DOE/ER/14079-35
CWP-146P
December 1994

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John Anderson, and Ilya Tsvankin

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Center for Wave Phenomena
Colorado School of Mines
Golden, Colorado 80401
303/273-3557

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Dip-moveout processing by Fourier transform in anisotropic media

John Anderson* and I. Tsvankin†

ABSTRACT

Conventional dip-moveout (DMO) processing is designed for isotropic media and cannot handle angle-dependent velocity. We show that Hale's isotropic DMO algorithm remains valid for elliptical anisotropy but may lead to serious errors for non-elliptical transversely isotropic models, even if velocity anisotropy is moderate.

Here, we present an extension of Hale's constant-velocity DMO method to anisotropic media. The DMO operator, to be applied to normal-moveout(NMO)-corrected data, is based on the analytic expression for NMO velocity for dipping reflectors given by Tsvankin (1995a). Since the anisotropic DMO depends on the elastic parameters of the medium, it should be preceded by an inversion procedure designed to obtain the NMO velocity as a function of ray parameter. This inversion technique for transversely isotropic media is discussed in detail by Alkhalifah and Tsvankin (1995). Another complication introduced by anisotropy is the influence of nonhyperbolic moveout not accounted for in our DMO operator. However, for spreads typical in conventional acquisition design, deviations from hyperbolic moveout for P -waves are not significant.

*Mobil Oil Company, MEPTEC, P.O. Box 650232, Dallas, TX 75265-0232

†Colorado School of Mines, Department of Geophysics, Golden, CO 80401-1887

Paper presented at the 64th SEG Annual Meeting, Los Angeles, 1994.

Impulse responses and synthetic examples for typical transversely isotropic models with a vertical symmetry axis (VTI) demonstrate the accuracy and efficiency of this DMO technique. Once the inversion step has been completed, the NMO-DMO sequence does not take any more computing time than that for the generic Hale's method in isotropic media. Our anisotropic DMO operator is not limited to VTI media; it can be applied in the same fashion in symmetry planes of more complicated models such as orthorhombic.

INTRODUCTION

Seismic anisotropy, widely recognized as a property of many subsurface formations of various origin (e.g., Thomsen, 1986), may have a strong influence on all seismic processing methods, including dip-moveout algorithms (Levin, 1990; Larner, 1993; Tsvankin, 1995a,b). Most constant-velocity DMO techniques are based on the cosine-of-dip dependence of moveout velocity on reflector dip, valid for homogeneous isotropic media (Levin, 1971):

$$V_{\text{nmo}}(\phi) = \frac{V_{\text{nmo}}(0)}{\cos \phi}, \quad (1)$$

where V_{nmo} is normal-moveout velocity, and ϕ is the dip angle. In isotropic media, equation (1) remains accurate for any spreadlength because, in the absence of inhomogeneity, moveout is purely hyperbolic.

For the purpose of DMO processing, moveout velocity is usually expressed through the ray parameter (horizontal slowness) $p(\phi)$ corresponding to the zero-offset reflection.

$$p(\phi) = \frac{1}{2} \frac{dt_0}{dy_0} = \frac{\sin \phi}{V}, \quad (2)$$

where $t_0(y_0)$ is the two-way travelttime on the zero-offset (or stacked) section, and y_0 is the midpoint position. Using equation (2) and taking into account that in isotropic media $V_{\text{nmo}}(0) = V$, equation (1) can be rewritten as

$$V_{\text{nmo}}(p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}}. \quad (3)$$

Tsvankin (1995a) (hereafter referred to as Paper I) derived an analytic equation for normal-moveout (NMO) velocity from dipping reflectors in anisotropic media and studied the error of equations (1) and (3) for transversely isotropic models. He concluded that the dip-dependence of the P -wave moveout velocity for transverse isotropy with a vertical symmetry axis (VTI media) is controlled, to a significant degree, by the difference between the parameters ϵ and δ introduced by Thomsen (1986). Most measurements made at seismic frequencies indicate that $\epsilon > \delta$ (Thomsen, 1986; Sayers, 1994; Tsvankin and Thomsen, 1994). If $\epsilon - \delta > 0$, the cosine-of-dip corrected moveout velocity may be significantly larger than the moveout velocity for a horizontal reflector. Even for moderate values of $\epsilon - \delta = 0.1$ and $\epsilon \leq 0.2$, the error of the isotropic equation (1) reaches 25 percent at 45-degree dip and 30-35 percent at a dip of 60 degrees. For a larger difference $\epsilon - \delta = 0.2$, the moveout velocity at 60-degree dip after the cosine-of-dip correction remains almost 60 percent higher than the zero-dip moveout velocity. The errors in equation (3) for positive $\epsilon - \delta$ turn out to be even higher than those in equation (1) (Paper I).

Another conclusion drawn in Paper I is that for inhomogeneous transversely isotropic media with positive values of the difference $\epsilon - \delta$ and a linear increase in the vertical velocity with depth, inhomogeneity reduces the error in constant-velocity DMO caused by anisotropy. However, in the absence of a substantial velocity gradient, conventional constant-velocity DMO algorithms can be expected to lead to pronounced errors in typical transversely isotropic media.

