1 of 2

CHARACTERIZATION OF NON-DARCY MULTIPHASE FLOW
IN PETROLEUM BEARING FORMATION

Final Report

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ABSTRACT

The productive capacity of oil and gas bearing rocks depends on various parameters characterizing the flow conditions in the reservoir. Among these, the non-Darcy flow coefficient specifically plays an important role for cases involving fluid accelerations or decelerations around the well bore and in the reservoir. However, most reservoir simulators used for reservoir management assume Darcy flow, and yield misleading results causing an incorrect analysis or projection of reservoir performance. A few attempts have been made to incorporate non-Darcy effect in reservoir models but many of these lack a reliable accuracy since they use simplified correlations which ignore the effects of the variation of the fluid and formation conditions.

The present study developed an accurate non-Darcy flow model that will lead to more accurate reservoir management decisions. First, a rigorous analysis and derivation of the porous media mass and momentum equations are presented considering the non-Darcy flow behavior. Second, steady-state and unsteady-state methods for simultaneous determination of relative permeability, capillary pressure, and interfacial drag during non-Darcy flow in laboratory cores are derived. This work results in several algebraic, integral, and differential interpretation methods. Third, correlations for the non-Darcy flow coefficient are investigated and improved. The study presented in this report provides new insights and formulations in the description of non-Darcy flow in oil and gas bearing formations.

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EXECUTIVE SUMMARY

1.1 Introduction

Slow flow is most common in oil and gas reservoirs and can be adequately described by Darcy's law. Although rapid fluid flow may occur only in certain limited locations, it can affect the productive capacity of in-situ reservoirs significantly. Therefore, for accurate predictability of reservoir productivity, accurate description of deviations from Darcy's law during rapid flow is important.

Although rapid flow in porous media, frequently referred to as non-Darcy flow, has been subject to numerous studies, there is still no consensus amongst the researchers as to the accurate way of describing the relevant processes. Therefore, this study has reviewed the previous studies and developed improved formulations and methodologies.

In the present study non-Darcy multiphase flow in porous materials is theoretically and experimentally investigated. Background material is presented which demonstrates a need for obtaining new theoretical and experimental results in order to better characterize this important porous media flow regime. Improved models, experimental data, and mathematical correlations are necessary for scientists and engineers to develop the technology that can be used to significantly reduce the cost of finding and producing natural gas and its associated liquids. Non-Darcy multiphase flow in various consolidated porous media and unconsolidated porous media characteristic of hydraulically created propped fractures and gravel pack systems is also investigated. The experimental research is carried out in the new non-Darcy Flow Research Laboratory recently constructed at the University of Oklahoma.

This work results in an improved correlation of the inertial flow coefficient and several algebraic, integral, and differential interpretation methods for simultaneous determination of relative permeability and capillary pressure using steady and unsteady state laboratory core flow data.

1.2 Objectives of the Research Program

The objectives of this research were to:

1. Develop a proper theoretical model for characterizing non-Darcy multiphase flow in petroleum bearing formations.
2. Develop an experimental technique for measuring non-Darcy flow coefficients under multiphase flow at in-situ reservoir conditions.
3. Develop dimensional consistent correlations to express the non-Darcy flow coefficient as a function of rock and fluid properties for consolidated and unconsolidated porous media.

1.3 Summary of Accomplishments

A summary of the work carried out during this three-year project are presented as follows.

1.3.1 Porous Media Mass and Momentum Equations

In this study, a local volume averaging approach has been carried out to derive the porous media mass and momentum equations rigorously from the basic principles. This exercise revealed that the currently used forms of the porous media mass and momentum equations omit several terms that may be of importance for multiphase fluid systems involving non-Darcy flow regimes. These new equations provide more accurate analysis of non-Darcy flow parameters because the additional terms resulting from rigorous local volume averaging provide more detailed information about the non-Darcy multiphase flow and the dependency of its parameters on porous structure and flow patterns. Therefore, a dimensional analysis between these additional features generates new dimensionless groups used to develop new correlations that may provide better predictions.

1.3.2. Extension of the JBN Method for Non-Darcy Flow

The ever popular JBN method has many limitations due to the inherent simplifying assumptions, including incompressible and constant viscosity fluids, and negligible interfacial drag, inertial flow, and capillary end-effect. Therefore, it is not applicable to non-Darcy flow associated with the expanding/compressing, accelerating/decelerating gases along with brine and oil. This study has focused on derivations of rigorous expressions to remove these limitations.

1.3.3. Correlation for the Inertia Flow Coefficient

Previous correlations such as Geertzma⁵¹, and Tek, et al¹⁰⁰ are oversimplified since these omit many variables pertinent to non-Darcy flow at reservoir conditions. The studies presented in this project have determined those pertinent variables and the new dimensionless groups between them in addition to a few used in the previous studies. The new dimensionless groups have been correlated with experimental data from various sources to develop a generalized correlation for the inertial flow coefficient.

1.3.4 Indirect Methods for Determination of Relative Permeability and Capillary Pressure from Laboratory Displacement Data

Recent efforts by various researchers^{80,86} indicate that "simulated annealing" is a more practical optimization method than the conventional methods, including the Levenberg-Marquardt method⁶⁸, which has been popular among the researchers dealing with relative permeability. Therefore, the simulated annealing method has been implemented to carry out a history matching of the results of laboratory core displacement data for non-Darcy flow to determine the best estimates of the non-Darcy flow parameters. As a result, the difficulties associated with the previous tedious and computationally intensive parameter estimation methods are conveniently alleviated. This approach is implemented for both the steady- and unsteady-state methods.

1.3.5 Direct Methods for Determination of Relative Permeability and Capillary Pressure from Laboratory Displacement Data

By and large the previous methods for determination of relative permeability and capillary pressure have relied upon the history matching method using a least squares optimization approach. This is an iterative method which demands extensive computational effort. Another shortcoming of this method is that it may not converge or it may not yield a unique set of values for the parameters. In addition, it may be subject to numerical instability and numerical conditioning problems. Therefore, a direct analytical method which conveniently avoids these problems has been developed. An additional bonus is the significant accuracy and uniqueness with minimal computational effort by direct calculations. The methods based on history matching use forward numerical solutions of differential equations to model the multi-phase flow problem. Therefore, such methods suffer from numerical instability, dispersion, oscillation, and grid point orientation problems. Whereas, the direct method developed herein is algebraic and does not use any numerical scheme. It does not involve possible errors arising from the use of numerical errors. Therefore, the direct method developed in the present study is an accurate method. This approach is implemented for both the steady- and unsteady-state methods.

1.3.6 Pore Level Modeling

Starting from the pore scale, flow equations are presented under different flow arrangements and flow regimes. An analytical solution of simultaneous dispersed bubble or spray flow of gas/liquid system in the pore throat is presented. Methodology of calculating the effective flow properties of pore network is described by using effective medium theory (E.M.T.). A procedure for calculating the Darcy and non-Darcy flow properties by comparing the effective flow properties of the network with macroscopic flow properties is presented. Finally, permeability and non-Darcy flow of a simple pore network is calculated and the shortcomings of the present methodology are discussed.

1.4 Conclusions

The present study leads to new insights into the mathematical description of the non-Darcy flow behavior in porous media. This work results in several instructive conclusions, as well as ideas and new directions for future research.

2. INTRODUCTION

Derivation of the conservation laws for describing the flow behavior in porous media has occupied many researchers because of a wide range of applications in engineering and the natural science. In spite of the many efforts reported in the literature, there appears no consensus among the multitude of researchers. Although the semiempirical equations presented by Darcy²⁶ for low velocity flow and by Forchheimer⁴⁹ for rapid flow have been popular, many alternative equations have also been proposed for porous media momentum conservation. Interestingly, the general derivation of the porous media continuity equation has been overlooked and is emphasized in the present study.

2.1 Modeling of Non-Darcy Flow in Porous Media

Darcy's law²⁶ given below has been widely accepted to hold satisfactorily for low velocity flow.

$$-dp/dx = (\mu/k)u \quad (2.1)$$

Forchheimer⁴⁹ modified Darcy's law for rapid flow as

$$-dp/dx = (\mu/k)u + \beta \rho u^n \quad (2.2)$$

for which a value of $n = 2$ was assumed following Muskat⁸³. Brinkman⁸ modified Darcy's law for the affect of containing walls for flow through confined porous media as

$$-\partial p/\partial x = (\mu/k)u - \mu \partial^2 u/\partial y^2 \quad (2.3)$$

where x denotes the flow direction and y the normal direction.

Vafai and Tien¹⁰³ proposed the following equation to include both Forchheimer's and Brinkman's corrections to Darcy's law:

$$-\partial p/\partial x = (\mu/k)u + \beta \rho u^2 - \mu \partial^2 u/\partial y^2 \quad (2.4)$$

Based on a capillary-orifice model Blick⁶ developed the following equation for rapid, unsteady flow through a constant porosity media:

$$-\partial p/\partial x = (\mu/k)u + \beta \rho u^2 + (\rho/\phi^2)u \partial u/\partial x + (\rho/\phi) \partial u/\partial t \quad (2.5)$$

Blick and Civan⁷ have shown that Forchheimer's equation is a good approximation to the full porous media momentum equation for rapid, unsteady flow.

Also, a large number of efforts have been made for the theoretical development of the porous media momentum equation by averaging the microscopic momentum equation. A review of these approaches are presented by Kaviany⁶⁷. As stated by de Boer and Ehlers²⁷ the averaging procedures are not clearly explained in many cases. Therefore, it has been difficult or impossible to judge the inherent limitations and/or validity of the various equations reported in the literature.

2.2 Correlation of Non-Darcy Flow Coefficients

Many research efforts related to the porous media momentum equation has been devoted to the development of empirical correlations for the parameters of the Darcy and Forchheimer equations. Klinkenberg⁷² pointed out that porous media permeability varies by pressure in the low pressure region according to

$$k = k_o \left(1 + \frac{b}{p} \right) \quad (2.6)$$

where k_o is the intrinsic permeability and b is an empirical constant. Blick and Civan⁷ pointed out that permeability must depend on both fluid and porous media conditions and properties.

Several researchers proposed various empirical correlations for the inertia resistance coefficient in Forchheimer's equation. Tek, et al¹⁰⁰ proposed

$$\beta[cm^{-1}] = \frac{3.21 \times 10^4}{[k, darcy]^{1.25} \phi^{0.75}} \quad (2.7)$$

Geertsma¹² proposed

$$\beta[cm^{-1}] = \frac{5.0 \times 10^1}{[k, darcy]^{0.5} \phi^{5.5}} \quad (2.8)$$

Ergun³³ has shown that k and β in Forchheimer's equation, Eq. 2.2, can be expressed, respectively, in darcy units as

$$k[darcy] = \frac{1}{180} \frac{\phi^3}{(1 - \phi)^2} d^2 [cm^2] \quad (2.9)$$

and

$$\beta[cm^{-1}] = \frac{1.75}{1.013 \times 10^6} \frac{(1 - \phi)}{\phi^3 d(cm)} \quad (2.10)$$

Thus, by eliminating d between Eqs. 2.9 and 2.10, we can obtain a new alternative expression as:

$$\beta[cm^{-1}] = \frac{1.43 \times 10^3}{[k, darcy]^{0.5} \phi^{1.5}} \quad (2.11)$$

Blick and Civan⁷ proposed:

$$\beta[cm^{-1}] = \frac{K_D}{8\sqrt{2}(k,darcy)^{0.5}\phi^{1.5}} \quad (2.12)$$

for which Blick and Civan suggested

$$K_D = 80/Re \text{ for } Re < 100 \quad (2.13)$$

$$K_D = 2.3 \text{ for } Re > 4,000 \quad (2.14)$$

However, no information is available in the $100 < Re < 4,000$ range. Note that the exponents of k and ϕ in Eq. 2.12 are exactly the same as that equation, Eq. 2.11, the present authors obtained using Ergun's equation.

Evans, et al^{36,37} modified Geertsma's correlation to account for the effect of an immobile liquid saturation, S_{wc} , as

$$\beta[cm^{-1}] = \frac{5.0 \times 10^1}{[k,darcy]^{0.5}\phi^{5.5}} \frac{1}{(1 - S_{wc})^{5.5}k_r^{0.5}} \quad (2.15)$$

k_r is the relative permeability of gas at immobile water saturation. For the effect of the overburden stress in the range of $1,000 < \sigma < 20,000$, Tiss and Evans¹⁰² proposed

$$\beta[cm^{-1}] = \frac{7.3 \log_{10}(\rho\sigma k/\mu) - 0.4324}{\phi(1 - S_{wc})k(cm^2)^{0.5}} \quad (2.16)$$

It is interesting to note that the predictions by the above correlations do not agree with each other. Because, as will be shown in the present study, first, Forchheimer's equation is a special equation which is only valid for certain flow conditions. In fact, Phipps and Khalil³⁹, and Kadi⁶⁵ stressed that the exponent "n" in Forchheimer's equation, Eq. 2.2, may deviate from 2 significantly. Second, to determine the β values, most researchers including Tek et al¹⁰⁰, Ahmed and Sunada², Green and Duwez⁵³, Geertsma⁵¹, and Firoczabadi, et al⁴⁸ relied upon the straight-line plots of the integral form of Forchheimer's equation over core length by treating β as being a constant value dependent solely on the porous media properties. Evans, et al^{36,37} showed it was a rock-fluid property. Although this may be approximately true, a close examination of the plots of experimental data presented by these authors reveals a peculiar, oscillatory behavior about the straight-lines common to all experimental data. Therefore, such oscillations may not be just noise but they must be reflecting some inadequacies in Forchheimer's equation. To the authors' knowledge, although it was not stated explicitly, Blick⁶ is the first to recognize such a feature, judging by the oscillating curve fit to data shown in Fig. 2.3 of his paper. All other authors attempted nonoscillating curve fits to experimental data by attributing such deviations to experimental errors. Third, in

addition, the β values determined by straight-line plots of Forchheimer's equation integrated over the core length must implicitly include the effect of the core length, although Firoozabadi, et al⁴⁸ dispute this point. In fact the study presented in Chapter 5 of this report indicates such a dependency.

Temeng and Horne¹⁰¹ concluded that lack of proportionality between pressure drop and flow rate does not imply a deviation from Darcy's law. This result is confirmed in the present study through a rigorous analysis by showing that nonlinearities can be incorporated into an apparent permeability concept. However, Temeng and Horne¹⁰¹ derived an expression for β as

$$\beta = \frac{\bar{c}L\mu^2}{2\rho k^2T} \quad (2.17)$$

which was proven invalid by Firoozabadi, et al⁴⁸.

2.3 The Present Studies

In the present study, new porous media continuity and momentum equations are derived by rigorous volume averaging of the microscopic conservation laws. The averaging rules are clearly stated for convenient evaluation of the procedure and the resulting equations. This exercise leads to several instructive conclusions and new directions for future research.

By-and-large, the type of experiments to be conducted and the data to be measured depend on the theory available to interpret the data. Previously reported methods for interpretation of immiscible fluids displacement data to generate relative permeability and capillary pressure curves relied upon external fluid data such as the effluent fluid rates and differential fluid pressure between the fluid influx and efflux. Examples of such methods are given by Johnson, Bossler, and Naumann⁶¹; Jones and Roszelle⁶⁴, Marle⁷⁹, and Civan and Donaldson¹⁹. Ramakrishnan and Cappiello⁹¹ have designed a special experimental method to guarantee that the capillary pressure is zero for the boundary condition at the core outlet so that a simplified theory can be used to predict only the nonwetting phase relative permeability. These methods allow for direct calculation of relative permeabilities and involve some limitations due to the inherent assumptions. There are also some indirect methods given by Kerig and Watson⁶⁸, and Ouenes et al⁸⁶ which rely on history matching methods. Indirect methods are prone to errors resulting from the numerical solution of the governing differential equations.

Recent research efforts appear to concentrate on developing methods to take advantage of the advancements in noninvasive techniques such as nuclear magnetic resonance imaging (NMRI) (Chen et al¹⁵), ultrasonic imaging (Chardaire-Riviere et al¹⁴), and x-ray tomography (Vinegar¹⁰⁵), to infer interior fluid data such as the fluid saturation distribution over the core length. However, most of these studies still rely upon the indirect method of history matching for interpretation, although such internal data are particularly suitable for direct interpretation as demonstrated in the present paper.

Enwere and Archer^{31,32} have shown that the saturation profiles dynamically measured by NMR imaging in conjunction with capillary pressure and relative permeability curves can be used to determine the capillary pressure gradients in a core. But the real objective is to determine capillary pressure and relative permeability from the saturation profiles.

In the present study two direct methods are presented for immiscible displacement of non-Darcy flow of variable density and viscosity fluids to determine the relative permeability and capillary pressure. These direct methods are more accurate and practical than the iterative history matching methods which involve numerical errors inherent in the numerical solution of the pertinent differential equations. The first method only uses the internal core data on saturation and saturation-weighted fluid pressure. The second method uses only the saturation-weighted fluid pressure and effluent fluid rates. Using saturation-weighted fluid pressure for analysis of core data is a new concept introduced in this study. For simplification purposes, the effects of fingering and fines migration, porous formation compressibility, and gas solubility are neglected. The properties of realistic cores are usually heterogeneous and anisotropic. Hence, the flow characteristics of fluids during flow through the cores are not uniform through the cross-sectional area of cores due to fingering and other pertinent phenomena. In addition, the fluids do not always move as continuous streams and the flow regime for a given phase may vary from Darcy to non-Darcy over a particular cross-sectional area. The measured flow rates at the core outlet end and the pressure and saturation profiles along the core bear the effects of the irregularities which occur inside the core during fluid displacement. Hence, the Darcy and non-Darcy relative permeabilities, the capillary pressure, and the interfacial drag calculated by the methods presented in this study are considered to be the apparent properties of the actual core represented with an ideal homogeneous and isotropic core.

