





**Analytic Formulation for the ac Electrical Conductivity in  
Two-temperature, Strongly Coupled, Overdense Plasma:  
FORTRAN Subroutine**

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**Abstract**

A FORTRAN subroutine for the calculation of the ac electrical conductivity in two-temperature, strongly coupled, overdense plasma is presented. The routine is the result of a model calculation based on classical transport theory with application to plasmas created by the interaction of short pulse lasers and solids. The formulation is analytic and the routine is self-contained.

Laser light absorption in short pulse laser irradiation of solids is difficult to calculate because the plasma is strongly coupled with respect to interparticle interactions. In order to find the absorption, the electrical conductivity or, equivalently, the electron-ion collision frequency must be known. The collision frequency is used in the inverse bremsstrahlung coefficient. [1] Since the driving mechanism is a laser with non-zero frequency, the electrical conductivity must be frequency-dependent, or, ac. In addition, in such plasmas, ions and electrons may have different temperatures, which leads to different absorption values over the usual single temperature case. [2]

We have expanded a strongly coupled transport theory model [3] to include the effects of frequency dependence and electron temperatures distinct from ion temperatures. The details of the calculation will appear elsewhere. [4] In the appendices are FORTRAN routines which contain the results of the calculation. Appendix A contains the subroutine (called 'rho') which performs the calculation given as input: ion density (in  $\text{cm}^{-3}$ ), electron temperature (in eV), ion temperature (in eV), mean ionization state, atomic weight of the material, and the wavelength of the laser light (in nanometers). Two additional arguments finish the argument list: a switch (default on) which modifies the calculation at low temperatures ( $\leq \sim 7$  eV for solid density aluminum) when an approximation used in the formulation begins to break down [The approximation involves the use of an effective interparticle potential which is not consistent with the Debye-Hückel particle

correlations also employed; as the plasma becomes more strongly coupled, this approximation becomes worse. Its use, however, allows a completely analytic result.]; and a switch (default on) which returns another analytic result due to Boercker, Rogers, and DeWitt [5] when the coupling is even lower, such that the electrons are degenerate [We have performed a classical calculation which brings quantum effects in only through the use of the effective potential.] Explanations for all variables are commented in the routine.

The routine returns: the electron plasma frequency,  $\omega_{pe}$ , (in Hz), the electron-ion collision frequency (in Hz), the electron-ion collision frequency divided by  $\omega_{pe}$ , the ac electrical conductivity (in Hz), the ac electrical conductivity divided by  $\omega_{pe}$ , the resistivity (in micro-ohm-cm), the electron-ion collision frequency divided by  $\omega_{pe}$  according to Ref. [5] (including a weak coupling multiplier referred to but not use in [5]), the resistivity according to Ref. [5] (in micro-ohm-cm), and the electron-ion collision frequency (divided by  $\omega_{pe}$ ) if only a 1-Sonine model were used [Most calculations are 1-Sonine equivalents. Our calculation is a 2-Sonine model (See Ref. [3]), which is necessary in strong coupling.] Two additional functions are appended to the 'rho' routine and are required. Function 'e1local' finds the exponential integral of the first kind for the independent variable  $x$ ,  $E_1(x)$ , which is used in function 'expe1,' which returns the value of  $e^x \times E_1(x)$ .

Appendix B is an example FORTRAN driver for the subroutine 'rho.' As part of the driver, a function 'zbar' is included. This is the coded version of More's Thomas-Fermi mean ionization result. [6] This is convenient since the 'rho' routine requires a

value of the average ionization. Appendix C contains sample output of the driver/subroutine for several temperatures of solid density aluminum.

This report specifies that these routines can be used by anyone who requests them. The authors request that if problems arise, users of the code report these problems to us. The approximations contained in the calculation, and thus in 'rho,' are known to break down when the plasma becomes too strongly coupled. This is primarily due to the fact that Debye-Hückel correlations were used. We anticipate that the use of correlations generated by, for example, hypernetted chain (HNC) theory [7] would produce more satisfactory results (at the cost of loss of analyticity.) HNC correlations have been shown to produce quite good results for strong coupling. [7] However, a low-temperature electron-ion HNC code does not seem to exist (yet). Clearly use of more appropriate correlations in the model would lead to better results at very strong coupling.