Since dip-moveout removal is an important step in the conventional processing flow (NMO-DMO-poststack migration), any errors in DMO will propagate into the final seismic image (e.g., dipping reflectors may be missed or dimmed due to misstacking). Clearly, there is a need to develop dip-moveout algorithms capable of handling transversely isotropic media and, in the future, more complicated anisotropic models.

Uren et al. (1990) showed that Forel-Gardner DMO (1988) can be updated and applied in a straightforward way to elliptically anisotropic models. However, elliptical anisotropy is no more than a special case of transverse isotropy that corresponds to $\epsilon = \delta$ and cannot be considered as typical for real rocks (Thomsen, 1986). Also, Paper I demonstrates that the P -wave NMO velocity is highly sensitive to deviations from elliptical anisotropy.

The main difficulty in devising efficient DMO algorithms for anisotropic media was the absence of closed-form expressions for normal-moveout velocity necessary to replace the isotropic equations (1) and (3). This gap was filled by the NMO equation from Paper I, which can be used in symmetry planes of anisotropic media. Here, we apply this equation to extend the dip-moveout method developed by Hale (1984) to anisotropic media. We show that the anisotropic correction to Hale’s DMO operator is mostly dependent on the difference $\epsilon - \delta$ and can make a significant contribution to DMO processing for non-elliptical models. In addition to NMO correction, the anisotropic DMO removal should be preceded by an inversion procedure intended to obtain the effective anisotropic parameter responsible for the dependence of NMO velocity on the ray parameter. We study the complications caused by anisotropy in the DMO process and illustrate the performance of our algorithm by calculating impulse responses and processing synthetic sections for typical transversely isotropic models.

ANALYTIC FORMULATION

Hale’s DMO method

First, we review the fundamentals of Hale’s (1984) frequency-wavenumber (F-K) DMO algorithm. For both isotropic and anisotropic media, the original event time can be expressed in terms of the zero-offset event time and the NMO velocity as a

function of dip.

$$t^2 = t_0^2 + \frac{4h^2}{V_{\text{nmo}}^2(\phi)}, \quad (4)$$

where t is the original event time, t_0 is the two-way zero-offset traveltimes and h is half the source-receiver offset. If the medium is isotropic, the common-midpoint (CMP) moveout curve for a dipping reflector is a hyperbola parametrized by the NMO velocity (1). The normal-moveout equation for horizontal reflectors, applied to the data prior to DMO processing, results in the NMO-corrected time t_n given by

$$t_n^2 = t^2 - \frac{4h^2}{V_{\text{nmo}}^2(0)}. \quad (5)$$

Combining equations (4) and (5) yields an expression for the zero-offset time in terms of the NMO-corrected time.

$$t_0^2 = t_n^2 + 4h^2 \left[\frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(\phi)} \right]. \quad (6)$$

This equation, valid both in isotropic media and for the hyperbolic portion of the moveout curve in anisotropic media, is required for the coordinate transformation used in Hale's DMO.

In a homogeneous, isotropic medium, we have from equations (6) and (1)

$$t_0^2 = t_n^2 + \frac{4h^2 \sin^2 \phi}{V^2}, \quad (7)$$

or, introducing the ray parameter [equations (2) and (3)],

$$t_0^2 = t_n^2 + 4h^2 p^2. \quad (8)$$

Hale's isotropic DMO corrects for the remaining dip-dependent moveout term in equations (7) and (8) ($4h^2 p^2$). Therefore, the process of generating the zero-offset section is split into the NMO and DMO steps, with the DMO operator being completely independent of velocity.

A convenient way to obtain the ray parameter is to transform the data into the frequency-wavenumber domain. Then, for the zero-offset wavefield we have

$$p = \frac{1}{2} \frac{dt_0}{dy_0} = \frac{k}{2\omega}, \quad (9)$$

where k is the horizontal spatial wavenumber over the common-midpoint axis, and ω is the angular frequency associated with t_0 .

Hale's DMO uses the following Fourier-transform relationships between the time-space domain and the frequency-wavenumber domain for the zero-offset wavefield P_0 .

$$P_0(t_0, y_0, h) = \int \frac{d\omega}{2\pi} \int \frac{dk}{2\pi} P_0(\omega, k, h) e^{-i(\omega t_0 - k y_0)}; \quad (10)$$

$$P_0(\omega, k, h) = \int dt_0 \int dy_0 P_0(t_0, y_0, h) e^{i(\omega t_0 - k y_0)}. \quad (11)$$

We define the data after normal-moveout correction as $P_n(t_n, y_n, h)$, establish the relation between the zero-offset and common-offset NMO-corrected data as $P_0(t_0, y_0, h) \equiv P_n(t_n, y_n, h)$, and change variables accordingly.

In Hale's (1984) original algorithm, it is assumed that $y_0 \equiv y_n$. A similar assumption is made in Paper I in the derivation of the equation (19) for $V_{\text{nmo}}(\phi)$ in anisotropic media. The identification $y_0 \equiv y_n$ is not used by the "true amplitude" isotropic DMO methods discussed below. However, as we discuss below, the only refinement due to a more accurate treatment of the relation between y_0 and y_n for isotropic DMO is a more complicated Jacobian for the coordinate transformations. The kinematics of the DMO operator remain the same for either choice of the Jacobian.

