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TITLE

AN EVALUATION OF THE REACTOR NEUTRON SPECTRUM

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AN EVALUATION OF THE REACTOR NEUTRON SPECTRUM

BY

W. L. Bunch

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AN EVALUATION OF THE REACTOR NEUTRON SPECTRUMINTRODUCTION

The neutrons in an operating nuclear reactor are generated primarily by the fission events which are taking place. The great bulk of these relatively high energy or fast neutrons are slowed down or thermalized by a series of elastic collisions with the moderator nuclei which comprise the bulk of the volume of the reactor core. Once slowed down, the neutrons diffuse through the reactor core until they are absorbed or eliminated by some other process. Each of these three groups of neutrons, i.e., the fast or source neutrons, the intermediate or slowing down neutrons, and the slow or thermal neutrons, has a characteristic energy distribution. At a constant power level or rate of fissioning, an equilibrium is soon established among the groups at any point in the reactor. If it is assumed that a smooth transition exists between the different energy groups, it is possible to evaluate the entire neutron spectrum at a point in the reactor by determining the parameters which characterize each of the three groups. This has been done in the F Reactor Quickie Facility using radioactinants.

SUMMARY

The neutron spectrum determined to exist in the F Reactor Quickie Facility midway between process tubes 2194 and 2294 is shown in Figures 1 and 2 at a specific power of 5 MW/AT (approximately 27 KW/ft). The equations describing the flux in the three characteristic energy regions are as follows:

neutrons/cm² second
per electron volt

neutrons/cm² second
per unit vigor

Thermal

$$\phi_{th}(E) = 1.1 \times 10^{16} E \exp(-E/0.0689)$$

$$\phi_{th}(V) = 2.54 \times 10^{16} E^2 \exp(-E/0.0689)$$

Intermediate

$$\phi_{int}(E) = 9.32 \times 10^{11}/E$$

$$\phi_{int}(V) = 2.15 \times 10^{12}$$

Fast

$$\phi_{fast}(E) = 6.3 \times 10^5 \exp(-10^{-6}E) \sinh$$

$$\phi_{fast}(V) = 1.45 \times 10^6 E \exp(-10^{-6}E) \sinh$$

$$\sqrt{2 E 10^{-6}}$$

$$\sqrt{2 E 10^{-6}}$$

where E is the neutron energy in electron volts. A smooth transition has been made between the energy regions. The true thermal flux in this location is 5.25×10^{13} neutrons per square centimeter per second, where the average velocity of the neutrons is assumed to be 4060 meters per second. The total intermediate flux in the energy interval between one electron volt and a million electron volts is approximately 1.3×10^{13} neutrons per square centimeter per second. The fast flux, defined as that with an energy greater than one million

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Figure 1

NEUTRON FLUX SPECTRUM IN THE QUICKIE FACILITY AS A FUNCTION
OF ENERGY IN ELECTRON VOLTS

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Figure 2

NEUTRON FLUX SPECTRUM IN THE QUICKIE FACILITY
AS A FUNCTION OF VIGOR

electron volts, is found to be approximately 9.6×10^{11} neutrons per square centimeter per second. The above three characteristic equations can be combined into a single equation describing the entire flux spectrum if an appropriate "joining function" is used. One such arbitrary joining function, to be applied to the intermediate flux equation, is:

$$J(E) = \left[1 - \exp(-200 E^4) \right] \exp(-E^4 10^{-24}).$$

The combined equations become:

$$\phi(E) = \phi_{th}(E) + J(E) \phi_{int}(E) + \phi_{fast}(E)$$

and

$$\phi(V) = \phi_{th}(V) + J(E) \phi_{int}(V) + \phi_{fast}(V)$$

where the components are the relations tabulated above. Neutron vigor is simply a convenient logarithmic method of expressing energy which makes possible a linear plot of the neutron spectrum. It is felt that this linear presentation will aid materially in the mental visualization and understanding of the neutron flux spectrum within the reactor and the relationship which exists between the various components.

DISCUSSION

A. Background

Neutron flux is a term which is used frequently, yet it is a term which is not fully understood and appreciated by a great many people. It is felt that the fact that neutron flux is differential in nature, rather than integral, causes much of the misunderstanding and confusion. Almost all conversations and calculations utilize the integral form of flux (neutrons per square centimeter per second) rather than the differential form (neutrons per square centimeter per second per unit energy interval); this use is justified and produces meaningful results only if the true, differential nature of the flux is understood so that appropriate values are used. The following brief discussions of neutron flux and reaction rates are included to indicate the importance of the differential nature of neutron flux.