## Appendix A

```
subroutine rho (ni,tev,tiv,z,aw,xwavel,wpe,  
> cfw0,cfw0wpe,econd,econdwpe,rhow0,  
> ppxdhbr,rhoxdhbr,cfppw0wp,iweesw,ibrds  
c  
implicit real*8(a-h,o-z)  
common tcond,xmult,omult,wowpe,gamei  
c  
real*8 me,ni,ne,mi,kdtkdb,lgeeb,lgeib,lgkdb  
real*8 kde2,kdi2,kd2,kee2,kei2,lgdel  
real*8 kde,kdi,kd2bar,kee2bar,kei2bar  
  
data pi,c/3.141592653589793238,2.99792458d10/  
data amu,one/1.6605655d-24,1.0d0/  
data sq2,forpiesq /1.4142135623731d0,2.898887d-18/  
c data esqr /2.3068617d-19/  
  
c For now, at least, restricted to T_e >= T_i  
c Returns equal temperature result at T_i otherwise  
  
if (tev . lt. tiv) tev = tiv  
  
me=0.9109534d-27  
hbar=1.05443d-27  
c  
c This version uses 2-temperature DH statics everywhere  
c Matrix elements are static - no propagators  
c  
c ni      - ion density in cm^-3  
c tev     - electron temperature (= ion temperature) in eV  
c tiv     - ion temperature (= ion temperature) in eV  
c z       - net ion charge (simple number)  
c aw      - atomic weight (simple number)  
c cfdh    - collision frequency / wpe  
c rhodh   - resistivity  
c xmult   - exact zero-coupling multiplier (divide 1 Sonine  
result - M_pp - to  
c          get approximate 2 Sonine result  
c omult   - actual multiplier  
c wpe     - electron plasma frequency  
c wowpe   - laser frequency / wpe  
c tcond   - thermal conductivity divided by k_boltzmann  
c cfw0    - ei collision frequency (Hz)  
c cfw0wpe - ei coll. freq. in units of wpe  
c cfppw0wp- M_pp ei coll freq in units of wpe  
c econd   - electrical conductivity  
c econdwpe- elect. cond in units of wpe  
c rhow0   - resistivity in micro-ohm-cm  
c rhow0wpe- restistivity in units of wpe  
c ppxdhbr- Boercker-Rogers-DeWitt coll. freq. ( units of wpe)  
c rhoxdhbr- BRD resistivity in units of wpe  
c iweesw  - switch on (=1) or off (=0) conversion to less  
pathological  
c          behavior at low temperature (default on)  
c ibrds   - switch on (=1) or off (=0) return of Boercker-  
Rogers-DeWitt
```

```

c          result at very low temperature (default on)
c
c          Given these params, the subroutine returns cf (electron-ion
collison  c          frequency) * wpe and rho (resistivity) in micro-ohm-cm.
See       c
c
c

```

```

ne = ni*z
aion=(3.0/4.0/pi/ni)**(1.0/3.0)
mi= amu*aw
beta=one/(tev*1.602021d-12)
betai=one/(tiv*1.602021d-12)
betaei=(me+mi)/(me/betai + mi/beta)
sbeta=dsqrt(beta)
sbetaei=dsqrt(betaei)
xmu=one/(one/me+one/mi)
gamei=z*forpiesq/aion*betaei/4./pi

```

```

constah=aion/hbar
kei2=constah*pi*2.*xmu*constah/betaei
kee2=constah*pi*me*constah/beta

```

```

const=aion*aion
kde2=forpiesq*ne*const*beta
kdi2=forpiesq*ni*const*z*z*betai
kd2=kde2+kdi2
kdi=dsqrt(kdi2)
kde=dsqrt(kde2)
wpe2=forpiesq*ne/me
wpe=dsqrt(wpe2)

```

```

wavel=xwavel*1.0d-7
wo=2.0*pi*c/wavel
wowpe=wo/wpe

```

```

rbeta=betaei**2/beta/betai
deltalr=one-rbeta
delabs=dabs(deltalr)
deltsqrt=dsqrt(deltalr)
kd2bar=kd2/kde/kdi
kei2bar=kei2/kde/kdi
kee2bar=kee2/kde/kdi

