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**EFFECTS OF CAPILLARITY ON MICROSCOPIC FLOW
IN POROUS MEDIA**

**Progress Report
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Abstract

The central theme of this proposal is to study the effects of capillarity on the motion of a fluid interface and to apply these results to flow in porous media. Here we report on several problems considered this year, the third year of the grant, and during the other years of the grant. In particular we have developed a numerical code to study the dynamics of a gas bubble in a pore in order to examine the fundamental mechanism for the generation of a foam in a porous material, we have investigated the stability of a foam lamella in order to understand the stability of foam flow in a porous material and we have derived systematically a slip coefficient for flow over a rough and coated surface, e.g., as in a pore. In addition we report on work on several other problems.

The primary direction during the current grant period has been to study the dynamics of foam flow in porous media and to study the fundamental mechanism associated with contact line motion. We have been able to model the snap-off process for the formation of foams and we have developed models for the dynamics of a foam lamella in a porous media. We have looked at models of the dynamics of an interface after snap-off, the derivation of the slip coefficient used in modeling contact line motion and the effect of van der Waals forces on the stability of contact line motion. We have also initiated a project on the motion of a fluid into an unsaturated porous material.

This work will have direct impact on problems involving flows in porous media and in particular to the areas of enhanced oil recovery and hazardous waste management, e.g., foams have been suggested for use in both of these applications and capillarity strongly influences the motion of a fluid into an unsaturated porous material as might occur in a spill of a liquid waste. In the following we will present a summary of the problems investigated.

Bubble snap-off: The main mechanism for the formation of a foam in a porous media is the snap-off process. This occurs when a gas bubble is driven past a constriction in a fluid saturated pore. The aim here is to model this process numerically and to study the effects of the physical parameters on the snap-off process. This is an important issue since it effects the texture, stability and effective viscosity of the foam and hence has a direct influence on how effective a foam is in forcing out another fluid trapped in a porous material, e.g., oil or a hazardous liquid waste.

In this study we use straight and constricted capillary tubes as a simplified model for a pore geometry. Capillary tubes have also been used in several experimental investigations of this process. In general, the bubble flows which occur in a porous material happen at very low Reynolds numbers, hence we use the Stokes equations to describe the fluid. So the problem is to determine the pressure driven motion of a bubble (or drop) in a straight or constricted capillary. We solve this moving boundary problem by using a boundary integral numerical method. The idea is to reformulate the problem as a system of integral-differential equations. Then using the kinematic boundary condition, we can formulate an iterative numerical method to evolve this initial value problem in time. We have been very careful so as to keep the numerical error as small as possible since changes in volume due to numerical error can have a significant effect on the dynamics of the drop. There have been previous investigations of the straight tube case. Hence our interest in the straight tube case is twofold: first to validate the code against the existing literature and second to study situations which have not been investigated before, e.g., small viscosity ratios (bubbles) and the transient motion of large finite bubbles (these can be compared to experiments in the literature). We have studied the dynamics of the bubbles as a function of the bubble volume, capillary number, viscosity ratio between the bubble and suspending fluid and the constriction size.

More specifically, we have been able to numerically determine the motion of a finite bubble in a straight capillary tube for all viscosity ratios. At large capillary numbers we find that a jet of suspending liquid enters the rear of the bubble leading to bubble breakup. These results compare very well with the recent experimental results on transient bubble motion in capillary tubes by Olbricht and Kung (Phys. Fluids A 1992). Also the motion of a bubble in a constricted capillary tube has been modeled and the snap-off instability has been identified. We are able to determine ranges of the parameters for which snap-off occurs. We find that bubbles can snap-off but drops will in general just pass through a given constriction. We find that there is a finite range of capillary numbers for which the bubbles will snap off.

This work has been submitted to the Journal of Fluid Mechanics in the paper, "Dynamics of a

Drop in a Constricted Capillary Tube" with my graduated student T.M. Tsai. Mr. Tsai will graduate with this Ph.D. under my supervision in September of 1994 and this work comprises part of his thesis.