1. Neutron flux

Neutron flux is defined as the product of neutron density (neutrons per cubic centimeter) and neutron velocity (centimeters per second). A collimated beam of monoenergetic neutrons provides the simplest picture of neutron flux. The neutron flux in such a collimated beam is simply the number of neutrons which pass through a unit area (square centimeter) perpendicular to the beam each second. The neutron flux associated with the beam can be varied in either of two ways: (1) by

increasing or decreasing the velocity of the neutrons in the beam, or (2) by increasing or decreasing the neutron density in the beam. The important point is that the neutron flux is the product of neutron density and neutron velocity and that changes in either may affect the value of the flux. In general, all of the neutrons in a beam will not be traveling with the same velocity and the neutron flux becomes the product of the neutron density and the average velocity of the neutrons. This is equivalent to the integral equation:

$$\phi = \int_0^{\infty} n(v) v dv$$

where $n(v)$ is the neutron density distribution as a function of velocity. The neutron flux in this case is a function of the neutron velocity distribution (the average velocity) and the neutron density.

Within a reactor the neutrons are not collimated in a beam moving in one direction but are moving randomly in all directions. The neutron flux is still defined as the product of the neutron density and velocity and is obtained from an evaluation of the above integral equation. However, the physical concept which must be attached to the flux within a reactor is not the same. In the general situation the neutron flux is the number of neutrons which pass through a sphere (not a plane) of unit cross sectional area each second. In an isotropic flux the number of neutrons which pass through a plane of unit area is just one half the flux. This is true because a plane area presents little or no surface to those neutrons passing parallel or nearly parallel to the plane. As an aid in remembering flux in terms of a sphere of unit area, one might think of a nucleus as a sphere whose cross section is equal in all directions. It should be noted that a thin foil is a valid flux monitor even though it has the macroscopic appearance of a plane surface; the interactions are with the individual nuclei on a microscopic level and the foil is essentially an isotropic detector.

2. Reaction rates

The total neutron flux value obtained from an evaluation of the integral equation presented above is seldom, if ever, of any value. This is true because it is generally the results or effects which the flux can produce which are of interest, and each unit of flux is not equally able to produce a specific result. Reaction rates are usually calculated by using an integral flux value and an average or effective cross section:

$$R = N \phi \sigma$$

where R is the reaction rate in a sample containing N atoms of an isotope having an effective cross section σ . However, in more general terms, the reaction rate is obtained from an evaluation of the integral:

$$R = N \int_0^{\infty} n(v) v \sigma(v) dv$$

where $\sigma(v)$ is the reaction cross section as a function of energy. Using the first of these equations it appears as if the reaction rate is proportional to the flux. Using the latter of these equations indicates that the relation between flux and reaction rate is complex. Remembering that the value of the flux is affected by changes both in the absolute neutron density and the neutron velocity distribution, the following examples are presented to show how the reaction might be affected by a change in the flux value.

Consider first a typical absorption process whose cross section is inversely proportional to the incident neutron velocity, i.e., $\sigma(v) = \sigma_0/v$. The integral equation for the reaction rate becomes:

$$R = N \sigma_0 \int_0^{\infty} n(v) dv = N n \sigma_0,$$

where n is the neutron density. In this case it is apparent that the reaction rate is directly proportional to the neutron density but is independent of the neutron velocity distribution. Therefore, changes in the flux which are effected by altering the velocity distribution do not affect the reaction rate. On the other hand, the reaction rate directly reflects changes in the flux caused by altering the absolute neutron density. To calculate the reaction rate of an isotope whose cross section is inversely proportional to neutron velocity one must know the neutron density, not the neutron flux. The neutron flux is of no value unless the average velocity is also known.

A second type of reaction rate which is not proportional to the neutron flux is the threshold reaction. A threshold reaction is one which requires that the neutron bring a certain minimum amount of kinetic energy into the compound nucleus to cause the particular event. As before, the reaction rate could be calculated by using an integral flux value and an effective cross section. More specifically, the reaction rate is given by an evaluation of the integral equation:

$$R = N \int_0^{\infty} n(v) v \sigma(v) dv = N \int_{v_t}^{\infty} n(v) v \sigma(v) dv$$

where v_t is the minimum or threshold velocity which the neutron must have to cause the reaction. It is once again apparent that this reaction rate is unrelated to the total neutron flux. In this case, however, the reaction rate is strongly dependent on the velocity distribution and is quite independent of the total neutron density--exactly the opposite of the $1/v$ reaction discussed above.

It can be seen that an integral flux value is frequently of little value and must be used with caution. In most cases it is required that the neutron velocity distribution, $n(v)$, be known, (or at least the neutron density, n) before meaningful results can be obtained. The purpose of the work presented here has been to measure this distribution in a particular test hole location.

