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ANALYSIS OF QUALITY CERTIFICATION DATA - WARP, T.F.C. AND DELTA OD₂ AS A FUNCTION OF REACTOR OPERATING CONDITIONS

INTRODUCTION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

This report is the second⁽¹⁾ in a series dedicated to the definition of performance characteristics of irradiated fuel elements as measured by the Quality Certification Program.⁽²⁾ The major objectives of this analysis are to establish mathematical relationships which fit the observed data from over 2000 irradiated fuel elements measurements for WARP, TFC, and DELTA OD₂, as a function of reactor conditions defined in terms of POWER, SURFACE TEMPERATURE, AND TIME.⁽³⁾

(1) HW-67120, "Analysis of Quality Certification Data - Effect of Warp on the Incidence of Hot Spots". L. T. Hagie and J. L. Jaech, October 14, 1960, (UNCLASSIFIED)

(2) HW-58454, "A Customer-Vendor Quality Certification Program for Fuel Elements", December 8, 1958, L. T. Hagie (CONFIDENTIAL-UNDOCUMENTED).

(3) HW-66536, "Machine Processing of Fuel Element Examination Data", August 18, 1960, L. T. Hagie (UNCLASSIFIED)

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Mathematical models are essentially numerical descriptions. They can frequently be used to define complex relationships which defy description by other means. The accuracy of such descriptions depend entirely on how well we understand that which is being defined. Like pictorial or word descriptions however, mathematical models may be incomplete initially because some important factors in the system may have been overlooked. Thus, the results reported here should be considered as an initial attempt toward a complete description of fuel performance. In actual practice however, even such an incomplete description is better than nothing because it establishes a common basis for future investigation.

This report presents the results of the empirically derived mathematical models in graphical and tabulated form so the reader may more easily visualize the effects of the variables under study.

It is expected that additional work may refine some of the values reported, but should not effect any of the major conclusions.

SUMMARY AND CONCLUSIONS

1. After removing the "reactor effects" of power, surface temperature, and time, a considerable amount of unexplained variation is still present. At the present time, we intend to define this residual variation as the "fuel element effect", recognizing it also contains measurement error, unknown reactor effects, and normal processing variation. This report discusses the variation predicted by power, surface temperature and time regardless of the underlying cause. Uranium core stress, released as a function of time and temperature, may appear as a "reactor effect".
2. Mathematical models for fuel element warp, TFC, and ΔOD_2 have been empirically developed which give excellent fits to the observed post-irradiation data. Extrapolations much beyond the limits of the data however should not be attempted at this time because the conclusions may be misleading.
3. Fuel element diametral change, as expressed by ΔOD_2 , is independent of surface temperature, and increases linearly with respect to time, power, and the interaction between these two variables.
4. The actual data suggest the ΔOD_2 may decrease shortly after charging. A more reasonable interpretation is that a 2 or 3 mil negative bias exists between the pre and post irradiation measurements, and that diametral enlargement actually begins with irradiation.
5. Fuel element warp and TFC values caused by reactor effects may be predicted by using a quadratic equation which includes surface temperature, time, and power terms. In general, higher warp and TFC values are expected at higher temperatures and powers, with this effect becoming relatively less important at longer time.
6. The application of these models to future problems should make it possible to evaluate the relative contribution of both "reactor" and "fuel" variables upon total fuel element performance. While new computer programs are being developed to refine these models by the addition of new data, the results are not expected to change any of the major conclusions above.
7. Pictorial and tabular representations of Warp, TFC and ΔOD_2 as a function of power, surface temperature and time are presented on pages 6, 7, and 8.

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8. Appendix A contains the assumptions and derivations used in developing the models in this document.
9. Appendix B discusses the need to transform warp and TFC data. Curves are included which are useful in visualizing the transformed averages in terms of measured dimensions. Additional curves show the per cent of the total population, centered at a given average, which exceeds a specified value of interest. Since these curves will be useful in the interpretation of future Quality Certification data, they have been issued as HW-67465 (Unclassified).

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DISCUSSION

In this report, the mathematical models which best describe the affects of reactor power, surface temperature, and time upon over 2000 fuel elements measured under the Quality Certification Program for warp, diametral change at the center (ΔOD_2) and T.F.C.⁽⁴⁾ are discussed. Specific cases have been tabulated in Table I for convenience. In order to visualize the relationship between the variables, a pictorial representation of warp, ΔOD_2 and T.F.C. are shown on pages 6, 7, & 8.

