AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910

301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

BATTLE-NORTHWEST
 RECEIVED
 APR 30 1971
 CIRCULATING
 GOPT
 SECTION

154

154

DOCUMENT NO.

HW-63157

SERIES AND COPY NO.

DATE _____

JULY 17, 1961

THIS DOCUMENT CONTAINS RESTRICTED DATA
DEFINED IN [REDACTED] ENERGY ACT
ITS TRANSFER OR THE DISCLOSURE OF ITS
CONTENT IN ANY MANNER TO ANY UNAUTHORIZED
PERSON IS PROHIBITED.

TITLE

PARAMETRIC ANALYSIS OF HANFORD SPEED OF CONTROL CRITERION

☐ OTHER OFFICIAL CLASSIFIED INFORMATION
THIS MATERIAL CONTAINS INFORMATION AFFECTING
THE NATIONAL DEFENSE OF THE UNITED STATES
WITHIN THE MEANING OF THE ESPIONAGE LAWS,
TITLE 18, U. S. C., SECS. 793 AND 794, THE TRANS-
MISSION OR REVELATION OF WHICH IN ANY MANNER
TO AN UNAUTHORIZED PERSON IS PROHIBITED BY
LAW.

AUTHOR

R. NILSON
R. E. TILLER
AND
J. C. PEDEN

ISSUING FILE

APR 28 1971 1961

THIS DOCUMENT MUST NOT BE LEFT UNATTENDED OR WHERE AN UNAUTHORIZED PERSON MAY HAVE ACCESS TO IT. WHEN NOT IN USE, IT MUST BE STORED IN AN APPROPRIATELY LOCKED REPOSITORY WITHIN AN APPROVED GUARDED AREA. WHEN IT IS YOUR POSSESSION, YOU HAVE OBTAINED A SIGNATURE FROM THE CLASSIFIED FILE. IT IS YOUR RESPONSIBILITY TO KEEP IT AND ITS CONTENTS WITHIN THE LIMITS OF THIS PROJECT AND FROM ANY UNAUTHORIZED PERSON. ITS TRANSMITTAL TO ANY OTHER STORAGE AT YOUR PLACE OF RESIDENCE IS PROHIBITED. IT IS NOT TO BE DUPLICATED. IF ADDITIONAL COPIES ARE REQUIRED, OBTAIN THEM FROM THE RELEASING OFFICE. ALL PERSONS RECEIVING THIS DOCUMENT ARE REQUESTED TO SIGN IN THE SPACE PROVIDED BELOW.

[illegible]

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

54-3000-340 (3-57) AEC-GE RICHLAND, WASH

(CLASSIFICATION)

DECLASSIFIED

Classification Cancelled and Changed To

DECLASSIFIED

By Authority of WA Snyder,

CG-PR-2, 1-7-94

By DK Hanson 1-13-94

Verified By JE Sandy 1-31-94

July 17, 1961

DISTRIBUTION:

This document contains restricted data as defined by the Atomic Energy Act of 1954. Its transmission or disclosure of its contents in any way to an unauthorized person is prohibited.

This document consists of
54 pages. No. 16 of
26 copies.

This document classified by:

R. Nilson

1. CW Allen
2. CE Bowers
3. JH Brown
4. WD Cameron
5. RD Carter
6. FG Dawson
7. GC Fullmer
8. CR Gallagher - HOO-AEC
9. OH Greager
10. AK Hardin
11. RA Harvey
12. RE Heineman
13. RH Meichle
14. DI Monnie
15. WS Nechodom
16. R Nilson
18. GF Owsley
19. RS Paul
20. RW Reid
21. RE Tiller
22. RE Trumble
23. FW VanWormer
24. AD Vaughn
25. 300 Files
26. Record Center

PARAMETRIC ANALYSIS OF HANFORD SPEED OF CONTROL CRITERION

DECLASSIFIED

July 17, 1961

PARAMETRIC ANALYSIS OF HANFORD SPEED OF CONTROL CRITERION

R. NILSON

R. E. TILLER

AND

J. C. PEDEN

IRRADIATION PROCESSING DEPARTMENT

REACTOR PHYSICS UNIT

DECLASSIFIED

INTRODUCTION

The initial designs of the Hanford reactors are such that the cold lattice is not fail-safe upon removal of coolant water. That is, the "dry" lattice is a more reactive configuration than the "wet" lattice. In the cold pile, the wet to dry reactivity difference is as much as 2.5 to 3% k in the 8-3/8" lattices and about 2% k in the 7-1/2" lattice. These figures are for current I&E geometry fuel elements.⁽¹⁾

On the other hand, in the hot operating reactor, under certain conditions of exposure and temperature, the wet pile can actually be more reactive than the dry pile, i.e., a fail-safe lattice. Consider, a green, hot reactor at the equilibrium power. The loss of coolant in a K reactor under such conditions would actually result in about a 1% loss in reactivity. The older reactors would gain a nominal 0.1 to .2% k under the same conditions.

However, a green, hot reactor is only a "denken" case of no practical interest, since due to the presence of Pu²³⁹ in the ripe pile the hot, dry reactor becomes again more reactive than the hot, wet reactor. For plutonium concentrations corresponding to a goal exposure of ~ 800 MWD/T⁽²⁾ the reactivity increase in a hot pile at equilibrium power is as large as 1 to 1.5% k (more exact figures are given elsewhere).⁴ This excess reactivity can be potentially added to the reactor in a relatively few seconds if the reactor suffers a maximum credible accident, i.e., rapid and complete loss of coolant pressure. A nuclear excursion resulting must be adequately extinguished to minimize damage from such an accident.

Therefore, the loss of coolant accident has always been recognized at Hanford as not only one in which a full pile meltdown could occur with the release of large quantities of radioactive fission products, but also one in which a rapid nuclear excursion might accompany the accident and serve to increase the consequences of the disaster. For these reasons, extreme precautions are taken to insure a continuous and reliably backed-up coolant supply, and in addition, under the assumption that rapid and complete loss of coolant is a credible accident, the operation of the reactors is governed by important control criteria. These criteria serve to guarantee no significant increase in the disaster consequences due to nuclear effects attending the accident.⁽³⁾

This report purposes to critically examine the many, interrelated, nuclear, physical and mechanical phenomena and actions which take place following a loss of coolant accident to a Hanford reactor operating at equilibrium power. The results are based on analytical studies in which each important variable and parameter relating to the sequence of events has been carefully examined to determine its importance on the overall calculation. The end result should help

1. OII, CII, KIII I&E Elements.
2. The average pile exposure would be close to 400 MWD/T.
3. These criteria are given in HW-64148, "Survey of Hanford Production Reactor Hazards", by R. E. Trumble, March 4, 1960 (Secret).
4. HW-70164, "Water Loss Reactivity Gain in Hanford Production Reactors," W. S. Mechodan and R. Nilson. To be issued (Secret).

DECLASSIFIED

answer the questions "How well are we assured that our reactors meet the important criterion relating to speed of control?", and "How sure are we that what we represent mathematically to be the most severe case is truly so?".

SUMMARY AND CONCLUSIONS

Analog simulations of the loss of coolant accident under various conditions of pile power have been made. The following parameters were varied in the study to assess the sensitivity of the results on uncertainties in the parameters.

VSR Strength
VSR Insertion Time
Doppler Effect
Instantaneous vs. Delayed Pressure Loss

The results of the analyses are presented as speed-of-control limit curves for the various combinations of assumptions. The following conclusions can be made.