```

```

eeteeb = kee2bar*kee2bar
eiteib = kei2bar*kei2bar
eeteib = kee2bar*kei2bar
eiteeb = eeteib
kdkdb = kd2bar*kd2bar
eitkdb = kei2bar*kd2bar
eetkdb = kee2bar*kd2bar
lgeeb = dlog(kee2bar)
lgeib = dlog(kei2bar)
lgkdb = dlog(kd2bar)

```

```

c
c          Approximate 2 Sonine / One Sonine ratio
          xmult=(13.0-9.0*sq2/(z+sq2))/4.0

```



```

c      BRD term with V=Coulomb
c      If ibrdsw = 1 and coupling is very large, this result is
returned

```

```

      balf=betaei/constah/8.0/me/constah*kd2
      dhlaminf=expel(balf)-expel(balf/2.)/2.
c      kmaxinf=2./3.*dhlaminf

```

```

      co=-ni/wpe*dsqrt(2.*pi)/(8.*pi**2*dsqrt(me)/sbeta)
      cxdxdh=- (forpiesq*z)**2*beta
      cxppxdh=co*cxdxdh

```

```

      ppxdhbr=cxppxdh*2.d0/3.d0*dhlaminf/xmult
      rhoxdhbr = 4.0*pi/wpe*ppxdhbr/xmult*9.0d17

```

```

c23456789112345678921234567893123456789412345678951234567896123456
789712

```

```

c      Calculate static integrals in V_Deutsch, c_DH, and
c      S_Debye-Huckel approximations for 1 or 2 temps

```

```

c      Recall below to negate all integrals since these are all
contributions

```

```

c      at zero; contribution at infinity is 0

```

```

c      *****
c      Delta integral - 1T & 2T are same if cD_ei is constructed
from S_ei

```

```

      dconst= -(forpiesq*z)*betaei*(forpiesq*z)*kei2/2.

```

```

c      1 / (x + kd^2) / (x + kei^2)

```

```

      deltain = dlog ( kd2 / kei2 ) / (kei2 - kd2)
      delta = - dconst*deltaint

```

```

c      *****

```

```

c      Omega_ei

```

```

      if (delabs .le. 1.e-8) then

```

```

c      1 / (x + kd^2) / (x + kei^2) [ from delta ]

```

```

      weil = deltain

```

```

c      1 / (y + kdbar^2) / (y + keibar^2) / (y + keebar^2)

```

```

c      A_INT

```

```

      comei=eiteib-eiteeb-eitkdb+eetkdb
      comee=eiteeb-eeteeb-eitkdb+eetkdb
      comkd=eiteeb-eetkdb-eitkdb+kdkdb
      aint=lgeib/comei-lgeeb/comee+lgkdb/comkd

```

```

      wei2=(one-kee2/kei2)/kdi2*aint

```

```

      wei = - dconst * (weil + wei2)

```

```

else

c      *****
c      We will need Arctan[kd2bar]/sqrt(4 delta1r - kd2bar^2)

      delsqr = 4.*delta1r - kdtkdb

      if (delsqr .lt. 0.) then
        del = dsqrt( -delsqr)
        arg = - kd2bar/del
        arc=dlog( (arg+1.)**2/(arg-1.)**2 ) / 4. / del

      elseif (delsqr .gt. 0.) then
        del = dsqrt( delsqr)
        arc = datan(kd2bar/del) / del
      else

        arc = -one / kd2bar

      endif

c      *****

      delteeb=delta1r*kee2bar
      delteib=delta1r*kei2bar
      deltkdb=delta1r*kd2bar
      lgdel = dlog(delta1r)

c      y / (y^2 + kdbar^2 + delta1r) / (y + keibar^2)
c      B_INT_IE

      term1 = (2.*delta1r - eitkdb) * arc
      term2 = -kei2bar*dlog( kei2bar/ deltsqrt )
      comcei= delta1r + eiteib - eitkdb
      binte1= (term1 + term2) / comcei

      weil = binte1/ kde / kdi

c      y / (y^2 + kdbar^2 + delta1r) / (y + keebar^2) / (y +
keibar^2)
c      C_INT

      ccom1d=(delta1r+eeteeb-eetkdb)
      ccom1d=ccom1d*(delta1r+eiteib-eitkdb)
      ccomn= 2.d0*delteib+2.d0*delteeb
>      -deltkdb-eetkdb*kei2bar

      term14= ( ccomn*arc+(-delta1r+eeteib)
>      *lgdel/2. ) / ccom1d

      ccom2d=delteib+kei2bar*eiteib-delteeb
>      +eiteib*(-kee2bar-kd2bar)
>      +eitkdb*kee2bar
      term2= kei2bar*lgdel/ccom2d

      ccom3d=-delteib+kee2bar*eeteeb+delteeb
>      +eeteeb*(-kei2bar-kd2bar)