In addition to the preceding work, studies have also been conducted on the development and implementation on the steady- and unsteady-state indirect methods for the purposes of comparison of these methods from the points of computational effort and accuracy. The steady-state flow description has resulted in two ordinary differential equations which were solved by the Runge-Kutta-Fehlberg⁴⁴ method. The unsteady-state method has resulted in two partial differential equations which were solved by an IMPES finite difference scheme. A simulated annealing method has been used to optimally match the model predictions to laboratory core flow measurements to determine the best estimates of the relative permeability and capillary pressure data. The applications of the methods have been illustrated by various fluid systems. A numerical example is presented to illustrate the application of the method developed.

3. NON-DARCY FLOW LABORATORY STUDIES

The following tasks have been completed in the Non-Darcy Flow Studies Laboratory:

3.1 Upgrading of the Non-Darcy Flow Apparatus

This earlier work included installing new fittings (some were heat damaged), pressure testing, and calibration. This work was necessary for the data acquisition phase which followed.

3.2 Acquisition of Data for the "Sensitivity of the Non-Darcy Flow Coefficient to Core Length Investigation"

We acquired non-Darcy flow data on the effects of varying the total flow path length of the core sample against the value of non-Darcy flow coefficient (and effective permeability) to determine a "characteristic value". Typical flow data are presented in Table 3.1.

3.3 The Non-Darcy Flow Data Acquisition/Control System Upgrade Phase

The schematic of the upgraded equipment design is shown in Fig. 3.1. Some of the features of the Data Acquisition/Control System are:

1. Real-time display of the Forchheimer plot (during data acquisition);
2. Automatic data writing to disc (during or after the test);
3. Test run times of less than five minutes (not including equipment setup time); and
4. Increased data accuracy.

3.4 Fabrication of the API Proppant Cell

The cell was designed and constructed to be used to examine the non-Darcy flow effect in propped hydraulic fracture (see Figure 3.2). The compressor, hydraulic fluid reservoir, and hydraulic press were all donated by Occidental Petroleum. The cell's features include:

- A. Complete compatibility with the (previously existing) non-Darcy flow apparatus (which utilizes a Hassler sleeve type coreholder), utilizing quick connects for rapid crossover. This system ties in with the automated (analogue) data acquisition system which was previously described in section 3.3 above.
- B. Direct determination of the dynamic fracture width during testing utilizing LVDT's, with automatic data acquisition.
- C. Direct determination of the applied load during testing utilizing a load cell, also with automatic (analogue) data acquisition.

E. Adaptation of the API Proppant Cell to high-rate gas flow:

1. A modified sealing system which includes a specialized O-ring system, and a highly polished metal sealing surface to accommodate sealing of a gas (the original system is for liquid flow only); and
2. A modified inlet-outlet flow port geometry to facilitate the extremely high (gas) flow-rate required to achieve non-Darcy flow (the original system was designed only for low (Darcy) flow-rates.

4. ESTIMATION OF THE NON-DARCY FLOW COEFFICIENT FOR FLUID FLOW THROUGH POROUS MEDIA - AN IMPROVED CORRELATION

4.1 Introduction

Darcy's law has long been used as a fundamental equation to describe fluid motion through porous media:

$$u = - \frac{k}{\mu} \frac{\partial P}{\partial L} \quad (4.1)$$

in which u represents superficial velocity of fluid, k is permeability, μ is fluid viscosity, P is pressure, and L is distance. Darcy's law represents a linear, empirical relationship between the pressure gradient and flow velocity. Many attempts have been made to theoretically derive Darcy's law from the Navier-Stokes equation and analogous to conduit flow. It has long been found that Darcy's law is valid only in the domain of low velocities. Flow deviated from Darcy's law at high flow velocities is referred to as non-Darcy flow. Forchheimer first proposed a nonlinear equation to describe additional pressure drop observed at high flow velocities by adding the term of velocity squared to the Darcy equation:

$$- \frac{\partial P}{\partial L} = \frac{\mu}{k} u + \beta \rho u^2 \quad (4.2)$$

in which β is the non-Darcy flow coefficient, and ρ is fluid density.

In the Forchheimer equation, the first term on the right-hand side given by Darcy's law represents the flow governed by viscous forces, whereas the second term introduced by Forchheimer denotes the flow governed by inertial forces. At low velocities the inertial term ($\beta \rho u^2$) is small and can be neglected. The Forchheimer equation (Eq. 4.2) is then reduced to the Darcy equation (Eq. 4.1). At high velocities deviations from Darcy's law become significant and inertial effects need to be considered.

Rearranging Eq. 4.2 in the form of Eq. 4.1 yields

$$u = -f_{nd} \frac{k}{\mu} \frac{\partial p}{\partial L} \quad (4.3)$$

in which f_{nd} denotes the non-Darcy factor defined as follows (Holditch and Morse, 1976):

$$f_{nd} = \frac{1}{1 + \frac{\rho \beta k u}{\mu}} \quad (4.4)$$

In Eq. 4.4 the second term in the denominator on the right-hand side represents the ratio of inertial force to viscous force - the Reynolds number for fluid flow in porous media (Geertsma⁵¹), i.e.

$$R_e = \frac{\rho \beta k u}{\mu} \quad (4.5)$$

Geertsma⁵¹ reasoned that the product βk represents a kind of characteristic length of the porous medium. Civan and Tiab²² called the βk product the apparent pore size. As pointed out by Geertsma⁵¹, the Reynolds number defined in Eq. 4.5 can be used to specify the limits of validity of Darcy's law in practical applications.

Non-Darcy flow effects are more pronounced in gas wells than oil wells because of higher flow velocities of gas resulting from the lower viscosity of gas and higher bottomhole pressure drop in gas wells. For high-capacity gas wells, pressure drop near the wellbore needs to be estimated from the Forchheimer equation. The non-Darcy flow coefficient in the Forchheimer equation can be measured from core samples in laboratories, or determined from analysis of multi-rate pressure tests in the field. Various correlations for the non-Darcy flow have been reported in the literature.

In the early research the flow of fluids through porous media was analogous to the laminar and turbulent flow in pipes (Fancher and Lewis⁹; and Brownell and Katz³⁸). Correlations for the flow of fluids through porous media were obtained by plotting a friction factor against a Reynolds number. The friction factor and the Reynolds number were defined similarly to those of fluid flow in pipes:

$$f = \frac{d}{2\rho u^2} \frac{dP}{dL} \quad (4.6)$$

$$R_e = \frac{\rho d u}{\mu} \quad (4.7)$$

in which d can be considered as the average grain or pore diameter, or some other characteristic length associated with the porous medium. These types of correlations are all subject to limitations since the Reynolds number depends significantly on the definition of a pore diameter, which can not be obtained properly. Furthermore, these correlations fail to explain the coexistence of the linear and nonlinear terms in the Forchheimer equation for high flow velocities in porous media because the linear term does not appear in the turbulent flow through pipes.

Classical fluid mechanics states that the friction factor for fluid flow through circular pipes in the laminar region is defined as

$$f = \frac{16}{R_e} \quad (4.8)$$

Combining Eqs. 4.6, 4.7, and 4.8 yields

$$\frac{dP}{dL} = \frac{\mu}{d^2/32} u \quad (4.9)$$

Eq. 4.9 is linear and known as the Hagen-Poiseuille equation. Analogous to Darcy's law for fluid flow in porous media, $d^2/32$ represents permeability. For fully developed turbulent flow in smooth pipe an empirical correlation for the determination of friction factor can be approximated by the Blasius equation,

$$f = \frac{0.0791}{R_e^{0.25}} \quad (4.10)$$

Substituting Eqs. 4.10 and 4.7 into Eq. 4.6 leads to

$$\frac{dP}{dL} = \frac{\rho^{0.75} \mu^{0.25}}{6.3211 d^{1.25} u^{1.75}} \quad (4.11)$$

Comparing Eq. 4.11 with the Forchheimer equation (Eq. 4.2), it can not represent either the Darcy or the non-Darcy flow term. The change from the laminar to turbulent flow occurs abruptly at the critical Reynolds number of about 2000. However, the deviation from Darcy's law occurs gradually with the increase of flow velocity. The critical Reynolds number is dimensionless and independent of flow geometry. For true turbulent flow through porous media, the critical Reynolds number must take the same value by properly defining pore diameter. The onset of turbulence in porous media requires enormously high flow velocity due to small pore diameters. The concerned flow of fluids through various porous media is usually in the laminar regime.

Various studies (Hubbert⁴⁷; Geertsma⁵¹; and Firoozabadi and Katz⁵⁸) indicate that the deviation from darcy's law is attributed to inertial forces arising from the laminar flow of fluid through curvilinear flow paths having periodical expansions and contractions of pore spaces. The inertial forces depend on the kinetic energy per unit volume of the fluid, i.e., ρu^2 and are independent of the fluid viscosity. The proportionality constant of inertial forces in the Forchheimer equation is β , called as the coefficient of non-Darcy flow or inertial flow resistance. The non-Darcy flow coefficient is determined by pore geometry in single-phase systems. Various variables such as pore size, shape and specific surface area can be used to describe the pore structure in pore-level scales. However, porosity, permeability and tortuosity can provide a useful evaluation of the nature of the pore structure and are routinely obtained from core analysis or formation evaluation in the petroleum industry.

The non-Darcy flow coefficient has been commonly correlated with the permeability and porosity of a porous medium. Ergun³³ developed a general equation for fluid flow in unconsolidated media based on the Kozeny theory of capillary tube model and the theory of Reynolds for resistance to fluid flow,

$$\frac{\Delta P}{L} = 150 \frac{(1-\phi)^2}{\phi^3 d^2} \mu u + 1.75 \frac{1-\phi}{\phi^3 d} \rho u^2 \quad (4.12)$$

Comparing the Ergun equation with the Forchheimer equation in Eq. 4.2, the following correlation for the non-Darcy flow coefficient is obtained for unconsolidated media,

$$\beta = \frac{0.143}{\phi^{1.5}\sqrt{k}} \quad (4.13)$$

in which k is permeability and has the form of

$$k = \frac{\phi^3 d^2}{(1 - \phi)^2} \quad (4.14)$$

In Eq. 4.13 the units of β and k need to be consistent, i.e., in the same unit of length because β has a unit of L^{-1} and k has a unit of L^2 . The Ergun equation may only apply to unconsolidated media because it was derived from the Kozeny equation which has been found valid in unconsolidated media, and the constants in the equation were determined by fitting the data of unconsolidated media.

Janicek and Katz⁶⁰ correlated the data from Cornell and Katz²⁵ for consolidated media, resulting in a plot for calculating the β -coefficient from the permeability and porosity of a rock. Janicek and Katz' correlation was later expressed by Tek et al¹⁰⁰ in the following form,

$$\beta = \frac{5.5 \times 10^9}{\phi^{0.75} k^{1.25}} \quad (4.15)$$

In this equation the non-Darcy flow coefficient and permeability are in units of ft^{-1} and md , respectively. Janicek and Katz' correlation may probably be the first reliable equation relating the non-Darcy flow coefficient to permeability and porosity of a porous medium. However, Gewers and Nichol⁶² applied the Janicek-Katz correlation to microvugular carbonates and found that the measured values of the β -coefficient could be as much as one order of magnitude higher than those obtained from the Janicek and Katz correlation.

Cooke²⁴ measured the non-Darcy flow coefficient for artificially propped fractures and developed an empirical relationship between the non-Darcy flow coefficient and permeability for different sand packs:

$$\beta = \frac{B}{k^A} \quad (4.16)$$

The constants A and B are determined by experiments for different types of proppants. A table of the constants A and B are included in the paper for various proppant sizes. Results in the paper show that non-Darcy flow effects are significant in propped fractures.

Geertsma⁶¹ developed a dimensionally consistent correlation between the non-Darcy flow coefficient, permeability and porosity. He related the Kozeny equation for permeability to the Ergun equation for the β -coefficient and obtained the following equation,

$$\beta = \frac{C}{\phi^{1.5}\sqrt{k}} \quad (4.17)$$

He further determined $C = 0.179$ from his experimental results for unconsolidated media. However, he found Eq. 4.17 does not hold for consolidated media with a wide range of porosity and permeability. Linear regression analysis of his and other experimental data of consolidated media has led to

$$\beta = \frac{0.005}{\phi^{5.5}\sqrt{k}} \quad (4.18)$$

Units of β and k in Eqs. 4.17 and 4.18 must be consistent because they both have the dimensions of length. To account for the effect of an immobile liquid saturation on the non-Darcy flow coefficient, Geertsma⁵¹ expected the following equation would hold:

$$\beta = \frac{0.005}{\phi^{5.5}\sqrt{K}} \left(\frac{1}{(1-s_w)^{5.5}\sqrt{k_r}} \right) \quad (4.19)$$

Maloney et al⁸² studied non-Darcy gas flow through propped fractures and proposed the following equation to correlate the β -coefficient:

$$\beta = \frac{A}{\phi^B \sqrt{k_R}} \quad (4.20)$$

Constants A and B are determined by fitting experimental data.

Evans and coworkers (Evans et al³⁷; and Tiss and Evans¹⁰²) have investigated the effects of immobile liquid saturation and effective stress on the non-Darcy flow coefficient. Based on their and other experimental data, they further developed the following equation to take into account the effects of liquid saturation s_w and effective stress σ :

$$\beta = \frac{C}{\phi(1-s_w)\sqrt{k}} \left(\frac{\sqrt{k\rho\sigma}}{\mu} \right)^m \quad (4.21)$$

in which C and m are empirical constants determined from linear regression analysis of experimental data.

Recently Kutasov⁷⁶ proposed an equation to predict the non-Darcy flow coefficient based on previous analytical investigations (Ergun³⁶) and compared his predictions with the experimental results of Evans and Evans³⁶ for gas flow in propped fractures. His equation is of the form

$$\beta = \frac{1432.6}{\sqrt{k_g} [\phi(1-s_w)]^{1.5}} \quad (4.22)$$

in which the units of β and k are cm^{-1} and darcy respectively.

4.2 Correlation Development

Previous studies indicate that the non-Darcy flow coefficient has been correlated with the permeability and porosity of a porous medium. Since the non-Darcy flow is basically caused by the inertial forces of the fluid flowing through tortuous pore spaces, it is logical to include the tortuosity factor in a general correlation of the β -coefficient. The tortuosity factor can be determined from the formation resistivity factor, which provides useful information about the nature of the pore structures. The appropriate evaluation of inertial effects for fluid flow through porous media requires some measure of the actual fluid velocity. However, superficial velocity is used in the Forchheimer equation. The tortuosity factor τ is usually defined as the ratio of the actual length of flow paths L_A to the length of the straight line along the overall flow direction L :

$$\tau = L_A/L \quad (4.23)$$

When the tortuosity is taken into account, the Dupuit relationship must be modified to calculate the actual velocity:

$$u_a = \tau \frac{u}{\phi} \quad (4.24)$$

Replacing the length and velocity in the Forchheimer equation (Eq. 4.2) by the actual length and velocity from Eqs. 4.23 and 4.24 leads to

$$-\frac{\partial P}{\partial(\tau L)} = \frac{\mu}{k} \left(\tau \frac{u}{\phi} \right) + \beta \rho \left(\frac{\tau u}{\phi} \right)^2 \quad (4.25)$$

or

$$-\frac{\partial P}{\partial L} = \frac{\mu}{k\phi/\tau^2} u + \frac{\beta\tau^3}{\phi^2} \rho u^2 \quad (4.26)$$

The above derivation clearly suggests that the non-Darcy flow coefficient is related with the tortuosity in addition to other rock properties. The actual value at the non-Darcy flow coefficient is expected to increase as the tortuosity increases. Various studies reveal that the non-Darcy flow coefficient decreases with increasing permeability and porosity. Based on previous experimental and analytical investigations, a power-law equation is then proposed to correlated the β -coefficient:

$$\beta = D \frac{\tau^C}{k^A \phi^B} \quad (4.27)$$

in which A, B, C and D are some constants to be determined from laboratory or field data. In this equation τ and ϕ are dimensionless variables, β has a unit of length reciprocal and k has a unit of length squared. If $A \neq 1/2$, Eq. 4.27 is not dimensionally consistent. Dimensional inhomogeneity may be necessary to correlate the β -coefficient.

Cornell and Katz²⁶ measured the non-Darcy flow coefficient for a large variety of rocks including sandstones, carbonates and dolomites. Their experimental results also include the permeability, porosity and tortuosity. We correlated the non-Darcy flow coefficient with permeability, porosity and tortuosity according to Eq. 4.27. We found that the non-Darcy flow coefficient is dependent strongly on permeability, but weakly on porosity and tortuosity. To keep the correlation concise without losing its accuracy, linear regression analysis displayed in Fig. 4.1 yielded the following correlation:

$$\beta = \frac{1.655 \times 10^9 \tau}{\phi k^{1.173}} \quad (4.28)$$

with correlation coefficient $R = 0.964$. $R = 1$ represents a perfect correlation. In Eq. 4.28 the units of β and k are expressed in ft^{-1} and md respectively. We also correlated Cornell and Katz' data in the same manner as Geertsma⁵¹ and obtained the following form:

$$\beta = \frac{1.451 \times 10^6}{\phi^{4.038} \sqrt{k}} \quad (4.29)$$

with $R = 0.290$. The plot shown in Fig. 4.2 indicates that Geertsma's correlation can not provide a good estimation although it is dimensionally consistent.