Substituting $P_n(t_n, y_n, h)$ into equation (11) yields

$$P_0(\omega, k, h) = \int dt_0 \int dy_0 P_n(t_n, y_n, h) e^{i(\omega t_0 - k y_0)}, \quad (12)$$

and

$$P_0(\omega, k, h) = \int dt_n \int dy_n P_n(t_n, y_n, h) J_T e^{i(\omega t_n A - k y_n)}, \quad (13)$$

where Hale defines $t_0 = t_n A$. To emphasize the form that is also valid for anisotropic media, we use equation (6) to represent the term A in equation (13) as

$$A = \sqrt{1 + \frac{4h^2}{t_n^2} \left[\frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)} \right]}. \quad (14)$$

For isotropic relationships, equation (9) for p enables us to put A in the form used by Hale (1984).

$$A = \sqrt{1 + \frac{4h^2 p^2}{t_n^2}} = \sqrt{1 + \frac{k^2 h^2}{\omega^2 t_n^2}}. \quad (15)$$

The Jacobian J_T of the transformation (13) in the original Hale's algorithm is defined as

$$J_T = \frac{\partial t_0}{\partial t_n} = \frac{t_n}{t_0} = A^{-1}. \quad (16)$$

Two “true-amplitude” variants of Hale's algorithm are of interest. Both variants correct Hale's identification $y_0 \equiv y_n$, make the corresponding correction to the t_0 as defined in equation (4), and thereby honor the movement of the surface location associated with a specular reflection point during the transformation to zero offset. The phase factors in equation (13) used by these “true-amplitude” algorithms coincide with Hale's phase factor; each differs from the original Hale's algorithm only in the value assumed for J_T . Both require a Jacobian based on $\frac{\partial(t_0, y_0)}{\partial(t_n, y_n)}$ rather than Hale's more simple Jacobian, $\frac{\partial t_0}{\partial t_n}$. Bleistein (1990) assumes that (1) the input data have not been corrected for spherical divergence, (2) NMO is done by resampling without any scale factors, and (3) the spectral density of the input wavelet is preserved. The corresponding value for J_T in Bleistein's algorithm is

$$J_T = A^{-1}(2A^2 - 1). \quad (17)$$

Black et al. (1993) assume that (1) the input data have been corrected for spherical divergence, (2) NMO is done by resampling without any scale factors, and (3) the peak amplitude of the input wavelet is preserved. This leads to the following value for J_T .

$$J_T = A^{-3}(2A^2 - 1). \quad (18)$$

In essence, both “true-amplitude” Jacobians correct a small amplitude error on the steeper dips associated with Hale’s Jacobian for a chosen sequence of data processing steps.

In anisotropic media, the specular reflection point moves differently than in isotropic media, and we do not account for this movement in our algorithm. However, for conventional spreads (close to the CMP-reflector distance or smaller) we do not expect this simplification to lead to noticeable distortions.

DMO for general anisotropy

Here, we modify Hale’s (1984) DMO operator using the anisotropic NMO equation given in Paper I. The model considered in Paper I consists of a plane dipping reflector beneath a homogeneous anisotropic medium (Figure 1). Anisotropy is not restricted to any specific type; however, the incidence (sagittal) plane is supposed to be a plane of symmetry. For transversely isotropic media (the most common anisotropic model), the incidence plane should contain the symmetry axis. If the medium is orthorhombic, the incidence plane is assumed to represent one of the mutually orthogonal symmetry planes.

If the symmetry assumption is satisfied, the normal-moveout velocity on a CMP line perpendicular to the strike of the reflector is given by (Paper I)

$$V_{\text{nmo}}(\phi) = \frac{V(\phi)}{\cos \phi} \frac{\sqrt{1 + \frac{1}{V(\phi)} \frac{d^2 V}{d\theta^2}}}{1 - \frac{\tan \phi}{V(\phi)} \frac{dV}{d\theta}}, \quad (19)$$

where $V(\theta)$ is phase velocity as a function of phase angle, and the derivatives are evaluated at the dip angle ϕ .

Relation (2) between the dip ϕ and ray parameter p on the zero-offset section continues to hold in anisotropic media (Paper I):

$$p(\phi) = \frac{1}{2} \frac{dt_0}{dy_0} = \frac{\sin \phi}{V(\phi)} = \frac{k}{2\omega}. \quad (20)$$

The replacement of the angle ϕ by the ray parameter p for a known anisotropic model can be done in a straightforward fashion. The phase angle ϕ and phase velocity $V(\phi)$ corresponding to a given value of p can be obtained from the Christoffel equation and used in formula (19) (Alkhalifah and Tsvankin, 1995).

