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SEMILEPTONIC DECAYS OF MESONS IN THE $SO(4)$ MODEL

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ABSTRACT

We present results of a calculation of semileptonic decays of mesons in the string-like $U(4) \otimes SU_f(n) \otimes SU_s(2) \otimes SU_c(3)$ model with $U(4) \supset SO(4)$ dynamic symmetry. The calculation describes the available data as accurately as the semi-relativistic quark model. We make predictions for several exclusive decays which can be tested by future experiments.

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Recently, it has been shown [1,2] that use of the spectrum generating algebra (SGA) $G \equiv U(4) \otimes SU_F(n) \otimes SU_S(2) \otimes SU_C(3)$, with $U(4) \supset O(4) \supset O_L(3) \supset O_{ML}(2)$ dynamic symmetry (DS) describes masses and electromagnetic decay widths of $(q\bar{q})$ mesons as accurately as the semi-relativistic quark model [3]. The purpose of this letter is twofold. First we want to show that the same conclusion is reached from an analysis of the semileptonic decays, and second we want to point out that the algebraic model provides a systematic framework for studying in detail these processes. The latter point is particularly important in view of the considerable interest in semileptonic decays as a way to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, which are fundamental quantities in the standard model.

Since the wave functions of all meson states can be explicitly written down in the $U(4)$ model with $O(4)$ symmetry, the calculation of the semileptonic decay rates and spectra is straightforward and only relies on the operator inducing the transition. This operator is then mapped onto elements of the algebra G and its matrix elements evaluated by using purely group theoretic methods.

The calculation is similar to that of the radiative decays and we sketch here its most important points. Within the standard model, the considered semileptonic decays of hadrons are due to the following weak quark current

$$J^\mu = \frac{g}{2\sqrt{2}} \left\{ [\bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij} d_j] + [\bar{d}_i V_{ij}^+ \gamma^\mu (1 - \gamma_5) u_j] \right\} \quad (1)$$

where $u_i = u, c, t$ and $d_j = d, s, b$ are the quark fields with $i, j \equiv 1, 2, 3$ being the generation indices, and the V_{ij} 's are the CKM matrix elements. The quark weak coupling constant is $g^2/8M_W^2 = G_F/\sqrt{2}$ with $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ and $M_W = 80.6 \text{ GeV}$. In this first step calculation, we consider, for simplicity, the lowest

order nonrelativistic form of J^μ , as in the corresponding quark model calculation of Isgur et al. [4], hereafter referred to as ISGW. Higher order terms can be included, if one wishes, as mentioned in the last paragraph (see Eq. (7)). The standard nonrelativistic limit of Eq. (1) yields the following form of quark weak current operator (deleting the generation indices and CKM matrix elements)

$$J^\mu = \frac{q}{2} [V^\mu - A^\mu]$$

with

$$V^0 = e^{-i\eta_2 \vec{q} \cdot \vec{r}} ,$$

$$\begin{aligned} \vec{V} = e^{-i\eta_2 \vec{q} \cdot \vec{r}} & \left[-\frac{\vec{q}}{2m_1'} + \left(\frac{1}{2m_1'} + \frac{1}{2m_1} \right) \vec{p} \right] \\ & + e^{-i\eta_2 \vec{q} \cdot \vec{r}} i\vec{\sigma}_1 \times \left[-\frac{\vec{q}}{2m_1'} + \left(\frac{1}{2m_1'} - \frac{1}{2m_1} \right) \vec{p} \right] , \end{aligned}$$

$$A^0 = e^{-i\eta_2 \vec{q} \cdot \vec{r}} \vec{\sigma}_1 \cdot \left[-\frac{\vec{q}}{2m_1'} + \left(\frac{1}{2m_1'} + \frac{1}{2m_1} \right) \vec{p} \right] ,$$

$$\vec{A} = e^{-i\eta_2 \vec{q} \cdot \vec{r}} \vec{\sigma}_1 . \quad (2)$$

Here we use the standard nonrelativistic relative position and momentum variables $\vec{r} = \vec{r}_1 - \vec{r}_2$, $\vec{p} = \eta_2 \vec{p}_1 - \eta_1 \vec{p}_2$ with $\eta_i = m_i / (m_1 + m_2)$. The index 1 labels the quark or antiquark at which the elementary weak process takes place. The initial and final quark masses are denoted respectively by m_1 and m_1' . The kinematics in laboratory frame is shown in Fig. 1: ℓ and k are respectively the momentum of electron and neutrino, q is the momentum transfer.