3. Energy units

The neutron density distribution and the flux have been discussed as a function of the neutron velocity. It is frequently convenient to utilize other energy units; the most common system employed is the electron volt. The kinetic energy of the neutron is related to its velocity by the equation:

$$K.E. = \frac{1}{2} m v^2$$

where m is the mass of the neutron. For the mass in grams and the velocity in centimeters per second, the kinetic energy is in ergs. One electron volt is equivalent to 1.6×10^{-12} ergs; the neutron energy E , in electron volts is related to the neutron velocity by:

$$E = 5.227 \times 10^{-13} v^2.$$

It is of interest to consider a couple of cases. For a neutron whose velocity is 2.2×10^5 centimeters per second (a little over a mile per second) the corresponding energy is 0.0253 electron volts. Conversely, a neutron whose energy is one million electron volts has a velocity of approximately 1.38×10^9 centimeters per second (about ten thousand miles per second). For significantly higher energies the relativistic form of the kinetic energy equation must be used as the neutron velocity approaches the speed of light. The relativistic form retains the feature of one to one correspondence between velocity and energy.

As indicated above, a large energy range must be considered in an evaluation of the neutron spectrum. It is convenient to use a logarithmic variable for plotting the spectrum over this large energy range. The logarithmic energy unit which is used has been given the name "vigor" and is defined by the equation:

$$V = (\ln E/0.001)/2.303$$

where E is the neutron energy in electron volts. Although the defining equation appears complex, vigor is quite easy to understand, and the comparison of neutron energy in electron volts and vigor is quite direct. The base energy in the vigor system has been set arbitrarily at 0.001 electron volt since there are essentially no neutrons below that energy in a practical system. A neutron with a kinetic energy of 0.001 electron volt is said to have no (zero) vigor. An energy of 0.01 electron volt corresponds to a vigor of 1, an energy of 0.1 electron volt corresponds to a vigor of 2, an energy of 1 electron volt corresponds to a vigor of 3, etc. Thus, each unit increase in vigor corresponds to a tenfold increase in energy. Those familiar with the logarithmic variable called lethargy used in reactor physics calculations will recognize the similarity between vigor and lethargy. Vigor has been used here because of this simple relation to energy to aid the mental visualization process; it is not necessarily a useful nor practical system for calculational purposes as is lethargy.

B. Theoretical Basis

The neutrons which are formed in the Hanford reactors are due primarily to the thermal fission of uranium 235. These relatively fast neutrons are slowed by interactions with the nuclei in the core, predominantly with the carbon nuclei in the moderator which form the bulk of the core. Once slowed down the neutrons diffuse through the reactor until they decay or leak out, or more likely, until they are absorbed to form a new isotope and in some cases to cause a fission event. Each of the three neutron groups, i.e., the virgin fission neutrons, the neutrons which are being slowed down, and the neutrons which have been slowed down, has a characteristic energy distribution. If it is assumed that a smooth transition exists between the groups, it should be possible to evaluate the entire spectrum by determining the proper parameters for each group. The characteristic energy distributions are described below.

1. Fission spectrum

The energy distribution of the neutrons which result from the thermal fission of uranium 235 has been determined experimentally. In the absence of a fundamental theory which predicts the energy spectrum, the data have been described by purely empirical relations. Although several such relations have been proposed, the one which has become most popular because of its combined simplicity and accuracy is the Watt fission spectrum:

$$d\phi/dE' = \phi(E') = 0.484 \phi_f \exp(-E') \sinh \sqrt{2 E'},$$

where $\phi(E')$ is in neutrons per square centimeter per second per Mev and E' is in Mev (million electron volts). This equation for the differential spectrum has been normalized so that the integral over all energies yields ϕ_f neutrons per square centimeter per second. A pure fission spectrum would be found only where none of the emitted neutrons has interacted with an intervening nucleus. Obviously a pure fission spectrum cannot exist anywhere in an operating reactor; however, the high energy portion of the reactor spectrum should consist primarily of unscattered fission neutrons or of neutrons whose distribution is fundamentally that of fission neutrons. By using threshold reactions sensitive only to these high energy neutrons, a fast fission coefficient, ϕ_f , can be determined for the Watt fission spectrum which should realistically describe the neutron flux above the threshold energy. This high energy tail must subsequently be joined to the slowing down spectrum.

2. Slowing down spectrum

An exact theoretical description of the slowing down neutron spectrum in a reactor, considering all of the known possible perturbing effects, is much beyond the scope of this paper. However, for a large graphite moderator, many simplifying assumptions can be made which make the spectrum tractable over most of the energy range. First, it will be assumed that the fast neutrons are being produced homogeneously and continuously throughout the volume under consideration. Actually,

the neutrons are "born" in the uranium, but if the production is considered to be at the point at which the neutron drops below a particular (high) energy, the source is distributed somewhat uniformly through the core moderator. Further assumptions which are made are: (a) there are no neutron losses in the slowing down spectrum due either to leakage from the system or absorption within the system, and (b) scattering is spherically symmetric in the center of mass system. In a large graphite system these two assumptions are reasonably valid; carbon has no significant resonance absorption cross sections and the neutron energies being considered are sufficiently low for the scattering to be isotropic in the center of mass system. Two other factors which are neglected for simplicity are: (a) the motion of the scattering nuclei, and (b) the energy distribution of the source. Neglecting the motion of the carbon nuclei means that the postulated spectrum is invalid at low energies where this motion due to the thermal energy of the system is important. Neglecting the spectral distribution of the source neutrons invalidates the postulated spectrum at or above the energy where significant numbers of neutrons are entering the system. It is in these two energy regions where it will be assumed that a smooth transition exists between the characteristic regions.