While equation (19) is valid for all wave types (except for mode conversions), we will consider DMO processing for only quasi- P -waves (in the following, we omit the qualifiers in “quasi- P -wave” and “quasi- S -wave” for brevity). In order to generalize Hale’s DMO for anisotropic media, we need to separate the term associated with dipping reflectors in the anisotropic NMO equation. Although P -wave reflection moveout even in homogeneous anisotropic media is generally nonhyperbolic (Tsvankin and Thomsen, 1994), it can still be approximated by a hyperbola on spreads with length comparable to the distance between the CMP and the reflector (Paper I). Hence, we start with

$$t^2 = t_0^2 + \frac{4h^2}{V_{\text{nmo}}^2(p)}, \quad (21)$$

with the NMO velocity given by equation (19). Expression (19) is too complicated to be separated analytically into the zero-dip velocity and the DMO correction factor, but we can accomplish this separation artificially by rewriting equation (21) in the same form as equation (6).

$$t_0^2 = t_n^2 + 4h^2 \left[\frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)} \right]. \quad (22)$$

If the medium were isotropic, equation (22) would reduce to the conventional NMO formula (8).

Therefore, the zero-offset section in anisotropic media can be produced by an NMO-DMO sequence similar to the one used in conventional processing. The time t_n is a result of the conventional NMO procedure that eliminates the zero-dip (NMO) term $4h^2/V_{\text{nmo}}^2(0)$. The influence of anisotropy in the normal-moveout correction is hidden in the value of $V_{\text{nmo}}(0)$. The only possible complication at this stage

is anisotropy-induced deviations from hyperbolic moveout. However, as mentioned above, P -wave moveout is sufficiently close to hyperbolic on conventional spreads where the source-receiver offset is less than or equal to the target depth.

In order to transform the NMO-corrected data to zero offset, the anisotropic DMO operator should compensate for the remaining moveout term $4h^2 D(p)$ in equation (8), with the factor $D(p)$ given by

$$D(p) = \frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)}. \quad (23)$$

The main difference between the anisotropic DMO factor $D(p)$ and its isotropic counterpart p^2 [equation (22)] is that $D(p)$ depends on the parameters of the anisotropic velocity field. Therefore, DMO processing should be preceded by an inversion procedure designed to reconstruct the NMO velocity as a function of ray parameter. Clearly, such an inversion is difficult to accomplish for general anisotropy. However, for transverse isotropy with a vertical symmetry axis (VTI), P -wave surface data alone may provide enough information to build the NMO velocity function (Alkhalifah and Tsvankin, 1995). The method developed by Alkhalifah and Tsvankin (1995) is used in our implementation of Hale's DMO in transversely isotropic media.

Assuming that the functions $V_{\text{nmo}}(p)$ and $D(p)$ have been recovered, the rest of Hale's DMO algorithm remains essentially unchanged. The DMO factor $D(p)$ should be substituted into the equation for A (14), yielding the expression valid for anisotropic media:

$$A = \sqrt{1 + \frac{4h^2}{t_n^2} D(p)}. \quad (24)$$

Equation (24) is then used in the integral transforms (13) and (10) designed to produce the zero-offset section from the NMO-corrected, constant-offset sections. This means that the entire anisotropic NMO-DMO sequence takes no more computing time than does Hale's method for isotropic media. The only extra step to be included due to the presence of anisotropy is the inversion procedure mentioned above.

Our algorithm ignores anisotropy-induced nonhyperbolic moveout, which seems to be a shortcoming compared to isotropic DMO methods. Although isotropic constant-velocity DMO methods are also based on the hyperbolic moveout equation, they are supposed to work at large offsets because reflection moveout in a homogeneous isotropic medium is purely hyperbolic. However, the homogeneous isotropic model is an idealization of realistic media, which are, at a minimum, vertically inhomogeneous. Any kind of inhomogeneity leads to nonhyperbolic moveout on long spreads that cannot be properly handled by conventional NMO and DMO algorithms.

Also, the results of Paper I and numerical examples discussed below suggest that deviations from hyperbolic moveout in homogeneous anisotropic media usually become less pronounced with dip, at least for transverse isotropy. Therefore, nonhyperbolic moveout can be expected to be more of a problem in the NMO correction for horizontal reflectors than in dip-moveout removal. Also, large offsets, at which deviations from a hyperbola are significant, are often muted out because of NMO stretch. If there is a need to preserve traces at large offsets, DMO processing should be performed by more accurate (but more costly) ray-tracing techniques.

Transversely isotropic media

While NMO equation (19) can be used in symmetry planes of any anisotropic medium, the main practical difficulty in the implementation of our DMO algorithm in complicated anisotropic models is the recovery of the factor $D(p)$ from seismic data. Therefore, in the following we concentrate on the most common anisotropic model, transverse isotropy with a vertical symmetry axis (VTI).

We describe VTI media by four Thomsen (1986) parameters: the vertical velocities V_{P0} and V_{S0} of P - and S -waves respectively and three dimensionless anisotropic coefficients ϵ , δ and γ . It should be emphasized that although the original Thomsen (1986) paper was devoted to weak transverse isotropy, the parameters ϵ , δ and

γ are convenient to use in TI media with arbitrary strength of velocity anisotropy (Tsvankin and Thomsen, 1994; Tsvankin, 1995b). In our DMO algorithm we use the exact equations for transverse isotropy without applying the weak-anisotropy approximation.

Although $P - SV$ propagation depends on four coefficients (V_{P0} , V_{S0} , ϵ , and δ), Tsvankin (1995b) demonstrates that P -wave velocities and reflection traveltimes are practically independent of the shear-wave vertical velocity V_{S0} , even for strong anisotropy. For practical purposes of dip-moveout processing, P -wave NMO velocity can be considered a function of just three parameters: V_{P0} , ϵ , and δ (Paper I).