Lamella stability: A foam in a porous media consists on a gas/liquid mixture where most of the liquid is in the form of a thin film which either coats the solid surface or separates large gas bubbles. The latter thin films are on the scale of a pore diameter and are called lamellae. The aim here is to model and study the stability of these thin films under a pressure gradient as would occur in an enhanced oil recovery process. As in the above, this microscopic stability question is related to the texture, stability and viscosity of the foam. One definition of a foam in a porous material is a gas-liquid mixture with a continuous liquid phase wetting the rock, and with part or all of the gas made discontinuous by thin liquid films called lamellae. Another possibility occurs if the rock were not wetted. Then the liquid phase would not be continuous and it still might be possible to have a succession of lamella progressing down the porous-rock channels. Here we consider the dynamics of a single lamella in the latter situation. The dynamics of the thin liquid film away from the wall will be governed by the effects of the surface tension of the liquid-gas interface plus the attractive effects of the two interfaces due to van der Waals forces. The motion of the liquid in the neighborhood of the wall is very complicated. Here we assume that each of the liquid interfaces intersects the wall at a finite contact angle. Hence the wall is only wetted along the thickness of the lamella. We use a Navier slip condition along the solid/liquid interface and we specify the relationship between the contact angle and the slip velocity at the contact line. These modeling assumptions have free parameters in them. They can be thought of as containing the effects of the plateau border which would occur in the neighborhood of the wall if the walls were completely wet. Under these assumptions we derive the evolution of the lamella in a capillary tube. By assuming that the film is thin we are able to use the method of matched asymptotic expansions to derive a system of evolution equations for the film. These are solved analytically in some special limits and numerically otherwise to determine the dynamics of the lamella. Linear stability of the steady states is also investigated.

We find that increasing the surface tension will stabilize the lamella while increasing in imposed pressure gradient will destabilize the lamella. The results are only weakly dependent on the slip velocity - contact angle relationship in that unless the slope in this relationship is very small, the stability is insensitive to its magnitude. Also the results are independent of the slip coefficient. The above two observations are positive since this says that the phenomena observed is insensitive to the modeling of the contact line, but depends more strongly on either the presence (as say with a plateau border) or absence of slip.

The results for the 2-d model have been submitted to the SIAM Journal of Applied Mathematics in the paper, "Dynamics of a Lamella in a Capillary Tube". This work was in collaboration with my graduate student Michael Ida. Michael is a third year graduate student and is expected to graduate in June of 1995. The results for the 3-d model has been submitted to the Physics of Fluids. One of the interesting results here is that we find that if the pressure drop across the lamella is order one, the film will break in the neighborhood of the tube wall, hence forming a large droplet after the instability occurs.

Slip over a rough surface: The no-slip boundary condition on the interface between a fluid and a solid is commonly accepted. For a smooth surface this is straight-forward to apply. For a rough surface, where the scale of the roughness is much smaller than the scale of interest,

this is in general a difficult condition to apply, e.g., in a pore. In particular, an accurate numerical solution of the Navier-Stokes equations would require a very fine computational grid in the neighborhood of the boundary. One means of overcoming this difficulty might be to replace the actual boundary by a smooth surface and hence introduce an effective-slip condition applied on the mean position of the interface. The aim here is to systematically derive such a effective boundary condition for flows over coated and rough surfaces. This will also have relevance to contact line motion flows where such conditions are also used.

We assumed that the roughness varies on a scale much smaller than the scale of interest in the macroscopic flow problem. The problem was studied analytically using asymptotic analysis. The method of multiple scales was applied in order to average the small scale roughness and the method of matched asymptotics is used to relate the macroscopic flow field to the roughness and the dynamics of the coating film. The result of this analysis was a Navier slip condition and slip coefficient for flow over a rough and/or coated surface. This condition replaces the actual boundary by a smooth surface and hence introduces an effective-slip on this smooth surface. The result of this work have been submitted to the Journal of Fluid Mechanics in the paper, "Slip over a rough and/or coated surface", and was in collaboration with Steve Davis of Northwestern University.