1. Under the presently used assumption that the maximum accident will involve coolant flow reduction to half of equilibrium in one second, operation of the reactors at present power levels is well within the speed-of-control limits.
2. The effect of uniform variation in safety rod strength is not great. For example, in C-Reactor, a uniform reduction of a factor of 1 1/2 in rod strength reduces the acceptable speed-of-control-limited power level by ~ 500 MW.
3. The effect of varying the insertion time of the VSR's is greater than the effect of varying their strength. The effect of slowing rod insertion by a factor of two is to reduce acceptable speed-of-control-limited power levels by ~ 800 MW, 1200 MW and 2000 MW in the BDF, C and K piles, respectively.
4. Conversely, the use of rods with more rapid insertion times would place the speed-of-control limits even farther above present operating levels.
5. The sensitivity of the speed-of-control power level curves to the Doppler effect is greater for the BDF piles. This is due to the reduced effect of the VSR's in these reactors caused by longer release times.

The results can be interpreted as showing that for the parameters considered, present uncertainties are not sufficient to materially affect the accuracy of the speed-of-control-limits. The results also show extreme sensitivity to the magnitude of the reactivity gain on water loss. A 2 mk error in the value assumed correct for the BDF equilibrium case would reduce the acceptable power level from 2700 MW to 2100 MW. It is therefore essential that conservative values be used for the reactivity effect of the water-loss in any evaluation of a particular speed-of-control situation.

The validity of the mathematical model used can only be partly assessed from these results. Further work is required to correct certain weaknesses in the model.

DECLASSIFIED

DISCUSSION

The continued power level ascensions in the Hanford reactors have quite naturally aroused questions as to any deleterious effects higher power levels might have on the consequences of a maximum credible accident. Recognition of the possibility that at sufficiently high power levels a nuclear excursion accompanying a water-loss might cause sufficient damage to a Hanford reactor to seriously magnify the inevitable consequences of the accident has led to the establishment of nuclear safety criteria. These criteria, if complied with, guarantee no adverse effects under water-loss accident conditions above the inevitable consequences of the loss of coolant.

Two general nuclear safety criteria have been formulated. The Speed-of-Control Criterion, which is of concern here, is presently stated as follows:

Speed of Control Criterion

The operation of the Hanford reactors shall be such that in the event of a power excursion from any credible combination of events including complete and permanent loss of coolant, the response of the reactor safety control system shall be sufficient to insure that no energy state will develop, while the reactor is supercritical during the excursion, which would significantly increase the consequences of the accident.

This criterion has been accepted by the various Government hazards evaluation groups.

Interpretation

Besides establishing values for the various reactor parameters which are utilized in calculations aimed at establishing limiting conditions imposed by the speed-of-control criterion (discussed in following sections), it is necessary to make major assumptions regarding a) the actual physical limit to the reactor power level during an excursion, below which the criterion is satisfied, and b) the manner and timing of a water-loss accident. These assumptions are:

- 1) The limiting fuel temperature during a power excursion shall not exceed the melting point of uranium.
- 2) The maximum credible rate of coolant flow reduction shall be 50 percent per second.

DECLASSIFIED

The validity of these assumptions is discussed elsewhere⁵. The assumptions have been accepted by the Government hazards evaluation groups.

Under the foregoing assumptions the equilibrium reactor power levels (for present fuel loadings and current goal exposures) are well within the limit set by the speed-of-control criterion. Only an extreme redesign of fuel elements in a manner which would increase the reactivity gain on loss of coolant (which is not anticipated), would cause the speed-of-control limits to approach reactor operating levels. Such a proposed change would require an examination of the reactor safety systems for modifications which would again guarantee compliance with the criteria.

Many factors naturally enter into the calculations required to assess that the reactor operations do indeed conform to the criterion. The material in this report summarizes very recent calculations which were aimed at defining (at least qualitatively) the uncertainties in the calculational methods, assumptions and results. The results of this study corroborate those of the early comprehensive work of Peden and Brown⁶. In addition, a significant updating and broadening of the problem has been achieved.

5. HW-53693, "Review of the Speed of Control Criterion," J. H. Brown, November 22, 1957 (Secret).
6. HW-52339, "Speed of Control in Hanford Reactors," J. C. Peden and J. H. Brown, September 16, 1957 (Secret).

DECLASSIFIED

THE PROBLEM

Kinetically, any nuclear reactor is a feedback system; a change in reactivity induces a change in neutron density and power level which in turn produces a change in temperature in the reactor. As a result the reactivity will be further changed. The particular problem here is a mathematical representation and solution of the time-dependent events occurring in a Hanford reactor following loss of coolant.

Insofar as is possible, actual experimental data are used in the problem. Where reliable information is lacking, insufficient or obscure for a particular phenomenon or parameter associated with the problem, a sufficiently conservative assumption, extrapolation or alternative is taken to assure that no misrepresentative conclusions are reached, and that the net error is on the safe side.

The Mathematical Model

1. Kinetic Equations

The space-independent, thermal group, reactor kinetic equations with nonlinear coefficients are used to describe the time-dependent behavior of the neutron density for a bare, finite, homogeneous reactor.

$$\frac{dn}{dt} = \frac{(\delta k - \beta) n}{l^*} - p e^{-B^2 \tau} \sum_i \lambda_i c_i \quad (1)$$

$$\frac{dc_i}{dt} = \frac{\beta_i n}{l^* p e^{-B^2 \tau}} - \lambda_i c_i. \quad (2)$$

In the above equations, n is proportional to the thermal neutron density.

p = resonance escape probability

τ = Fermi age to thermal

β_i = i th delayed neutron group fraction ($\beta = \sum_i \beta_i$)

λ_i = decay constant of precursor of i th delayed neutron group

B^2 = geometric buckling of reactor

c_i = i th group precursor concentration (nuclei/cm³)

l^* = mean neutron lifetime in finite reactor

k = effective multiplication factor

DECLASSIFIED

2. Fuel Temperature Equations

A key portion of the problem is the calculation of the fuel temperature as a function of instantaneous power. The calculation of the fuel temperature distribution requires transforming the partial differential heat diffusion equation into an ordinary differential equation. This is accomplished by describing the spatial temperature distribution by the method of finite differences. The fuel element is divided into five regions of equal width as shown in Figure I-2 (Appendix I). The final fuel temperature equations are as follows (derivation is in Appendix I).

$$\frac{dT_1}{dt} = \frac{1}{c_1} \left[A_1 n(t) + B_1 n(e_g) + k_{12} A_1^+ (T_2 - T_1) + DA_1^+ (T_w - T_1) \right] \quad (3)$$

$$\frac{dT_2}{dt} = \frac{1}{c_2} \left[A_2 n(t) + B_2 n(e_g) + k_{23} A_2^+ (T_3 - T_2) - k_{12} A_2^- (T_2 - T_1) \right] \quad (4)$$

$$\frac{dT_3}{dt} = \frac{1}{c_3} \left[A_3 n(t) + B_3 n(e_g) + k_{34} A_3^+ (T_4 - T_3) - k_{23} A_3^- (T_3 - T_2) \right] \quad (5)$$

$$\frac{dT_4}{dt} = \frac{1}{c_4} \left[A_4 n(t) + B_4 n(e_g) + k_{45} A_4^+ (T_5 - T_4) - k_{34} A_4^- (T_4 - T_3) \right] \quad (6)$$

$$\frac{dT_5}{dt} = \frac{1}{c_5} \left[A_5 n(t) + B_5 n(e_g) + A_5^+ (T_w - T_5) D - k_{45} A_5^- (T_5 - T_4) \right] \quad (7)$$

The various terms and constants in Equations (3) to (7) have the following meanings:

- c_i = specific heat of each uranium region
- T_i = temperature (average) of each region
- $k_{i(i+1)}$ = thermal conductivity (average between regions i and $i + 1$).

