispersion. It is unknown how general this might be.

Figure 6.

Band-limited IPR of target buried 5 meters below surface as a function of transmitted bandwidth.

- Delays and IPR widths that are specified in time units are related to distance units via the velocity of propagation in the dielectric, which is, of course, frequency dependent.

SYNTHETIC APERTURE RADAR PROCESSING

This development will be for a spotlight mode SAR with the geometry of figure 7, where the CRP is the central reference point,

Figure 7.

Geometry for SAR phase history model

and defines the origin for this geometry. The dielectric boundary is the x-y plane, with free space above it. The radar is located in free space with vector r_c , and has cartesian coordinates

$$(\|r_c\| \cos \psi_c \sin \alpha, -\|r_c\| \cos \psi_c \cos \alpha, \|r_c\| \sin \psi_c). \quad (8)$$

The parameters α and ψ_c represent SAR azimuth and elevation angles, respectively. At the center of a spatial aperture, $\alpha=0$ and $\psi_c=\psi_{c0}$. The target is an isotropic point reflector buried s_d meters below the surface of the lossy half space, and located at cartesian coordinates $(s_x, s_y, -s_d)$. The vector s identifies the surface projection of the target position, and points to cartesian coordinates $(s_x, s_y, 0)$. The vector r_s is the difference between r_c and s , and ψ_s is the angle between r_s and the x-y plane. We will assume that the assumptions made for equation (2) still apply over all target positions of interest.

Now consider a transmitted signal from the radar described by

$$X_T(t) = A_T(t) \cos\left(\omega_0 t + \frac{\gamma}{2} t^2\right), \quad (9)$$

where $A_T(t)$ is transmitted pulse envelope, ω_0 is the transmitted center frequency, and γ is the transmitted chirp rate. For long pulses compared to the depths of interest, the received echo from the point target can be approximated with

$$X_R(t) \approx \sigma A_r G_\eta(\omega_0 + \gamma(t - \tau_s)) X_T(t - \tau_s) \quad (10)$$

where A_r is the two-way loss in signal strength to the surface above the target, and σ is a measure of the target's reflectivity. Mixing this signal with a local oscillator of the form

$$X_{LO}(t) = \exp \left\{ -j \left(\omega_0 \left(t - \frac{2}{c} \|r_c\| \right) + \frac{\gamma}{2} \left(t - \frac{2}{c} \|r_c\| \right)^2 \right) \right\} \quad (12)$$

and following with low-pass filtering, yields a complex video signal that is approximated by

$$X_V(t) \approx \sigma A_r A_T(t - \tau_s) G_\eta(\omega_0 + \gamma(t - \tau_s)) \exp \left\{ j \frac{2}{c} \left(\omega_0 + \gamma \left(t - \frac{2}{c} \|r_c\| \right) \right) \right\} \left(\cos \psi_c (s_x \sin \alpha - s_y \cos \alpha) - s_d \text{Real} \left[\eta \sqrt{1 - \left(\frac{\cos \psi_c}{\eta} \right)^2} \right] \right) \quad (13)$$

Note that we have also made the small patch assumption that $\psi_s \approx \psi_c$. For $s_d=0$ and $\Gamma=0$, equation (13) reduces to the expression solved in Polar-Format processing. The intent in image formation is to extract the target coordinates (s_x, s_y, s_d) and assign σ to it. This paper, however, is not directly concerned with image formation algorithms, but rather in how the information is carried, and the effects of the sub-surface dielectric on that information. To this end we make the following observations.

- The complex refraction index η is generally a slowly varying function of frequency, but since frequency is related to time via the chirp rate γ , then η becomes a function of time as well.
- Apertures in the azimuth parameter α can be used to resolve s_x . If apertures in time t are used to resolve s_y , there will be an ambiguity due to the influence of target depth s_d . This is the 'layover' problem in traditional 2-D SAR images.
- Target coordinates s_y and s_d may be separated with an aperture in elevation parameter ψ_c , since $\cos(\psi_c)$ multiplies s_y and not s_d . The image formation processing could use an aperture in ψ_c to locate s_y , and leave the aperture in time (bandwidth) to locate s_d . This is the strategy for 3-D SAR processing.
- Resolution in s_x and s_y are unaffected by η . However, resolution in s_d is made finer by a factor

$$\left(\text{Real} \left[\eta \sqrt{1 - \left(\frac{\cos \psi_c}{\eta} \right)^2} \right] \right)^{-1}.$$

The resolution enhancement in the depth direction may be some consolation for any misfocus caused by dispersion and loss in the dielectric. By the same argument used for resolution, the patch size in the x and y directions is largely unaffected by target depth or dielectric properties.