Surface tension driven motion of a wedge of viscous fluid: When a bubble snaps-off, the motion of the two interfaces is initially driven by surface tension. Also when two drops or bubbles come into contact, it is surface tension which drives the motion of the drop surface towards an equilibrium position. If one would like to model the formation of a foam after the snap-off process, knowing the local dynamics of the interfaces is important since computationally one cannot just compute through the snap-off time because of the difficulty in keeping an accurate numerical solution. Also, guessing the solution after snap-off can under certain conditions have a nontrivial effect on the dynamics of the two new bubbles. The aim of this work was to determine the dynamics of an interface which was initially in the shape of a wedge. The snap-off problems studied in this project use the Stokes equations. So the question posed here is can one solve for the motion of a viscous wedge of liquid as a function of time including the effect of surface tension. At time zero the wedge has finite wedge angle and the sides are straight to infinity. This problem has a similarity solution which reduces it to solving in the similarity plane for the free boundary of a region determined by the bi-harmonic equation with nonlinear boundary conditions along the interface. Where initially there was a finite wedge angle and hence a discontinuity in slope of the interface, for time greater than zero the boundary condition is continuity of the interface slope. This problem is then solved numerically by using a boundary integral method. This requires that the problem be reformulated as a nonlinear integral-differential system of equations for the unknown interface. These nonlinear equations are discretized and solve numerically by using Newton's method. This work was done in collaboration with my graduate student T.M. Tsai and was submitted to the Physics of Fluids A in the paper titled, "The Effect of Surface Tension on the Motion of a Wedge of Viscous Fluid".

Stability of a driven contact line: A basic question in understanding the dynamics of contact line motion is how does the contact angle at the contact line influence the bulk motion of a fluid. In a partially saturated porous material or at the interface between a saturated and unsaturated porous material this study could lend insight into how the liquid is distributed within the material. Our aim here is to investigate the above question in a related problem. A simple situation in which one can ask this question and for which there has been experimental

work is for the motion of a fluid film with a leading edge down an inclined plane. Here it is observed that in some cases the interface becomes unstable and forms fingers which advanced down the plane and in other cases a sawtooth shape developed at the leading edge with both the points and the bases of the triangular protuberances moving downwards. The contact angle has been suggested as a reason for this observation.

When a liquid film with a leading edge is flowing down an inclined plane it is observed that a ridge of fluid forms near the leading edge. Away from the edge and up the inclined plane, the fluid is much thinner than at the ridge. Hence it was suggested by Hocking (JFM 1990) that it would be reasonable to study the dynamics of the ridge alone. A ridge

has a contact line at its leading edge and at its trailing edge. This problem was studied in the lubrication limit. A Navier slip condition was given along the liquid/solid interface and a linear relationship between the contact angle and the slip velocity was specified. Applying these assumptions, one can derive a nonlinear evolution equation for the interface height which is first order in time and fourth order in space. By looking for quasi-steady solutions (low capillary number limit) the problem can be reduced to solving a Poisson equation for the interface along with the boundary condition at the contact lines. This was solved numerically. The linear stability of the ridge can be studied by linearizing the full nonlinear evolution equation about the steady states and solving the resulting linear equation numerically. Unfortunately, for small values of the slip coefficient in the Navier slip condition, the numerics give poor results. Since small values of the slip coefficient are more physically interesting, improvement is necessary. This can be done by using the method of matched asymptotics in the neighborhood of the contact line. The outer solution away from the contact line must still be determined numerically but by analyzing the behavior of the solution in the neighborhood of the contact line (the inner solution), the outer solution can be matched numerically to the inner. This work was in collaboration with Leslie Hocking of the University College London and was published in the Journal of Fluid Mechanics, vol. 247, pp. 157-177, 1993, "Stability of a Fluid Ridge".