DECLASSIFIED

The A_1 , B_1 , and A_1^+ coefficients are defined in Appendix I. $\eta(t)$ and $n(eq)$ are numbers proportional to the time dependent and equilibrium neutron densities, respectively (See Appendix I).

The D-function is a time dependent function which represents the rate at which water boils out. This function is discussed in detail later.

Analog Representation

Figure 1 is a schematic sketch of the analog reactor simulation. The main section solves the neutron kinetics equations using six groups of delayed neutrons. Available analog equipment makes it necessary to simulate the average neutron population - a single-point reactor model is used. The input to the neutron kinetics section is the effective multiplication factor of the reactor which is changing constantly due to the various simultaneous phenomena. The reactivity contribution of vertical safety rod insertion (delayed according to the delay and release times discussed later) is simulated by a separate section. The duration of the excursion is too short to make it necessary to consider reactivity effects due to graphite heating, xenon buildup or horizontal rod insertion.

The output of the reactor dynamics section is in terms of reactor power which both drives the water-boilout function generator and determines the heat source for the calculation of the uranium temperature. The uranium temperature section uses a five-node model to compute fuel temperature (Equations 3-7). A second input to the uranium temperature section is the function describing loss of heat transfer due to the water boilout. The output of the fuel temperature section is used to calculate the Doppler effect which is fed back to the reactor dynamics equations as part of the total reactivity input. The reactivity effect due to coolant loss is also fed to the neutron kinetic sections. See Reference⁷ for analog circuit details.

Assumptions and Parameters

In any mathematical representation of a complex physical process, it is necessary to make certain simplifying assumptions. Every attempt should be made to insure conservative assumptions. The referenced work of Brown and Peden⁶ used a model which we shall call Speed-of-Control Model II. (Model I differed from Model II in that instantaneous pressure loss was assumed in the very initial studies). Model II simulated only the solid fuel geometry as far as the heat generation and heat transfer section of the problem were concerned. The work described here (hence called Model III) is similar in most details to that of Model II. Future models may be desirable to incorporate new ideas (such as a non-linear Doppler effect) and will be studied and reported later.

1. Assumptions

- a. It is considered that, even for the worst accident, instantaneous loss of coolant pressure is not possible. A reasonable assumption is that the pressure can decay at a rate no faster than fifty percent in one second.

7. HW-69940, "Analog Program in Reactor Speed of Control Studies," W. D. Cameron and R. E. Tiller (To be issued).

DECLASSIFIED

SCHEMATIC SKETCH OF REACTOR SIMULATION ON ANALOG COMPUTER

FIGURE 1

UNCLASSIFIED

-10-

HM-63157

UNCLASSIFIED

- b. The detection of the loss of pressure is accomplished by one of many sensitive pressure gages (one on each tube) whose measured response time to such a pressure decay is 0.1 to 0.4 seconds. To be conservative it is assumed it requires the full 0.4 seconds for the trip.
- c. The boilout time of the coolant has been experimentally determined for solid fuel rods under simulated accident conditions⁸. The same time behavior for coolant boilout is assumed for the I&E fuel element. This is reasonable since although there is an additional heat transfer surface, there is more water per tube.
- d. The reactivity-effect of water-loss is assumed to vary linearly with water volume.
- e. VSR response is based on measured relay responses and rod drop times⁹. The insertion rate is cosine-squared weighted to obtain the reactivity-effect (See Reference 6).
- f. The Doppler effect is a linear function of the volume-averaged fuel temperature.
- g. Loss of heat transfer is assumed as water is boiled out.

2. Parameters Varied in Study

To provide as general study as possible and to assess the effect of uncertainties in various key-parameters on the speed-of-control, power-level limits, the following parameters were varied:

- a. VSR control strength
- b. VSR insertion time including accelerated rods
- c. Doppler coefficient
- d. Instantaneous vs. delayed pressure loss

The extent of the variation in these parameters is shown in Table I for each reactor type.

3. Values of Fixed Parameters Used in Study.

The values of the fixed parameters used in the study are shown in Table II.

- 8. HW-52340, "Process Tube Boilout Experiments," D. C. Pound and G. J. Busselman, August 3, 1957 (Secret).
- 9. HW-61478, "Safety Circuit and VSR Delay Times," P. C. Althoff, October 12, 1959 (Confidential).

DECLASSIFIED

TABLE I
RANGE OF VARIATION IN PARAMETERS

<u>Parameter</u>	<u>BDF</u>	<u>C</u>	<u>KE, KW</u>
VSR Strength (mk)	0 - 52	0 - 52	26 - 52
VSR Insertion Time (Sec)	0.8 - 3.2	0.8 - 3.2	0.8 - 3.0
Doppler Coeff. (mk/C)	.0093 - .028	.0094 - .031	.011 - .044
Pressure Loss	Instantaneous and one second delay		
Water Loss dk/k (mk)	0 - 26	0 - 28	8 - 30

DR and H Reactors differ slightly in their characteristics from the above cases in that they have faster VSR release times than the other older reactors and H has the same VSR system as C.

TABLE II
FIXED PARAMETERS

<u>Parameter</u>	<u>BDF</u>	<u>C</u>	<u>KE, KW</u>
VSR Release Time ^a (Sec.)	1.4	1.0	0.9
Neutron Lifetime ^b (Sec.)	9.4×10^{-4}	9.4×10^{-4}	9.4×10^{-4}
Delayed Neutron Constants ^c			
β_1, λ_1	0.0002103, 0.0125		
β_2, λ_2	0.001437, 0.0306		
β_3, λ_3	0.001316, 0.1125		
β_4, λ_4	0.002697, 0.3037		
β_5, λ_5	0.0008702, 1.1470		
β_6, λ_6	0.0003106, 3.0533		
$\Sigma \beta_i$	0.00684		

^a Includes time between Panellit gauge opening to rod tip entering active zone (See Reference 6 for details).

^b Peden and Brown used 1.4×10^{-3} sec. Recent calculations of the lifetime in the reactors indicate that the 9.4×10^{-4} seconds used is conservative for all but the K-piles. For these reactors the minimum theoretical thermal lifetime is $\sim 8.5 \times 10^{-4}$ seconds. If the slowing down time is added, the value is 10^{-3} seconds, however.

^c Natural uranium "Based on Keepin's results," (Reference 10)

10. LA-2110, "Delayed Neutrons From Fissionable Isotopes of Uranium, Plutonium and Thorium," Keepin et al, July 23, 1957 (Unclassified).

DECLASSIFIED

4. Values of Fixed Constants

The KIIIN, CIIN and OIIN I&E fuel geometries were used in the studies. The dimensions of these fuel elements and the pertinent other reactor dimensions can be found in Appendix I along with the various hydraulic constants, numerical constants in the heat equations and radii of the sectionalized fuel models.

Details of the Calculation

It is necessary to detail in some degree some of the more important aspects of the calculation. Of particular importance are the effects associated with coolant loss, VSR insertion, and uranium heating.

1. Coolant Loss Functions

The two events that occur during the power excursion associated with water loss are the gain in lattice reactivity and the loss of heat transfer from the fuel. These are described by two coolant-loss functions. One of these functions, called the reactivity-weighted, coolant-loss function (Figure 2) describes the gain in reactivity due to coolant loss. The other, called the D-function (Figure 3) describes the loss of heat transfer from the fuel when the coolant boils out. Both are dependent upon the energy generation after flow-stoppage.

The basic water boilout data used in deriving these functions were obtained from Reference 8. The weighting procedure that was applied to these data to obtain the desired functions is discussed in Reference 6, and need not be repeated here.