Our analysis of experimental data from consolidated media indicates that the Geertsma equation can not satisfactorily apply to general cases because it overestimates the effect of porosity and underestimates the effect of permeability on the non-Darcy flow coefficient. Correlations having dimensional inconsistency between the β -coefficient and permeability are necessary to better estimate the non-Darcy flow coefficient. The coefficients A and D in Eq. 4.27 are then dimensional and reflect other factors which affect the pore structure and can not be represented by permeability, porosity and tortuosity. All constants in the general correlation are expected to be identical for different porous media. The correlation we obtained in Eq. 4.28 gives better estimation than a dimensionally consistent correlation for consolidated media.

Effect of Immobile Liquids on the β -Coefficient

Gewers and Nichol⁵² investigated the effect of an immobile liquid on the non-Darcy flow coefficient, and found that the non-Darcy flow coefficient showed a substantial increase at liquid saturation of 20 to 30%. Wong¹⁰⁷ extended the work of Gewers and Nichol to include the effect of a mobile liquid on the non-Darcy flow coefficient. Wong observed that an increase in mobile liquid saturation from 40 to 70% could lead to an increase in the β -

coefficient by as much as eight-fold. He also found that the β -coefficient could be approximated by using the effective permeability to gas at a given saturation and dry core non-Darcy flow coefficient-permeability relationship for the rock of consideration. Evans and coworkers (Evans et al³⁶; Evans and Evans³⁷; and Tiss and Evans¹⁰²) have also investigated the effect of two-phase fluids on the β -coefficient. Their experimental results confirm Wong's finding that the β -coefficient in the presence of immobile liquid can be estimated using dry-core β -coefficient with the replacement of absolute permeability by effective permeability at given saturation. However, in the correlations they developed, porosity ϕ is replaced by $\phi(1 - S_l)$, which is similar to Geertsma's hypothesis⁵¹.

We found that the immobile liquid saturation should not be included in the general correlation of the non-Darcy flow coefficient as long as the effective permeability of gas in the presence of immobile liquid is used. Results of our experimental studies of the effect of immobile liquid saturation on the β -coefficient in consolidated media were used. We first correlated the β -coefficient with the porosity of rock and the effective gas permeability at given saturation, and obtained the following relationship from regression analysis (Fig. 4.3):

$$\beta = \frac{7.147 \times 10^8}{\phi k_r^{0.980}} \quad (4.30)$$

with $R = 0.932$.

When we replaced porosity ϕ by the fractional pore volume occupied by gas $\phi(1 - S_l)$, we obtained a worse correlation (Fig. 4.4):

$$\beta = \frac{1.785 \times 10^8}{\phi(1 - s_w) k_r^{0.774}} \quad (4.31)$$

with $R = 0.811$.

We also tried replacing ϕ by $\phi(1 - S_w)^n$ to see how the liquid saturation affects the β -coefficient. We found that the correlation improves as the exponent n goes to zero. Our analysis reveals that the variable of liquid saturation does not need to be included in the β correlation. The reason is that the effect of immobile liquid saturation is already considered implicitly in terms of the effective permeability.

Effect of Stress on the β -Coefficient

The non-Darcy flow coefficient in single-phase systems is a property of the pore structure. Any change of the pore structure due to stress will affect the non-Darcy flow coefficient. It has been known that pore deformation depends only on the effective stress — the difference between the externally applied stress (overburden pressure) and the internal stress (pore fluid pressure). The concept of effective stress was first introduced for saturated soils and has proven sufficiently accurate for engineering purposes (Jaeger and Cook⁵⁹).

The effect of overburden pressure on the absolute permeability of a rock was probably

first investigated by Fatt and Davis⁴³. They found that the permeability of sandstones decreases as the confining pressure applied on core samples increases. Later Fatt⁴⁰ systematically studied the effect of the effective stress on porosity, permeability and formation resistivity factor of sandstones. He found a reduction in porosity by 1-5%, an increase in formation resistivity factor by up to 35%, and a decrease in permeability by as much as 50% by applying the effective confining pressure of 5000 psi. Fatt further explained less change in porosity than permeability and formation factor due to rock compression as follows: porosity depends only on pore volume while permeability and formation factor are determined by the volume and the shape of pores.

The change of porosity and formation resistivity factor of a rock under stress also reflects the change of its tortuosity. In fact tortuosity is usually determined from formation resistivity factor. According to Carman¹⁰ the relationship between tortuosity and formation factor is given by

$$\tau = F\phi \quad (4.32)$$

Cornell and Katz²⁵ developed a different relationship between tortuosity and formation resistivity factor,

$$\tau = \sqrt{F\phi} \quad (4.33)$$

For example, 5% reduction in porosity and 35% increase in formation resistivity factor leads to an increase in tortuosity by 28% according to Eq. 4.32.

Tiss and Evans¹⁰² studied the effect of overburden pressure on the non-Darcy flow coefficient and found that the β -coefficient increased with increasing the effective confining pressure on core samples. The increase of the non-Darcy flow coefficient results from the decrease in permeability and porosity and the increase in tortuosity as the effective stress increases for rocks in elastic or plastic deformation regime. Once the rock fails, an increase in confining pressure results in an increase in porosity and permeability as a consequence of micro cracking processes. Effective stress is not a primary variable in determining the non-Darcy flow coefficient. The general correlation for the β -coefficient developed in Eq. 4.27 holds for the porous medium under high overburden pressure as long as values of porosity, permeability and tortuosity are obtained from the rock under the same overburden pressure. This approach is realistic because petrophysical properties such as porosity, permeability and formation resistivity are often obtained from well logging and well tests under reservoir conditions.

In fact, effective stress influences rock properties in a predictable manner. Wyble¹⁰⁸ developed an empirical equation based on his experimental observations to describe the effect of applied pressure on the porosity, permeability and electrical conductivity of sandstones. Jones and Owens⁶² proposed another convenient form of equation to correlate permeability with net confining pressure for low-permeability sandstones. Fatt⁴² experimentally studied pore volume compressibilities of sandstone reservoir rocks. Observing his experimental results we find out that the pore volume compressibility decreases exponentially as the net applied pressure increases. This suggests the following analytical expression:

$$c = c_o \exp[-\alpha(\sigma - \sigma_o)] \quad (4.34)$$

in which c is compressibility at stress σ , c_o is the compressibility at initial stress σ_o , and α is a constant determined by fitting experimental data. According to the definition of pore volume compressibility we have:

$$c = - \frac{1}{\phi} \frac{d\phi}{d\sigma} \quad (4.35)$$

Solving Eqs. 4.34 and 4.35 yields

$$\phi = \phi_o \exp \left(\frac{c_o}{\alpha} [e^{-\alpha(\sigma - \sigma_o)} - 1] \right) \quad (4.36)$$

The relationship between porosity and permeability can be expressed in the Kozeny-Carman equation (Carman¹⁰):

$$k = \frac{\phi^3}{c_K \tau^2 S^2} \quad (4.37)$$

in which c_K is the Kozeny constant and S is specific surface area. If we assume the change of c_K and S is negligible due to stress, the following expression can be obtained:

$$\frac{k}{k_o} = \left(\frac{\phi}{\phi_o} \right)^3 \left(\frac{\tau_o}{\tau} \right)^2 \quad (4.38)$$

in which k_o and τ_o denote permeability and tortuosity at initial stress σ_o . Eq. 4.38 provides a useful evaluation of permeability of the rock under stress. If permeability and porosity are measured under stress, Eq. 4.38 can be used to determine the effect of stress on tortuosity. The capability of Eq. 4.38 relies on the validity of the Kozeny-Carman equation and our assumption. Although theoretical values of the exponents in Eq. 4.38 are three and two, this formula can be generalized by allowing the exponents as variables:

$$\frac{k}{k_o} = \left(\frac{\phi}{\phi_o} \right)^m \left(\frac{\tau_o}{\tau} \right)^n \quad (4.39)$$

A General Correlation for the β -Coefficient

In the preceding sections, we first demonstrated that the non-Darcy flow coefficient could be correlated in terms of permeability, porosity and tortuosity, and then suggested that the non-Darcy flow coefficient could be estimated using dry-core correlation and effective permeability in the presence of immobile liquid, and using effective rock properties for formations under overburden stress. In this section, we present a general correlation for the

non-Darcy flow coefficient using a large variety of data from consolidated and unconsolidated media including the effects of multiphase fluids and overburden stress. Due to lack of tortuosity factors in most experimental measurements, we will correlate the β -coefficient in terms of permeability and porosity of a porous medium.

We collected a total of 183 data points in this work. In addition to the data discussed in the preceding sections, we also employed data from Geertsma⁵¹ in consolidated media, and from Evans and Evans³⁶ and Whitney¹⁰⁶ for the effects of immobile liquid saturation and closure stress on the β -coefficient in propped fractures. The units of β and k in all the data are expressed in ft^{-1} and md respectively. The regression line through all the data is shown in Fig. 4.5 and yielded the following general correlation:

$$\beta = \frac{1.485 \times 10^9}{\phi k^{1.021}} \quad (4.40)$$

with correlation coefficient $R = 0.974$. Since this correlation is obtained from a large variety of porous media under different conditions, it is expected to provide a reasonable estimation for the β -coefficient.

4.3 Dimensional Analysis of Non-Darcy Flow Coefficient in Two-Phase Flow

Extensive literature is available on the non-Darcy flow behavior of a single-phase fluid in porous media. At high flow velocities the Forchheimer equation is usually considered as a momentum equation to include inertial effects. Similar to the extension of the Darcy equation to multiphase flow, the Forchheimer equation can also be extended to multiphase flow. For two-phase flow of gas-brine or gas-oil, the viscosities and velocities of gas and liquid phase are usually different. An interfacial drag force between gas and liquid then arises, which can be significant at high flow velocities. Therefore, a more realistic approach requires the interfacial drag force to be included in the momentum equation for each phase. Following Schulenberg and Muller⁹⁷, the extended momentum equations can be written as:

$$-\frac{\partial P_l}{\partial L} = \frac{\mu_l}{k k_{rl}} u_l + \frac{\beta}{\eta_{rl}} \rho_l u_l^2 - \frac{F_i}{s_l} \quad (4.41)$$

$$-\frac{\partial P_g}{\partial L} = \frac{\mu_g}{k k_{rg}} u_g + \frac{\beta}{\eta_{rg}} \rho_g u_g^2 + \frac{F_i}{1-s_l} \quad (4.42)$$

k_{rl} and k_{rg} are relative permeability to liquid and gas phase. η_{rg} and η_{rl} can be termed as relative inertial coefficient for liquid and gas. F_i is the interfacial drag force.

The concept of relative permeability has long been used in the petroleum industry. The concepts of relative inertial coefficient and interfacial drag are not widely spread because most previous studies of non-Darcy flow behavior deal with single-phase fluids. Absolute permeability and inertial coefficient (or non-Darcy flow coefficient) in single-phase flow systems have been well studied and commonly accepted as functions of macroscopic properties of porous media. Correlations for the inertial coefficient have been presented in previous sections. Major objectives of this section are to study the relative inertial coefficient

and interfacial drag in two-phase flow of gas and liquid through porous media and to obtain correlations for estimating the inertial coefficient from rock and fluid properties by means of theoretical and dimensional analysis.

Interfacial Drag Between Gas and Liquid

Tung and Dhir¹⁰³ developed a hydrodynamic model for two-phase flow through porous media by evaluating the particle-gas drag, particle-liquid drag and liquid-gas interfacial drag forces. The interfacial drag forces are induced by the relative motion between two phases. Flow patterns are important in modeling interfacial drag. They proposed the flow patterns be similar to those observed in tubes, i.e., bubble flow, slug flow and annular flow. Although expressions for interfacial drag are different in different flow regimes, studies by Tung and Dhir¹⁰³ reveal that interfacial drag comes from viscous and inertial effects at various flow velocities. Their model equation can be summarized as follows:

$$F_i = c_v u_r + c_i u_r^2 \quad (4.43)$$

where u_r is the relative velocity,

$$u_r = \frac{u_g}{1 - s_l} - \frac{u_l}{s_l} \quad (4.44)$$

The coefficients c_v and c_i are functions of pore structure, flow pattern and properties of gas and liquid.

By examining Eq. 4.43 we can interpret that the interfacial drag also consists of viscous and inertial terms. For two-phase flow of gas and liquid through porous media, inertial effect is usually pronounced for gas phase because of its low viscosity and high velocity. If we neglect the effect of the flow velocity of liquid on gas flow, the momentum equation for gas can be simplified by substituting Eqs. 4.43 and 4.44 into Eq. 4.42:

$$-\frac{\partial P_g}{\partial L} = \frac{\mu_g}{k k_{rg}} u_g + \frac{\beta}{\eta_{rg}} \rho_g u_g^2 \quad (4.45)$$

Now the relative permeability k_{rg} and relative inertial coefficient η_{rg} in Eq. 4.45 are different from those in Eq. 4.42 because the effect of interfacial drag is included in the parameters in Eq. 4.45. The advantage of Eq. 4.45 over Eq. 4.42 is that the momentum equation is simplified and the interfacial drag is considered implicitly in terms of relative permeability and relative inertial coefficient for gas phase.

Correlation Development for Relative Inertial Coefficient

Saez and Carbonell⁹⁶ investigated simultaneous two-phase flow of gas and liquid in packed beds by extending the concept of relative permeability to the inertial regime. They proposed empirical correlations for estimating the relative permeability and relative inertial coefficient of the gas and liquid phases as functions of the saturation of the liquid phase. By fitting experimental data, they found that the relative inertial coefficient can be treated equal to the relative viscous permeability. Later Saez et al⁹⁶ considered the case of two-phase flow

through a periodically constricted tube. In this model the liquid phase contacts the tube surface while the gas as a nonwetting phase stays in the center of the tube. The problem was solved by means of the finite-element method. Analytical solutions were obtained for the relative permeabilities of the gas and liquid phases flowing through a straight tube. The relative permeability to gas phase in the straight tube model is given by:

$$k_{rg} = s_g \frac{\Omega \left[s_g + 2 \frac{\mu_g (1 - s_g)}{\mu_l} \right] + \left(1 - \frac{\rho_g}{\rho_l} \right) s_g (1 - s_g)^2 \left(1 - 2 \frac{\mu_g \ln s_g}{\mu_l} \right)}{\Omega + \left(1 - \frac{\rho_g}{\rho_l} \right) (1 - s_g)^2} \quad (4.46)$$

where

$$\Omega = -72 \frac{\mu_l \mu_l (1 - \phi)^2}{\rho_g g d^2 \phi^3} - 2 s_g \left(1 - \frac{\rho_g}{\rho_l} \right) (1 - s_g + s_g \ln s_g) \quad (4.47)$$

Eqs. 4.46 and 4.47 indicate that the relative permeability to gas phase is a function of properties of the porous medium (ϕ, d) and properties of fluids ($s_g, \mu_g, \mu_l, \rho_g, \rho_l$). In their study, Saez et al⁸⁶ assumed that the relative permeability of each phase is the same in the viscous and inertial flow regimes.

Although the straight tube model is oversimplified to represent the nature of two-phase flow through porous media, it can provide physical quantities involved in the problem. If the variables involved in a physical phenomenon are known and the relationships among them are unknown, dimensional analysis can be performed to establish the relationships among a set of dimensionless groups of the variables. In the following, a correlation for inertial coefficient is obtained by the procedure of dimensional analysis.

It is considered that the relative inertial coefficients of gas phase, η_{rg} , in Eq. 4.45 is related to properties of a porous medium and fluids as well as interfacial tension between liquids. The properties of the porous medium can be represented by the permeability, porosity and tortuosity of the porous medium. Therefore, the relative inertial coefficient, η_{rg} , is dependent on:

1. permeability, $k[L^2]$;
2. porosity, ϕ ;
3. tortuosity, τ ;
4. saturation, s_g ;
5. gas density, $\rho_g[ML^{-3}]$;
6. liquid density, $\rho_l[ML^{-3}]$;
7. gas viscosity, $\mu_g[ML^{-1}T^{-1}]$;
8. liquid viscosity, $\mu_l[ML^{-1}T^{-1}]$;
9. interfacial tension, $\gamma[MT^{-2}]$.

Among these dimensional variables, the following dimensionless groups can be formed:

$$\pi_1 = \frac{\mu_g^2}{\sqrt{k} \rho_g \gamma} \quad (4.48)$$

$$\pi_2 = \rho_g / \rho_l \quad (4.49)$$

$$\pi_3 = \mu_g / \mu_l \quad (4.50)$$

For a given rock-fluid system, only fluid saturation varies during simultaneous two-phase flow. We can then propose the following expression for the relative inertial coefficient:

$$\eta_{rg} = f(s_g) \left(\frac{\mu_g^2}{\sqrt{k} \rho_g \gamma} \right)^{m_1} \left(\frac{\rho_g}{\rho_l} \right)^{m_2} \left(\frac{\mu_g}{\mu_l} \right)^{m_3} \phi^{M_4} \tau^{m_4} \quad (4.51)$$

The function f and exponents m_1 to m_4 in Eq. 4.51 need to be determined by analyzing experimental data.