- The loss function $G_\eta(\cdot)$ behaves as a depth dependent range window function in limiting usable bandwidth, and hence resolution.
- Even with minimal bandwidth, $\gamma=0$, we are fully capable of locating s_x and s_y with high precision and resolution. This mode is sometimes called 'holographic SAR'. If the 2-D aperture is an arc, then this is called 'Narrow Band SAR'. With such a system, depth resolution is limited by radar bandwidth instead of 'soil' bandwidth.

Examples

Consider a point target buried 2 meters below the surface in San Antonio Clay Loam with 5% water content by weight. Let a SAR be used to image the target with a nominal 100 MHz center frequency, and 150 MHz chirp, at a nominal range of 2 km.

Traditional SAR: Figure 8 illustrates the 3-Dimensional IPR of the point target with data collected along a linear flight path with nominal 30° elevation angle.

Figure 8.

3-D IPR of point target buried 2 meters below surface in San Antonio Clay Loam with 5% water content by weight. Traditional SAR with 1-Dimensional spatial aperture at 30° elevation angle. Image sequence represents 0.5-meter depth slices beginning at surface.

3-D SAR: Figure 9 illustrates the 3-Dimensional IPR of the point target with data collected over a 2-Dimensional spatial aperture made of 3 linear apertures arranged in an asterisk pattern directly above the target area.

Figure 9.

3-D IPR of point target buried 2 meters below surface in San Antonio Clay Loam with 5% water content by weight. 3-Dimensional SAR with 2-Dimensional spatial aperture at 90° elevation angle. Image sequence represents 0.5-meter depth slices beginning at surface.

DYNAMIC RANGE ISSUES

Dynamic range is defined as a maximum value divided by a minimum value, and is usually expressed in dB. For a ground penetrating SAR image, the values are radar cross section. The maximum value is the largest radar cross section that still keeps processing components in a sufficiently linear operating region, usually associated with a surface return, and then often defined as a surface clutter return. The minimum value is the return from the smallest radar cross section that still maintains some nominal signal-to-noise ratio (typically 0 dB, or 6 dB) in a processed image, usually associated with some target at some depth. The deepest such depth is sometimes called the 'depth horizon'. Dynamic range for an image can be calculated as follows:

$$DR_{\text{Image,dB}} = \left(\begin{array}{c} \text{brightest} \\ \text{surface target} \\ \text{cross section} \end{array} \right) - \left(\begin{array}{c} \text{dimmiest} \\ \text{buried target} \\ \text{cross section} \end{array} \right) \\ + (\text{propagation loss}) + (\text{transmissivity loss})$$

To calculate the raw dynamic range required of the radar front-end hardware, including the analog-to-digital converters, we need to subtract the coherent processing gain from the processed image dynamic range, and reference the noise floor by adding the minimum required signal-to-noise ratio (SNR) in the final image.

$$DR_{\text{Raw,dB}} = DR_{\text{Image,dB}} - \left(\begin{array}{c} \text{processing} \\ \text{gain} \end{array} \right) + \text{SNR}_{\text{min}}$$

What becomes readily apparent is that when searching for a particular target, say a 55-gallon metal drum, at an unknown depth, the image dynamic range required will be at least as large as the propagation loss to the deepest depth being considered. This may be many tens of dBs. Furthermore, the coherent processing gain will need to be in this range in order to not require an unreasonable number of bits in the analog-to-digital converters.