The coolant-loss functions used for the five older reactors and C-Reactor are those given in Reference 6. The same D-function used in the solid-slug analyses⁶ was used to describe the loss of heat transfer for both the inside and outside water annuli of the I&E fuel elements studies in the present work. The functions used were also extrapolated to simulate the boilout of all the water in the reactor which is a departure from the earlier work (Model II, Reference 6). The boilout data presented in Reference 6 indicate approximately ten percent of the water remained in the tube after any experimental boilout run. However, the maximum energy generation in the boilout experiments were 5000 kw-sec per tube which is considerably under energy generations obtained in the excursion calculations. The data were therefore extrapolated linearly to obtain complete boilout. (See Figure 3). The extrapolated $D \rightarrow 0$ point for C and the old reactors is 16,000 MW-sec for complete boilout.

There are no similar boilout data available for the K-pile geometry. The functions assumed for K-pile were based on the most severe of the C-pile boilout data.

CLASSIFIED

REACTIVITY WEIGHTED WATER LOSS FUNCTION

INTEGRATED POWER - MEGAWATT SECONDS X

FIGURE 2

DECLASSIFIED³

FIGURE 3

DECLASSIFIED

The pile-weighting procedure for the K-pile cases was the same as that used by Peden and Brown⁶ for the old piles. The radial flux-distribution outside the flattened zone of the reactor used in the weighting is

$$\phi(r) = -2.257 J_0(.0085 r) + 0.749 Y_0(.0085 r) \text{ for}$$

$$R_1 < r < R_2, \text{ where}$$

$$R_1 = 412 \text{ cm} - \text{radius of flattened zone, and}$$

$$R_2 = 610 \text{ cm} - \text{equivalent outside radius of reactor.}$$

This describes a flux distribution corresponding to an ECT of 2500. Again, the same D-function was used to describe the loss of heat transfer for both the inside and outside water annuli.

The water loss functions given in Figures 2 and 3 are for an instantaneous stoppage of flow. In order to consider the effect of a more realistic loss of flow (for example 50 percent reduction in flow in one second), the water loss functions were delayed by one second relative to when the pressure loss is signaled to the panellit guage. The process tube boilout experiments⁸ considered both instantaneous and slower cutoff cases. The slower cutoff, water-loss transients were very similar to those of the instantaneous cutoff cases with the exception that the time was effectively displaced. The boilout curves coincided if zero time was taken at about the half-flow point

2. Vertical Safety Rods (VSR)

The vertical safety rod functions for the old piles and C-pile are discussed in Reference 6, and are shown in Figure 4. The rod drop characteristic for K-pile was weighted the same as that for C-pile and is also presented in Figure 4.

Two methods were used to initiate a VSR trip. A panellit pressure gauge trip was used that included a 0.4 second delay in the trip signal, and a power level trip that generated a trip signal when the power achieved 115% of its equilibrium level.

The accelerated rod function shown in Figure 4 corresponds to an air-accelerated rod and was used in some of the calculations to determine the effect of accelerated rods.

The VSR release times listed in Table II are based on Althoff's measurements⁹ and include all relay release times, any built-in delays to override power fluctuations, clutch demagnetization times and the time for the rods to drop from their initial positions to the top of the active zone of the reactor.

DECLASSIFIED

VERTICAL SAFETY ROD FUNCTIONS

FIGURE 4

DECLASSIFIED

3. Uranium Temperature Coefficient

The heating of the uranium in the excursions provides a negative reactivity effect by the well-known Doppler effect. Although the Doppler effect was one of the parameters varied in the calculations, it is necessary to discuss the manner in which the effect was treated.

There have been no measurements of the Doppler effect for non-uniform changes in fuel temperatures. In any actual water-loss excursion, the initial fuel surface temperature is lower than the average temperature in the fuel. As coolant is lost, the loss of heat transfer causes the surface temperature of the fuel element to rise considerably more than the temperature within the fuel. Thus, the metal heating is not uniform over the fuel element, and a proper averaging of the Doppler effect (based on uniform heating experiments) over the volume of the fuel element is necessary. Furthermore, the resonance capture of neutrons in U^{238} is a pronounced surface effect. Figure 5 shows measured resonance absorptions in a CIIN fuel element¹¹. One might, therefore, expect a larger Doppler effect in the uranium near the fuel surfaces. In an attempt to take into account both non-uniform heating and the greater resonance absorption on the surfaces of the fuel, the local Doppler effect on the multiplication factor was computed as follows:

$$\delta k_m = \alpha_m \frac{\int_{\text{fuel}} \delta T(r) \psi(r) r dr}{\int_{\text{fuel}} \psi(r) r dr} \quad (8)$$

where

α_m = uniform temperature Doppler coefficient (mk/c)

$T(r)$ = radial temperature distribution in fuel

$\psi(r)$ = radial distribution of resonance absorption in fuel
(See Figure 5)

In the sectionalized fuel model, the integral is replaced by a sum over each section.

$$\delta k_m = \alpha_m \sum_i \delta T_i W_i, \quad (9)$$

11. Unpublished work of R. Nilson

DECLASSIFIED

DISTRIBUTION OF U238 RESONANCE ABSORPTIONS IN CIIN SLUG

DECLASSIFIED

where the weighting factor W_1 is given by

$$W_1 = \frac{\int_{r_1}^{r_1 + 1} \psi(r) r dr}{\int_{r_1}^{r_6} \psi(r) r dr},$$

and the T_1 's are the axially-averaged fuel temperature in the maximum powered tube.

The weighting factors are very nearly identical for each fuel type considered and are tabulated below.

TABLE III
DOPPLER COEFFICIENT WEIGHTING FACTORS

r_1	$r_1 + 1$	W_1
1	2	.1205
2	3	.1139
3	4	.1407
4	5	.1973
5	6	.4277

The T_1 's appearing in Equation 9 are calculated as those representing the average fuel temperature (averaged axially) in the maximum powered tube. Actually, to obtain a more representative overall pile effect, the T_1 's should be also averaged over the radial dimension of the pile; this would reduce the Doppler effect. However, a compensating effect, which is also ignored in the calculation, is the fact that (under the assumption that the flux preserves its chopped-cosine distribution throughout the excursion) the δT in the center of the reactor is considerably greater than the δT averaged front to rear and hence the Doppler effect is greater in the center of the pile where the neutron importance is greater. If it is assumed that both the axial fuel temperature distribution and the axial flux distribution follows the cosine law, then the proper averaging of the change in fuel temperature should be weighted by the cube of the flux. The combined effect of these two considerations results in an expression for δk_m which exceeds that of Equation 9.

$$\delta k_m = 1.2 \alpha_m \sum_i T_i W_i \quad (10)$$

Equation 9 was used in the calculations which has the effect of underestimating the Doppler effect - a conservative factor in the calculations.

RESULTS

The power level limit curves presented in this report were obtained by determining, on a trial-and-error basis, the combinations of equilibrium power levels and reactivity-worths of water which would just delay uranium melting at the surface of the hottest fuel element in the reactor until the excursion was terminated. The excursion was considered terminated when the power level returned to its former equilibrium value. This is a more restrictive interpretation than the current statement of the speed-of-control criterion which states that uranium melting shall be delayed until the pile is subcritical. As the curves in Figure 6 show, the point in time when the reactor returns to its equilibrium power is later than the point where the reactor goes subcritical.

The actual temperature calculated by Equations 3 - 7 represents the average fuel temperature (averaged along the reactor axis) for a maximum-powered tube. If a cosine temperature distribution is assumed along the pile axis, the average temperature that would correspond to a peak temperature of 1133 C is 721 C. Thus, the limit curves given herein describe those combinations of equilibrium powers and reactivity-worths of the water which would cause the average surface fuel temperature to reach 721 C at the time when the power returned to its former equilibrium level.