Moreover, Alkhalifah and Tsvankin (1995) demonstrate that P -wave NMO velocity in VTI media depends on only *two* combinations of these three parameters: the zero-dip NMO velocity $V_{\text{nmo}}(0)$ and an effective parameter, denoted as η .

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta} = \frac{1}{2} \left[\frac{V_p^2(90)}{V_{\text{nmo}}^2(0)} - 1 \right], \quad (25)$$

where $V_p(90)$ is the horizontal P -wave velocity.

The velocity-analysis technique by Alkhalifah and Tsvankin (1995) is designed to recover the parameters $V_{\text{nmo}}(0)$ and η using P -wave NMO velocities from two different dips. If one of the reflectors is horizontal (the most common case), then $V_{\text{nmo}}(0)$ is obtained directly by conventional velocity (e.g., semblance) analysis. The NMO velocity for the dipping reflector is then used to determine η and build the normal-moveout velocity as a function of the ray parameter.

It is convenient to list the relationships between the anisotropic parameters and velocities in the following form as a memory aid:

$$V_{\text{nmo}}(0) = V_{P0} \sqrt{1 + 2\delta}, \quad (26)$$

$$V_P(90) = V_{P0} \sqrt{1 + 2\epsilon} = V_{\text{nmo}}(0) \sqrt{1 + 2\eta}. \quad (27)$$

To understand the performance of the anisotropic DMO algorithm, we have to examine the dependence of the factor $D(p)$ [equation (23)] on the parameters of the

anisotropic velocity field. First, we consider elliptically anisotropic models, in which wavefronts and slowness surfaces are elliptical for P -waves and spherical for SV -waves. Elliptical anisotropy is a special (and uncommon) case of transverse isotropy that occurs if the parameters ϵ and δ are equal to each other.

P -wave phase velocity in elliptically anisotropic media ($\epsilon = \delta$, $\eta = 0$, and $V_P(90) = V_{\text{nmo}}(0)$) is given by

$$V_P(\theta) = V_{P0} \sqrt{1 + 2\delta \sin^2 \theta}. \quad (28)$$

The NMO velocity (19) as a function of ray parameter then becomes (Alkhalifah and Tsvankin, 1995)

$$V_{\text{nmo}}(p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}}. \quad (29)$$

Equation (29) shows that, for elliptical anisotropy, P -wave NMO velocity is the same function of the ray parameter and zero-dip moveout velocity as in isotropic media (3), with no explicit dependence on the coefficient $\epsilon = \delta$. The difference between the isotropic and elliptically anisotropic expressions for $V_{\text{nmo}}(p)$ is only in the value of the zero-dip velocity $V_{\text{nmo}}(0)$ [equation (26)], which is equal to the horizontal velocity (but is different from the vertical velocity) if the medium is elliptically anisotropic.

Substituting formula (29) into the expression for $D(p)$ (23), we obtain

$$D(p) = p^2, \quad (30)$$

which coincides with the DMO factor in Hale's isotropic algorithm.

We conclude that constant-velocity Hale's DMO is perfectly suitable not only for isotropic, but also for elliptically anisotropic media. This result is not intuitively clear because the NMO velocity as a function of dip in elliptically anisotropic media does deviate from the cosine-of-dip dependence (Paper I).

Since reflection moveout for elliptical anisotropy is purely hyperbolic, the entire conventional NMO-DMO sequence should encounter no problems in elliptical models.

Furthermore, isotropic poststack time migration remains entirely valid for elliptical anisotropy (Alkhalifah and Tsvankin, 1995). The difference between isotropy and elliptical anisotropy becomes important only in time-to-depth conversion or poststack depth migration of the zero-offset section. Since for elliptical anisotropy the zero-dip NMO velocity, conventionally used in time-to-depth conversion or migration, does not coincide with the true phase or group velocity in any direction except for horizontal, isotropic poststack depth migration will result in the wrong depth of reflectors, including horizontal ones (Alkhalifah and Larner, 1994).

While the elliptical assumption leads to a considerable simplification in the analysis of all aspects of wave propagation, elliptical anisotropy is hardly typical for subsurface formations (Thomsen, 1986). Next, we consider a general transversely isotropic model with no fixed relation between ϵ and δ .

The most convenient way to get analytic insight into the dependence of $D(p)$ on the anisotropic coefficients is to use the weak-anisotropy approximation ($\epsilon \ll 1$, $\delta \ll 1$). The P -wave phase velocity, linearized in the parameters ϵ and δ , is given by (Thomsen, 1986)

$$V_P(\theta) = V_{P0} (1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta). \quad (31)$$

Substitution of equation (31) into the NMO formula (19) and replacement of the phase angle by ray parameter yields (Alkhalifah and Tsvankin, 1995)

$$V_{\text{nmo}}(p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}} [1 + (\epsilon - \delta) f(p V_{\text{nmo}}(0))], \quad (32)$$

$$f \equiv \frac{y(4y^2 - 9y + 6)}{1 - y}; \quad y \equiv p^2 V_{\text{nmo}}^2(0).$$

The weak-anisotropy approximation (32) reduces to the exact NMO formula for elliptically anisotropic models [$\epsilon = \delta$, equation (29)] but, clearly, can deviate from the exact result for non-elliptical media.