Large dynamic range systems have several difficulties associated with them. The first is with the antenna. Low-frequency high-resolution airborne SARs require wide-beam, and hence, low-gain antennas. Such antennas may have only a few dB of sidelobe attenuation, but will be used to form images with perhaps many tens of dB of dynamic range. The implication is that for practical purposes, the antenna should be considered isotropic. This leads to a problem with ambiguous ranges. The pulsed nature of radar signals allow distant target echoes from previous pulses to arrive at the same time as more recent echoes from nearer targets. Typically, the antenna directivity is used to attenuate the undesired echoes, but this may no longer be an option for large dynamic range systems with wide-beam antennas. Instead, we need to rely on the range loss which is only 40 dB per decade for point targets. For example, a surface target at a 100 km range may have a stronger return than a fairly shallow target in a lossy soil at only 10 km range. One answer to this problem might be to decrease the PRF as much as possible to extend the ambiguous ranges as far as possible, but this in turn reduces azimuth integration gain, which needs to be made up with transmitter power to maintain adequate SNR.

Another problem with large dynamic range systems is that the dynamic range may not always be usable. Dynamic range numbers typically ignore the interaction of target sidelobes. The finite spatial and time apertures (bandwidth) generate undesirable impulse response shapes and sidelobes in the image. In typical 2-D SAR the deeper target may be masked by the surface clutter return. In this case the target-signal to surface-clutter ratio is the important measure. Even in 3-D SAR, where the depth-horizon may only be a few resolution cells down, sidelobes from the surface clutter can easily mask deeper targets, in spite of the dynamic range. In this

case, the important measure becomes the target-signal to surface-clutter-sidelobe ratio. The question becomes "How useful is, say, 100 dB of dynamic range if clutter sidelobes can only be reduced by, say, 50 dB?" There do exist low-sidelobe window functions with sidelobe attenuation of many tens of dB, but this comes at a cost of significant broadening of the mainlobe. One intriguing idea stems from the observation that sidelobes are only really a problem in the down direction, and then only to the depth-horizon. Noting this allows us to consider complex window functions that push down sidelobes only in the important regions at the expense of unimportant sidelobes elsewhere (Doerry, 1994). This technique broadens the mainlobe less than traditional low-sidelobe window functions. In any case, most low-sidelobe techniques are often very sensitive to system nonlinearities, placing great burden on the hardware, and setting practical limits on the ultimate level of sidelobe reduction.

REMARKS

Forming an image of sub-surface targets requires consideration and mitigation of the ground's effects. The dielectric will impact the phase of the radar echo, as well as the amplitude in a frequency dependent manner. In an image this impacts the depth-resolution, making it finer at shallow depths, and coarser at deeper depths. The attenuation of deep targets impacts the necessary dynamic range of a SAR image, but the bigger problem is layover of surface clutter in traditional SAR, and clutter sidelobes in 3-D SAR. The model presented in this paper is useful for exploring the behavior of radar signals in dielectrics representative of soils.

ACKNOWLEDGEMENTS

This work performed at Sandia National Laboratories is supported by the U.S. Department of Energy under contract DE-AC04-94AL85000.

REFERENCES

- [1] Balanis, C.A. "Advanced Engineering Electromagnetics", New York: John Wiley & Sons, Inc., 1989, pp 185-214.
- [2] Brock, B. C., W. E. Patitz "Factors governing selection of operating frequency for subsurface-imaging synthetic-aperture radar", SPIE's International Symposium on Optical Engineering in Aerospace Sensing, SPIE Proceedings Vol. 2217, April 1994.
- [3] Brock, B. C., W. E. Patitz "Optimum Frequency for Subsurface-Imaging Synthetic-Aperture Radar, Sandia National Laboratories Report SAND93-0815, May 1993.
- [4] Daniels, D.J., D.J. Gunton, H.F. Scott, "Introduction to subsurface radar", IEE Proceedings, Vol. 135, Pt. F, No. 4, August 1988, pp. 278-320.
- [5] Doerry, A.W. "A Model for Forming Airborne Synthetic Aperture Radar Images of Underground Targets", Sandia National Laboratories Report SAND94-0139, January 1994.
- [6] Hipp, J.E., "Soil Electromagnetic Parameters as Functions of Frequency, Soil Density, and Soil Moisture", Proceedings of the IEEE, Vol. 62, No. 1, January 1974, pp. 98-103.
- [7] Xu, R., Z. Cao, F. Su, "Synthetic Aperture Radar Imaging for Objects Imbedded in a Half Space Lossy Medium at Close Range", SPIE Vol. 1875 Ultrahigh Resolution Radar, 1993, pp. 146-155.

DATE

FILMED

7/7/94

END