Figure 6 shows in detail typical transient data obtained in the calculations. The curves represent K-Reactor cases in which water was lost at 6000 MW equilibrium power (Case I) and at 4000 MW equilibrium power (Case II). The reactivity worth of the water was taken as 20 mk, VSR strength as 37 mk, VSR insertion time as 1.6 sec., and Doppler coefficient as -.022 mk/C. Pressure loss was assumed to drop to half-value in one second. The remaining parameters are as given in Table II.

The top curves in Figure 6 show the excess pile reactivity during the excursion. In both cases prompt criticality is exceeded for a short period. The reactor returns to a subcritical state at .8 - .9 sec. on the time scale shown. The middle curves show the behavior of the uranium surface temperature of the hottest fuel element in the reactor during the excursion. In Case I, melting did occur, which indicates that for this set of assumptions (6000 MW equilibrium power and 20 mk for the reactivity worth of coolant loss), the speed-of-control criterion is exceeded. Case II which represents nearly present power conditions in the K-Reactors does not exceed the criterion. The bottom figures show the power of the reactor. Note that the melting occurs in Case I before the power returns to its pre-excursion value. As the top curve shows, the reactor is subcritical before the fuel melts.

DECLASSIFIED

TYPICAL TRANSIENT CURVES - K PILES

TIME - SECONDS

FIGURE 6

REF. ASSIFIED

The continued rise in temperature in Case I after the excursion is terminated is due to decay heat generation in the then-insulated fuel. The decay heat generation is held constant during the entire calculation, which tends to overestimate the power level for times beyond the time the reactor goes subcritical.

The speed-of-control limit curves obtained in these calculations are shown in Figures 7 through 19. These curves are interpreted as follows: A point to the left and below the curves represents a combination of equilibrium power level (at which the water-loss accident occurs) and reactivity worth of the water-loss which meets the speed-of-control criterion. In other words, operation in this region would insure that no uranium melting would occur during excursion caused by loss of coolant. The effects of the various parameters varied in the study are discussed below.

Effect of Instantaneous vs. Delayed Water Loss

Figures 7 and 8 show the large effect delaying the water loss by one second makes on the curves for the BDF and C Piles. The assumed parameters are shown in the figures. Under earlier assumptions (pre-1957) of instantaneous water loss, the present operation of the BDF and C Reactors would be outside of the criterion. The relaxation afforded by the presently-used assumption of coolant flow reduction to half level in one second is quite large, enabling operation well within the revised criterion for all reactors.

Effect of Varying Rod Strength

Figures 9, 10 and 11 show the effects of varying the strength of the VSR system in the BDF, C and K Piles. The variation is assumed to be a uniform variation over the entire pile. Local variations, for example that caused by certain rods inoperable in a localized region, are not represented by these curves. The solid curve in each figure represents the curve for the "best values" of the parameters. It is noted that the effect of reducing rod strength is not very great. For example, for C Reactor, a uniform reduction of a factor of $1\frac{1}{2}$ in rod strength reduces the acceptable speed-of-control-limited power level by ~ 500 MW. It is also interesting to note that there is an acceptable area of operation (at low values of water-loss reactivity gain) for no rods. This attests to the excursion-terminating effect of the Doppler effect in those cases where the loss of coolant does not result in a very large reactivity gain (< 10 mk).

Effect of Varying Rod Insertion Time

Figures 12, 13, and 14 show the effect of varying the insertion time of the VSR's. The VSR response curves (Figure 4) were merely stretched or compressed in time to simulate the effect of slower or faster rods. The effect of slowing the rod insertion by a factor of two is to reduce the acceptable speed-of-control-limited power level by ~ 800 MW in the BDF piles, ~ 1200 MW in C Pile and ~ 2000 MW in the K piles. The reason the effect is smaller in the BDF piles is that in these reactors the rod release time is considerably longer than in the newer piles and hence the effectiveness of the rod is reduced by their later arrival into the active zone of the reactor.

FIGURE 7
BDF PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

FIGURE 8
C PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

FIGURE 9
BDF PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

FIGURE 10
C PILE LIMIT CURVES - 1133 °C MAXIMUM SURFACE TEMPERATURE

DECLASSIFIED

FIGURE 11
K PILE LIMIT CURVES - 1133 °C MAXIMUM SURFACE TEMPERATURE

FIGURE 12
BDF PILE LIMIT CURVES - 1133 °C MAXIMUM SURFACE TEMPERATURE

FIGURE 13
C PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

DECLASSIFIED

FIGURE 14
K PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

A case was calculated for K-Reactor in which 7.8 mk of accelerated rods was added to the 37 mk of normal VSR's. The results are shown in Figure 15. The effect of the accelerated rods serves to increase the speed-of-control-limited power level considerably.

Effect of Varying Doppler Coefficient

The effect of varying the Doppler coefficient, α_m , is shown in Figure 16, 17, and 18. The sensitivity of the curves to the Doppler effect is greatest for the BDF pile since the VSR effect is reduced by the longer VSR release times in these reactors, hence making the termination of the excursion more dependent on the negative Doppler effect.

INTERPRETATION OF RESULTS

The results of this study can be used to partly answer the questions raised in the introduction. The question of how well the reactors meet the speed-of-control criterion depends greatly on how well the values of the parameters assumed in the calculations match the actual cases. As pointed out on the discussion, several conservative assumptions are made to assure a result which would still be conservative even when uncertainties in the calculational parameters are considered. Some of the weak points in the mathematical model can similarly be evaluated from these results to help answer the question on the trueness of the mathematical representation.

Degree to Which Reactors Meet Criterion

It is sufficient to answer the first question by considering the results for the reactors which would be most limited by the speed-of-control criterion. These are the three oldest reactors, B, D and F Reactors. The speed-of-control curve for these reactors can be obtained from any of the previous figures using the best values of the parameters. For convenience the curve is reproduced in Figure 19.

For a value of reactivity worth of water of 15 mk^4 which is conservatively representative of the reactivity worth of the water in a hot BDF pile with an average fuel exposure of 500 MWD/T, the speed-of-control limit is seen to be 2700 MW, which is considerably in excess of present power levels in these reactors.

The best value for the BDF VSR strength is 39 mk. Assuming (to be very conservative) that this value is too large by a factor of $1 \frac{1}{2}$ (for example if certain rods fail to operate or if there are sufficient horizontal control rods in the reactor to shadow the VSR's appreciably) would reduce the speed-of-control limit by 200 MW. Furthermore, assuming that the Doppler coefficient is smaller in magnitude than that assumed by a factor of $1 \frac{1}{2}$ (to take into consideration a possible temperature-dependence of the Doppler coefficient which would make it smaller at higher temperatures) would reduce the limit by 250 MW.

DECLASSIFIED

FIGURE 15
K PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

FIGURE 16
BDF PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

FIGURE 17
C PILE LINE CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

DECLASSIFIED

FIGURE 18
K PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

DECLASSIFIED

FIGURE 19

BDF PILE LIMIT CURVES - 1133° C MAXIMUM SURFACE TEMPERATURE

On the other hand, the assumption of a 0.4 second Panellit response time may be long by 0.3 second. If a 0.2-second Panellit response were used, the speed-of-control limit would be raised by at least 150 MW. This is based on the results of varying rod insertion times - actually, shortening the Panellit response time would be more effective than these results show since the rods would be available for control sooner. Also, the procedure of using a Doppler effect based on the average fuel temperature of the maximum process tube was shown to be conservative; a pile-average effect would be 1.2 times larger. The effect of this change would increase the speed-of-control limit by 100 MW.