Consider $\beta \eta_{rg}$ in Eq. 4.45 as an effective inertial coefficient of gas phase,

$$\beta_g = \beta / \eta_{rg} \quad (4.52)$$

Since β_g has a unit of reciprocal length, one more dimensionless group can be constructed:

$$\pi_4 = \sqrt{k} \beta_g \quad (4.53)$$

Then β_g can be expressed as

$$\beta_g = f(s_g) \frac{1}{\sqrt{k}} \left(\frac{\mu_g^2}{\sqrt{k} \rho_g \gamma} \right)^{m_1} \left(\frac{\rho_g}{\rho_l} \right)^{m_2} \left(\frac{\mu_g}{\mu_l} \right)^{m_3} \phi^{m_4} \tau^{m_4} \quad (4.54)$$

Relationship Between Relative Permeability and Relative Inertial Coefficient

Generally speaking, the relative permeability is also a function of rock and fluid properties and wettability. If we assume the variables involved in the relative permeability are the same as those in the relative inertial coefficient, we can obtain an expression for the relative permeability similar to Eq. 4.51 by means of dimensional analysis:

$$k_{rR} = g(s_R) \left(\frac{\mu_R^2}{\sqrt{k} \rho_R \gamma} \right)^{n_1} \left(\frac{\rho_R}{\rho_l} \right)^{n_2} \left(\frac{\mu_R}{\mu_l} \right)^{n_3} \phi^{n_4} \tau^{n_5} \quad (4.55)$$

However, the function g and exponents n_1 to n_5 in Eq. 4.55 may be different from those in Eq. 4.51 because the relative permeability and inertial coefficient represent different flow phenomenon.

Since both the relative permeability and relative inertial coefficient are proposed to be dependent on the same set of variables, the following relationship between the relative permeability and inertial coefficient is obtained:

$$\eta_{rR} = k_{rR} \frac{f(s_R)}{g(s_R)} \left(\frac{\mu_R^2}{\sqrt{k} \rho_R \gamma} \right)^{n_1 - n_1} \left(\frac{\rho_R}{\rho_l} \right)^{n_2 - n_2} \left(\frac{\mu_R}{\mu_l} \right)^{n_3 - n_3} \phi^{n_4 - n_4} \tau^{n_5 - n_5} \quad (4.56)$$

If $f(s_R) = g(s_R)$ and $m_i = n_i$ ($i = 1$ to 5), then Eq. 4.56 reduces to the following form:

$$\eta_{rR} = k_{rR} \quad (4.57)$$

Chung and Catton¹⁸ assumed Eq. 4.57 in their study. Eqs. 4.56 and 4.57 can be used to estimate the relative inertial coefficient from the relative permeability.

Being a function of the properties of rock and fluids, the relative permeability is strongly dependent on the saturation of fluid. Although there have been attempts to calculate relative permeability on theoretical grounds, it is most commonly obtained as a function of fluid saturation by experimental measurements. This is justified because the relative permeability is not a strong function of fluid properties and also properties of rock and fluids do not change for a given rock-fluid system. A review of various correlations for calculating the relative permeability as a function of fluid saturation has been presented by Honarpour et al⁵⁶.

If we assume that the relative permeability is only a function of fluid saturation in a given rock-fluid system, the relative inertial coefficient can also be expressed in terms of fluid saturation. Saez and Carbonnel⁹⁵ proposed relationships for calculating the relative permeability and inertial coefficient as functions of fluid saturation. By fitting the empirical relationships to experimental data, they found that the relative inertial coefficient is approximately equal to the relative permeability. Schulenberg and Muller⁹⁷ also assumed that the relative permeability equals the relative inertial coefficient. However, further investigations are required to demonstrate whether this assumption is valid.

4.4 Conclusions

This investigation presents attempts to determine the non-Darcy flow or inertial coefficient for fluid flow through porous media. Previous theoretical and current dimensional analyses indicate that the non-Darcy flow coefficient in two-phase flow is dependent upon

both rock and fluid properties. However, non-Darcy flow in single-phase flow is caused by inertial forces of the fluid flowing at high velocity through curved flow paths having periodical expansions and contractions of pore spaces. Based on this study the following conclusions can be drawn:

1. The non-Darcy flow coefficient in single-phase systems is determined by rock properties, and can be estimated from the permeability, porosity and tortuosity of a given rock.
2. The β -coefficient is more sensitive to permeability than other variables.
3. The β -coefficient of rock under stress can be estimated using the same correlation provided that rock properties are measured from the rock under stress.
4. The β -coefficient in multiphase systems can be estimated from the correlation in single-phase system by replacing the rock permeability by the effective permeability at given saturation.

5. DEPENDENCY OF β AND k ON CORE LENGTH L

Effect of core length on the permeability and non-Darcy flow coefficient has been a controversial debate amongst the various researchers (Temeng and Horne¹⁰¹, and Firoozabadi et al⁴⁸). To investigate this matter, a series of different length cores have been used to generate pressure differential vs. mass flow data at steady-state conditions. These data are plotted for each different length core (Fig. 5.1) and the permeability and non-Darcy coefficients are determined (Tables 5.1 and 5.2) which are then plotted against core length to determine the sensitivity due to the core length. The results given in Fig. 5.2 indeed indicated that the core length average permeability and inertial flow coefficient are dependent on the core length. The actual values of permeability and the inertial flow coefficient are obtained by extrapolation of the core length average values to zero length core according to Civan¹⁷ as follows:

$$k_{Local} = \lim_{L \rightarrow 0} k_L = 125md \quad (5.1)$$

$$\beta_{Local} = \lim_{L \rightarrow 0} \beta_L = 7.2 \times 10^5 cm^{-1} \quad (5.2)$$

An examination of the plots given in Fig. 5.1 and those reported in the literature reveals an oscillatory behavior about the straight-lines common to all experimental data. Therefore, such oscillations may not be just noise but they must be reflecting some feature of flow of compressible fluids through porous media. The straight-line plots are based on the integral of Forchheimer's equation over the core length. Unless both density and viscosity variations over the core length are considered such straight-line plots are only an approximation. To account for the variations of density and viscosity along the core, a numerical integration of the ordinary differential equations representing the steady-state mass and momentum equations as a function of the distance from the core inlet to outlet should be resorted.

This, however, requires further studies for an implicit determination of permeability and the inertial flow coefficient as a function of the dimensionless groups derived in Chapter 6. Although several correlations have been derived in Chapter 4 for unconsolidated and consolidated porous media, these correlations also facilitated the core length average permeability and inertial flow coefficient determined by straight-line plots.

6. FORMULATION OF POROUS MEDIA MASS AND MOMENTUM EQUATIONS

6.1 Introduction

New equations for porous media mass and momentum conservation for variable fluid and porous media properties are derived by representative volume averaging of the microscopic laws. The new mass conservation equation contains an extra term interpreted as mass transfer by spatial dispersion in porous media. The new momentum conservation equation contains only one empirical parameter, referred to as an apparent permeability. It is shown that the Klinkenberg and Forchheimer equations can be derived readily from the apparent permeability concept. Therefore, it is concluded that future research should be devoted to developing correlations for the apparent permeability rather than an absolute permeability, inertia resistance coefficient, and other parameters that the special equations Darcy, Forchheimer, and others may include. It is also shown that apparent permeability is a function of the porous media Reynolds number which implicitly includes the information about the fluid conditions through density, viscosity, and flow rate, and the information about the porous media properties through the use of an apparent pore size diameter concept. The present study, thus, leads to new insights into the mathematical description of the flow behavior in porous media.

6.2 Porous Media Local Volume Averaging

The methodology that is facilitated for derivation of porous media volume averaged equations from microscopic equations is described in this section.

The bulk volume of a porous material saturated with a fluid is given as a sum of the volumes of the porous matrix and the fluid contained in the pore space.

$$V_r = V_s + V_f \quad (6.1)$$

The porosity of the porous media is defined by

$$\phi = V_f / V_r \quad (6.2)$$

The local volume average of a property, f , of a fluid in porous media is defined by (Slattery⁹⁸):

$$\langle f \rangle = \frac{1}{V_f} \int_{V_f} f dV \quad (6.3)$$

V_f is the volume of fluid within the volume, V , considered for averaging.

Thus, a microscopic quantity can be expressed as a sum of the volume average, $\langle f \rangle$, and the spatial deviation from the average, f' , as (Hsu and Cheng⁵⁷):

$$f = \langle f \rangle + f' \quad (6.4)$$

Since the average of an average is equal to itself, an average of Eq. 6.4 leads to the following

expression when applied with Eq. 6.3:

$$\langle f' \rangle = \frac{1}{V} \int_V f' dV = 0 \quad (6.5)$$

Eq. 6.5 implicitly defines the representative elemental volume, V , necessary for volume averaging defined by Eq. 3.

When applied over the representative elemental volume of the porous media, V_r , and over the volume, V_f , of the fluid contained within the elemental volume, Eq. 3 leads, respectively, to:

$$\langle f \rangle_r = \frac{1}{V_r} \int_{V_r} f dV \quad (6.6)$$

and

$$\langle f \rangle_f = \frac{1}{V_f} \int_{V_f} f dV \quad (6.7)$$

Therefore, using Eqs. 6.2, 6.6, and 6.7 porous media volume averages can be converted to fluid volume averages by

$$\langle f \rangle_r = \phi \langle f \rangle_f \quad (6.8)$$

Although Eqs. 6.1 through 6.8 are straight forward, implementation of these rules for volume averaging of intricate equations containing nonlinear terms and composite operators is rather challenging.

Slattery⁹⁸ derived the following rule for averaging the gradients of a property, f :

$$\langle \nabla f \rangle = \nabla \langle f \rangle + \frac{1}{V} \int_{A_i} f n dA \quad (6.9)$$

A is a pore surface area variable. A_i is the solid-fluid interface within the representative elemental volume. n is the unit vector normal to the pore surface area element.

When the average of a product of two variables, f and g , are required, following the procedure by Hsu and Cheng⁵⁷ and Kaviany⁹⁷ first these variables are decomposed according to Eq. 6.4, as

$$f = \langle f \rangle + f' \quad (6.10)$$

$$g = \langle g \rangle + g' \quad (6.11)$$

for which it is assumed that there exists a common representative elemental volume such that both

$$\langle f' \rangle = \langle g' \rangle = 0 \quad (6.12)$$

Then, the average of the product is expressed as, using Eqs. 6.10-6.12:

$$\begin{aligned} \langle fg \rangle &= \langle (\langle f \rangle + f')(\langle g \rangle + g') \rangle \\ &= \langle \langle f \rangle \langle g \rangle + \langle f \rangle g' + f' \langle g \rangle + f' g' \rangle \\ &= \langle f \rangle \langle g \rangle + \langle f \rangle \langle g' \rangle + \langle f' \rangle \langle g \rangle + \langle f' g' \rangle \\ &= \langle f \rangle \langle g \rangle + \langle f' g' \rangle \end{aligned} \quad (6.13)$$

This procedure can be extended to averaging the products of multiple variables. For example,

$$\begin{aligned} \langle fgh \rangle &= \langle f \rangle \langle g \rangle \langle h \rangle + \langle f \rangle \langle g' h' \rangle + \langle g \rangle \langle f' h' \rangle \\ &\quad + \langle h \rangle \langle f' g' \rangle + \langle f' g' h' \rangle \end{aligned} \quad (6.14)$$

Thus, applying Eqs. 6.13 and 6.14 we obtain the following expressions used in the present chapter:

$$\langle \rho v \rangle = \langle \rho \rangle \langle v \rangle + \langle \rho' v' \rangle \quad (6.15)$$

$$\langle \rho v v \rangle = \langle \rho \rangle \langle v \rangle \langle v \rangle + \langle \rho \rangle \langle v' v' \rangle + 2 \langle \rho' v' \rangle \langle v \rangle + \langle \rho' v' v' \rangle \quad (6.16)$$

An examination of Eqs. 6.13-6.16 reveals that another challenging aspect of volume averaging involves the development of methods for closure of the averages of the products of the spatial deviations. The gradient closure model given below offers a simplistic approach (Hsu and Cheng⁵⁷, and Kaviany⁶⁷).

$$\langle f' v' \rangle = - \alpha \cdot \nabla \langle f \rangle \quad (6.17)$$

in which α is an empirical tensorial coefficient.

In the following, we derive the two new expressions used in the present chapter. The first is:

$$\begin{aligned}
\langle \rho \nabla \psi \rangle &= \langle (\langle \rho \rangle + \rho') \nabla (\langle \psi \rangle + \psi') \rangle \\
&= \langle \langle \rho \rangle \nabla \psi + \langle \rho \rangle \nabla \psi' + \rho' \nabla \langle \psi \rangle + \rho' \nabla \psi' \rangle \\
&= \langle \rho \rangle \langle \nabla \langle \psi \rangle \rangle + \langle \rho \rangle \langle \nabla \psi' \rangle + \langle \rho' \nabla \langle \psi \rangle \rangle + \langle \rho' \nabla \psi' \rangle
\end{aligned} \tag{6.18}$$

in which

$$\langle \nabla \langle \psi \rangle \rangle = \nabla \langle \psi \rangle + \frac{1}{V} \int_{A_i} \langle \psi \rangle n dA \tag{6.19}$$

The second is:

$$\begin{aligned}
\langle \mu \nabla \cdot (\nabla v) \rangle &= \langle (\langle \mu \rangle + \mu') \nabla \cdot [\nabla (\langle v \rangle + v')] \rangle \\
&= \langle (\langle \mu \rangle + \mu') [\nabla \cdot (\nabla \langle v \rangle) + \nabla \cdot (\nabla v')] \rangle \\
&= \langle \langle \mu \rangle \nabla \cdot (\nabla \langle v \rangle) + \langle \mu \rangle \nabla \cdot (\nabla v') \\
&\quad + \mu' \nabla \cdot (\nabla \langle v \rangle) + \mu' \nabla \cdot (\nabla v') \rangle \\
&= \langle \mu \rangle \langle \nabla \cdot (\nabla \langle v \rangle) \rangle + \langle \mu \rangle \langle \nabla \cdot (\nabla v') \rangle \\
&\quad + \langle \mu' \nabla \cdot (\nabla \langle v \rangle) \rangle + \langle \mu' \nabla \cdot (\nabla v') \rangle
\end{aligned} \tag{6.20}$$

in which

$$\langle \nabla \cdot (\nabla v) \rangle = \nabla \cdot \langle \nabla v \rangle + \frac{1}{V} \int_{A_i} (\nabla v) n dA \tag{6.21}$$

where

$$\langle \nabla v \rangle = \nabla v + \frac{1}{V} \int_{A_i} v n dA \tag{6.22}$$

A similar derivation is carried out for the $\langle (\lambda + \mu) \nabla (\nabla \cdot v) \rangle$ term.

6.3 Formulation

The development presented in this section is carried out in vector-tensor convention. The volume averaging facilitates the rules derived in the appendix. The microscopic description of Newtonian fluid flow through porous media can be represented by the

continuity and momentum equations given by Slattery⁹⁹ which can be written, respectively, in the conservation form as:

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho v) = 0 \quad (6.23)$$

and

$$\begin{aligned} \frac{\partial}{\partial t} (\rho v) + \nabla \cdot (\rho v v) = & - \rho \nabla \Psi + (\lambda + \mu) \nabla (\nabla \cdot v) \\ & + \mu \nabla \cdot (\nabla v) \end{aligned} \quad (6.24)$$

in which t is time, and ∇ is the gradient operator. ρ , μ , and λ are the mass density, the shear coefficient of viscosity, and an empirical coefficient involving the bulk coefficient of viscosity, respectively. v is the velocity vector; Ψ is the flow potential defined by

$$\Psi = \int_{p_0}^p \frac{dp}{\rho} + gz \quad (6.25)$$

where p is pressure, p_0 is the pressure at a reference vertical distance $z=0$, g is the gravitational acceleration, and z is the vertical distance.