```

```

>      +eitkdb*kee2bar
term3= kee2bar*lggeb/ccom3d
cint = term14 + term2 + term3

wei2 = (one - kee2/kei2) * cint / kdi2

wei = - dconst * (wei1 + wei2)

endif

c *****
c      Omega_ee

weeconst=-forpiesq*beta*forpiesq*kee2/2.0

if (delabs .le. 1.e-8) then

c      1 / (x + kd^2) / (x + kee^2)

wee1 = dlog ( kd2 / kee2 ) / (kee2 - kd2)

c      1 / (y + kdbar^2) / (y + keibar^2) / (y + keebar^2) [ from
wei ]
c      A_INT

wee2=(one - kei2/kee2)/kde2*aint

wee = - weeconst * (wee1 + wee2)

c ****
c      The following statement is a compromise. Kei != kee which
causes
c      problems with the wee2 integral. wee1 is negative; wee2
is positive.
c      As Tev decreases, wee2 becomes large enough to make wee
positive.
c      As Tev further decreases, this sign change + increasing
absolute
c      value of wee causes 'bigd' to change sign. Since bigd is
a
c      determinant, this causes a "resonance" to occur in nu_ei.
c      Not permitting wee to be > 0, say, by absolute-valuing wee
doesn't work
c      as well as just zeroing out the entire wee term
c
c      iweesw must equal 1 for this to occur

      if ((wee .gt. 0.) .and. (iweesw .eq. 1)) wee = 0.
c ****

else

c      y / (y^2 + kdbar^2 + delta1r) / (y + keebar^2)
c      B_INT_EE

term1 = (2.*delta1r - eetkdb) * arc
term2 = -kee2bar*dlog( kee2bar/ deltsqrt )
comcee= delta1r + eeteeb - eetkdb
bintee= (term1 + term2) / comcee

```

```

wee1 = bintee/ kde / kdi

c      y / (y^2 + kdbar^2 + delta1r) / (y + keebar^2) / (y +
keibar^2)
c      C_INT

wee2 = (one - kei2/kee2*betaei/betai) * cint / kde2

c      1 / (y^2 + kdbar^2 + delta1r) / (y + keebar^2) / (y +
keibar^2)
c      D_INT [ similar to C_INT]

c**      absorb (1-betaei/betai) = delta1r into result

dcomng= delta1r+eiteib-eitkdb
dcomnh= delta1r+eeteeb-eetkdb
dcomngh= kei2bar-kee2bar

comn1x=2.d0*(-delta1r+eiteeb)-eitkdb
comn1=delta1r*(comn1x-eetkdb+kdkdb)
comn2=-delteib-delteeb+deltkdb
term12= (comn1*arc+comn2*lgdel/2.d0)/dcomng/dcomnh

term3= - delta1r*lgeib/dcomng/dcomngh

term4= delta1r*lgeeb/dcomnh/dcomngh

dint = term12 + term3 + term4

wee3 = kei2bar * dint / kde2

c      Now get extra terms with delta1r in front due to cD_ee
c**      absorb (1-betaei/betai) = delta1r into all results

c      y / (y^2 + kdbar^2 + delta1r) / (y + kdbar^2) / (y +
keebar^2)
c      F_INT

termn1=2.d0*delteeb+deltkdb-kee2bar*kdkdb
term1=termn1*arc

term2=(-delta1r+eetkdb)*lgdel/2.d0

term3=delteeb*lgeeb / (kee2bar-kd2bar)

term4=kd2bar*lgkdb/(-kee2bar+kd2bar)

fint = (term1+term2+term3)/dcomnh + term4

wee4 = -fint / kde2

c      1 / (y^2 + kdbar^2 + delta1r) / (y + kdbar^2) / (y +
keebar^2)
c      G_INT_EE