The net effect of the four factor-variations just discussed is to reduce the speed-of-control limit for the BDF reactors by only 200 MW, leaving the limit still well above the present power level.

Another variable to which the limit is quite sensitive is the reactivity worth of the water. For the case just considered, a 2 mk error in the value assumed to be correct would reduce the power level limit from 2700 MW to 2100 MW, very near the maximum level obtainable in the BDF reactors with 95 C bulk limit and 85,000 gpm flow. Hence, it is very essential that conservative values be assumed for the reactivity effect of the water-loss. Experimental, loss-of-coolant, reactivity changes are available for the cold, green, clean piles, but the determination of the values for the hot, operating piles with plutonium and fission products present must rely on theoretical methods.⁴

Evaluation of Mathematical Model

The second question is more difficult to answer, and further study is needed. The following weaknesses in the mathematical model are mentioned.

1. Linear Doppler Effect. The effect of assuming a Doppler effect which is independent of temperature is non-conservative. An idea of the seriousness of this assumption was obtained in the previous discussion where a reduction of the Doppler effect by a factor of 1 1/2 (which is more than ample to cover any decrease with increasing temperature) was shown to be only mildly important. A temperature-dependent Doppler coefficient will be used in future models.
2. Delayed Neutron Constants for Natural Uranium. The delayed neutron fraction from Pu²³⁹ is smaller than that for natural uranium. The use of delayed neutron constants for uranium is therefore non-conservative. The effect of using more representative constants has been looked at only cursorily and will be included in future models.
3. One-Point Reactor Model. The unsatisfactory nature of a single-point model for the neutron-kinetics equations cannot be fully assessed. For near-uniform reactor loadings and a uniform response of the VSR system the approximation should be satisfactory. For, non-uniform loadings and non-uniform VSR response, a space-dependent model is necessary. Such a model has been developed and results are to be reported at a future date.

DECLASSIFIED

4. Heat Transfer Model. The details of loss of heat transfer due to coolant boilout may not be adequately simulated in the present model. Additional assumptions can be made which would yield a known-conservative model.
5. Boilout Function. The use of the boilout function obtained for the solid fuel geometry should be carefully evaluated. The fact that there are two opposing effects which differentiate the two geometries in so far as coolant boilout is concerned (more coolant to be removed vs. more heat-transfer surfaces) does not lead one to too great an alarm. However, additional boilout test would be valuable for the newer fuel geometries.
6. Flux Distribution. The flux distribution in the reactor is assumed to preserve its chopped-cosine distribution axially and its fairly-flat radial distribution throughout the excursion. The flux depression induced by VSR-insertion in the center of the reactor is not taken into account. A spatial-dependent model could help to determine the effects of this assumption.

FUTURE STUDIES

Future studies will use a model which is more-representative of the actual cases. Temperature-dependent Doppler effect, delayed neutron constants for exposed fuel, up-to-date neutron lifetimes, and other changes will be incorporated. Their effect should not significantly alter results, however. Additional studies using a space-dependent kinetics model will also be carried out.

R. Nilson

Reactor Physics
Process and Reactor Development Subsection

R.E. Tiller

Reactor Physics
Process and Reactor Development Subsection

J.C. Peden By R.E.T.

Reactor Physics
Process and Reactor Development Subsection

R Nilson:RE Tiller:JC Peden:eck

DECLASSIFIED

APPENDIX IDerivation of the Fuel Temperature Equations

The simulation of the radial temperature distribution across the uranium fuel element requires solving the heat diffusion equation.

$$c \rho_u \frac{\partial T}{\partial t} = \Sigma + \vec{\nabla} \cdot k_u \vec{\nabla} T, \quad (1)$$

where c = the specific heat of the uranium,

ρ_u = the density of the uranium,

k_u = the thermal conductivity of the uranium,

Σ = the heat source.

The second term on the right of Equation (1) can be expanded as follows:

$$\vec{\nabla} \cdot k_u \vec{\nabla} T = k_u \nabla^2 T + \vec{\nabla} T \cdot \vec{\nabla} k_u.$$

If k_u is assumed linear with temperature (See Figure I-1), then

$$\vec{\nabla} \cdot k_u \vec{\nabla} T = (A + BT) \nabla^2 T + B(\vec{\nabla} T)^2. \quad (2)$$

Assuming $(\vec{\nabla} T)^2$ to be small compared to the other terms in Equation (2), Equation (1) becomes:

$$c \rho_u \frac{\partial T}{\partial t} = \Sigma + k_u \nabla^2 T. \quad (3)$$

This equation must be transformed into an ordinary differential equation to be solved by analog computer means. Therefore, the spatial temperature distribution $T(r)$ is described by finite differences, and the fuel element is divided into five annular regions of equal width as shown in Figure I-2.

DECLASSIFIED

FIGURE I-1
THERMAL CONDUCTIVITY OF URANIUM

DECLASSIFIED

FIGURE I-2 GEOMETRY OF FUEL ELEMENT

DECLASSIFIED

For an annular region 1, with volume V_1 , Equation (3) takes the form:

$$C_i \rho_i V_i \frac{\partial \bar{T}_i}{\partial t} = \sum_i V_i + k_i \nabla^2 \bar{T}_i, \quad (4)$$

where $\bar{T}_i$ is the average temperature of the i th region. The second term on the right side of Equation (4) represents the net rate of heat flow in volume V_i . If we restrict the problem only to radial heat flow with no angular dependence, the heat flow for region 1 is

$$k_i \nabla^2 \bar{T}_i = \frac{\bar{T}_{i+1} - \bar{T}_i}{R_{i(i+1)}} + \frac{\bar{T}_{i-1} - \bar{T}_i}{R_{i(i-1)}}, \quad (5)$$

where $R_{i(i+1)}$ and $R_{i(i-1)}$ are the resistances to heat flow between region i and $(i+1)$ and i and $(i-1)$, respectively.

For regions of like material of equal width Δr , the resistance term is of the form

$$R_{i(i+1)} = \frac{\Delta r}{k_{i(i+1)} S_{i(i+1)}}, \quad (6)$$

where $S_{i(i+1)}$ is the average surface area for heat transfer between sections i and $(i+1)$,

and $k_{i(i+1)}$ is the average conductivity between sections i and $(i+1)$.

The heat transfer areas $S_{i(i+1)}$ are given by

$$S_{i(i+1)} = 2\pi (r_{i+1}) L,$$

where L is the total fuel charge length, including end caps.

The effective conductivity between the two regions is given by

$$k_{i(i+1)} = \frac{k_i + k_{i+1}}{2}.$$

DECLASSIFIED

With the above definitions of $S_1(i+1)$ and $k_1(i+1)$, the resistance terms have the form:

$$R_{i(i+1)} = \frac{\Delta r}{[k_i + k_{i+1}] \pi L r_{i+1}} \quad (7)$$

$$R_{i(i-1)} = \frac{\Delta r}{[k_i + k_{i-1}] \pi L r_i} \quad (8)$$

Substitution of Equations (5), (7) and (8) into Equation (4) and dividing by $\rho_u V_i$, yields the following:

$$C_i \frac{dT_i}{dt} = \frac{\Sigma_i}{\rho_u} + h_{i(i+1)} A_i^+ (T_{i+1} - T_i) - h_{i(i-1)} A_i^- (T_i - T_{i-1}) \quad (9)$$

where the constants are defined as follows

$$A_i^+ = \frac{2 r_{i+1} L}{\rho_u r_i^2 (2 r_i + \Delta r) L_f} \quad (i = 1 \text{ to } 4)$$

$$A_i^- = \frac{2 r_i L}{\rho_u r_i^2 (2 r_i + \Delta r) L_f} \quad (i = 2 \text{ to } 5)$$

where L_f is the actual uranium length as contrasted to the fuel charge length L .