Table I: This table summarizes the warp, T.F.C. and ΔOD_2 in mils for the combinations of power, surface temperature, and time at 4 levels of interest. These data have been used in plotting the response surfaces shown in Figures 1 through III. The term "Days" is actually $\frac{MWD}{MW}$, and thus is neither actual residence

time or "days operated". Temperature does not enter the equation for ΔOD_2 , and therefore the same data is reported for each temperature level. The negative ΔOD_2 values for low power fuel elements is best explained as a measurement bias between pre and post irradiation measurements.⁽⁵⁾ This does not exclude the unlikely possibility however that an actual reduction in fuel diameter does occur immediately after charging. If we assume that measurement bias is the cause of the negative ΔOD_2 values, and that 58 "days" is essentially zero change, then a better estimate of the true ΔOD_2 can be found by adding 2.15 mils to each ΔOD_2 value. This table may be used to predict dimensional instability caused by power, surface temperature, and time over a wide range of actual reactor conditions.

Figure I: This figure represents a graphical display of fuel element warp at 58 and 103 "days". It is interesting to note that at equal power outputs, the "time effect" is negligible at higher temperatures, while quite important for low temperature operation. This result confirms some of our earlier measurements regarding warp in upstream sections of isolated tubes.

Figure II: This figure represents a graphical display of fuel element T.F.C. under the same set of conditions as used in Figure I. Since T.F.C. is influenced to some extent by changes in ΔOD_2 , the response surface differs from that of Warp.

Figure III: As indicated in Appendix "A", surface temperature terms were found ineffective and not included in the ΔOD_2 model. This simplified model adequately describes diametral change in terms of "days" and power.

(4): T.F.C. = Tube Filling Capacity. A calculated statistic which combines diametral change and core bending to give an estimate of projected diameter.

(5) Letter: H. P. Kramer to K. E. Fields, June 24, 1960, "Comparison of Fuel Element Measurement Methods" (Unclassified).

TABLE I

SUMMARY OF PLOTTED DATA

(Reported in Mills)

"DAYS" (MMD) (MM)		POWER - KW/SLUG																																																	
		6										15										33										51										60									
		58	73	88	103	58	73	88	103	58	73	88	103	58	73	88	103	58	73	88	103	58	73	88	103	58	73	88	103																						
ce	WARP	2.7	4.2	4.9	6.2	5.0	5.5	6.5	8.2	6.7	7.4	8.7	10.8	6.4	7.1	8.5	10.8	6.4	7.1	8.5	10.8	5.5	6.2	7.5	9.7																										
	T.F.C	6.7	7.3	8.4	10.1	9.5	10.0	10.8	12.9	14.2	15.1	16.4	19.4	16.3	17.7	19.8	22.9	16.3	17.7	19.8	22.9	16.3	17.7	19.0	19.9																										
	Delta OD ₂	-2.15	-1.59	-1.04	-.49	-1.47	-1.17	-.87	-.57	-.12	.43	.99	1.54	1.23	2.04	2.84	3.65	1.91	2.46	3.01	3.57																														
	WARP	4.7	4.8	5.2	6.2	6.1	6.4	7.1	8.4	8.5	9.7	9.7	11.5	8.5	9.8	9.8	12.0	7.6	8.1	9.1	11.0																														
	TFC	8.5	8.8	9.6	11.0	11.6	11.7	12.5	13.9	17.1	17.5	19.0	21.0	19.8	20.4	22.3	24.9	19.8	20.4	21.5	23.8																														
	Delta OD ₂	-2.15	-1.59	-1.04	-.49	-1.47	-1.17	-.87	-.57	-.12	.43	.99	1.54	1.23	2.04	2.84	3.65	1.91	2.46	3.01	3.57																														
	WARP	4.9	4.7	4.9	5.4	6.7	7.2	6.7	7.6	9.5	9.3	9.7	11.0	9.9	9.8	10.4	11.8	9.0	9.0	9.7	11.1																														
	TFC	8.8	8.8	9.3	10.4	12.2	11.9	12.3	13.4	17.9	18.0	18.6	20.3	20.8	21.1	22.2	24.3	20.8	21.1	21.5	23.3																														
	Delta OD ₂	-2.15	-1.59	-1.04	-.49	-1.47	-1.17	-.87	-.57	-.12	.43	.99	1.54	1.23	2.04	2.84	3.65	1.91	2.46	3.01	3.51																														
	WARP	4.4	4.0	3.9	4.1	6.2	5.6	5.5	5.9	9.3	8.6	8.6	9.3	10.0	9.5	9.6	10.5	9.5	9.0	9.1	10.0																														
C	TFC	7.8	7.4	7.6	8.3	10.8	10.3	10.2	10.8	16.4	15.8	16.1	17.2	19.3	19.0	19.6	21.0	19.3	19.0	19.1	20.2																														
	Delta OD ₂	-2.15	-1.59	-1.04	-.49	-1.47	-1.17	-.87	-.57	-.12	.43	.99	1.54	1.23	2.04	2.84	3.65	1.91	2.46	3.01	3.57																														