With the above assumptions and limitations it is shown by Glasstone and Edlund* that the equilibrium flux distribution in the slowing down energy region is given by:

$$\phi(E) = Q/\xi \Sigma_s E = \phi_i/E$$

where Q is the neutron source strength in neutrons per cubic centimeter per second which enter the system, Σ_s is the macroscopic scattering cross section of the moderator in reciprocal centimeters, ξ is the average logarithmic energy loss per collision with a moderator nucleus, and the energy E is in electron volts. The function $\phi(E)$ is in neutrons per square centimeter per second per electron volt while the intermediate energy flux coefficient ϕ_i is in neutrons per square centimeter per second.

3. Thermal Spectrum

The neutrons which have been slowed down come into thermal equilibrium with the moderator nuclei. These neutrons may either gain or lose energy in a given collision with a moderator atom, but there is no net energy exchange between the neutrons and the moderator when the system as a whole is considered. Under these conditions the kinetic energy of the neutrons is distributed statistically according to the Maxwell-Boltzman gas equation:

* Glasstone and Edlund, "The Elements of Nuclear Reactor Theory," (D. Van Nostrand, New York) 1952, Chapter VI.

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$$n(v) = dn/dv = \frac{4}{\sqrt{\pi}} m \frac{v^2}{v_0^3} \exp(-v^2/v_0^2)$$

where v_0 is the most probable neutron velocity in the distribution (i.e., the velocity at which $n(v)$ is a maximum). As before, n is the neutron density in neutrons per cubic centimeter and v is the velocity in centimeters per second. The flux distribution as a function of velocity can be obtained by multiplying the velocity distribution by velocity:

$$\phi(v) = n(v) v = \frac{4}{\sqrt{\pi}} m \frac{v^3}{v_0^3} \exp(-v^2/v_0^2).$$

In order that the thermal flux spectrum be joined to the slowing down spectrum, it is necessary that the energy variable be converted from velocity to electron volts. This can be done readily by using the equation relating velocity and electron volts ($E = 5.227 \times 10^{-13} v^2$) and utilizing the fact that the most probable neutron velocity is related to the absolute temperature of the system by the equation:

$$2 k T = m v_0^2$$

where k is the Boltzman constant (8.617×10^{-5} electron volts per Kelvin degree). The thermal flux distribution as a function of energy in electron volts and system temperature in degrees Kelvin becomes:

$$\phi(E) = \frac{2}{\sqrt{\pi}} \phi_0 \frac{E}{(kT)^2} \exp(-E/kT)$$

where ϕ_0 has been used for the product of the neutron density, n , and the most probable velocity, v_0 . It has been found that the effective temperature of the flux distribution is somewhat higher than the absolute temperature of the system. This phenomenon is attributed to hardening of the spectrum due to the preferential absorption of the lower energy neutrons by nuclei whose absorption cross sections are inversely proportional to the neutron velocity. Because of the small absorption cross section of graphite, hardening of the spectrum in a graphite moderator is not great; an optimum choice for the effective neutron temperature makes the above equation a good approximation of the actual distribution.

If the above thermal neutron flux spectrum is integrated over all energies, it will be found that the thermal neutron flux is $\frac{2}{\sqrt{\pi}} \phi_0$ neutrons per square centimeter per second. This value of $\frac{2}{\sqrt{\pi}}$ the thermal flux is frequently called the "true" neutron flux and is equivalent to the neutron density times the average neutron velocity. It is apparent that the average neutron velocity in a Maxwell-Boltzman distribution is $2/\sqrt{\pi} = 1.128$ times as great as the most probable velocity. To utilize the true neutron flux in calculations it would be necessary to use cross sections which corresponded to this average velocity. The true thermal neutron flux is the best simple approximation to use in diffusion calculations where the behaviour of the average neutron is the predominant factor.

Since the effective neutron temperature must be known in order to evaluate the true neutron flux, it is also possible to obtain the neutron density, n . As was shown previously, the activation of most isotopes is proportional to the neutron density, not the flux; therefore, it frequently is desirable to know the neutron density. An artificial but convenient system of thermal neutron flux has been derived based on this importance of neutron density in activation and heat generation problems. The system is called "conventional thermal neutron flux" and is based on the product of the neutron density and an arbitrary velocity of 2200 meters per second--the most probable velocity in a Maxwell-Boltzman distribution at room temperature. Even at room temperature, the conventional thermal neutron flux is only (1/1.128) as great as the true thermal neutron flux; at higher temperatures the difference is progressively greater. However, the "Chart of the Nuclides", the tables in the "Barn Book" (BNL-325) and most other tabulations of "thermal" neutron cross sections are for 2200 meter per second neutrons and must be used with conventional thermal neutron flux values if meaningful absorption rates are to be calculated. The extensive but profitable and practical use of this artificial integrated flux system undoubtedly contributes to the confusion which too frequently is associated with neutron flux.