This work is currently being extended to a semi-infinite sheet of liquid which now accounts for the effect of van der Waals forces in the neighborhood of the contact line. The inclusion of van der Waals forces does not eliminate the need for a slip velocity boundary condition (Hocking, Phys. Fluids A, 1993). In Fig. 1 we sketch the dimensionless steady state profile of a thin film moving down (toward negative x is down) an inclined plane with and without the effect of van der Waals forces. All the other constants are the same. The leading edge of the film is placed at $x = -1$ and the film extends to $x = \infty$ at which the height h has a dimensionless value of $h = 1$. The film is assumed to be thin so the scaling differs in the vertical and horizontal directions. Note that in inclusion of van der Waals forces has decreased the height of the steady ridge. Questions of stability are currently being investigated.

Additional problems and activity has occurred during the current grant period. In particular:

- "Interaction of Panel Oscillations and Acoustic Waves", (with L. Ting), This has been submitted for publication to Applied Math Letters. It is a theoretical study on how a incident plane acoustic wave is reflected off of a constricted panel in the limit where the ratio of the sound speed in the fluid to the sound speed in the panel is small. We find that this three dimensional acoustic scattering problem can be reduced to solving a forced two dimensional partial differential equation on the surface of the panel.

- "Wave Propagation in a Bubbly Liquid at Small Volume Fraction", (with L. Ting), Chem. Eng. Comm., 118, 59-73, 1992. A system of effective equations for wave propagation in a bubbly liquid with a small gas volume fraction is obtained. The derivation consists of averaging the basic equations of the two-phase flow and then using the method of matched asymptotic expansions in order to define the microscopic problem. The resulting system consists of a continuity equation with a source term and a momentum equation with forcing terms. These terms are coupled to a canonical microscopic problem for the motion of a single bubble. The system accounts for the nonlinear effects of the translation of the bubble centers and the oscillation of the bubble interfaces including the spherically symmetric mode and all the asymmetric modes. The system remains valid regardless of the order of magnitude of the gas volume fraction relative to the reciprocal of the bubble number density.
- "Bounds for Mobility of Foams Flowing Through Porous Media", Phys. Fluids A, 3, 2541-2545, 1991. Variational bounds have been used to estimate the fluid permeability of one-phase flow in porous media. Here an extension of this idea to foam flow in porous media is discussed.
- "Effective Equations for Multiphase Flows - Waves in a Bubbly Liquid", (with L. Ting), Advances in Applied Mechanics, 28, eds. J. W. Hutchinson and T. Y. Wu, 141-260, 1991 (published in 1992). This is a review article concerning the derivation of effective nonlinear equations in multiphase flows.
- "Effects of Capillarity on Microscopic Flow in Porous Media", (with T.M. Tsai), Proceedings of the Eleventh Symposium on Energy Engineering Sciences, May 3-5, 1993, Argonne, IL. Here we study the effect of capillarity on a microscopic problem related to foam flow in porous media: the transport of a bubble or drop through a constricted capillarity tube. The aim is to study the snap off process which occurs during the formation of a foam in a porous media. We illustrate the dynamics of the bubble for different initial data and for several different sets of the physical parameters.

Finally we note that in collaboration with my graduate student Michael Ida, we have initiated a project to study the motion of a fluid into an unsaturated porous media. This will be the final project in Mike's Ph.D. Thesis (expected 1995). The idea here is to model the motion of the interface including the contact line (the three phase line at the solid/liquid/gas interface). The singularity at the contact line must be identified analytically and used in a numerical solution of this problem. Additional details of this project are given later in this proposal.

The principal investigator expects that all funds will be spent by the end of the current grant period.

During this period I have taken two summer month in salary plus 5% academic year. I plan to take 1.2 summer months in salary during 94-95.

Figure 1

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