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PAGE 6

RESPONSE SURFACE

WARP AS A FUNCTION
OF "DAYS", KW/SLUG &
SURFACE TEMP.

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RESPONSE SURFACES

TFC AS A FUNCTION OF
"DAYS", KW/SLUG, AND
SURFACE TEMP.

103 "DAYS"

58 "DAYS"

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RESPONSE SURFACE

OD₂ CHANGE AS A FUNCTION
OF "DAYS" AND KW/SLUG

OD₂ CHANGE SCALE
-3 -2 -1 0 1 2 3 4

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APPENDIX A

DERIVATION OF MATHEMATICAL MODELS FOR WARP, TFC, AND DELTA OD₂

This appendix contains a discussion of the techniques used in determining the prediction equations expressing dimensional distortion as functions of reactor operating conditions.

Theoretically, the behavior of any system is governed by ascertainable laws so that one should perhaps be able to express warp, for example, as a function of reactor operating conditions on a theoretical basis. In practice, the underlying mechanisms of the system are often so complicated that an empirical approach may first be necessary. This is not by way of defense of the empirical approach, since the practical importance of being able to express distortion as a function of reactor operating conditions is such that we don't really care how the prediction equations were derived, but rather, do they truly predict well? There are also shortcomings associated with the empirical approach which we must consider and govern our actions accordingly. First, and most important, we hesitate to extrapolate beyond the range of the data for obvious reasons. A well-formed theoretically approached model would not have this limitation. Secondly, with respect to understanding why fuel elements distort as they do under a given set of conditions, the empirically derived equations are of little value. In spite of these shortcomings, in the absence of a theoretical model, we choose to develop one empirically.

Speaking in general terms for a moment, we let x_1 , x_2 , and x_3 represent values associated with the reactor variables under study, and let η represent the true value of warp (or TFC, ΔOD_2 , etc.) under the given set of conditions. The relationship between η and (x_1, x_2, x_3) is

$$\eta = \phi(x_1, x_2, x_3)^* \quad (1)$$

The function ϕ is what we would like to know. Theoretically, its form may be, and probably is, extremely complicated. Empirically, this doesn't bother us since we make use of the fact that if the true functional relationship results in a smooth "response surface", which is a generalization of a smooth curve in two dimensions, we may represent this surface to any required degree of approximation by including a sufficient number of terms in a polynomial series. For example, if we had only the one independent variable x_1 , we could express η to any degree of approximation by including sufficient terms in the series:

$$\eta = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2 + \beta_{111} x_1^3 + \dots \quad (2)$$

* We let η represent the true value, and y the observed value. y is equal to η plus some random error.

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With the three variables x_1 , x_2 , and x_3 , the terms through the quadratic give the model:

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 \quad (3)$$

Addition of cubic terms to the model would result in nine more terms.

The approach consisted in first fitting a quadratic response surface to the data of the form (3). If this fit were not adequate, additional terms would have been necessary. On the other hand, it could be that some of the terms in (3) contribute little to the adequacy of fit, and hence could be ignored.

It is convenient for statistical reasons to work with transformed values of the variables. The data were grouped into 48 categories whose mid-points were approximately:

Temperature $x_1 = 30, 60, 90, 120^\circ \text{C}$

Time $x_2 = 58, 73, 88, 103$ days

Power $x_3 = 15, 33, 51$ kw.