4. Spectrum summary

Equations have been presented above which describe the neutron flux spectrum in each of the three characteristic groups as a function of neutron energy in electron volts. As indicated previously, it is also desirable to express the equations as a function of neutron vigor so that a simple graphic presentation can be made which will aid in the understanding of the relative fluxes in the different groups and energy ranges. The conversion of the above equations is based on the fact that the flux in an infinitesimally small energy interval must be equal to the flux in an infinitesimally small vigor interval:

$$\phi(V) dV = \phi(E) dE.$$

Using the defining equation for vigor as a function of energy we find that:

$$\phi(V) = 2.303 E \phi(E).$$

The three characteristic flux equations as a function of energy:

$$\text{fast,} \quad \phi(E) = 0.484 \phi_f 10^{-6} \exp(-10^{-6}E) \sinh \sqrt{2 E 10^{-6}}$$

$$\text{intermediate,} \quad \phi(E) = \phi_i / E$$

$$\text{thermal,} \quad \phi(E) = \frac{2}{\sqrt{\pi}} \phi_o \frac{E}{(kT)^2} \exp(-E/kT)$$

become the following three equations as a function of vigor:

$$\text{fast,} \quad \phi(V) = 1.11 \phi_f E 10^{-6} \exp(-10^{-6}E) \sinh \sqrt{2 E 10^{-6}}$$

$$\text{intermediate,} \quad \phi(V) = 2.303 \phi_i$$

$$\text{thermal,} \quad \phi(V) = 2.303 \phi_o \frac{2}{\sqrt{\pi}} \frac{E^2}{(kT)^2} \exp(-E/kT)$$

where the energy is in electron volts in all of the equations. A plot of the vigor flux as a function of vigor can be made easily on linear paper over the entire flux spectrum so that a visual comparison and integration of the neutron fluxes within different groups and energy ranges can be made readily. Because of the simple relation between vigor and energy (each increment of vigor is a factor of ten in energy) the comparison to electron volts is also straightforward.

C. EXPERIMENTAL BASIS

The experimental problem consists of evaluating the three parameters, ϕ_o , ϕ_i , and ϕ_f , and of estimating or establishing the effective equilibrium neutron temperature, T . The thermal and intermediate energy flux parameters were obtained by comparative techniques using the Hanford Standard Pile as a basis. The Hanford Standard Pile is a block of graphite containing a radium-beryllium neutron source of known strength so that the thermal and slowing down fluxes are known in the graphite as a function of distance from the source. Gold was used as a radioactivant for the comparisons. Accuracy of the determination is limited by the standardization of the fluxes in the Hanford Standard Pile; some uncertainty is also introduced by the comparison technique because of the tremendous difference in the neutron densities in the two locations, Standard Pile and experimental facility. The fast neutron flux parameter cannot be obtained with the same precision as the lower energy fluxes. The fast neutron flux parameter is based on the absolute activity of elements having threshold reactions; the paucity of cross section data which exists for these reactions severely limits the accuracy of the determination which can be made at this time. Many of the experimental techniques which were employed and which are described briefly below were developed by E. G. Peterson of the Hanford Laboratories Operation for neutron flux measurements in reactor shield materials.

1. Thermal flux coefficient

The thermal flux parameter is based on the activation of gold foils. The neutron activation cross section of gold is essentially inversely proportional to velocity in the thermal region even though several large resonances exist in the intermediate energy region. It can be shown that the difference in activity between a bare gold foil and a cadmium covered gold foil irradiated in the same location is proportional to the thermal neutron density except for a small perturbation due to the absorption of intermediate flux in the cadmium cover. The purpose of the cadmium cover is to eliminate the thermal flux getting to the foil while not materially affecting the intermediate

energy flux. Actually, a small error is introduced because of perturbation of the intermediate energy flux, but if the ratio of the activity of the bare foil to the cadmium covered foil is large, the error due to the perturbation is small. Assuming that the neutron density in the Hanford Standard Pile is known, then the neutron density at some experimental location is given by the relation:

$$n = n_s \Delta R / \Delta R_s$$

where ΔR is the difference in count rate between the bare and cadmium covered foil for that location and the subscript s denotes the standard pile values. The parameter ϕ_0 is the product of the neutron density and the most probable velocity, v_0 . Therefore, the thermal neutron flux parameter for the experimental location is related to the neutron density in the standard pile by:

$$\phi_0 = n v_0 = (n v_s / v_s) v_0 (\Delta R / \Delta R_s) = \phi_s (v_0 / v_s) (\Delta R / \Delta R_s)$$

where the most probable velocities for both the standard pile location (v_s) and for the experimental location (v_0) must be estimated based on the local moderator temperature.