Note that since $\vec{r}$ and $\vec{p}=i\vec{\nabla}$ do not commute, the order of the operators in (2) is important. The current operator Eq. (2), can be simply mapped into elements of G. The spin operator, $\vec{\sigma}$, is already in SU_2 . The coordinate and momentum operators are mapped as in Ref. [2],

$$\vec{r} \rightarrow \frac{\beta}{N} \vec{\sigma} , \quad \vec{p} \rightarrow \frac{\zeta}{N} \vec{\sigma} . \quad (3)$$

Here β and ζ represent the scale factors of coordinates and momenta (not determined by the $O(4)$ symmetry). With $SO(4)$ notations of Refs. [1,2], the wave functions of the initial and final meson states in the process $M \rightarrow M' + e + \nu_e$ can be written as

$$| [N] \omega L S J M_J \rangle = \sum_{M_L, M_S} \langle L M_L S M_S | J M_J \rangle | [N] \omega L M_L \rangle | S M_S \rangle . \quad (4)$$

where N is the dimensions of representation, ω is the principal quantum number, L , S , and J are respectively the orbital spin and total angular momentum.

For evaluation of the matrix elements of the operators in (2) one needs, in addition to the matrix elements of the spin operators, which are trivial, those of the exponentials. The matrix elements relevant to the present calculation are given by [5]

$$\begin{aligned} \tilde{f}_0^{(L)} &\equiv \langle [N], \omega=N, L, 0 | e^{-i\eta \vec{q} \cdot \vec{r}} | [N], \omega=N, 0, 0 \rangle \\ &= (-1)^L (-1)^{\frac{L}{2}} \sqrt{(2L+1)} j_L(\eta q \beta) , \end{aligned} \quad (5a)$$

$$\langle [N], \omega=N, 0, 0 | e^{-i\eta \vec{q} \cdot \vec{r}} p_m | [N], \omega=N, L, -M_L \rangle \quad (5b)$$

$$= \delta_{m, M_L} \left(\frac{S}{N} \right) \left\{ \begin{aligned} & (-1)^{L+1} (-1)^{\frac{L}{2}} \sqrt{\frac{L(L+1)}{2(2L+1)}} [-(L+1)j_{L+1}(\eta q \rho) + Lj_{L-1}(\eta q \rho)] , m = \pm 1 \\ & (-1)^{L+1} (-1)^{\frac{L}{2}} \sqrt{\frac{1}{2(2L+1)}} [-(L+1)^2 j_{L+1}(\eta q \rho) - L^2 j_{L-1}(\eta q \rho)] , m = 0 \end{aligned} \right.$$

where $j_L(X)$ is the spherical Bessel function.

We will only consider mesons with orbital angular momentum $L=0$ and the decaying state is always a state with $L=0$, $S=0$, $J=0$, $M_J=0$. Combining the matrix elements in (5) and those of the spin operators, one obtains the needed matrix elements

$$\mathcal{P}^0(S J M_J) = \langle [N] \omega(0S) J M_J | V^0 - A^0 | [N] \omega(00) 00 \rangle ,$$

$$\vec{\mathcal{P}}(S J M_J) = \langle [N] \omega(0S) J M_J | \vec{V} - \vec{A} | [N] \omega(00) 00 \rangle , \quad (6)$$