The DMO correction factor $D(p)$ (23) for weak transverse isotropy becomes

$$D(p) = p^2 [1 + 2(\epsilon - \delta) (4y^2 - 9y + 6)]. \quad (33)$$

Note that the anisotropic correction to the DMO operator in the weak-anisotropy approximation contains only the difference $\epsilon - \delta$, with no separate dependence on either of the coefficients. The predominant influence of $\epsilon - \delta$ on the dip-dependence of P -wave NMO velocity was shown in Paper I. It is interesting, however, that the weak-anisotropy NMO formula derived in Paper I as a function of the dip angle did contain a separate contribution of the parameter δ . As demonstrated by equations (32) and (33), this contribution is absorbed by the ray parameter p .

The numerical analysis by Alkhalifah and Tsvankin (1995) shows that the structure of the factor $D(p)$ (23) for transverse isotropy with arbitrary strength of velocity anisotropy is similar to that in equation (33), with the influence of the anisotropic coefficients ϵ and δ represented by the effective parameter η . Clearly, η [equation (25)] reduces to $\epsilon - \delta$ in the limit of weak anisotropy.

Although it might be dangerous to use the weak-anisotropy approximation for quantitative estimates, it is clear from equation (33) that the anisotropy makes a significant contribution to the DMO factor. For instance, for $y = 0.25$, which roughly corresponds to a dip of 30 degrees, the anisotropic term $2(\epsilon - \delta) (4y^2 - 9y + 6)$ becomes $8(\epsilon - \delta)$, which means that the isotropic and anisotropic parts of the DMO factor are comparable even for relatively small $(\epsilon - \delta)$. This conclusion is in good agreement with results of Paper I.

It is also important that the influence of the anisotropy cannot be ignored even at mild dips (small y); in fact, the relative magnitude of the anisotropic term *decreases* with dip. Of course, the contribution of the dip term as a whole at mild dips is relatively small. Our analysis also shows that, although the anisotropic DMO operator does depend on the zero-dip NMO velocity through $y \equiv p^2 V_{\text{nmo}}^2(0)$, the influence of errors in $V_{\text{nmo}}(0)$ becomes pronounced only for large p corresponding to steep events.

NUMERICAL IMPLEMENTATION

The DMO technique for VTI media discussed above has been implemented using a variant of the conventional Hale F-K DMO algorithm. First, $V_{\text{nmo}}(\phi)$ and $p = \sin \phi / V(\phi)$ for each homogenous interval are tabulated from 0 to 89 degrees in constant increments of ϕ using equation (19). The input parameters to be determined beforehand are the NMO velocity for a horizontal reflector $V_{\text{nmo}}(0)$ and the coefficient η [equation (25)]. We use the exact equations for NMO and phase velocity for transverse isotropy, so there are no restrictions requiring weak anisotropy in the generation of this table. The table is then interpolated back to a new table in constant increments in p . Alternatively, the function $V_{\text{nmo}}(p)$ can be built directly using phase-velocity equations for transverse isotropy. Based upon this table, $V_{\text{nmo}}(p)$ and $D(p)$ can be readily obtained.

Input NMO-corrected common-offset data are Fourier-transformed over the mid-point axis providing complex-valued traces as a function of wavenumber. The time integral over t_n is computed numerically, yielding the zero-offset section in the F-K ($\omega - k$) domain [equation (13)].

$$P_0(\omega, k, h) = \int dt_n \frac{P_n(t_n, k, h)}{A} e^{i\omega t_n A}, \quad (34)$$

where $p = \frac{k}{2\omega}$, and

$$A(t_n, k, \omega) = \sqrt{1 + \frac{4h^2}{t_n^2} D(p)} = \sqrt{1 + \frac{4h^2}{t_n^2} \left[\frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)} \right]}.$$

Finally, we apply a two-dimensional inverse Fourier transform to produce the output DMO-corrected, common-offset gather in the $t - x$ domain.

The direct numerical integration over t_n [equation (34)], currently implemented in our code, is the most time-consuming part of the algorithm. In the future, we plan to study the possibility of reducing the integral (34) to a Fourier transform by the log-stretch method (Notfors and Godfrey, 1987; Liner, 1990), which is widely

used to speed up application of Hale DMO in isotropic media. The log-stretch temporal relationships, which are exact for homogeneous isotropic media, become only approximate for homogeneous transversely isotropic media.

SYNTHETIC EXAMPLES

Figures 2-10 demonstrate basic aspects of the transversely isotropic DMO operator compared with its isotropic counterpart. As expected, the TI DMO algorithm produces results (Figure 3) identical to conventional Hale DMO (Figure 2) if the medium is isotropic. In agreement with the analytic results discussed above, the transversely isotropic operator reduces to the conventional Hale's operator not only for isotropic but also for elliptically anisotropic media ($\epsilon = \delta$, Figure 4). Although velocity anisotropy for the model in Figure 4 is quite pronounced ($\epsilon = \delta = 0.2$), it has no influence on the kinematics of the DMO process. The only difference between the two DMO operators is that in the Hale algorithm the NMO velocity is generated analytically while the VTI DMO response uses the values of V_{nmo} from table lookup. As discussed above, our VTI DMO code is currently based on Hale's Jacobian. Bleistein's "true amplitude" operator (Figure 5) has the same shape as Hale's operator but leads to higher amplitudes for the larger dips.