Very little heat is generated in the Hanford Standard Pile; therefore, it is assumed that the most probable neutron velocity in the graphite is 2200 meters per second. At a position 10.64 centimeters from the neutron source, the flux in the pile has been determined to be 8,518 neutrons per square centimeter per second based on this velocity assumption. Using the experimental techniques employed in the present experiment, it was found that the ratio, $\phi_s / \Delta R_s$ for this location was 0.899.* The above equation relating the thermal flux parameter to the difference in foil activity becomes:

$$\phi_0 = 0.899 \Delta R (v_0 / v_s).$$

The conventional thermal neutron flux in the experimental location, defined previously as the product of the neutron density and an arbitrary velocity of 2200 meters per second, would be simply $0.899 \Delta R$. This conventional neutron flux value could be used with tabulated cross sections for calculating the thermal activation of most isotopes. On the other hand, the true neutron flux would be 1.128 times as great as ϕ_0 and would be equal to the product of the neutron density and the average neutron velocity. The true thermal neutron flux value would be needed for calculations pertaining to the diffusion of neutrons where the use of the average velocity approximates the actual behaviour. Either of these useful integral flux values can be obtained from the differential thermal neutron flux spectrum.

2. Intermediate flux coefficient

The activity of a cadmium covered gold foil is proportional to the neutron flux which penetrates the cadmium. Because of the large

* Letter, E. G. Peterson to J. DePangher, dated 12-29-59.

thermal absorption cross section of cadmium, covers of appropriate thickness absorb essentially all of the incident thermal neutron flux while permitting almost all of the intermediate flux to get to the enclosed foil. In a moderating medium such as graphite, where it is postulated that the intermediate energy differential flux spectrum is inversely proportional to the energy, the activity of a cadmium covered foil is a measure of the slowing down density which in turn is a measure of the entire intermediate neutron energy spectrum. The intermediate flux coefficient at the experimental location would be:

$$\phi_i = \phi_{is} R_c / R_{cs} = 0.185 R_c$$

where the subscript c indicates count rates of the cadmium covered foils. The ratio ϕ_{is}/R_{cs} was found to be 0.185 in the Hanford Standard Pile using the same or similar proportional counting equipment, foils, and covers as that employed in the present experiment.

3. Fast flux coefficient

The fast flux coefficient is based on a determination of the absolute activity of an isotope produced by a particular threshold reaction. The saturated activity of such a threshold detector is given by the equation:

$$A = N \int_0^{\infty} \phi(E) \sigma(E) dE$$

where A is the absolute saturated activity of the sample in disintegrations per second, N is the number of atoms of the threshold isotope in the sample, and $\phi(E)$ and $\sigma(E)$ are the energy dependent flux spectrum and cross section respectively. If it is assumed that the energy spectrum is known, an effective cross section for that spectrum can be obtained. For example, if it is assumed that the fast flux spectrum above the threshold energy is of the same form as a fission spectrum, the equation may be written:

$$A = N \phi_f \int_{E'_T}^{\infty} 0.484 \exp(-E') \sinh \sqrt{2 E'} \sigma(E') dE' = N \phi_f \sigma_f$$

where the integral can be evaluated using experimental values for the cross section as a function of energy. The effective cross section, σ_f , is the cross section weighted over the fission spectrum. For those reactions in which the threshold energy is above the most probable energy in the fission spectrum, the cross section is not very sensitive to the entire distribution but only to those neutrons immediately above the threshold energy. This is because of the rapid decay of the spectrum in the high energy tail. However, a series of threshold detectors with different threshold energies should indicate whether or not the assumption of a fission spectrum tail is valid.

Although most isotopes have threshold reactions, only a few elements have distinctive products whose half life and decay spectrum make them useful detectors. Of these elements, the cross section data published to date is sketchy, incomplete, and inconsistent.^{1,2,3} Sulfur appears to be the most reliable detector at the present time. The cross section for the $S^{32}(n,p)P^{32}$ reaction has been measured many times and the product half life (14.5 days) is convenient. Using the latest data tabulated by Howerton, the effective cross section of the reaction was found to be 62 millibarns from a graphical integration (see Figure 3). Another useful element and reaction is the $Al^{27}(n,\alpha)Na^{24}$ reaction which has a half life of 15.0 hours. The threshold for this reaction is at a much higher energy; therefore, the effective cross section was found to be only 0.57 millibarns (see Figure 4). On the basis of the referenced data, it appears that a factor of two uncertainty would not be unlikely in these values. The use of threshold detectors for accurate spectral measurements in the high energy region must await better cross section measurements.