Observe that these levels are equally spaced on an arithmetic scale. This permits us to represent the linear and quadratic terms in the model as follows:

$$\left. \begin{aligned} x_{1L} &= \frac{x_1 - 75}{15} \\ x_{1Q} &= \frac{x_{1L}^2 - 5}{4} \\ x_{2L} &= \frac{2(x_2 - 80.5)}{15} \\ x_{2Q} &= \frac{x_{2L}^2 - 5}{4} \\ x_{3L} &= \frac{x_3 - 33}{18} \\ x_{3Q} &= 3x_{3L}^2 - 2 \end{aligned} \right\} \quad (4)$$

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Using the transformations given by (4), we get the identification between the original and transformed values:

x_1	x_{1L}	x_{1Q}	x_2	x_{2L}	x_{2Q}	x_3	x_{3L}	x_{3Q}
30	-3	1	58	-3	1	15	-1	1
60	-1	-1	73	-1	-1	33	0	-2
90	1	-1	88	1	-1	51	1	1
120	3	1	103	3	1			

Model (3) is then conveniently written:

$$\eta = \alpha_0 + \alpha_1 x_{1L} + \alpha_2 x_{2L} + \alpha_3 x_{3L} + \alpha_{11} x_{1Q} + \alpha_{22} x_{2Q} + \alpha_{33} x_{3Q} + \alpha_{12} x_{1L} x_{2L} + \alpha_{13} x_{1L} x_{3L} + \alpha_{23} x_{2L} x_{3L} \quad (5)$$

where one can determine the α 's in (5) and then express (5) in terms of the original x values in (3) if he desires. However, it is much more convenient to work with the transformed variables given in (4) and use (5) also for prediction purposes. In addition to working with smaller numbers when using the transformed values, the mechanics of fitting the data are much easier, and the statistical interpretation of the results more clear cut. The technique is a standard statistical technique.

One other transformation is made. In the case of warp and TFC, we observe that the data are more variable as the values increase. In order to stabilize this variability, a transformation is employed. For this reason, the response is taken to be the square root of warp and TFC rather than the actual values.

As far as the estimation procedure is concerned, the α 's are estimated using least squares techniques. The equations which need be solved are ten in number, and are solved by inverting a matrix of order 10. Now, if all 48 combinations of x_1 , x_2 , and x_3 had data associated with them, all but the diagonal elements in the matrix would be zero as a result of using the transformations (4). However, there is data missing in some cases, since, for example, there are no fuel elements at the combinations of high power and low temperature. In all, average dimensional data were available for only 40 of the 48 combinations. Were the diagonal elements the only nonzero ones, the solutions for the α 's would be immediate, since the inverse matrix in this case would be the matrix with diagonal elements equal to the reciprocals of the diagonal elements in the original matrix, and zeroes elsewhere. Further, in this case, the estimates of the various α 's would be statistically independent of one another such that one could be removed from the model without affecting the others. These properties are what one achieves in a designed experiment.

In this particular application, there is enough balance in the data such that some nondiagonal values are zero. As a result, some of the α estimates are independent of some of the others, but complete independence is not achieved. Specifically, the equations which must be solved to determine the α 's can be divided into three sets as follows:

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$$\begin{aligned}
 &40 \alpha_0 + 16 \alpha_1 - 8 \alpha_{33} + 8 \alpha_{13} = \Sigma y \\
 &16 \alpha_0 + 200 \alpha_1 + 8 \alpha_3 + 8 \alpha_{11} + 16 \alpha_{33} - 32 \alpha_{13} = \Sigma x_{1L} y \\
 &\quad 8 \alpha_1 + 24 \alpha_3 - 8 \alpha_{11} + 16 \alpha_{13} = \Sigma x_{3L} y \\
 &\quad 8 \alpha_1 - 8 \alpha_3 + 40 \alpha_{11} + 16 \alpha_{13} = \Sigma x_{1Q} y \\
 &- 8 \alpha_0 + 16 \alpha_1 + 88 \alpha_{33} + 8 \alpha_{13} = \Sigma x_{3Q} y \\
 &\quad 8 \alpha_0 - 32 \alpha_1 + 16 \alpha_3 + 16 \alpha_{11} + 8 \alpha_{33} + 120 \alpha_{13} = \Sigma x_{1L} x_{3L} y
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} &40 \alpha_0 + 16 \alpha_1 - 8 \alpha_{33} + 8 \alpha_{13} = \Sigma y \\ &16 \alpha_0 + 200 \alpha_1 + 8 \alpha_3 + 8 \alpha_{11} + 16 \alpha_{33} - 32 \alpha_{13} = \Sigma x_{1L} y \\ &\quad 8 \alpha_1 + 24 \alpha_3 - 8 \alpha_{11} + 16 \alpha_{13} = \Sigma x_{3L} y \\ &\quad 8 \alpha_1 - 8 \alpha_3 + 40 \alpha_{11} + 16 \alpha_{13} = \Sigma x_{1Q} y \\ &- 8 \alpha_0 + 16 \alpha_1 + 88 \alpha_{33} + 8 \alpha_{13} = \Sigma x_{3Q} y \\ &\quad 8 \alpha_0 - 32 \alpha_1 + 16 \alpha_3 + 16 \alpha_{11} + 8 \alpha_{33} + 120 \alpha_{13} = \Sigma x_{1L} x_{3L} y \end{aligned}} \right\} (6) a$$