A porous media volume averaging of Eq. 6.23 according to the procedure described in Section 6.2 leads to the following porous media continuity equation:

$$\frac{\partial}{\partial t} \langle \rho \rangle_r + \langle \nabla \cdot (\rho v) \rangle_r = 0 \quad (6.26)$$

$\langle \rangle$ denotes the averaging over a representative bulk volume of the porous media. Based on the derivations given in Section 6.2, Eq. 6.26 can be expanded as follows:

$$\frac{\partial}{\partial t} \langle \rho \rangle_r + \nabla \cdot \langle \rho v \rangle_r + \frac{1}{V_{rA_i}} \int (\rho v) \cdot n dA = 0 \quad (6.27)$$

The last term is zero assuming no mass flux, ρv , across the solid-fluid interface, A_i . Then, expressing the bulk volume averages in terms of the fluid volume averages Eq. 6.27 becomes:

$$\frac{\partial}{\partial t} (\phi \langle \rho \rangle_f) + \nabla \cdot (\phi \langle \rho v \rangle_f) = 0 \quad (6.28)$$

Finally, expressing the average of the product of ρ and v in terms of the averages and spatial deviations of these quantities, Eq. 6.28 becomes:

$$\frac{\partial}{\partial t}(\phi\langle\rho\rangle_r) + \nabla \cdot (\phi\langle\rho\rangle_r\langle v\rangle_r) + \nabla \cdot (\phi\langle\rho'v'\rangle_r) = 0 \quad (6.29)$$

Similarly, the porous media momentum equation can be obtained by averaging Eq. 6.24 to yield:

$$\begin{aligned} \frac{\partial}{\partial t}\langle\rho v\rangle_r + \langle\nabla \cdot (\rho v v)\rangle_r &= -\langle\rho\nabla\psi\rangle_r \\ &+ \langle(\lambda + \mu)\nabla(\nabla \cdot v)\rangle_r + \langle\mu\nabla \cdot (\nabla v)\rangle_r \end{aligned} \quad (6.30)$$

Eq. 6.30 can be expanded using the expressions given in Section 6.2 as follows:

$$\begin{aligned} &\frac{\partial}{\partial t}\langle\rho v\rangle_r + \nabla \cdot \langle\rho v v\rangle_r + \underbrace{\frac{1}{V_r A_i} \int (\rho v v) \cdot ndA}_{\text{underlined}} \\ &= - \left\{ \langle\rho\rangle_r \nabla\langle\psi\rangle_r + \langle\rho\rangle_r \frac{1}{V_r A_i} \int \langle\psi\rangle_r ndA + \langle\rho'\nabla\psi'\rangle_r \right\} \\ &\quad + \left\{ \langle\lambda\rangle_r + \langle\mu\rangle_r \left[\nabla \left(\nabla \cdot \langle v\rangle_r + \frac{1}{V_r A_i} \int \langle v\rangle_r ndA \right) \right. \right. \\ &\quad \left. \left. + \frac{1}{V_r A_i} \int (\nabla \cdot \langle v\rangle_r) ndA \right] + \langle\lambda\rangle_r + \langle\mu\rangle_r \nabla(\nabla \cdot v') \right\} \\ &\quad + \langle(\lambda' + \mu')\nabla(\nabla \cdot \langle v\rangle_r)\rangle + \langle(\lambda' + \mu')\nabla(\nabla \cdot v')\rangle \\ &\quad + \left\{ \langle\mu\rangle_r \left[\nabla \cdot \left(\nabla\langle v\rangle_r + \frac{1}{V_r A_i} \int \langle v\rangle_r ndA \right) + \frac{1}{V_r A_i} \int (\nabla\langle v\rangle_r) ndA \right] \right. \\ &\quad \left. + \langle\mu\rangle_r \nabla \cdot (\nabla v') \right\} + \langle\mu'\nabla \cdot (\nabla v')\rangle + \langle\mu'\nabla \cdot (\nabla v')\rangle \end{aligned} \quad (6.31)$$

The underlined terms are zero assuming no flow across and no slip along the solid-fluid interface. Rearranging, Eq. 6.31 can be written in the following form for convenience:

$$\frac{\partial}{\partial t} \langle \rho v \rangle_r + \nabla \cdot \langle \rho v v \rangle_r = - \langle \rho \rangle_r \nabla \langle \psi \rangle_r \quad (6.32)$$

$$+ (\langle \lambda \rangle_r + \langle \mu \rangle_r) \nabla (\nabla \cdot \langle v \rangle_r) + \langle \mu \rangle_r \nabla^2 \langle v \rangle_r - R_1$$

R_1 is the sum of all terms other than those shown in Eq. 6.32. Expressing the porous media bulk volume averages in terms of the fluid volume averages, Eq. 6.32 leads to:

$$\frac{\partial}{\partial t} (\phi \langle \rho v \rangle_r) + \nabla \cdot (\phi \langle \rho v v \rangle_r) = - \phi \langle \rho \rangle_r \nabla (\phi \langle \psi \rangle_r) + \phi (\langle \lambda \rangle_r \quad (6.33)$$

$$+ \langle \mu \rangle_r \nabla [\nabla \cdot (\phi \langle v \rangle_r)] + \phi \langle \mu \rangle_r \nabla^2 (\phi \langle v \rangle_r) - R_1$$

Expanding $\langle \rho v \rangle_r$ and $\langle \rho v v \rangle_r$ according to the procedure given in Section 6.2 and collecting the terms associated with the spatial deviations ρ' and v' with the R_1 term as the R_2 term, Eq. 6.33 becomes

$$\begin{aligned} \frac{\partial}{\partial t} (\phi \langle \rho \rangle_r \langle v \rangle_r) + \nabla \cdot (\phi \langle \rho \rangle_r \langle v \rangle_r \langle v \rangle_r) = \\ - \phi \langle \rho \rangle_r \nabla (\phi \langle \psi \rangle_r) + \phi (\langle \lambda \rangle_r + \langle \mu \rangle_r) \nabla [\nabla \cdot (\phi \langle v \rangle_r)] \\ + \phi \langle \mu \rangle_r \nabla^2 (\phi \langle v \rangle_r) - R_2 \end{aligned} \quad (6.34)$$

Using the continuity equation, Eq. 6.29; dividing by ϕ , and substituting $\langle v \rangle_r = \phi \langle v \rangle_f$ and $\langle \Psi \rangle_r = \phi \langle \Psi \rangle_f$, Eq. 6.34 can be written as:

$$\begin{aligned} \langle \rho \rangle_f \left[\frac{\partial}{\partial t} \left(\frac{\langle v \rangle_r}{\phi} \right) + \left(\frac{\langle v \rangle_r}{\phi} \right) \cdot \nabla \left(\frac{\langle v \rangle_r}{\phi} \right) \right] = \\ - \langle \rho \rangle_f \nabla \langle \psi \rangle_r + (\langle \lambda \rangle_f + \langle \mu \rangle_f) \nabla (\nabla \cdot \langle v \rangle_r) + \langle \mu \rangle_f \nabla^2 \langle v \rangle_r - F/\phi \end{aligned} \quad (6.35)$$

in which F represents a collection of all the terms other than those which appear in Eq. 6.35. To determine this term in a practical manner, we resort to the method of dimensional analysis.

Assume that F in Eq. 6.35 is a function of the superficial velocity $\langle v \rangle_r$, the density $\langle \rho \rangle_r$ and viscosity $\langle \mu \rangle_r$ of the fluid, and the mean pore diameter tensor d given by (Collins²³)

$$d^2 = C k_o / \phi \quad (6.36)$$

where C is an empirical constant, k_o is the intrinsic permeability tensor, and ϕ is the porosity. A dimensional analysis between these variables reveals that two dimensionless groups can be formed.

$$\pi = \frac{F \cdot d^2 \cdot \langle v \rangle_r}{\langle \mu \rangle_f \langle v \rangle_r \cdot \langle v \rangle_r} \quad (6.37)$$

$$Re = \frac{\langle \rho \rangle_f d \cdot \langle v \rangle_r}{\langle \mu \rangle_f} \quad (6.38)$$

Eq. 6.38 is the porous media Reynolds number. Hence, a functional relationship between the two dimensionless groups given by Eqs. 6.37 and 6.38 can be considered as:

$$\pi = \tilde{f}(Re) \quad (6.39)$$

Substituting Eqs. 6.36 and 6.37, Eq. 6.39 can be written as:

$$F = \phi \langle \mu \rangle_f k^{-1} \langle v \rangle_r \quad (6.40)$$

in which K is an apparent permeability tensor expressed as:

$$k = k_o f(Re) \quad (6.41)$$

Eq. 6.41 confirms the claim made by Blick and Civan⁷ that permeability is a function of fluid and porous media properties. As a result, substituting Eq. 6.40 into Eq. 6.35 and adopting $\rho \equiv \langle \rho \rangle_f$, $\mu \equiv \langle \mu \rangle_f$, $\lambda \equiv \langle \lambda \rangle_f$, $u \equiv \langle v \rangle_r$, and $\Psi = \langle \Psi \rangle_f$ for simplicity, a generalized porous media momentum equation is obtained as follows:

$$\rho \left[\frac{\partial}{\partial t} \left(\frac{u}{\phi} \right) + \left(\frac{u}{\phi} \right) \cdot \nabla \left(\frac{u}{\phi} \right) \right] = - \rho \nabla \Psi \quad (6.42)$$

$$+ (\lambda + \mu) \nabla (\nabla \cdot u) + \mu \nabla^2 u - \mu k^{-1} u$$

Adopting the similar symbols and using a gradient model given by

$$\langle \rho' v' \rangle_f = - \alpha \cdot \nabla \rho \quad (6.43)$$

where α is a spatial hydraulic dispersivity tensor (Civan¹⁸), the porous media continuity equation, Eq. 6.29, becomes:

$$\frac{\partial}{\partial t} (\phi \rho) + \nabla \cdot (\rho u) - \nabla \cdot (\phi \alpha \cdot \nabla \rho) = 0 \quad (6.44)$$

For an incompressible fluid and constant porosity case Eq. 6.44 simplifies to

$$\nabla \cdot u = 0 \quad (6.45)$$

and Eq. 6.42 simplifies to

$$\frac{\rho}{\phi} \frac{\partial u}{\partial t} + \frac{\rho}{\phi^2} u \cdot \nabla u = -\rho \nabla \psi + \mu \nabla^2 u - \mu k^{-1} u \quad (6.46)$$

Having derived the porous media momentum equations, we next focus on Eq. 6.41, expressing the apparent permeability as a product of the intrinsic permeability, k_o and the unknown function of the Reynolds number, $f(Re)$. If this function is approximated by a power series of the inverse Reynolds number as

$$f(Re) = \sum_{i=0}^{\infty} (a_i^{1/i} Re^{-1})^i = I + a_1 Re^{-1} + a_2 Re^{-2} + \dots \quad (6.47)$$

where I is a unit tensor.

Substituting

$$Re = \frac{\rho d \cdot u}{\mu} \quad (6.48)$$

and

$$\rho = \frac{MP}{zRT} \quad (6.49)$$

And neglecting the second and higher order terms, Eq. 6.47 can be approximated by

$$f(Re) \cong I + \frac{b}{p} \quad (6.50)$$

in which

$$b = \frac{a_1 \mu z RT}{M} (d \cdot u)^{-1} \quad (6.51)$$

Hence, substituting Eq. 6.51, Eq. 6.41 yields the Klinkenberg⁷² equation as:

$$k = k_o \left(I + \frac{b}{p} \right) \quad (6.52)$$

Eq. 6.51 implies that the coefficient b in Eq. 6.52 is a variable in contrast to the conventional treatment of it as a constant.

To obtain the Forchheimer correction for non-Darcy flow, we must resort to an alternative form for the $f(Re)$ function as:

$$[f(Re)]^{-1} = \sum_{i=0}^{\infty} (a_i)^{1/i} Re^i \quad (6.53)$$

$$= I + a_1 Re + a_2 Re^2 + \dots$$

Substituting Eq. 6.48 and neglecting the second and the higher order terms Eq. 6.53 can be approximated by

$$[f(Re)]^{-1} = I + a_1 \frac{\rho d \cdot u}{\mu} \quad (6.54)$$

Thus, invoking Eq. 6.54, Eq. 6.41 becomes

$$k = k_o \left(I + a_1 \frac{\rho d \cdot u}{\mu} \right)^{-1} \quad (6.55)$$

On the other hand, assume that Eqs. 6.42 or 6.46 can be approximated as:

$$0 = -\rho \nabla \psi - k^{-1} \mu u \quad (6.56)$$

Then, substituting Eq. 6.55, Eq. 6.56 can be rearranged as:

$$-\rho \nabla \psi = \mu k_o^{-1} u + \rho (\beta \cdot u) u \quad (6.57)$$

in which β is an inertial flow coefficient tensor given by

$$\beta = a_1 k_o^{-1} \cdot d \quad (6.58)$$

from which the apparent pore diameter is obtained as

$$d = (1/a_1) k_o \cdot \beta \quad (6.59)$$

which confirms the definition of the apparent pore diameter given by Civan and Tiab²².

6.4 Conclusion

The preceding exercises demonstrate that two special approximations to the unknown $f(Re)$ function can be proposed, one leading to the Klinkenberg equation and the other to the Forchheimer equation. In general, inferred by the conclusions of Blick and Civan⁷, the $f(Re)$ function is related to the porous media friction factor. However, further research is necessary to characterize the $f(Re)$ function in a manner to predict the oscillating nature observed in experimental data.

7. DIRECT INTERPRETATION METHOD FOR DETERMINATION OF RELATIVE PERMEABILITY AND CAPILLARY PRESSURE FROM UNSTEADY-STATE NON-DARCY DISPLACEMENT DATA

7.1 Introduction

Most of the previously reported work for extracting relative permeability and capillary pressure data has been indirect and based on reservoir core history matching which requires iterative numerical solutions of inverse problems until a satisfactory match is obtained. The direct methods are based on integral formulations of the equations describing the fluid pressures and saturations for laboratory cores and they do not require numerical solution methods for differential equations involved in the indirect methods. Therefore, the numerical stability and accuracy and the history matching problems associated with the indirect methods are eliminated.

In the present study, a new direct method for determination of relative permeability and capillary pressure data from unsteady-state non-Darcy fluid displacement in laboratory cores is presented. The new method takes advantage of the internal fluid displacement data that can be obtained by noninvasive techniques in addition to the usual external fluid rate and pressure data. The mathematical interpretation method developed in this study enables the determination of relative permeability and capillary pressure simultaneously without the inherent limitations of the previous methods. It is also computationally more convenient, based on a semi-analytic solution of an algebraic formulation of the flow through porous media. Because it is an algebraic formulation it also provides a numerically more accurate interpretation of laboratory core data. The method is applied with typical gas/brine data to demonstrate its capability.

7.2 Basic Relationships

In this section, one-dimensional, isothermal and unsteady rapid flow of two immiscible and compressible fluids in an inclined, homogeneous and isotropic porous media is considered and the relationships essential for determination of relative permeabilities, non-Darcy flow coefficients, capillary pressure, and the interfacial drag force are presented. The calculation procedure is described. In the following, Phase 1 and Phase 2 refer to the wetting and nonwetting fluids, respectively. The core length is L , the cross-sectional area A , the porosity ϕ , and the permeability k .

The mass balance equations for Phase 1 and Phase 2 are given, respectively, by

$$\frac{\partial(\rho_1 u_1)}{\partial x} + \phi \frac{\partial(\rho_1 S_1)}{\partial t} = 0 \quad (7.1)$$

$$\frac{\partial(\rho_2 u_2)}{\partial x} + \phi \frac{\partial(\rho_2 S_2)}{\partial t} = 0 \quad (7.2)$$

The saturations add up to one; i.e.

$$S_1 + S_2 = 1.0 \quad (7.3)$$

The momentum balance equations given by Tutu et al (see Schulenberg and Muller⁸⁷), respectively are

$$-\frac{\partial p_1}{\partial x} + \rho_1 g \sin \theta = \frac{\mu_1}{kk_{r1}} u_1 + \frac{\rho_1}{\eta \eta_{r1}} u_1^2 + \frac{F_{12}}{S_1} \quad (7.4)$$

$$-\frac{\partial p_2}{\partial x} + \rho_2 g \sin \theta = \frac{\mu_2}{kk_{r2}} u_2 + \frac{\rho_2}{\eta \eta_{r2}} u_2^2 + \frac{F_{21}}{S_2} \quad (7.5)$$

In Eqs. 7.4 and 7.5 k is the absolute permeability and η is the reciprocal of the inertial flow coefficient, β . F_{12} and F_{21} are the interfacial drag forces experienced by Phase 1 and Phase 2 which are the opposite and equal of each other; i.e.

$$F_{21} = -F_{12} \quad (7.6)$$

A correlation for F_{12} is given in Appendix 7.B. k_r is the relative permeability and η_r is the relative reciprocal inertial flow coefficient.

The capillary pressure is given by

$$p_c(S_1) = p_2 - p_1 \quad (7.7)$$

Applying Eqs. 7.6 and 7.7 and the chain rule, Eq. 7.5 can be expressed as:

$$-\frac{\partial p_1}{\partial x} + \rho_2 g \sin \theta = \frac{\mu_2}{kk_{r2}} u_2 + \frac{\rho_2}{\eta \eta_{r2}} u_2^2 - \frac{F_{12}}{S_2} + \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} \quad (7.8)$$

The relationships between volumetric fluxes and cumulative volumes of Phase 1, Phase 2, and the total fluid, are given respectively by:

$$u_1 = \frac{1}{A} \frac{dQ_1}{dt} \quad (7.9)$$

$$u_2 = \frac{1}{A} \frac{dQ_2}{dt} \quad (7.10)$$

$$u = \frac{1}{A} \frac{dQ}{dt} \quad (7.11)$$

The total volumetric flux and total cumulative volume are given by

$$u = u_1 + u_2 \quad (7.12)$$

and

$$Q = Q_1 + Q_2 \quad (7.13)$$

The relationship between volumetric flow rate and volumetric fluxes are given by:

$$q_1 = Au_1 \quad (7.14)$$

$$q_2 = Au_2 \quad (7.15)$$

The total volumetric flow rate is given by:

$$q = q_1 + q_2 = Au \quad (7.16)$$

The fractional flow of the phases are given by

$$f_1 = u_1/u = dQ_1/dQ \quad (7.17)$$

$$f_2 = u_2/u = dQ_2/dQ \quad (7.18)$$

Thus, the fractional flows add up to one:

$$f_1 + f_2 = 1 \quad (7.19)$$

Equations 7.4 and 7.8 can be manipulated for convenience. First, subtracting Eq. 7.4 from Eq. 7.8, using Eq. 7.3, and rearranging yields:

$$-\frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} + (\rho_2 - \rho_1)g \sin \theta = \frac{\mu_2}{kk_2} u_2 - \frac{\mu_1}{kk_{r_1}} u_1 + \frac{\rho_2}{\eta \eta_{r_2}} u_2^2 - \frac{\rho_1}{\eta \eta_{r_1}} u_1^2 - \frac{F_{12}}{S_1 S_2} \quad (7.20)$$

By means of Eqs. 7.12, 7.17, 7.18 and 7.19, Eq. 7.20 can be transformed into a fractional flow equation as:

$$-\frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} + (\rho_2 - \rho_1)g \sin \theta = \quad (7.21)$$

$$\frac{\mu_2}{kk_{r_2}} u - \left(\frac{\mu_1}{k_{r_1}} + \frac{\mu_2}{k_{r_2}} \right) \frac{u}{k} f_1 + \frac{u_2}{\eta} \left[-\frac{\rho_1 f_1^2}{\eta_{r_1}} + \frac{\rho_2 (1 - f_1)^2}{\eta_{r_2}} \right] - \frac{F_{12}}{S_1 S_2}$$

Second, rearranging Eqs. 7.4 and 7.8 one obtains

$$\left(-\frac{\partial p_1}{\partial x} + \rho_1 g \sin \theta \right) \frac{kk_{r_1}}{\mu_1} = u_1 + \frac{kk_{r_1} \rho_1}{\eta \eta_{r_1} \mu_1} u_1^2 + \frac{kk_{r_1}}{\mu_1} \frac{F_{12}}{S_1} \quad (7.22)$$

$$\left(-\frac{\partial p_1}{\partial x} + \rho_2 g \sin \theta \right) \frac{kk_{r_2}}{\mu_2} = u_2 + \frac{kk_{r_2} \rho_2}{\eta \eta_{r_2} \mu_2} u_2^2 + \frac{kk_{r_2}}{\mu_2} \left(-\frac{F_{12}}{S_2} + \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} \right) \quad (7.23)$$

Adding Eqs. 7.22 and 7.23 and substituting Eqs. 7.12, 7.17, 7.18 and 7.19 yields the following pressure equation:

$$-\frac{\partial p_1}{\partial x} \left(\frac{k_{r_1}}{\mu_1} + \frac{k_{r_2}}{\mu_2} \right) + g \sin \theta \left(\frac{k_{r_1}}{v_1} + \frac{k_{r_2}}{v_2} \right) = \quad (7.24)$$

$$\frac{u}{k} + \frac{u^2}{\eta} \left[\frac{k_{r_1} f_1^2}{\eta_{r_1} v_1} + \frac{k_{r_2} (1 - f_1)^2}{\eta_{r_2} v_2} \right] + F_{12} \left(\frac{k_{r_1}}{\mu_1 S_1} - \frac{k_{r_2}}{\mu_2 S_2} \right) + \frac{k_{r_2}}{\mu_2} \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x}$$

in which $v_1 = \mu_1/\rho_1$ and $v_2 = \mu_2/\rho_2$. Note that Eq. 7.21 and 7.24 collapse to Marle's⁷⁹ equations when $u \rightarrow 0$, $p_c = 0$, and $F_{12} = 0$.