4. Description of test facility

The Quickie Facility in the F Reactor is located in the E test hole which is 83 inches downstream from the centerline of the graphite stack. The facility, which penetrates the reactor between process tube rows 21 and 22, consists of four concentric aluminum tubes with cooling water passing in and out of the reactor in the small annular spaces between the outer three of these tubes. The dry, innermost tube of the four contains a push rod with eighteen sample pockets spaced one and three quarters inches apart center to center. Each sample pocket is an inch long and a quarter inch in diameter. A long, external lead-filled shield attached to the reactor makes it possible to charge and discharge samples during reactor operation. The manually operated push rod is indexed so that any pocket can be aligned beneath the charge slot or over the discharge chute. Because of space restrictions on the experimental level the length of the external shielding is limited which in turn limits the extent of the penetration within the reactor which is practical. At the present time, pocket number one can be inserted to a position between tubes 2194 and 2294; at the same time the outermost pocket, number eighteen, is approximately at the midpoint of the reflector. Only two or three seconds of transit time are required to push the rod from the shield (zero flux region) to the maximum flux position; hence, irradiations of only a few minutes duration can be made and timed quite accurately.

E. DATA

A total of four separate irradiations were made in which detectors were placed in all of the available sample pockets (some of the pockets could

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1. UCRL-5226, "Tabulated Neutron Cross Sections, Part 1", R. J. Howerton, May 1958.
 2. Nucleonics, "Fission-Neutron Cross Sections for Threshold Reactions", R. S. Rochlin, January 1959.
 3. BNL-325, "Neutron Cross Sections", D. J. Hughes and R. B. Schwartz, July 1, 1958.

CROSS SECTION -

Millibarns per Mev

Figure 3

EFFECTIVE CROSS SECTION OF SULFUR 32 (n,p) REACTION
FOR FISSION NEUTRONS

Points are product $0.484 \exp(-E) \sinh \sqrt{2E} \sigma_E$.

σ_E values are from: Phys. Rev. 107, 1363, (1957), Allen, Biggers,
Prestwood and Smith.

Area under curve = $\sigma_f = 62$ millibarns

not be used because of long term irradiations which were in progress). In each of these irradiations, one or more bare gold foils was used; in three of the irradiations, cadmium covered gold foils were used; and in two of the irradiations, cadmium covered sulfur was exposed. In addition to these four traverses, two aluminum irradiations were made in the innermost pocket (number one) as a check on the fast neutron flux spectrum at that point. The gold and sulfur activity were measured in proportional counters; the aluminum activity was determined with a gamma spectrometer. The normalized activation data of the four traverses are shown in Figure 5, along with the heat generation rates of the adjacent process tubes. Although each traverse which was obtained was found to be internally consistent, a significant absolute difference appeared between the traverses of similar foils. An analysis of the count rate data showed that in every case the longer the decay time which occurred before the activity was measured, the higher the extrapolated saturated activity. This discrepancy leads to the conclusion that longer lived isotopes were present. No reasonable explanation has been found for either the gold or sulfur. By normalizing all of the data to the earliest counting date, the effect of the longer lived isotopes is minimized but not necessarily eliminated. The standard deviation of the thermal and intermediate fluxes obtained from the data should be 10% based on uncertainties in the Hanford Standard Pile flux values, counting techniques, etc. However, the longer half life impurity which was present means that the experimental flux values are probably within plus 10% and minus 20% of the actual values. The fast flux values are limited by the unknown uncertainty in the cross sections.

F. RESULTS

The neutron flux spectrum in any of the test hole pockets can be estimated by evaluating the flux coefficients using the experimental data presented in Figure 5. As an example, the following determination is made for the maximum flux location (pocket one) which appears to be representative of the core region. The thermal and intermediate flux coefficients can be obtained directly from the count rate data using the Hanford Standard Pile factors obtained previously for the foils and counters:

$$\begin{aligned}\phi_0 &= 0.899 \Delta R (v_0/v_s) = 0.899 (3.65 - 0.51) 10^{13} (3600/2200) \\ &= 4.65 \times 10^{13}\end{aligned}$$

where the velocity (3600 meters per second) is the most probable velocity for a neutron temperature distribution at 800°K. The conventional thermal neutron flux would be simply $0.899 \Delta R$ which is equal to 2.84×10^{13} neutrons per square centimeter per second. The true flux would be $1.128 \times \phi_0$ which is equal to 5.25×10^{13} neutrons per square centimeter per second, nearly twice the value of the conventional flux.