In terms of kinematical variables shown in Fig. 1, it is straightforward to show that the electron spectrum of the semileptonic decay process can then be calculated by

$$\begin{aligned} \frac{d\Gamma}{dE_\ell} &= \frac{1}{(2\pi)^5} G_F^2 \int d\Omega_\ell \int d\Omega_k F(\vec{\ell}, \vec{k}) \\ &\times \sum_{M_J} \{ 2\text{Re}[(\vec{\mathcal{P}}(S J M_J) \cdot \vec{k}) (\vec{\mathcal{P}}(S J M_J) \cdot \vec{\ell})^* \\ &- (\vec{\ell} \cdot \vec{k}) \vec{\mathcal{P}}(S J M_J) \cdot \vec{\mathcal{P}}^*(S J M_J) \\ &- i\epsilon_{\alpha\beta\gamma\delta} \ell^\alpha \mathcal{P}^{\beta*}(S J M_J) \ell^\gamma \mathcal{P}^\delta(S J M_J)] \} , \end{aligned}$$

$$F(\vec{\ell}, \vec{k}) = \frac{\ell^2 k^2}{E_\ell} \frac{1}{\left| 1 + \frac{\vec{\ell} \cdot \vec{k} + k^2}{k \sqrt{M^2 + \vec{q}^2}} \right|} ,$$

$$|\vec{k}| = k = \frac{(M - E_\ell)^2 - M'^2 - |\vec{\ell}|^2}{2\vec{\ell} \cdot \vec{k} + 2(M - E_\ell)} . \quad (7)$$

The total rate is $\Gamma = \int d\Gamma/dE_e$. M and M' are respectively the mass of the initial and final mesons.

We have used Eqs. (4-6) to calculate electron spectra and rates for all known semileptonic decays [6]. The calculated integrated rates are compared with the data in the upper part of Table I. The calculation is done with the values of the CKM matrix given in the last column. The flavor part of the meson wave function is the same as in Refs. [1,2]. In addition to the values of the CKM matrix elements, our results depend only on two quantities, the scale of coordinate, β , and of momenta, ζ , which appears in the form (ζ/N) . The results of Table I are obtained by using the $\beta=1.4$ fm and $\zeta/N=2.537$ GeV for all mesons. Variations of the size of mesons with flavor, as done for example in Table II of Ref. [4], would obviously improve the description. Furthermore, we have used values of β determined from a fit to the electromagnetic decay widths ($\beta(L=0) = 1.4$ fm) [2]. Thus the only variational parameter is the momentum scale (ζ/N) . This too could be fixed from the electromagnetic decays of mesons. Unfortunately, there are not enough experimental data on meson electromagnetic decays with $\Delta L=1$ to determine this parameter uniquely [2]. We emphasize the fact that a combination of weak and electromagnetic decays of mesons allows a parameter independent calculation of the semileptonic decays and thus a stringent test of hadronic structure models. Table I shows also some of the predictions that we are able to make based on the $SO(4)$ model.

We also note that measurements of semileptonic decays from the same initial state to two or more final states belonging to the same family provide a test of the model calculations which is independent of the value of the CKM matrix elements. There is one of them [7] known experimentally $\Gamma(B^0 \rightarrow D^{*}(2010)^- \ell^+ \nu) / \Gamma(B^0 \rightarrow D^- \ell^+ \nu) = 3.3 \pm 1.7$. We calculate for this ratio 4.3 while ISGW have 2.2. A second one can be inferred from the available data: $\Gamma(D^0 \rightarrow K^{*}(892)^- \ell^+ \nu) / \Gamma(D^0 \rightarrow K^- \ell^+ \nu) = 0.40 \pm 0.04$. We obtain 0.90 for this ratio while ISGW have 1.09. For further test of the $SO(4)$ model, we present in the lower part of Table I our predictions of the decays of D and B mesons into light mesons. In Fig. 2, we show that the calculated e^+ spectrum (solid curve) for $D \rightarrow X e^+ \gamma_e$ is in good agreement with the data. The dotted curve is from the decays into light mesons $\pi, \eta, \eta', \rho, \omega, \phi$. The dashed curve is obtained by adding the contribution from the decay into K meson. The solid curve is obtained by further including the decay into $K^{*}(892)$ meson. Clearly, the largest contributions are from the decays into $X = K$ and $K^{*}(892)$ mesons. This is in agreement with Ref. [4].