Inferred by Chung and Catton¹⁸, and Schulenberg and Muller⁹⁷, first we assume

$$\eta_r = k_r \quad (7.25)$$

Substituting Eqs. 7.12, 7.17, 7.18, 7.19 and 7.25 into Eqs. 7.4 and 7.8, one obtains:

$$k_{r_1} = \frac{u}{k} \mu_1 f_1 \left(1 + \frac{\rho_1 u f_1 k}{\mu_1 \eta} \right) + \left[-\frac{\partial p_1}{\partial x} + \rho_1 g \sin \theta - \frac{F_{12}}{S_1} \right] \quad (7.26)$$

and

$$k_{r_2} = \frac{u}{k} \mu_2 (1 - f_1) \left(1 + \frac{\rho_2 u (1 - f_1) k}{\mu_2 \eta} \right) + \left[-\frac{\partial p_1}{\partial x} - \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} + \rho_2 g \sin \theta + \frac{F_{12}}{S_2} \right] \quad (7.27)$$

Similarly, Eqs. 7.21 and 7.24 yield the following alternative expressions, respectively:

$$k_{r_1} = \frac{u}{k} \left[-\mu_1 f_1 \left(1 + \frac{\rho_1 u f_1 k}{\mu_1 \eta} \right) + \left(\frac{k_{r_1}}{k_{r_2}} \right) \mu_2 (1 - f_1) \left(1 + \frac{\rho_2 u (1 - f_1) k}{\mu_2 \eta} \right) \right] + \left[-\frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} + (\rho_2 - \rho_1) g \sin \theta + \frac{F_{12}}{S_1 S_2} \right] \quad (7.28)$$

$$\frac{1}{k_{r_2}} = \left\{ -\frac{\partial p_1}{\partial x} \left[\left(\frac{k_{r_1}}{k_{r_2}} \right) \frac{1}{\mu_1} + \frac{1}{\mu_2} \right] + g \sin \theta \left[\left(\frac{k_{r_1}}{k_{r_2}} \right) \frac{1}{v_1} + \frac{1}{v_2} \right] - F_{12} \left[\left(\frac{k_{r_1}}{k_{r_2}} \right) \frac{1}{\mu_1 S_1} - \frac{1}{\mu_2 S_2} \right] - \frac{1}{\mu_2} \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} \right\} + \left\{ \frac{u}{k} + \frac{u^2}{\eta} \left[\frac{f_1^2}{v_1} + \frac{(1 - f_1)^2}{v_2} \right] \right\} \quad (7.29)$$

Note that Eqs. 7.26-7.29 can be simplified for Darcy flow by substituting $\eta \rightarrow \infty$ and $F_{12} = 0$.

The solution of Eqs. 7.28 and 7.29 is given in Appendix 7.C. Also, a special solution neglecting the capillary end-effect during rapid flow through porous media is given in Appendix 7.D.

7.3 Relationships for Processing Experimental Data

The calculation of relative permeability and capillary pressure values from fluid displacement data requires additional relationships derived in this section. For this purpose, we consider two cases of measurements. In the first case, we assume that the dynamic saturation profiles along the core can be measured by well established methods such as NMR imaging, x-ray tomography, or ultrasonic imaging methods. In the second case we assume that only the effluent fluid volumes can be measured. In both cases, we assume that the dynamic total fluid pressures along the core are measurable by appropriate methods such as by means of pressure transducers mounted along the core.

Evaluation of S_1 and $\partial S_1 / \partial x$

If S_1 vs x are measured, then $\partial S_1 / \partial x$ can be directly evaluated from these data. If only the effluent fluid rates are measured, then S_1 and $\partial S_1 / \partial x$ can be evaluated indirectly by the method described as follows.

Define average density and saturation over the core length according to the mean value theorem, respectively, by

$$\bar{\rho}_1 = \frac{1}{x} \int_0^x \rho_1 dx \quad (7.30)$$

and

$$\bar{\rho}_1 \bar{S}_1 = \frac{1}{x} \int_0^x \rho_1 S_1 dx \quad (7.31)$$

For an incompressible fluid, Eq. 7.31 becomes

$$\bar{S}_1 = \frac{1}{x} \int_0^x S_1 dx \quad (7.32)$$

Differentiating Eq. 7.31 twice, with respect to x , gives

$$\rho_1 S_1 = \bar{\rho}_1 \bar{S}_1 + x \frac{\partial(\bar{\rho}_1 \bar{S}_1)}{\partial x} \quad (7.33)$$

and

$$\frac{\partial(\rho_1 S_1)}{\partial x} = 2 \frac{\partial(\bar{\rho}_1 \bar{S}_1)}{\partial x} + x \frac{\partial^2(\bar{\rho}_1 \bar{S}_1)}{\partial x^2} \quad (7.34)$$

Note that ρ_1 is calculated at the pressure p_1 determined by Eq. 7.56 given later. Following the procedure given in Appendix 7.A we can express Eqs. 7.33 and 7.34 at $x = L$ in terms of the cumulative injection, Q^* , of the displacing phase at $x = 0$, as

$$\rho_1 S_1 = \bar{\rho}_1 \bar{S}_1 - Q^* \frac{d(\bar{\rho}_1 \bar{S}_1)}{dQ^*} \quad (7.35)$$

$$\frac{\partial(\rho_1 S_1)}{\partial x} = \left(\frac{Q^*}{L} \right) \frac{d^2(\bar{\rho}_1 \bar{S}_1)}{dQ^{*2}} \quad (7.36)$$

For an incompressible fluid Eqs. 7.35 and 7.36 becomes

$$S_1 = \bar{S}_1 - Q^* \frac{d\bar{S}_1}{dQ^*} \quad (7.37)$$

$$\frac{\partial S_1}{\partial x} = \left(\frac{Q^*}{L} \right)^2 \frac{d^2 \bar{S}_1}{dQ^{*2}} \quad (7.38)$$

* indicates the conditions at $x = 0$.

To express the second order derivative on the right of Eq. 7.36 we use Eq. 7.1. Integrating Eq. 7.1 over the core length yields:

$$\rho_1 u_1 - \rho_1^* u_1^* + \phi \frac{d}{dt} \int_0^L \rho_1 S_1 dx = 0 \quad (7.39)$$

Substituting Eqs. 7.30 and 7.31 into Eq. 7.39 one obtains

$$\rho_1 u_1 - \rho_1^* u_1^* + \phi L \frac{d(\bar{\rho}_1 \bar{S}_1)}{dt} = 0 \quad (7.40)$$

Substituting Eq. 7.9 for u_1 and u_1^* and dividing by dQ^* , Eq. 7.40 leads to

$$\frac{d(\bar{\rho}_1 \bar{S}_1)}{dQ^*} = \frac{1}{A\phi L} \left[\rho_1^* \frac{dQ_1^*}{dQ^*} - \rho_1 \frac{dQ_1}{dQ^*} \right] \quad (7.41)$$

Integrating Eq. 7.41 for a cumulative injection from zero to Q^* yields:

$$\bar{\rho}_1 \bar{S}_1 = \bar{\rho}_1 \bar{S}_1 \Big|_{(Q^*=0)} + \frac{1}{A\phi L} \int_0^{Q^*} \left(\rho_1^* \frac{dQ_1^*}{dQ^*} - \rho_1 \frac{dQ_1}{dQ^*} \right) dQ^* \quad (7.42)$$

Differentiating, Eq. 7.41 yields:

$$\frac{d^2(\bar{\rho}_1 \bar{S}_1)}{dQ^{*2}} = \frac{1}{A\phi L} \frac{d}{dQ^*} \left[\rho_1^* \frac{dQ_1^*}{dQ^*} - \rho_1 \frac{dQ_1}{dQ^*} \right] \quad (7.43)$$

For an incompressible fluid $Q = Q^*$ and Eqs. 7.41-7.43 become:

$$\frac{d\bar{S}_1}{dQ^*} = \frac{1}{A\phi L} \left[\frac{dQ_1^*}{dQ^*} - \frac{dQ_1}{dQ^*} \right] = \frac{f_1^* - f_1}{A\phi L} \quad (7.44)$$

$$\bar{S}_1 = \bar{S}_1 \Big|_{(Q^*=0)} + \frac{1}{A\phi L} (Q_1^* - Q_1) \quad (7.45)$$

$$\frac{d^2 \bar{S}_1}{dQ^{*2}} = \frac{1}{A\phi L} \left[\frac{df_1^*}{dQ^*} - \frac{df_1}{dQ^*} \right] \quad (7.46)$$

If the core is initially saturated by Phase 2 to the immobile residual saturation of Phase 1, S_{1r} , and, then, only Phase 1 is injected, then

$$Q^* = Q_1^* \quad (7.47)$$

and

$$\rho_1 S_1|_{t=0} = \rho_1|_{t=0} S_{1r} \quad (7.48)$$

Thus, Eqs. 7.41-7.43 become

$$\frac{d(\bar{\rho}_1 \bar{S}_1)}{dQ^*} = \frac{1}{A\phi L} \left[\rho_1^* - \rho_1 \frac{dQ_1}{dQ^*} \right] \quad (7.49)$$

$$\bar{\rho}_1 \bar{S}_1 = \rho_1|_{t=0} S_{1r} + \frac{1}{A\phi L} \left[\rho_1^* Q^* - \int_0^{Q^*} \rho_1 \frac{dQ_1}{dQ^*} dQ^* \right] \quad (7.50)$$

and

$$\frac{d^2(\bar{\rho}_1 \bar{S}_1)}{dQ^{*2}} = - \frac{1}{A\phi L} \frac{d}{dQ^*} \left(\rho_1 \frac{dQ_1}{dQ^*} \right) \quad (7.51)$$

Again, assuming an incompressible fluid Eqs. 7.49-7.51 become:

$$\frac{d\bar{S}_1}{dQ^*} = \frac{1}{A\phi L} \left(1 - \frac{dQ_1}{dQ^*} \right) = \frac{1}{A\phi L} (1 - f_1) \quad (7.52)$$

$$\bar{S}_1 = S_{1r} + \frac{1}{A\phi L} (Q^* - Q_1) = S_{1r} + \frac{Q_2}{A\phi L} \quad (7.53)$$

$$\frac{d^2 \bar{S}_1}{dQ^{*2}} = - \frac{1}{A\phi L} \frac{d^2 Q_1}{dQ^{*2}} = - \frac{1}{A\phi L} \frac{df_1}{dQ^*} \quad (7.54)$$

Evaluation of p_1 and $\partial p_1/\partial x$

Consider a saturation weighted two-phase fluid pressure defined by

$$p = S_1 p_1 + S_2 p_2 \quad (7.55)$$

The derivation of the equation is presented in Lewis and Schrefler⁷⁷. The equation is based upon a volumetric average although a more appropriate average for pressure would be an average over the total solid contact surface as shown in Appendix 7.D. Since the measurement of the fraction of the total area contacted by each fluid phase is difficult, the saturation weighting given in Eq. 7.55 represents an approximation.

We assume p can be measured along the core. By means of Eqs. 7.3 and 7.7, Eq. 7.55 can be written as

$$p_1 = p - (1 - S_1)p_c \quad (7.56)$$

from which we obtain $\partial p_1/\partial x$ directly by a numerical method. Alternatively, differentiating Eq. 7.56 with respect to x and rearranging we can obtain:

$$\frac{\partial p_1}{\partial x} = \frac{\partial p}{\partial x} + \left[p_c - (1 - S_1) \frac{\partial p_c}{\partial S_1} \right] \frac{\partial S_1}{\partial x} \quad (7.57)$$

Evaluation of Dynamic Capillary Pressure

p_c and $\partial p_c/\partial S_1$, can be calculated as follows. Consider an alternative definition of average saturation according to

$$\bar{S}_1 = \frac{\int_{p_c^*}^{p_c} \rho_1 S_1 dp_c}{\int_{p_c^*}^{p_c} \rho_1 dp_c} \quad (7.58)$$

and define

$$\bar{\rho}_1 = \frac{\int_{p_c^*}^{p_c} \rho_1 dp_c}{p_c - p_c^*} \quad (7.59)$$

We assume Eqs. 7.58 and 7.59 yield the same averages obtained by Eqs. 7.30 and 7.31. Hence, Eqs. 7.58 and 7.59 can be combined to obtain:

$$\bar{\rho}_1 \bar{S}_1 = \frac{\int_{p_c^*}^{p_c} \rho_1 S_1 dp_c}{p_c - p_c^*} \quad (7.60)$$

p_c^* is the capillary pressure at the inlet conditions of the core. Since immediately after the injection begins, the immobile residual saturation of Phase 2 is reached, therefore p_c^* is the capillary pressure at

$$S_1 = 1 - S_{2r}, x = 0, t > 0 \quad (7.61)$$

Differentiating Eq. 7.60 yields

$$\rho_1 S_1 = \bar{\rho}_1 \bar{S}_1 + (p_c - p_c^*) \frac{d(\bar{\rho}_1 \bar{S}_1)}{dp_c} \quad (7.62)$$

Comparing Eqs. 7.35 and 7.62 yields:

$$\frac{dp_c}{p_c - p_c^*} = -\frac{dQ^*}{Q^*} \quad (7.63)$$

Integrating Eq. 7.63 results in the following expression for the dynamic capillary pressure:

$$p_c = \frac{C}{Q^*} + p_c^* \quad (7.64)$$

C is an integration constant which can be determined using a known condition. From Eq. 7.64

$$\frac{\partial p_c}{\partial S_1} = -\frac{C}{Q^{*2}} \frac{\partial Q^*}{\partial S_1} \quad (7.65)$$

Evaluation of Static Capillary Pressure

Kalaydjian⁶⁶ proposed the following expression for correction of static capillary pressure to obtain the dynamic capillary pressure:

$$p_c = p_{c_{static}} + \frac{1}{q} \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial t} \quad (7.66)$$

q is the total flow rate given by Eq. 7.16. An alternative similar expression including the variation of porosity is given by Pavone⁸⁷. By means of Eqs. 7.11 and 7.16, Eq. 7.66 can be written as:

$$p_c = p_{c_{static}} + \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial Q^*} \quad (7.67)$$

or simply

$$p_c = p_{c_{static}} + \frac{dp_c}{dQ^*} \quad (7.68)$$

Substituting Eq. 7.64 into Eq. 7.68 gives

$$p_{c_{static}} = C \left(\frac{1}{Q^*} + \frac{1}{Q^{*2}} \right) + p_c^* \quad (7.69)$$

7.4 Applications

The method developed in the preceding section is applied with a typical nitrogen/brine displacement data for illustration. For this purpose, a vertical core is saturated fully with brine and then the displacement process is initiated by injecting nitrogen gas from the upper end of the core. The core, fluid, and displacement data by Ouenes⁸⁶ are presented in Table 1. Using these data with Eqs. 7.B3-5, 7.13, 7.17, 7.26, 7.27, 7.37, 7.38, 7.52, 7.53, 7.54, 7.57, 7.64, 7.65, and 7.69, the relative permeability and capillary pressure data for the nitrogen/brine system are calculated. Typical numerical results are presented in Table 7.1. Figs. 7.1 and 7.2 show a comparison of the present results with those given by Ouenes, et al⁸⁶.