```

```

termn1= 2.d0*delta1r-eetkdb
term1= -termn1*arc

term2= -kee2bar*lgdel / 2.d0

term3= delta1r*lgeeb/(-kee2bar+kd2bar)
term4= lgkdb/(kee2bar-kd2bar)

gintee= (term1+term2+term3)/dcomnh + term4

wee5 = -kdi/kde/kde2 * gintee

c      1 / (y^2 + kdbar^2 + delta1r) / (y + keibar^2) / (y +
kdbar^2)
c      G_INT_IE

termn1= 2.d0*delta1r-eitkdb
term1= -termn1*arc

term2= -kei2bar*lgdel / 2.d0

term3= delta1r*lgeib/(-kei2bar+kd2bar)
term4= lgkdb/(kei2bar-kd2bar)

gintie= (term1+term2+term3)/dcomng + term4

wee6 = kdi/kde/kde2*betaei/betai*kei2/kee2 * gintie

wee = - weeconst * (wee1+wee2+wee3+wee4+wee5+wee6)
c ****
c      if ((wee .gt. 0.) .and. (iweesw .eq. 1)) wee = 0.
c ****

endif

c      Now get the matrix elements.
c      Set up constants:
c      pi^(-3/2)*one/me^(-1/2) & dsqrts of beta's
c      this excludes a factor of ni; put in later

constm=one/(pi**1.5)/(dsqrt(me))
sbeta=dsqrt(beta)
sbetaei=dsqrt(betaei)

c      M_pp

xnum1 = one/6.0/dsqrt(2.0d0)
ppw0 = -constm*sbetaei*xnum1*delta

c      M_rr

xnum1 = 3.0/28.0
xnum2 = 433./280./6./dsqrt(2.0d0)
rrw0 = -constm*z*(sbeta*xnum1*wee+sbetaei*xnum2/z*wei)

```

```

c           M_qq

xnum1 = one/15.
xnum2 = 13./60./dsqrt(2.0d0)
qqw0 = -constm*z*(sbeta*xnum1*wee+sbetaei*xnum2/z*wei)

c           M_qr

xnum1 = one/5./dsqrt(28.0d0)
xnum2 = 23./20./dsqrt(2.0d0)/dsqrt(28.0d0)
qrw0 = constm*z*(sbeta*xnum1*wee+sbetaei*xnum2/z*wei)

c           M_pq

xnum1 = one/8./dsqrt(5.0d0)
pqw0 = constm*sbetaei*xnum1*(delta+wei)

c           M_pr

xnum1 = 5./16./dsqrt(35.0d0)
prw0 = -constm*sbetaei*xnum1*(delta+wei)

c           Put together the combinations

c           Each Mx element is missing a factor of ni in front
c           Do same for frequency here (all to prevent overflows)

woni = wo / ni
wonisq = woni * woni

bigd=  rrw0*qqw0 - qrw0*qrw0

bigrr= rrw0*pqw0 - qrw0*prw0
bigqq= qqw0*prw0 - qrw0*pqw0

c           thermal conductivity const excludes k_boltzmann
c           reported as purely static

tcond=2.5*ne/(bigd*ni)/beta/me*rrw0

cfppw0 = ppw0*ni
cfppw0wp = cfppw0 / wpe / xmult

Dsqr=(bigd-wonisq)**2+wonisq*(qqw0+rrw0)**2
psil=(bigd-wonisq)/Dsqr*(pqw0*bigrr+prw0*bigqq)
psi2=wonisq/Dsqr*(qqw0+rrw0)*(pqw0**2+prw0**2)

psi = (psil + psi2) * ni
cfw0 = cfppw0 - psi
omult = cfppw0 / cfw0
cfw0wpe = cfw0 / wpe

c           Return BRD result if CR result is bad (large coupling) and
c           ibrdsw = 1