$$40 \alpha_{22} = \Sigma x_{2Q} y \quad \left. \vphantom{40 \alpha_{22} = \Sigma x_{2Q} y} \right\} (6) b$$

$$\begin{aligned}
 &200 \alpha_2 + 80 \alpha_{12} = \Sigma x_{2L} y \\
 &80 \alpha_2 + 1000 \alpha_{12} + 52 \alpha_{23} = \Sigma x_{1L} x_{2L} y \\
 &52 \alpha_{12} + 120 \alpha_{23} = \Sigma x_{2L} x_{3L} y
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} &200 \alpha_2 + 80 \alpha_{12} = \Sigma x_{2L} y \\ &80 \alpha_2 + 1000 \alpha_{12} + 52 \alpha_{23} = \Sigma x_{1L} x_{2L} y \\ &52 \alpha_{12} + 120 \alpha_{23} = \Sigma x_{2L} x_{3L} y \end{aligned}} \right\} (6) c$$

Notice that solutions for $\alpha_0, \alpha_1, \alpha_3, \alpha_{11}, \alpha_{33},$ and α_{13} are found from set (6)a, for α_{22} from (6) b, and for $\alpha_2, \alpha_{12},$ and α_{23} from (6) c. Estimates of each of the α 's with each set are dependent on one another. For example, the estimates of α_1 and $\alpha_{33},$ both in set (6) a are statistically dependent, whereas the estimates for α_{22} and α_{23} are not. This means that if $\alpha_{33},$ for example, is estimated to be "small", it cannot be removed from the model without adjusting the other α 's in (6) a. However, $\alpha_{22}, \alpha_2, \alpha_{12},$ and α_{23} would not need adjustment. As indicated before, in a designed experiment, all equations are of the form (6)b with the advantages of this being obvious.

The solutions to equations (6) were found using the 7090 electronic computer. Without giving the details of computation involved, the estimates of the various α 's together with their standard deviations are listed:

	$\sqrt{\text{Warp}}$		$\sqrt{\text{TFC}}$		ΔOD_2	
	estimate	standard dev.	estimate	standard dev.	estimate	standard dev.
α_0	2.7006	.0341	3.8223	.0383	.7130	.1243
α_1	.0182	.0162	.0024	.0182	.0216	.0591
α_2	.0611	.0146	.0625	.0165	.2850	.0535
α_3	.2927	.0487	.6056	.0548	1.7607	.1778
α_{11}	-.0966	.0366	-.1419	.0412	.1201	.1339
α_{22}	.0575	.0321	.0510	.0362	.0675	.1176
α_{33}	-.0760	.0225	-.0748	.0253	.0575	.0821
α_{12}	-.0202	.0066	-.0156	.0074	-.0222	.0242
α_{13}	.0349	.0225	.0089	.0254	.0019	.0824
α_{23}	.0116	.0188	.0172	.0211	.1287	.0687

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For warp and TFC, it might be possible to omit certain terms such as α_{23} in warp and α_{13} in TFC. However, this would necessitate changing estimates of some of the other α 's, and it was thought advisable to keep the models as complete quadratic responses. For ΔOD_2 , on the other hand, there were a sufficient number of nonsignificant α 's to justify re-computing the α 's in a reduced and simpler model. Just as good a fit was achieved in retaining only the linear terms for power and time plus their interaction, disregarding the squared terms and neglecting temperature entirely. The adjusted model for ΔOD_2 then becomes

$$\Delta OD_2 = .7230 + .2761 x_{2L} + 1.7292 x_{3L} + .1257 x_{2L}x_{3L} \quad (7)$$

To demonstrate the adequacy of these models, Figures I, II, and III were prepared showing the relationships between predicted and observed values for warp, TFC, and ΔOD_2 values respectively. The restricted model for ΔOD_2 given by (7) was used. The points that are the farthest removed from the lines of perfect agreement are, in general, based on a fewer number of fuel elements. In view of the closeness of fit plus the lack of bias (i.e., tendency to over-predict small values or under-predict large ones), it was concluded that these models give adequate empirical representations of the true responses.