7.5 Conclusions

The important features of the present model are:

- 1) It includes the inertial, capillary, and interfacial drag effects.
- 2) It facilitates the pressure and/or saturation distribution along the core that can be measured by implementing new technology such as NMR, CT scanner, ultrasonics, and pressure measurements in addition to the conventional measurements of effluent fluid rates and pressure differential over the core length.
- 3) The previous methods assumed that the core was initially saturated by one phase up to the immobile saturation of the second phase and then only the second phase was injected to displace the first phase. The present study presents formulae for this specific case as well as for an initially partially saturated core and for simultaneous injection of the two immiscible phases at variable and constant rates, or a single phase.
- 4) The previous methods assumed constant density and viscosity for the immiscible phases. The present formulation considers fully variable properties. However, the formulae for the constant properties case are also provided. Therefore, the present formulation is applicable for gas/liquid and liquid/liquid systems.
- 5) The present method allows for determination of relative permeabilities and capillary

pressure simultaneously. However, it requires that the capillary pressure value be known at a saturation value to construct the capillary pressure curve.

The methods presented in this paper remove the limitations of the previous direct methods and reveal the type of data needed to be gathered from laboratory experiments. Thus, core experiments should be carried out to extract the type of data required by these methods. Using such experimental data, the validation of the methods presented in this paper can be accomplished with confidence.

APPENDIX 7.A

Inferring Function and Function Derivative Values from Average Function Values

The following is an extension of the procedure given by Jones and Roszelle⁶⁴ to determine $\partial f/\partial x$ from $\bar{f}$.

According to the mean value theorem

$$\bar{f} = \frac{1}{x} \int_0^x f dx \quad (7.A1)$$

from which we obtain by differentiating

$$f = \bar{f} + x \frac{\partial \bar{f}}{\partial x} \quad (7.A2)$$

and

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(\bar{f} + x \frac{\partial \bar{f}}{\partial x} \right) \quad (7.A3)$$

Eq. 7.A2 can be reformulated as follows:

$$f = \bar{f} + x \frac{\partial \bar{f}}{\partial x} \frac{\partial(Q/x)}{\partial(Q/x)} \quad (A4)$$

$$f = \bar{f} + x \frac{\partial \bar{f}}{\partial(Q/x)} \frac{\partial(Q/x)}{\partial x}$$

Similarly, Eq. 7.A3 leads to:

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial(Q/x)}{\partial(Q/x)} \frac{\partial}{\partial x} \left(\bar{f} + x \frac{\partial \bar{f}}{\partial x} \frac{\partial(Q/x)}{\partial(Q/x)} \right) \\ &= \frac{\partial(Q/x)}{\partial x} \frac{\partial}{\partial(Q/x)} \left(\bar{f} + x \frac{\partial \bar{f}}{\partial(Q/x)} \frac{\partial(Q/x)}{\partial x} \right)\end{aligned}\quad (7.A5)$$

Now consider the following:

$$\partial \left(\frac{Q}{x} \right) = \frac{1}{x} \partial Q - \frac{Q}{x^2} \partial x \quad (7.A6)$$

Thus

$$\frac{\partial(Q/x)}{\partial x} = \frac{1}{x} \frac{\partial Q}{\partial x} - \frac{Q}{x^2} \quad (7.A7)$$

Since $Q = Q(t)$ only, Eq. 7.A7 yields

$$\frac{\partial(Q/x)}{\partial x} = - \frac{Q}{x^2} \quad (7.A8)$$

Substituting Eq. 7.A8, Eqs. 7.A4 and 7.A5, respectively, become:

$$f = \bar{f} - \frac{Q}{x} \frac{\partial \bar{f}}{\partial(Q/x)} \quad (7.A9)$$

$$\frac{\partial f}{\partial x} = - \frac{Q}{x^2} \frac{\partial}{\partial(Q/x)} \left[\bar{f} - \frac{Q}{x} \frac{\partial \bar{f}}{\partial(Q/x)} \right] \quad (A10)$$

Eq. 7.A10 can be expanded to obtain

$$\frac{\partial f}{\partial x} = \frac{Q^2}{x^3} \frac{\partial^2 \bar{f}}{\partial(Q/x)^2} \quad (7.A11)$$

Applying Eq. 7.A6 at $x = L$ yields

$$\partial(Q/x)_{x=L} = \frac{1}{L} \partial Q \quad (7.A12)$$

Thus, substituting Eq. 7.A12, Eqs. 7.A9 and 7.A11 at $x = L$, respectively, become:

$$f|_{x=L} = \bar{f}_L - Q \frac{d\bar{f}}{dQ} \quad (7.A13)$$

$$\left. \frac{\partial f}{\partial x} \right|_{x=L} = \frac{Q^2}{L^2} \frac{d^2 \bar{f}}{dQ^2} \quad (7.A14)$$

APPENDIX 7.B

Inertial flow coefficient, β or η

The relationship between β and η is simply

$$\eta = 1/\beta \quad (7.B1)$$

in which β is given by Tek et al¹⁰⁰.

$$\beta = \frac{3.21 \times 10^4}{k^{1.25} \phi^{0.75}} \quad (7.B2)$$

Interfacial Drag Force, F_{12}

Schulenberg and Muller⁹⁷ derived an empirical correlation for the interfacial drag force, F_{12} , considering that the relevant quantities are the buoyant force, $(\rho_1 - \rho_2)g$; the viscous force in the liquid phase with respect to the relative velocity of the phases, $(\mu_1/k) (u_2/S_2 - u_1/S_1)$; the inertial force in the liquid phase with respect to the phases, $(\rho_1/\eta) (u_2/S_2 - u_1/S_1)^2$; and the capillary force, σ/k . Their equation can be written, using Eqs. 7.17 and 7.18, as:

$$F_{12} = u^2 \bar{F}_{12} \quad (7.B3)$$

where

$$\bar{F}_{12} = \frac{kg}{\eta\sigma} \left(1 - \frac{\rho_2}{\rho_1} \right) \left(\frac{f_2}{S_2} - \frac{f_1}{S_1} \right)^2 W(S_1) \quad (7.B4)$$

$W(S_1)$ is an empirically determined dimensionless function of the liquid phase saturation S_1 . Schulenberg and Muller⁹⁷ determined this function for air with different liquids and porous media of packed grains to be

$$W(S_1) = W_o S_1^m (1 - S_1) \quad (7.B5)$$

with $W_o = 350$ and $m = 7$.

APPENDIX 7.C

Solution of Eqs. 7.28 and 7.29

$$k_{r_1} = a + b \left(\frac{k_{r_1}}{k_{r_2}} \right) \quad (7.C1)$$

$$\frac{k_{r_1}}{k_{r_2}} = \frac{-(ad + be - 1) \pm [(ad + be - 1)^2 - 4abde]^{1/2}}{2bd} \quad (7.C2)$$

in which

$$a = \frac{-\mu_1 f_1}{c} \left(1 + \frac{u f_1 k}{v_1 \eta} \right) \quad (7.C3)$$

$$b = \frac{\mu_2(1 - f_1)}{c} \left(1 + \frac{u(1 - f_1)k}{v_2 \eta} \right) \quad (7.C4)$$

$$c = \frac{k}{u} \left[-\frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} + (\rho_2 - \rho_1) g \sin \theta + \frac{F_{12}}{S_1 S_2} \right] \quad (7.C5)$$

$$d = \frac{1}{h} \left[\left(-\frac{\partial p_1}{\partial x} \right) \frac{1}{\mu_1} + g \sin \theta \frac{1}{v_1} - \frac{F_{12}}{\mu_1 S_1} \right] \quad (7.C6)$$

$$e = \frac{1}{h} \left[\left(-\frac{\partial p_1}{\partial x} \right) \frac{1}{\mu_2} + g \sin \theta \frac{1}{v_2} + \frac{F_{12}}{\mu_2 S_2} - \frac{1}{\mu_2} \frac{\partial p_c}{\partial S_1} \frac{\partial S_1}{\partial x} \right] \quad (7.C7)$$

$$h = \frac{u}{k} + \frac{u^2}{\eta} \left[\frac{f_1^2}{v_1} + \frac{(1 - f_1)^2}{v_2} \right] \quad (7.C8)$$

APPENDIX 7.D

Two Phase Average Pressure in a Pore Space

According to the mean value theorem the average fluid pressure p is given by

$$p = \frac{\int_A p dA}{\int_A dA} = \frac{p_1 A_1 + p_2 A_2}{A_1 + A_2} \quad (7.D1)$$

where $A = A_1 + A_2$ denotes the total pore surface area in contact with the fluid. Simplifying Eq. 7.D1, one obtains

$$p = p_1(A_1/A) + p_2(A_2/A) \quad (7.D2)$$

If we assume that the solid/fluid contact area of each phase is proportional to the phase saturations

$$A_1/A = C_1 S_1 \quad (7.D3)$$

$$A_2/A = C_2 S_2 \quad (7.D4)$$

where C_1 and C_2 are some proportionality constants, then,

$$p = p_1 C_1 S_1 + p_2 C_2 S_2 \quad (7.D5)$$

Since $S_1 + S_2 = 1$ then to obtain the Lewis and Schrefler⁷⁷ relationship $C_1 = C_2 = 1$. Lewis and Schrefler⁷⁷ recognized this more correct method but they did not present the mathematical details.

8. MODIFICATION OF THE MARLE'S METHOD FOR PERMEABILITY DETERMINATION FROM UNSTEADY-STATE NON-DARCY DISPLACEMENT DATA

8.1 Introduction

Relative permeability data are among the essential information required for simulation of multiphase fluid flow in oil and gas reservoirs. These data are usually acquired through the interpretation of laboratory core fluid flow and/or displacement data with the aid of the porous media momentum and mass balance equations. However, the interpretation methods should be suitable for the conditions of multiphase fluid system during the laboratory tests which are usually determined by the physical properties (density, viscosity, surface tension) of the fluids and the permeability of the core used. When the core and the fluid conditions require lower flow rates, then the capillary end effect should be considered. On the other hand, for higher flow rates, non-Darcy flow conditions prevail and therefore inertial effects become important while the capillary end-effects are suppressed. The popular Johnson-Bossler-Naumann (JBN)⁶¹ method and its analytical analogue given by Marle⁷⁴ fail to satisfy the either extreme cases.

Most of the previous efforts towards correcting the JBN method has been for the capillary end-effect. Among the various approaches the history matching method has been popular (Kerig and Watson⁶⁸). Almost all the attempts which facilitated this method relied upon some variations of empirical expressions, the empirical coefficients of which are determined to match the predicted flow data to those of the measured (Kerig and Watson⁶⁸). Civan and Donaldson¹⁹ modified the Marle's⁷⁴ analytical analogue of the JBN method to account for the capillary end-effect in the formulation. This method is, therefore, applicable to interpret the displacement data for low permeability cores and heavy fluids for which the flow rates are relatively low and the capillary end-effects are important.

Although a less frequent case, an interpretation method is also required for the rapid flow that may result for cases of light fluids and/or high permeability materials. Examples include flow through proppant packed fractures and flow of light fluids such as the brine/gas systems. It appears that there is no method available in this category judging from the literature. Therefore, in this section, Marle's analytical analogue of the JBN method is reformulated to include the inertial flow effects by replacing Darcy law with the Forchheimer equation. For rapid flow the capillary end-effect is negligible and therefore is omitted in the derivation. The application of the new method is demonstrated using rapid flow core data for the gas/brine systems.

8.2 Solution Neglecting the Capillary End-Effect for Constant Fluid Properties

Case I - Constant Injection Rate

Substituting $p_e = 0$ and $p_1 = p_2 = p$ and $F_{12} = u^2 F_{12}$ (Eq. 7.B3), Eq. 7.29 simplifies as:

$$-\frac{\partial p}{\partial x} = \left\{ \frac{u}{k} - g \sin \theta \left(\frac{k_{r1}}{v_1} + \frac{k_{r2}}{v_2} \right) + u^2 \left[\frac{1}{\eta} \left(\frac{f_1^2}{v_1} + \frac{f_2^2}{v_2} \right) + \hat{F}_{12} \left(\frac{k_{r1}}{\mu_1 S_1} - \frac{k_{r2}}{\mu_2 S_2} \right) \right] \right\} \div \left(\frac{k_{r1}}{\mu_1} + \frac{k_{r2}}{\mu_2} \right) \quad (8.1)$$

On the other hand, the overall pressure differential for a core length of L is given by:

$$\Delta p = \int_0^L (-\partial p / \partial x) dx \quad (8.2)$$

in which $(-\partial p / \partial x)$ is given by Eq. 8.1. For constant physical properties, ρ_1 , ρ_2 , μ_1 , and μ_2 , and $p_c = 0$, the saturation is the only variable in Eq. 8.1. Therefore, changing from variable x to S_1 in Eq. 8.1, the Buckley-Leverett equation expressing the velocity of Phase 1 at a given saturation can be expressed as (Marle⁷⁸):

$$(dx/dt)_{S_1} = (u/\phi)(df_1/dS_1)_{S_1} \quad (8.3)$$

Substituting Eq. 7.11, Eq. 8.3 becomes:

$$dx_{S_1} = (df_1/dS_1)_{S_1} dQ/(A\phi) \quad (8.4)$$

Since $f_1 = f_1(S_1)$ only, then $(df_1/dS_1)_{S_1}$ is a fixed value. Thus, integrating and applying the initial condition that

$$x_{S_1} = 0, \quad Q = 0, \quad t = 0 \quad (8.5)$$

Eq. 8.4 leads to an expression for the location of point with a given saturation value as:

$$x_{S_1} = \frac{Q}{A\phi} \left(\frac{df_1}{dS_1} \right)_{S_1} \quad (8.6)$$

from which

$$dx_{S_1} = \frac{Q}{A\phi} d \left(\frac{df_1}{dS_1} \right)_{S_1} \quad (8.7)$$

If u is a constant, then Eq. 7.11 simplifies to

$$Q = uAt \quad (8.8)$$

Substituting Eq. 8.8 into Eqs. 8.6 and 8.7 yields:

$$x_{S_1} = \frac{ut}{\phi} \left(\frac{df_1}{dS_1} \right)_{S_1} \quad (8.9)$$

and

$$dx_{s_1} = \frac{ut}{\phi} d\left(\frac{df_1}{dS_1}\right)_{s_1} \quad (8.10)$$

Substituting Eqs. 8.9 and 8.10 into Eq. 8.2, and dividing by t leads to:

$$\frac{\Delta p}{t} = \int_0^{\frac{df_1}{dS_1} = \frac{\phi L}{ut}} \left(-\frac{\partial p}{\partial x} \right) \frac{u}{\phi} d\left(\frac{df_1}{dS_1}\right) \quad (8.11)$$

Taking a derivative with respect to t and applying the Leibniz rule and substituting Eq. 8.1 into 8.11 yields the following expression which applies at $x = L$:

$$\begin{aligned} \frac{1}{L} \left(\Delta p - t \frac{d\Delta p}{dt} \right) = & \left\{ \frac{u}{k} - g \sin \theta \left(\frac{k_{r_1}}{v_1} + \frac{k_{r_2}}{v_2} \right) \right. \\ & \left. + u^2 \left[\frac{1}{\eta} \left(\frac{f_1^2}{v_1} + \frac{f_2^2}{v_2} \right) + \bar{F}_{12} \left(\frac{k_{r_1}}{\mu_1 S_1} - \frac{k_{r_2}}{\mu_2 S_2} \right) \right] + \left(\frac{k_{r_1}}{\mu_1} + \frac{k_{r_2}}{\mu_2} \right) \right\} \end{aligned} \quad (8.12)$$

Eq. 8.12 has two unknown relative permeabilities, kr_1 and kr_2 .