```

```

ppxdhbr      if ((cfw0wpe .lt. ppxdhbr).and.(ibrdsw .eq. 1)) cfw0wpe =
cfw0 = cfw0wpe * wpe

psii1=woni/Dsqr*(qqw0+rrw0)*(pqw0*bigrr+prw0*bigqq)
psii2=-woni/Dsqr*(bigd-wonisq)*(qqw0+rrw0)**2
psii = (psii1 + psii2) * ni
psiiw = woni + psii

econd = wpe2/4./pi*cfw0/(cfw0*cfw0+psiiw*psiiw)

psiiwpe = psiiw/wpe
econdwpe = cfw0wpe/4./pi/(cfw0wpe*cfw0wpe+psiiwpe*psiiwpe)

rhow0 = 4.*pi/wpe2*cfw0*9.0e17

return
end

```

```

c *****

```

```

      real*8 function expel(x)
      implicit real*8 (a-h,o-z)

c      returns the value of exp(x) * E1(x)
c      (have to watch out for large x)

      if(x .gt. 50.0) then
      x2=x*x
      x4=x2*x2
      poly=1.0-1.0/x+2.0/x2-6.0/x2/x+24.0/x4
      expel=poly/x
      return
      endif

      expel=dexp(x)*ellocal(x)
      return
      end

```

```

c *****
  real*8 function ellocal(x)
    implicit real*8 (a-h,o-z)

c
c   Returns exponential integral of the first kind. Abramowitz
& Stegan p. 231.
c
  dimension a(4),b(4),c(6)

  data c /-.57721566, .99999193,-.24991055,
>         .05519968,-.00976004, .00107857/
  data a / 8.5733287401,18.0590169730,
>         8.6347608925, .2677737343/
  data b / 9.5733223454,25.6329561486,
>         21.0996530827, 3.9584969228/

  if(x.le. 0.0) then
    write(*,100)
100 format(' argument of E1 < or = zero)')
    stop
  endif

  if(x.le. 1.0)then
    e=c(6)*x+c(5)
    do 10 i=4,1,-1
      e=e*x+c(i)
10 continue
    ellocal=e - dlog(x)
    return
  endif

  enum=x+a(1)
  eden=x+b(1)
  do 20 i=2,4
    enum=enum*x+a(i)
    eden=eden*x+b(i)
20 continue
  ellocal=enum/eden*dexp(-x)/x

  return
end

```



## Appendix B

```
program rhotest

  implicit real*8(a-h,o-z)
  real*8 ni
  common tcond,xmult,omult,wowpe,gamei

  write(*,2)
  2 format(' Restricted to T_electron => T_ion',/
  >       ' If Zbar < 0, calc Zbar',/)
  z=-9.
  iweesw = 1
  ibrdsw = 1

  99 write(*,100)
  100 format(' Input n_ion (cm-3), kT_e (eV),kT_i, Zbar',/,
  >        '      Atomic Weight, Wavelength (nm)')
  read(*,*) ni,tev,tiv,z,aw,xwavel

c
  if (ni .le. 0.0) stop

  write(*,101)
  101 format(' Input Low T switch, BRD switch')
  read(*,*) iweesw,ibrdsw

  if (z .le. 0.) then
    write(*,98)
    98 format(' Zbar, input Z_nuc and Temperature (T_e Default)')
    temp=tev
    read(*,*) znuc,temp
    z = zbar(ni,temp,znuc)
    write(*,501) z
  501 format(' Zbar = ',f7.3)
  endif

  call rho (ni,tev,tiv,z,aw,xwavel,wpe,
  >  cfw0,cfw0wpe,econd,econdwpe,rhow0,
  >  ppxdhbr,rhoxdhbr,cfppw0wp,iweesw,ibrdsw)

  write(*,699) wpe,z,xmult,omult,gamei
  write(*,700) tcond
  write(*,701) cfw0,cfw0wpe,econd,econdwpe,rhow0
  write(*,702) cfppw0wp,ppxdhbr,rhoxdhbr
  699 format(' wpe = ',e12.6,' Z = ',f6.3,' t_mult = ',f6.3,
  >        ' o_mult = ',f6.3,' Gamma_ei = ',f7.3)
  700 format(' thermal conductivity/k_B: ',e12.6)
  701 format(' nu_ei: ',e12.6,2x,e12.6,/,
  >        ' econd: ',e12.6,2x,e12.6,/,
  >        ' rho: ',e12.6,2x,e12.6)
  702 format(' M_pp only: ',8x,e12.6,/,
  >        ' BRD nu_ei: ',8x,e12.6,/, ' BRD rho: ',10x,e12.6)

  go to 99
end
```