However, even relatively small deviations from elliptical anisotropy have strong influence on the DMO response. It is believed that transversely isotropic subsurface formations, such as shales, typically have positive values of $\epsilon - \delta$ (Tsvankin, 1995b). Figure 6 demonstrates the broadening of the DMO ellipse that occurs for a transversely isotropic model with large positive $\epsilon - \delta$. For small negative values of $\epsilon - \delta$, the DMO response becomes more narrow as shown in Figure 7. Figure 8 shows that for more strongly negative values of $\epsilon - \delta$, the DMO response curves downward with a shape reminiscent of the isotropic inverse DMO operator.

The weak-anisotropy approximation was used to compute the impulse response

in Figure 10 for the same anisotropic parameters as in Figure 9. For this model with moderate values of ϵ and δ , the DMO responses are similar whether or not the weak-anisotropy equation is used. However, the accuracy of the weak-anisotropy approximation decreases for larger values of $\epsilon - \delta$, and we use the exact VTI operator in our routine numerical work.

The next example shows the performance of the VTI DMO algorithm in a homogeneous VTI fan model with four reflectors and dips that range up to 75 degrees (Figure 11). The Alkhalifah and Tsvankin model parameters are $V_{\text{nmo}}(0)=3.15$ km/s and $\eta=0.14$; the Thomsen parameters are $V_{P0}=3$ km/s, $\epsilon=0.2$, and $\delta=0.05$. Synthetic common-offset data for offsets ranging from 0 to 3 km in offset increments of 0.1 km were generated by a ray-tracing code. Figure 12 shows the zero-offset section which may be considered as the ideal output for NMO followed by DMO. The vertical line at midpoint location 2.75 km marks where CMP gathers were selected for the comparison of isotropic DMO versus VTI DMO. The key model events on the CMP gathers may be identified with the same events on this zero-offset section.

The NMO-corrected common-offset data were processed once by the conventional isotropic Hale's DMO method (Figure 13), and a second time by our new VTI DMO algorithm (Figure 14). In Figure 14 after VTI DMO, the events out to a fairly large offset are well-aligned, irrespective of dip, allowing CMP stack to sum without distortion. In contrast, after application of isotropic DMO in Figure 13, all dipping events remain misaligned. It should be emphasized that the synthetic data were generated by a ray-tracing algorithm that is not related to the analytic formula used in our anisotropic DMO correction.

Figure 14 shows that the horizontal events still have significant nonhyperbolic moveout and will require muting on the longer offsets prior to the stack. The dipping events also have nonhyperbolic moveout, but to a far lesser degree than the horizontal events. If it is necessary to include uncommonly long offsets in the stack, an NMO/DMO sequence based on anisotropic ray tracing will be necessary to correct

for nonhyperbolic moveout. It should be noted that this example exaggerates the influence of nonhyperbolic moveout because standard processing rarely includes offsets out to 3 km at times less than 1.5 s.

DISCUSSION AND CONCLUSIONS

Hale's DMO method has been extended to anisotropic media using the analytic expression for NMO velocity given by Tsvankin (1994) (Paper I). For elliptical anisotropy ($\epsilon = \delta$), the isotropic DMO algorithm remains entirely valid, although isotropic poststack depth migration based on the zero-dip moveout velocity may result in depth errors. However, elliptical anisotropy is just a special kind of transverse isotropy that cannot be considered as typical for subsurface formations, and even relatively small deviations from the elliptical model may result in significant DMO errors.

The anisotropic correction to the DMO operator for transversely isotropic media with a vertical symmetry axis is determined by two effective parameters: the NMO velocity for reflections from a horizontal interface (obtained by conventional velocity analysis) and the anisotropic coefficient denoted by Alkhalifah and Tsvankin (1995) as η [$\eta \equiv (\epsilon - \delta)/(1 + 2\delta)$]. As shown by Alkhalifah and Tsvankin, the parameter η can be recovered in a stable fashion from the P -wave NMO velocity for a single dipping reflector. After the inversion step has been completed, an analytic moveout equation (Paper I) is used to tabulate the NMO velocity as a function of the ray parameter. Then we carry out the transformation of constant-offset sections to zero offset essentially in the same fashion as in isotropic media. Except for the inversion procedure, the entire NMO-DMO sequence takes no more computing time than NMO-DMO in isotropic media.

Alkhalifah and Tsvankin (1995) have proved that knowledge of η and $V_{\text{nmo}}(0)$ is sufficient not only for DMO, but for all time-related processing methods including

prestack and poststack migration. However, accurate time-to-depth conversion requires the individual values of the vertical velocity V_{P0} and anisotropic parameters ϵ and δ .