The intermediate energy flux coefficient for pocket one is:

$$\phi_1 = 0.185 \times R_c = 0.185 \times 0.51 \times 10^{13} = 9.32 \times 10^{11}.$$

Figure 5

NORMALIZED SATURATED ACTIVITY OF FLUX MONITORS IRRADIATED
IN THE F REACTOR QUICKIE FACILITY

The fast flux coefficient is obtained from the absolute activity of the sulfur, not the count rate as was the case for the gold foils where the Hanford Standard Pile had been used to obtain conversion factors. The activity A in disintegrations per second is related to the count rate R in counts per minute per gram by:

$$A = R/60 (\text{counter efficiency})(\text{burning efficiency})$$

where the counter and burning efficiency factors were determined experimentally to be 44.79% and 93.5% respectively. (The irradiated sulfur is burned to eliminate the sulfur and leave only the radioactive phosphorous for counting purposes. Some of the phosphorous is lost in the burning process). The number of atoms of S32 in a gram of sulfur is:

$$N = 0.95 \times 6.023 \times 10^{23}/32 = 1.79 \times 10^{22} \text{ atoms per gram.}$$

The fast flux factor is:

$$\begin{aligned} \phi_f &= A/N \sigma_f = R/60 (0.4479)(0.935)(1.79 \times 10^{22})(62 \times 10^{-27}) \\ &= 35.9 R = 35.9 \times 3.35 \times 10^{10} = 1.2 \times 10^{12}, \end{aligned}$$

based on the activity of the sulfur. To confirm this value of the fast flux coefficient, two different aluminum irradiations were made in pocket one and the specific activity of the sodium measured with a gamma spectrometer. The average value of the fast flux coefficient based on these two measurements was 1.39×10^{12} , which is reasonably good agreement considering the difference in the threshold energies of the sulfur and aluminum reactions (see Figures 3 and 4). The "best" value of the fast flux coefficient for pocket one was assumed to be 1.3×10^{12} .

For the assumed temperature of 800°K (kT = 0.0689) the three characteristic energy groups in pocket one become (as a function of energy):

$$\begin{aligned} \text{Thermal:} \quad \phi(E) &= \frac{2}{\sqrt{\pi}} 4.65 \times 10^{13} \frac{E}{(0.0689)^2} \exp(-E/0.0689) \\ &= 1.1 \times 10^{16} E \exp(-E/0.0689). \end{aligned}$$

$$\text{Intermediate:} \quad \phi(E) = 9.32 \times 10^{11}/E$$

$$\begin{aligned} \text{Fast:} \quad \phi(E) &= 0.484 \times 1.3 \times 10^{12} \times 10^{-6} \times \exp(-10^{-6} E) \\ \sinh \sqrt{2 E 10^{-6}} &= 6.3 \times 10^5 \exp(-10^{-6} E) \\ \sinh \sqrt{2 E 10^{-6}} & \end{aligned}$$

These equations can be combined into a single expression for the flux as a function of energy by the use of an arbitrary joining function. Both the thermal and fast components are finite in their present form and essentially go to zero at the desired juncture point; therefore, the joining function need only apply to the intermediate component which is

otherwise unbounded. A simple exponential joining function which is unity throughout the energy region in which the intermediate flux predominates and which rapidly approaches zero at the desired juncture points is:

$$J = [1 - \exp(-200E^4)] \exp(-E^4 10^{-24}).$$

The flux spectrum then becomes:

$$\begin{aligned} \phi(E) = & 1.1 \times 10^{16} E \exp(-E/0.0689) + [1 - \exp(-200E^4)] \\ & \exp(-E^4 10^{-24}) 9.32 \times 10^{11}/E + 6.3 \times 10^5 \exp(-10^{-6} E) \\ & \sinh \sqrt{2 E 10^{-6}}. \end{aligned}$$

When this relation is plotted, it is noted that the transition is not smooth in the vicinity of one million electron volts; this is due to an inadequacy in the joining function and is not intended to reflect variations in the flux which actually might exist. The flux spectrum as a function of vigor can be obtained by multiplying the above equation by 2.303 E, since $\phi(V) = 2.303 E \phi(E)$.

G. CONCLUSIONS

Radioactivants provide a method of evaluating the neutron flux spectrum within a reactor if the spectrum is assumed to be composed of three simple characteristic energy groups. The flux spectrum in the lower energy groups, i.e., below about a hundred thousand electron volts, should be determined quite accurately both because of the experimental methods available and the limited number of assumptions which need to be made. The flux spectrum above about two million electron volts can be obtained only as accurately as threshold cross sections have been measured. At the present time these cross sections do not seem to be available with an accuracy much better than a factor of two. The problem of joining the fast and intermediate components undoubtedly introduces somewhat greater uncertainty than a factor of two into the energy region between about one-tenth and two million electron volts.

It is felt that the flux spectrum which was found to exist in the innermost pocket of the F Reactor Quickie Facility should be representative of the spectrum in many of the test facilities and in much of the graphite within the reactor (when normalized on an adjacent power basis). A measurement needs to be made adjacent to a process tube or next to the uranium to determine the extreme variation of the flux spectrum in the graphite.

W. L. Bunch

W. L. Bunch
Irradiation Testing

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