C

\*\*\*\*\*

```
real*8 function zbar (deni,tev,znuc)
implicit real*8 (a-h,o-z)
```

C  
C calculates average charge in a plasma given the  
C temp, dens, z, and atomic number of the plasma  
C ions. taken from r. m. more, ucrl-84991 (5 march,  
C 1981), p. 61 dens is the ion density.  
C

```
data av/6.022e23/
data al,be/14.3139,0.6624/
data a1,a2,a3,a4/0.003323,0.971832,9.26148e-5,3.10165/
data b0,b1,b2/-1.763,1.43175,0.315463/
data c1,c2/-0.366667,0.983333/
```

C

```
to=tev/znuc**(4.0/3.0)
tf=to/(1.0+to)
r=deni/znuc/av
aa=a1*to**a2+a3*to**a4
bb=-exp(b0+b1*tf+b2*tf**7)
cc=c1*tf+c2
q1=aa*r**bb
q=(r**cc+q1**cc)**(1.0/cc)
x=a1*q**be
zbr=znuc*x/(1.0+x+sqrt(1.0+2.0*x))
zbar=zbr
return
end
```

## Appendix C

```
Restricted to T_electron => T_ion
If Zbar < 0, calc Zbar

Input n_ion (cm-3), kT_e (eV),kT_i, Zbar,
      Atomic Weight, Wavelength (nm)
  6.e+22 , 10.0 , 10.0, 3.0 , 27.0, 400.0
Input Low T switch, BRD switch
/
wpe = .239334e+17 Z = 3.000 t_mult = 2.529 o_mult = 2.662
Gamma_ei = 2.726
nu_ei: .735262e+16 .307212e+00
econd: .619514e+15 .258849e-01
rho: .145173e+03
M_pp only: .323319e+00
BRD nu_ei: .886300e-01
BRD rho: .165598e+02

Input n_ion (cm-3), kT_e (eV),kT_i, Zbar,
      Atomic Weight, Wavelength (nm)
  6.e+22 , 50.0 , 10.0, -9.0 , 27.0, 1.0E20
Input Low T switch, BRD switch
1,1
Zbar, input Z_nuc and Temperature (T_e Default)
13.0,50.0
Zbar = 5.233
wpe = .316084e+17 Z = 5.233 t_mult = 2.771 o_mult = 2.779
Gamma_ei = .951
nu_ei: .279881e+16 .885463e-01
econd: .284068e+17 .898710e+00
rho: .316826e+02
M_pp only: .887862e-01
BRD nu_ei: .339936e-01
BRD rho: .438901e+01

Input n_ion (cm-3), kT_e (eV),kT_i, Zbar,
      Atomic Weight, Wavelength (nm)
  6.e+22 , 50.0 , 10.0, 3.0 , 27.0, 1.0E20
Input Low T switch, BRD switch
/
wpe = .239334e+17 Z = 3.000 t_mult = 2.529 o_mult = 2.560
Gamma_ei = .545
nu_ei: .159570e+16 .666725e-01
econd: .285659e+17 .119356e+01
rho: .315061e+02
M_pp only: .674836e-01
BRD nu_ei: .358051e-01
BRD rho: .668989e+01

Input n_ion (cm-3), kT_e (eV),kT_i, Zbar,
      Atomic Weight, Wavelength (nm)
-9. /

stop
```

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