Tests of the algorithm on synthetic data for transversely isotropic media show that it efficiently flattens P -wave moveout on conventional spreads, independent of dip. Two interesting points are clarified by the examples. First, reflections from steep interfaces are less influenced by nonhyperbolic moveout and appear more flat after DMO than are those from horizontal reflectors. This conclusion is in good agreement with the results reported by Tsvankin (1995a). Therefore, we expect nonhyperbolic moveout (ignored in our algorithm) to be more of a problem in normal-moveout correction than in DMO. Second, the DMO response of this algorithm in strongly anisotropic media is not restricted to the region between the source and the receiver.

Application of the table lookup makes our algorithm no slower than the generic Hale DMO method in isotropic media. However, direct numerical integration over the NMO-corrected time (also implemented in our code) makes Hale DMO relatively slow. The conventional way to speed up the Hale's isotropic DMO algorithm is to use the log-stretch method (Notfors and Godfrey, 1987; Liner, 1990); however, generalization of this method to anisotropic media is not straightforward as only the kinematics of the small-offset near-elliptical portion of the DMO impulse response can be preserved. The variations of amplitude and triplications seen along the anisotropic DMO response can not be fully duplicated by straightforward stretching and squeezing in the log-stretch domain. In the future, we will explore the possibility of applying the log-stretch method in VTI media to increase the computational efficiency of this new DMO technique while simultaneously attempting to estimate the size of the associated errors and the range of validity.

The algorithm can be upgraded to perform $V(z)$ DMO in transversely isotropic media by replacing the single-layer NMO formula with a more general equation for layered media developed by Alkhalifah and Tsvankin (1995). However, since our

“homogeneous” VTI DMO operator has the degree of freedom provided by the parameter η , in many cases it can be adapted for $V(z)$ media by applying an effective value of η that absorbs the influence of both the anisotropy and inhomogeneity. Also, our anisotropic DMO operator can be applied in a straightforward way in symmetry planes of more complicated models such as orthorhombic. The main problem in DMO processing for anisotropic models with a large number of independent parameters is the reconstruction of the dip-dependence of NMO velocity from seismic data.

ACKNOWLEDGMENTS

We are grateful to Ken Lerner (CSM), Tariq Alkhalifah (CSM), and Denis Schmitt (Mobil) for helpful discussions and suggestions. This work was performed during J. Anderson’s tenure as Mobil Visiting Scientist in the Department of Geophysics at the Colorado School of Mines. I. Tsvankin acknowledges the support provided by the members of the Consortium Project on Seismic Inverse Methods for Complex Structures at the Center for Wave Phenomena, Colorado School of Mines, and by the United States Department of Energy, Grant Number DE-FG02-89ER14079 (this support does not constitute an endorsement by DOE of the views expressed in this paper).

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FIGURES

FIG. 1. Common-midpoint gather over a homogeneous anisotropic medium. $\vec{V}_{gr}$ and $\vec{V}_{ph}$ are the group- and phase-velocity vectors, respectively. The phase-velocity vector corresponding to the zero-offset (“normal-incidence”) ray is perpendicular to the reflector.

FIG. 2. Isotropic Hale DMO impulse-response curves on a common-offset gather. Here, and for Figures 3-10 offset equals 400 m and the trace spacing is 25 m.

FIG. 3. Isotropic Tsvankin (VTI) DMO impulse-response curves on a common-offset gather. The VTI DMO response is identical to the Hale response for isotropic models.

FIG. 4. VTI DMO impulse-response curves on a common-offset gather for an elliptically anisotropic medium with $\delta = \epsilon = 0.2$ ($\eta=0$). The elliptically anisotropic DMO operator is kinematically identical to the isotropic DMO operator for any strength of the anisotropy.

FIG. 5. Isotropic Bleistein DMO impulse-response curves on a common-offset gather. The Bleistein “true-amplitude” DMO response has the same kinematics but stronger amplitudes at larger dips.

FIG. 6. VTI DMO impulse-response curves on a common-offset gather for $\epsilon = 0.4$ and $\delta = 0.0$ ($\eta=0.4$). Relative to the operator for isotropic DMO, the anisotropic DMO operator is more broad for positive values of η and may have triplications.

FIG. 7. VTI DMO impulse-response curves on a common-offset gather for $\epsilon = 0.0$ and $\delta = 0.1$ ($\eta = -0.08$). The operator becomes more narrow for small negative values of η .

FIG. 8. VTI DMO impulse-response curves on a common-offset gather for $\epsilon = 0.0$ and $\delta = 0.2$ ($\eta = -0.14$). For larger negative values of η , the operator can curve downward rather than upward.

FIG. 9. VTI DMO impulse-response curves on a common-offset gather for $\epsilon = 0.2$ and $\delta = 0.1$ ($\eta=0.083$).

FIG. 10. VTI DMO impulse-response curves for the model from Figure 9, but this time based on the weak-anisotropy approximation [equation (33)].

FIG. 11. VTI model used for generating synthetic data.

FIG. 12. Zero-offset synthetic data generated by ray tracing. The vertical line at 2.75 km marks the common-midpoint location of gathers used for comparing isotropic DMO versus VTI DMO.

FIG. 13. CMP gather after NMO and isotropic DMO.

FIG. 14. CMP gather after NMO and VTI DMO.

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