Equation (9) can now be used to describe the temperature in regions 2 through 4 of Figure I-2. Regions 1 and 5 must be treated differently. If the specific heat of the aluminum is assumed to be zero, then the heat flow from region 1 to the inside water annulus is:

$$\text{Heat to inside} = \frac{T_i - T_w}{R_{in}} \quad ,$$

where $R_{in} = R_u + R_{AL} + R_h$.

$$R_u = \frac{\Delta r}{k_u 2\pi (r_i + \frac{\Delta r}{4}) L} = \text{resistance from the node of region 1 to the uranium-aluminum interface}$$

$$R_{AL} = \frac{(r)_{AL}}{k_{AL} 2\pi (r_o + \frac{(r)_{AL}}{2}) L} = \text{resistance of aluminum}$$

UNCLASSIFIED

$$R_h = \frac{L}{h_{in} 2\pi r_o L} = \text{water film resistance.}$$

Then,

$$R_{in} = \left[\frac{ar}{2k_{Al}(r_o - \frac{ar}{2})} + \frac{\frac{(ar)k_u}{2}}{k_{Al}(r_o + \frac{(ar)k_u}{2})} + \frac{1}{h_{in} r_o} \right] \frac{1}{2\pi L} \quad (10)$$

The temperature dependence of the uranium conductivity k_u from the node of region 1 to the aluminum-uranium interface is ignored and the aluminum conductivity is considered constant.

By similar reasoning, the resistance from the node of region 5 to the outer water annulus is given by

$$R_{out} = \left[\frac{ar}{2k_{Al}(r_o - \frac{ar}{2})} + \frac{\frac{(ar)k_u}{2}}{k_{Al}(r_o + \frac{(ar)k_u}{2})} + \frac{1}{h_{out} r_o} \right] \frac{1}{2\pi L} \quad (11)$$

The water film coefficients h are obtained as follows.¹

$$h = M(1.0 + .012 T_b) G^{.8} F \times 1.356 \times 10^{-4} \text{ cal (sec-cm}^2\text{-}^\circ\text{C)}^{-1},$$

where for the outside and inside annuli, respectively, the constant M is

$$M = .02566 \left(\frac{\rho'_w}{s} \right)^{.8} \frac{1}{(d_c)^{.2}} \left(\frac{d_2}{d_1} \right)^{.5}, \text{ and}$$

$$M = .02566 \left(\frac{\rho'_w}{s} \right)^{.8} \frac{1}{(d_c)^{.2}}.$$

The various parameters and terms are defined as follows:

$$d_c = \frac{4s}{E}$$

G = Flow rate (gpm)

F = Viscosity correction factor

ρ'_w = Water density (lbs/ft³)

1. HW-45048, "Technical Note on Water Film Coefficients", S. S. Jones, August 27, 1956 (Confidential)

DECLASSIFIED

- s = Free cross sectional area of flow channel (ft²)
 d_g = Equivalent diameter of annular flow channel (ft)
 d_2 = Outer diameter of canned fuel element (ft)
 d_1 = Inside diameter of process tube (ft)
 E = Wetted perimeter (ft)
 T_b = Local bulk water temperature (F)

Although the film coefficient will vary when the coolant flow is lost, no attempt was made to calculate this change. Instead the entire resistance terms, Equations (10) and (11), were divided by a function (called the D function) which represented the rate at which the water boils out. The D function is dependent on the energy generation after water loss and varies from unity at full flow to zero when all water has been expelled from the reactor, at which point the fuel element is essentially an insulated cylinder. The D function has been discussed in the body of the report

From Equations (9), (10) and (11) the temperature equations for the five region fuel element are then:

$$\frac{dT_1}{d\tau} = \frac{1}{C_1} \left[\frac{\Sigma_1}{\rho} + R_{12} A_1^+ (T_2 - T_1) - A_1^- (T_1 - T_w) D \right], \quad (12)$$

$$\frac{dT_2}{d\tau} = \frac{1}{C_2} \left[\frac{\Sigma_2}{\rho} + R_{23} A_2^+ (T_3 - T_2) - R_{12} A_2^- (T_2 - T_1) \right], \quad (13)$$

$$\frac{dT_3}{d\tau} = \frac{1}{C_3} \left[\frac{\Sigma_3}{\rho} + R_{34} A_3^+ (T_4 - T_3) - R_{23} A_3^- (T_3 - T_2) \right], \quad (14)$$

$$\frac{dT_4}{d\tau} = \frac{1}{C_4} \left[\frac{\Sigma_4}{\rho} + R_{45} A_4^+ (T_5 - T_4) - R_{34} A_4^- (T_4 - T_3) \right], \quad (15)$$

$$\frac{dT_5}{d\tau} = \frac{1}{C_5} \left[\frac{\Sigma_5}{\rho} + A_5^+ (T_w - T_5) D - R_{45} A_5^- (T_5 - T_4) \right]. \quad (16)$$

The constants A_1^- and A_5^+ are defined as:

$$A_1^- = (\rho_u R_{in} V_1)^{-1}, \text{ and}$$

$$A_5^+ = (\rho_u R_{out} V_5)^{-1}.$$

DECLASSIFIED

The values for the aluminum and uranium conductivity contained in A_1^- and A_5^+ were taken as:

$$k_u = .059 \text{ cal (sec-cm-C)}^{-1}$$

$$k_{Al} = 0.5 \text{ cal (sec-cm-C)}^{-1}$$

Also,

$$\rho_u = 18.9 \text{ gm cm}^{-3} \text{ for uranium.}$$

From Figure I-1, the variable uranium conductivity is obtained as

$$k_{1(1+1)} = .955 + 3.35 \times 10^{-5} (T_1 + T_1 + 1)$$

Figure I-3 shows the uranium specific heat as a function of temperature and the approximation to it which was used in the simulation.

The source terms ($\dot{q}_i$) contained in Equations (12) through (16) are a function of the radial thermal flux distribution across the fuel element. A typical flux distribution is given in Figure I-4. Since the heat generation is proportional to the flux, the fraction of the total heat generation in the fuel element that is generated in region i will be

$$f_i = \frac{\int_{r_i}^{r_{i+1}} \phi dV}{\int_{r_0}^{r_N} \phi dV}$$

If q_T is the total heat generation in the fuel element, then the heat generated in region i will be

$$q_i = f_i q_T$$

The source density is then

$$\Sigma_i = \frac{q_i}{V_i} = \frac{f_i q_T}{V_i} \quad (17)$$

Of the heat generated in a cell, 93.25% is released as prompt heat and 6.75% is delayed. Further, 95.5% of the total prompt heat and 88% of the total delayed heat is released in the fuel, the remainder being released in the graphite. Considering an equilibrium state, where Q_c represents the total

DECLASSIFIED

FIGURE I-3
SPECIFIC HEAT OF URANIUM

DECLASSIFIED

-19-
FIGURE I-1
THERMAL FLUX DISTRIBUTION

HM-63157

equilibrium heat generation in the cell, if q_T^P is prompt equilibrium heat generation in the fuel, and q_T^d is the delayed equilibrium heat generation in the fuel, then

$$Q_c^P = .9325 Q_c = \text{total prompt heat,}$$

$$Q_c^d = .0675 Q_c = \text{total delayed heat, where}$$

$$q_T^P = .955 Q_c^P, \text{ and}$$

$$q_T^d = .88 Q_c^d = .0594 Q_c.$$

The source density, Equation (17), can then be given as a sum of two parts.