In conclusion, we have presented a method based on the SGA $U(4) \otimes SU_f(n) \otimes SU_s(2) \otimes SU_c(3)$ that allows a systematic investigations of all semileptonic exclusive decays of mesons. We have used this method in its lowest, non-relativistic, approximation for the current operator. This approximation can be improved to any order. Altomari and Wolfenstein [8] for example, have pointed out that p^2 correction terms contribute about 10% to the decay rate. The inclusion of these terms can be done, if one wishes, within the SGA approach. One needs to evaluate only matrix elements of the type

$$F_{mn}^{(L)} \equiv \langle [N], \omega=N, L, M_L | P_m P_n e^{-i\vec{\eta} \cdot \vec{r}} | [N], \omega=N, 0, 0 \rangle, \quad (8)$$

but this can be done (almost) trivially using the same group theoretic techniques that allow the evaluation of terms linear in the momentum, Eq. (4b). The corresponding formulas will be presented in a longer publication. Finally, our calculations can be turned around and used as a way to extract the CKM matrix, V_{ij} , as done for example using other models in Ref. [7].

This work was supported in part by U.S. Department of Energy grant number DE-FG02-91ER40608 and Nuclear Physics Division under contract W-31-109-ENG-38.

TABLE I. Comparison of semileptonic decay rates [6] with those computed in the $U(4) \supset O(4)$ model and in the semi-relativistic ISGW model [4]. The values of β are taken from electromagnetic decays, $\beta(L=0) = 1.4$ fm. The variational parameter (ζ/N) is 3.5 GeV. Quark masses are $m_u=m_d = 270$ MeV, $m_s = 427$ MeV, $m_c = 1554$ MeV, and $m_b = 4830$ MeV.

Decay	Experiment	$\Gamma(\text{sec}^{-1})$ O(4) Symmetry	ISGW ^a	V_{ij}
$\pi^+ \rightarrow \pi^0 e^+ \nu_e$	(0.39 ± 0.01)	0.38	-	0.97
$K^+ \rightarrow \pi^0 e^+ \nu_e$	$(3.90 \pm 0.05) \times 10^6$	3.40×10^6	-	0.22
$D^+ \rightarrow K^*(892)^0 e^+ \nu_c$	$(3.6 \pm 0.7) \times 10^{10}$	4.87×10^{10}	9.20×10^{10}	0.97
$D^0 \rightarrow K^- e^+ \nu_e$	$(3.0 \pm 1.1) \times 10^{10}$	5.39×10^{10}	8.44×10^{10}	0.97
$D^0 \rightarrow \pi^- e^+ \nu_e$	$(0.9 \pm 0.5) \times 10^{10}$	0.44×10^{10}	0.56×10^{10}	0.27
$B^0 \rightarrow D^- e^+ \nu_3$	$(0.75 \pm 0.4) \times 10^{10}$	0.56×10^{10}	1.74×10^{10}	0.04
$B^0 \rightarrow D^*(2010)^- e^+ \nu_e$	$(4.15 \pm 0.1) \times 10^{10}$	2.36×10^{10}	4.33×10^{10}	0.04
$D^+ \rightarrow \eta^+ e^+ \nu_e$	-	0.36×10^{10}	-	0.27
$D^+ \rightarrow \eta'^+ e^+ \nu_e$	-	0.29×10^{10}	-	0.27
$D^+ \rightarrow \rho^0 e^+ \nu_e$	-	0.27×10^{10}	-	0.27
$D^+ \rightarrow \omega^+ e^+ \nu_e$	-	0.24×10^{10}	-	0.27
$D^+ \rightarrow \phi^+ e^+ \nu_e$	-	0.49×10^{10}	-	0.27
$B^0 \rightarrow \pi^- e^+ \nu_e$	-	0.21×10^9	-	0.0035
$B^0 \rightarrow \rho^- e^+ \nu_e$	-	0.18×10^9	-	0.0035

^aExtracted from Ref. [4] using the values of V_{ij} in the last column.

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FIGURE CAPTIONS

Fig. 1 Kinematic variables in the process $M \rightarrow M' + e + \nu_e$.

Fig. 2 Comparison between calculated and experimental spectral shapes for $D \rightarrow X e^+ \nu_e$. The dotted curve includes the contribution from the decay into $\pi, \eta, \eta', \rho, \omega, \phi$. The dashed (solid) curve is obtained by adding the contribution of decays into $X = K (K, K^*(892))$. The experimental spectrum [9] is normalized as to give the same total integrated rate from the calculation.

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Fig. 1

Fig. 2

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