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WARP - PREDICTED
VERSUS OBSERVED VALUES

FIGURE I

OBSERVED AVERAGE $\sqrt{WARP}$ (MILS)

PREDICTED AVERAGE $\sqrt{WARP}$ (MILS $\frac{1}{2}$)

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T. A. C. - PREDICTED
VERSUS OBSERVED VALUES

FIGURE II

TECHNICAL

OBSERVED AVERAGE $\sqrt{WARP}$ (MILS)

5.0
4.6
4.2
3.8
3.4
3.0
2.0

PREDICTED AVERAGE $\sqrt{WARP}$ (MILS^{1/2})

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OD₂ CHANGE PREDICTED
VERSUS OBSERVED VALUES

FIGURE III

TECHNICAL
OBSERVED OD₂ CHANGE (MILS)

PREDICTED OD₂ CHANGE (MILS)

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APPENDIX B

INTERPRETATION OF VARIOUS DATA TRANSFORMATIONS USED
IN THE ANALYSIS OF QUALITY CERTIFICATION DATA

Figures I, II, and III were prepared as part of HW-67465 (Unclassified) to aid in the interpretation of average ΔOD_2 values, and average square roots for warp and TFC. The averages are meaningful primarily inasmuch as they reflect how many fuel elements will distort beyond a given set of values. With respect to ΔOD_2 , we estimate from the data that the standard deviation associated with this value for a single fuel element is 5.1 mils. This is after correcting for reactor conditions, but includes all ranges of metal quality or other unmeasured variables. Thus, to find the probability that a given fuel element which comes from a population having an average ΔOD_2 of 2 mils will exceed 5 mils, we want

$$\text{Prob} (\Delta OD_2 > 5)$$

$$= \text{Prob} \left(\frac{\Delta OD_2 - 2}{5.1} > \frac{5-2}{5.1} \right)$$

$$= .278 \text{ using a table of the normal distribution.}$$

Thus, 27.8% of the fuel elements having an average ΔOD_2 of 2 mils will exceed 5 mils in ΔOD_2 .

Similar results hold for warp and TFC, except that results are first calculated for the square roots, and then squared to give actual warp and TFC. Specifically, we determine that on a square root basis, the standard deviation is 1.0 mils^{1/2} for warp, and 1.3 mils^{1/2} for TFC. Therefore, if we want to find the percent of fuel elements having warp greater than 25 mils given that the average $\sqrt{\text{warp}}$ is 3.2, we find

$$\text{Prob} (\text{warp} > 25)$$

$$= \text{Prob} (\sqrt{\text{warp}} > 5)$$

$$= \text{Prob} \left(\frac{\sqrt{\text{warp}} - 3.2}{1} > \frac{5-3.2}{1} \right) = .036, \text{ or } 3.6\% \text{ of all fuel elements.}$$

Finally, since we prefer to think in terms of warp and TFC rather than $\sqrt{\text{warp}}$ and $\sqrt{\text{TFC}}$, we convert average of the square roots to arithmetic averages. We have, for either case,

$$E(x) = E[(\sqrt{x})^2] = \text{Variance } \sqrt{x} + \{E(\sqrt{x})\}^2 \quad (8)$$

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Since variance $\sqrt{\text{warp}} = (1.0)^2 = 1.00$, and variance $\sqrt{\text{TFC}} = (1.3)^2 = 1.69$, we see from (8) that

$$\text{average warp} = 1.00 + (\text{average } \sqrt{\text{warp}})^2 \quad (9)$$

and

$$\text{average TFC} = 1.69 + (\text{average } \sqrt{\text{TFC}})^2 \quad (10)$$

These relationships are also plotted on the following pages to aid in interpreting square root values.

INTERPRETATION OF
WARP DATAFIG. I

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INTERPRETATION OF
TFC DATA
(TUBE FILLING CAPACITY)

FIG. II

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INTERPRETATION OF
OD₂ CHANGE DATA
FIG. III

% OF TOTAL POPULATION EXCEEDING X MILS

X = 5 MILS

X = 10 MILS

OD₂ CHANGE - MILS

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