$$\Sigma_i = \Sigma_i^P + \Sigma_i^d = \frac{f_i}{V_i} (q_T^P + q_T^d),$$

where

$$\Sigma_i^P = .955 \frac{f_i}{V_i} Q_c^P, \text{ and}$$

$$\Sigma_i^d = .0594 \frac{f_i}{V_i} Q_c.$$

The power excursion during coolant loss is very rapid, several seconds, compared with the rate at which the delayed power varies. Therefore it can be assumed that the delayed heat generation remains constant during an excursion, equal to what it was at equilibrium. Letting the time dependence of the prompt heat be described by the kinetic equations, then

$$\Sigma_i^P = .955 \frac{f_i}{V_i} Q_c^P(\tau), \text{ and} \quad (18)$$

$$\Sigma_i^d = .0594 \frac{f_i}{V_i} Q_c (\text{Equilibrium}). \quad (19)$$

In order to write the temperature equations in terms of total reactor power, one must assume a relationship between tube power and reactor power.

$$\text{Tube Power} = \frac{\text{Reactor Power}}{\text{ECT}} \quad (20)$$

where ECT is the number of effective central tubes. ECT is calculated from

$$\text{ECT} = \frac{\text{Total Reactor Power}}{\text{Power of Highest Powered Tube}}$$

Thus, Q_c in the source density equations corresponds to the maximum power tube in the reactor. The reactor power is proportional to n , the neutron density.

$$\text{Reactor Power} = P_R = \sigma n \text{ (MW)}, \text{ and} \quad (21)$$

$$\text{Tube Power} = P_T = \frac{\sigma n}{\text{ECT}} \text{ (MW)}, \quad (22)$$

where σ = proportionality constant in MW per unit of n .

Using the conversion factor, $1 \text{ MW} = 2.389 \times 10^5 \frac{\text{cal}}{\text{sec}}$,

$$Q_c^P(r) = 2.389 \times 10^5 \frac{\sigma n(r)}{\text{ECT}}, \text{ and} \quad (23)$$

$$Q_c(\text{Equilibrium}) = 2.389 \times 10^5 \frac{\sigma n(\text{Equilibrium})}{\text{ECT}} \quad (24)$$

Then Equations (18) and (19) become

$$\sum_i^P = 2.2815 \times 10^5 \frac{f_i}{V_i} \frac{\sigma n(r)}{\text{ECT}}, \text{ and} \quad (25)$$

$$\sum_i^d = 1.4191 \times 10^4 \frac{f_i}{V_i} \frac{\sigma n(\text{Equilibrium})}{\text{ECT}}. \quad (26)$$

The f_i 's for the three types of fuel elements, CIIN, KIIIN, OIIN, that were used in this analysis were essentially the same, and are given below for the CIIN slug using the flux distribution given in Figure I-4. The change in the flux distribution going from the wet to dry slug was neglected.

$$\begin{aligned} f_1 &= .1121 \\ f_2 &= .1472 \\ f_3 &= .1891 \\ f_4 &= .2417 \\ f_5 &= .3099 \end{aligned}$$

The five region differential equations are then written as follows:

$$\frac{dT_1}{d\tau} = \frac{1}{C_1} \left[A_1 m(\tau) + B_1 m(\tau) + A_{12} A_1^+ (T_2 - T_1) - A_1^- (T_1 - T_w) D \right] \quad (27)$$

$$\frac{dT_2}{d\tau} = \frac{1}{C_2} \left[A_2 m(\tau) + B_2 m(\tau) + A_{23} A_2^+ (T_3 - T_2) - A_{12} A_2^- (T_2 - T_1) \right] \quad (28)$$

$$\frac{dT_3}{d\tau} = \frac{1}{C_3} \left[A_3 m(\tau) + B_3 m(\tau) + A_{34} A_3^+ (T_4 - T_3) - A_{23} A_3^- (T_3 - T_2) \right] \quad (29)$$

$$\frac{dT_4}{d\tau} = \frac{1}{C_4} \left[A_4 m(\tau) + B_4 m(\tau) + A_{45} A_4^+ (T_5 - T_4) - A_{34} A_4^- (T_4 - T_3) \right] \quad (30)$$

$$\frac{dT_5}{d\tau} = \frac{1}{C_5} \left[A_5 m(\tau) + B_5 m(\tau) + A_5^+ (T_w - T_5) D - A_{45} A_5^- (T_5 - T_4) \right] \quad (31)$$

Where $A_1 = 2.2815 \times 10^5 \frac{r_1 \sigma}{Q_{V_1} \text{ECT}}$, and

$B_1 = 1.419 \times 10^4 \frac{r_1 \sigma}{Q_{V_1} \text{ECT}}$

Numerical values of constants are given in the following tables.

Fuel Dimensions

	<u>K I I I N</u>	<u>C I I N</u>	<u>O I I N</u>
r_0 (cm)	0.48895	0.47625	0.39365
r_1 (cm)	0.62357	0.61087	0.5283
r_2 (cm)	0.84684	0.83667	0.7671
r_3 (cm)	1.0701	1.06247	1.0059
r_4 (cm)	1.2934	1.2883	1.2447
r_5 (cm)	1.5167	1.5141	1.4835
r_6 (cm)	1.7399	1.7398	1.7223
r_7 (cm)	1.8542	1.8540	1.8350
r (cm)	0.2233	0.2258	0.2388
L (cm)	855.65	728.64	728.64
L_f (cm)	803.51	676.67	676.67

Hydraulic Constants

<u>Constant</u>	<u>Fuel Geometry</u>					
	<u>KIIN</u>		<u>CIIN</u>		<u>OIIN</u>	
	<u>Inner</u>	<u>Outer</u>	<u>Inner</u>	<u>Outer</u>	<u>Inner</u>	<u>Outer</u>
F	1.053	1.053	1.053	1.053	1.053	1.053
ρ_w (lb/ft ³)		61.6		61.6		61.6
s (ft ³)	.000808	.00371	.00076	.00303	.000812	.00253
d _e (ft)	.03208	.0388	.0312	.0317	.02583	.02674
E (ft)		.3822		.3820		.3784
G (gpm)	18.	42.	18.	42.	18.	42.
(d ₂ /d ₁) ^{1/2}	1.00	1.072	1.00	1.060	1.00	1.052
M	410.84	126.9	431.4	151.2	427.4	209.0
T _b (°F)	131.	131.	131.	131.	131.	131.
h(cal/sec-cm ² °C)	1.516	.918	1.592	1.094	1.577	1.295

DECLASSIFIED

Numerical Constants

	<u>KIIN SLUG</u>	<u>CIIN SLUG</u>	<u>OIIN SLUG</u>
A_1^+	1.3084	1.2918	1.1836
A_2^+	1.26304	1.2504	1.1338
A_3^+	1.23751	1.2248	1.10504
A_4^+	1.22003	1.2074	1.08663
A_5^+	0.084452	0.08829	0.084177
A_1^-	0.072052	0.07452	0.064548
A_2^-	0.998584	0.9847	0.86463
A_3^-	1.03007	1.0101	0.89303
A_4^-	1.04042	1.0274	0.91172
A_5^-	1.05285	1.0403	0.92471
A_1	0.5194	0.5225	0.5488
A_2	0.5197	0.5261	0.5264
A_3	0.5396	0.5485	0.5329
A_4	0.5783	0.5895	0.5617
A_5	0.6390	0.6524	0.6131
B_1	0.03230	0.03250	0.03413
B_2	0.03232	0.03272	0.03274
B_3	0.03356	0.03411	0.03314
B_4	0.03597	0.03666	0.03494
B_5	0.03974	0.04058	0.03813
ECT	2500.	1500.	1500.

DECLASSIFIED

DATE

FILMED

7 / 13 / 94

END