Eq. 7.28 provides the second equation needed to obtain solutions for kr_1 and kr_2 . A solution of Eq. 7.28 for $p_c = 0$ simultaneously with Eq. 8.12 yields

$$k_{r_1} = a + b \left(\frac{k_{r_1}}{k_{r_2}} \right) \quad (8.13)$$

$$\frac{k_{r_1}}{k_{r_2}} = \frac{-(ad + be - 1) \pm [(ad + be - 1)^2 - 4abde]^{1/2}}{2bd} \quad (7.14)$$

in which

$$a = \frac{-\mu_1 f_1}{c} \left(1 + \frac{u f_1 k}{v_1 \eta} \right) \quad (8.15)$$

$$b = \frac{\mu_2(1-f_1)}{c} \left(1 + \frac{u(1-f_1)k}{v_2\eta} \right) \quad (8.16)$$

$$c = \frac{k}{u} \left[(\rho_2 - \rho_1)g \sin \theta + \frac{F_{12}}{S_1 S_2} \right] \quad (8.17)$$

$$e = \frac{1}{h} \left[\frac{r}{u_2} + g \sin \theta \frac{1}{v_2} + \frac{F_{12}}{u_2 S_2} \right] \quad (8.19)$$

$$h = \frac{u}{k} + \frac{u^2}{\eta} \left[\frac{f_1^2}{v_1} + \frac{(1-f_1)^2}{v_2} \right] \quad (8.20)$$

$$r = \frac{1}{L} \left(\Delta p - t \frac{d\Delta p}{dt} \right) \quad (8.21)$$

Case II - Variable Injection Rate

When the total volumetric flux, u , is a variable, a solution similar to the preceding one can be obtained for a horizontal core, $\theta = 0$. Extending the procedure by Marle⁷⁹ and substituting Eqs. 7.11, 8.1, 8.6, and 8.7 into Eq. 8.2 and rearranging one obtains:

$$\frac{\Delta p}{Q \frac{dQ}{dt}} = \frac{1}{A^2 k \phi} \int_0^{\frac{df_1}{dS_1} = \frac{A \phi L}{Q}} \varphi d \left(\frac{df_1}{dS_1} \right) + \frac{1}{A^3 \phi} \frac{dQ}{dt} \int_0^{\frac{df_1}{dS_1} = \frac{A \phi L}{Q}} \psi d \left(\frac{df_1}{dS_1} \right)_{S_1} \quad (8.22)$$

in which

$$\varphi = \left(\frac{k_{r1}}{\mu_1} + \frac{k_{r2}}{\mu_2} \right)^{-1} \quad (8.23)$$

$$\psi = \left[\frac{1}{\eta} \left(\frac{f_1^2}{v_1} + \frac{f_2^2}{v_2} \right) + \bar{F}_{12} \left(\frac{k_{r1}}{\mu_1 S_1} - \frac{k_{r2}}{\mu_2 S_2} \right) \right] \phi \quad (\text{D24})$$

Taking a derivative with respect to Q and applying the Leibniz rule and rearranging, Eq. 8.22 yields:

$$\zeta = \frac{\frac{df_1}{dS_1} - \frac{A\phi L}{Q}}{\int_0^1} \psi d \left(\frac{df_1}{dS_1} \right) \quad (8.25)$$

in which

$$\zeta = A^3 \phi \frac{d}{dQ} \left[\frac{\Delta p}{Q \frac{dQ}{dt}} \right] + \frac{A^2 \phi L}{k} \frac{1}{Q^2} \left[\phi + \frac{k}{A} \frac{dQ}{dt} \psi \right] + \frac{d}{dQ} \left(\frac{dQ}{dt} \right) \quad (8.26)$$

Taking another derivative with respect to Q , Eq. 8.25 yields:

$$\frac{d\zeta}{dQ} = \psi \left(- \frac{A\phi L}{Q^2} \right) \quad (8.27)$$

In Eq. 8.26

$$\frac{d}{dQ} \left(\frac{dQ}{dt} \right) = \frac{dt}{dQ} \frac{d}{dt} \left(\frac{dQ}{dt} \right) = \left(\frac{dQ}{dt} \right)^{-1} \frac{d^2 Q}{dt^2} \quad (8.28)$$

Eqs. 7.28 and 8.27 apply at the core outlet end (i.e. $x = L$), and can be solved numerically for kr_1 and kr_2 .

9. RELATIVE PERMEABILITY AND CAPILLARY PRESSURE DATA FROM STEADY NON-DARCY FLOW OF GAS/LIQUID SYSTEMS IN LABORATORY CORES

9.1 Introduction

The recent work for interpretation of laboratory core floods has focused primarily on unsteady-state methods (Ouenes et al⁸⁶, and Richmond and Watson⁹³). However, steady-state methods are more advantageous for gas/liquid systems involving non-Darcy flow both from the point of laboratory core floods and interpretation of the data to determine relative permeabilities and capillary pressure.

Unsteady-state methods require numerical solutions of partial differential equations for immiscible flow description. Numerical solution of partial differential equations for the pertinent immiscible displacement is highly challenging and computationally intensive when accurate solutions are required. Therefore, the utilization of a forward solution for the iterative inverse problem of determining relative permeabilities and capillary pressure by history matching of laboratory core data is highly tedious and it may suffer from severe numerical stability problems unless sufficiently small time increments and grid point spacings are used. In addition, gas/liquid systems involving non-Darcy flow, unsteady-state core floods are so rapid it is difficult to collect accurate data. These difficulties are conveniently alleviated by the steady-state method. Steady-state methods are advantageous over unsteady-state methods for gas/liquid systems for several reasons. First, steady-states can be attained rapidly in the laboratory with gas/liquid systems in contrast to liquid/liquid systems. Second, steady-state allows for accurate data acquisition such as saturation profiles by NMR and CAT scanner, etc., using modern laboratory techniques. Third, the flow model on which the steady-state interpretation is based is formed from ordinary differential equations which can be solved much more accurately than the partial differential equations involved in unsteady-state methods as demonstrated by Civan and Evans²⁰ previously. Combining the convenience and accuracy of the steady-state interpretation method with history matching results in a practical and rapid method for interpretation of steady-state non-Darcy gas/liquid core flood data.

9.2 Formulation

Consider the schematic of a steady-state multiphase flow of a gas/oil/brine system through a reservoir core sample shown in Figure 9.1. In this section a steady-state fluid flow model is developed by considering inertial flow and variable physical properties for which the model parameters such as relative permeabilities, capillary pressures, and interfacial drag can be determined by history matching of the model response to laboratory core flow data obtained by a system shown in Figure 9.1. Typically, the fluid phases are injected into the core at prescribed rates. Then, the fluid pressures before the core inlet end and after the core outlet end, and the average fluid saturations in the core, or the fluid saturations along the core length, are measured at steady-state. The various fluid phases may be injected at different pressures into the distribution chamber that feeds in the fluids through the core inlet port. Since the capillary pressure is zero before the fluids enter the core, all the fluid phases assume the same pressure in the distribution chamber. However, as soon as the fluids enter the core certain saturation values, depending on the flowrates, are established

at the core side of the inlet section which in turn cause the fluid phases to attain different pressure value corresponding to the governing saturation values. Therefore, the overall pressure differential measured as the difference between the external pressures of the fluids (same for all) before they enter and after they leave the core is not the same as the internal pressure differentials of various fluid phases inside the core. Figure 9.2 is a schematic comparison of the internal and external fluid pressures. Consequently, the pressure differential data measured in this conventional way are not useful in the interpretation of steady-state core flow data. However, if the average fluid pressure along the core could be measured, then the pressures of individual phases could be estimated according to Civan and Evans²¹, by facilitating history matching as demonstrated in this study.

The steady-state mass and momentum balance equations for multiphase flow through porous media are given, respectively, by

$$m_l = \rho_l u_l = \rho_l^0 u_l^0, \quad l \in (g, o, w) \quad (9.1)$$

and

$$-\frac{dp_l}{dx} + \rho_l g \sin \theta = \frac{\mu_l}{kk_r} u_l + \frac{\rho_l}{\eta \eta_r} u_l^2 + \frac{F_l}{S_l} \quad (9.2)$$

Eq. 9.2 is given by Schulenberg and Muller⁹⁷. l denotes a phase. m and u are mass and volume fluxes, ρ and μ are density and viscosity, k_r and η_r are relative permeability and relative inertial flow, respectively. p is pressure. k is the absolute permeability. η is the inertial flow coefficient. g is the gravitational acceleration. θ is the angle of inclination of the core. F_l is the interfacial drag between the l_{th} phase and the other phases. The superscript ⁰ on the right of Eq. 9.1 denotes the conditions of the fluids at the core inlet, $x = 0$.

Invoking Eq. 9.1, Eq. 9.2 can be written as

$$-\frac{dp_l}{dx} + \rho_l g \sin \theta = \frac{\mu_l}{kk_r} \frac{m_l}{\rho_l} + \frac{\rho_l}{\eta \eta_r} \left(\frac{m_l}{\rho_l} \right)^2 + \frac{F_l}{S_l} \quad (9.3)$$

The saturations, S_l , of the various fluid phases add up to one:

$$\sum_{l=1}^N S_l = 1.0 \quad (9.4)$$

N denotes the number of phases.

The capillary pressures between various phases are defined by empirical functions of the fluid saturations as:

$$p_{c_{in}}(S_l, l=1, \dots, N-1) = p_n - p_l \quad , \quad l \in (g, o, w) \text{ and } n \neq l \quad (9.5)$$

P_n and P_l denote the nonwetting and wetting phase pressures, respectively.

The relative permeabilities are defined in terms of the fluid saturations as:

$$k_{r_l} = k_{r_l}(S_l, l=1, \dots, N-1) \quad (9.6)$$

At isothermal conditions, the fluid density and viscosity are only functions of pressure:

$$\rho_l = \rho(p_l) \quad (9.7)$$

$$\mu_l = \mu(p_l) \quad (9.8)$$

The fluid phase pressures can be estimated from the saturation weighted fluid pressure measured along the core as follows by generalizing the Lewis and Schrefler⁷⁷ approach. Consider a saturation weighted multiphase fluid pressure defined by

$$p = \sum_{l=1}^N S_l p_l \quad (9.9)$$

$$p_{c_{in}} = p_n = p_l \quad , \quad l = 1, 2, \dots, (N-1) \quad (10)$$

Eliminating all but one p_l by using Eqs. 9.10 and 9.9 leads to

$$p_l = p - \sum_{\substack{n=1 \\ n \neq l}}^N S_n p_{c_{in}} \quad , \quad l = 1, 2, \dots, (N-1) \quad (9.11)$$

from which the following can be obtained:

$$\frac{dp_l}{dx} = \frac{dp}{dx} - \sum_{\substack{n=1 \\ n \neq l}}^N \frac{\partial S_n}{\partial x} \left(p_{c_{in}} + S_n \frac{dp_{c_{in}}}{dS_n} \right) \quad (8.12)$$

Note that p_c is the static capillary pressure.

A solution of Eqs. 9.1, 9.3, 9.4, 9.5, and 9.6 for p_l , S_l , and u_l requires two boundary conditions. At the inlet, the fluids are fed into the core at prescribed pressures, p_l^0 , and volumetric flow rates, q_l^0 , as shown in Fig. 9.1. At the outlet, the fluids are discharged into a prescribed outlet pressure, P^L .

The boundary conditions at the core inlet are given as follows.

Before the choke in Fig. 9.1:

$$p_l = p_l^0, \quad u_l = q_l^0/A, \quad l=1,\dots,N \quad (9.13)$$

Thus, by means of Eqs. 9.1, 9.7 and 9.13, the steady-state mass flux of fluids through the core can be quantified as:

$$m_l = \rho_l^0 u_l^0 \quad (9.14)$$

After the choke in Fig. 9.1, the fluid phases attain the same pressure in the distribution chamber before entering the core:

$$p_l = p_{in}, \quad l=1,\dots,N, \quad x=0^+ \quad (9.15)$$

The boundary condition at the core outlet is given by the same pressure for all the fluid phases in the collection chamber.

$$p_l = p_{out}, \quad l=1,\dots,N, \quad x=L \quad (9.16)$$

The equations represented by Eq. 9.3 are first order ordinary differential equations that can be solved simultaneously for the fluid pressures via an appropriate numerical method if the flow parameters (relative permeabilities, capillary pressures, and interfacial drag data) are given. Then, the fluid saturations can be calculated implicitly by means of Eq. 9.5. This is referred to as the forward solution. However, the primary objective of the present study is to estimate the flow parameters by matching the model response to a set of experimental data. The latter is referred to as the inverse problem.

Experimental data can be obtained in a variety of forms depending on the availability of facilities. In a conventional approach a series of steady-state flow tests are carried out using a range of inlet and outlet conditions and the quantities defined by Eqs. 9.13 and 9.16, and the average fluid saturations, S_i , or the saturation profiles along the core, S_i vs. s , are measured. Then, the flow parameters (relative permeability and capillary pressure) are determined to match the model predictions to the measured data.

The advantage of using the internal core data such as the fluid pressure and saturation distributions along the core by means of NMR, Catscanner, and ultrasonic imaging methods is that one set of data obtained by a single flow test may be sufficient to estimate all the flow parameters. In this case, there are two choices to determine the flow parameters. The first is the numerical solution of the ordinary differential equations, Eq. 9.3 for history matching. The second is a direct method in which Eq. 9.3 is solved algebraically upon the evaluation of the derivatives dp/dx , directly from the measured data.

9.3 Indirect and Direct Methods for Two-Phase Systems

For purposes of illustration, consider a brine/gas system. Let the subscripts ₁ and ₂ denote the brine and gas phases, respectively, for which Eq. 9.3 becomes

$$-\frac{dp_1}{dx} + \rho_1 g \sin \theta = \frac{\mu_1}{kk_{r_1}} \frac{m_1}{\rho_1} + \frac{\rho_1}{\eta \eta_{r_1}} \left(\frac{m_1}{\rho_1} \right)^2 + \frac{F_{12}}{S_1} \quad (9.17)$$

and

$$-\frac{dp_2}{dx} + \rho_2 g \sin \theta = \frac{\mu_2}{kk_{r_2}} \frac{m_2}{\rho_2} + \frac{\rho_2}{\eta \eta_{r_2}} \left(\frac{m_2}{\rho_2} \right)^2 + \frac{F_{21}}{S_2} \quad (9.18)$$

Note that the interfacial drag forces are equal but opposite. Thus,

$$F_{12} = -F_{21} \quad (9.19)$$

Eq. 9.5 becomes

$$p_c = p_2 - p_1 \quad (9.20)$$

In the cases where only the pressure and saturation of one of the fluid phases can be measured, it is advantageous to use the following equation obtained from Eqs. 9.17 - 9.20:

$$\begin{aligned} \frac{dp_c}{dS_1} \frac{dS_1}{dx} + (\rho_1 - \rho_2) g \sin \theta &= \frac{\mu_1}{kk_{r_1}} \frac{m_1}{\rho_1} + \frac{\rho_1}{\eta \eta_{r_1}} \left(\frac{m_1}{\rho_1} \right)^2 \\ &- \frac{\mu_2}{kk_{r_2}} \frac{m_2}{\rho_2} - \frac{\rho_2}{\eta \eta_{r_2}} \left(\frac{m_2}{\rho_2} \right)^2 + \frac{F_{12}}{S_1 S_2} \end{aligned} \quad (9.21)$$

In the following the indirect and direct applications of Eqs. 9.17 and 9.21 are presented for interpretation of steady-state core flow data.

9.3.1 Indirect Method

This method is used for determination of relative permeability and capillary pressure by history matching.

For convenience in forward solution, Eqs. 9.17 and 9.21 are rearranged, respectively, as:

$$\frac{dp_1}{dx} = - \frac{v_1 m_1}{kk_{r_1}} \left(1 + \frac{m_1 k}{\mu_1 \eta} \right) - \frac{F_{12}}{S_1} + \rho_1 g \sin \theta \quad (9.22)$$

and

$$\frac{dS_1}{dx} = \left(\frac{dp_c}{dS_1} \right)^{-1} \left[\frac{v_1 m_1}{k k_{r_1}} \left(1 + \frac{m_1 k}{\mu_1 \eta} \right) - \frac{v_2 m_2}{k k_{r_2}} \left(1 + \frac{m_2 k}{\mu_2 \eta} \right) + \frac{F_{12}}{S_1 S_2} - (\rho_1 - \rho_2) g \sin \theta \right] \quad (9.23)$$

The saturation, S_1 , at the inlet side of the core can be determined using the boundary condition given by Richardson et al⁹²:

$$dS_1/dx=0 \quad , \quad x = 0 \quad (9.24)$$

Applying Eq. 9.23 leads to the following implicit expression for the saturation value at the core inlet:

$$f_1(S_1, S_2, p_1, p_2) = \frac{v_1 m_1}{k k_{r_1}} \left(1 + \frac{m_1 k}{\mu_1 \eta} \right) - \frac{v_2 m_2}{k k_{r_2}} \left(1 + \frac{m_2 k}{\mu_2 \eta} \right) + \frac{F_{12}}{S_1 S_2} - (\rho_1 - \rho_2) g \sin \theta = 0 \quad , \quad x=0 \quad (9.25)$$

On the other hand, the static capillary pressure is given as a function of saturation as:

$$f_2(S_1, p_1, p_2) = p_c(S_1) - (p_2 - p_1) = 0 \quad (9.26)$$

By definition of saturations, there exists

$$f_3 = 1 - S_1 - S_2 = 0 \quad (9.27)$$

The p_1 value at $x = 0$ is guessed.

Since the external and internal fluid pressures are different at the core inlet and outlet faces the internal pressures at the core sides are not known. That is why p_1 is guessed until the measured saturations are matched by the solution of the model. Simultaneously, Eq. 9.22 must be satisfied at $x = 0$.

Typical sketches of pressure and saturation profiles are given in Fig. 9.3.

The interfacial drag relationship by Schulenberger and Muller⁹⁷ is given as:

$$F_{12} = W(S_1) \frac{k}{\eta \sigma} \left(\frac{u_2}{S_2} - \frac{u_1}{S_1} \right)^2 \left(1 - \frac{\rho_2}{\rho_1} \right) g \quad (9.28)$$

where W is a dimensionless number given by

$$W(S_1) = W_0 S_1^m (1 - S_1) \quad (9.29)$$

with $W_0 = 350$ and $m = 7$.

The inertial flow coefficient is given by Tek, et al¹⁰⁰ as:

$$\beta = \frac{1}{\eta} = \frac{3.21 \times 10^4}{k^{1.25} \phi^{0.75}} \quad (9.30)$$

A solution of Eq. 9.25-9.27 using the Newton-Raphson method by Carnahan et al¹¹ yields the saturation value, S_1 , at the core inlet. This value is then used as the boundary value for the solution of Eq. 9.20. Then, Eqs. 9.22 and 9.23 can be integrated numerically using an appropriate method. In the present study, a Runge-Kutta-Fehlberg Four (Five)⁴⁴ method has been used. Typical saturation profiles are sketched in Fig. 9.4 which shows the capillary end effect for low, medium and high flow rates.

The method described above is for solving the forward problem to calculate the steady-state saturation and pressure profiles along the core if the data on $k_{r,l}$ are $\eta_{r,l} l = 1, 2$, and F_{12} are known.

However, the most interesting problem is the solution of the inverse to determine $k_{r,l}$ and $\eta_{r,l} l = 1, 2$ and F_{12} data such that the pressure and saturation values predicted by the forward solution of Eqs. 9.22 and 9.23 match those measured by laboratory core flood tests. In a conventional steady-state test the pressure differential across the core, Δp , and the average saturation, S_1 , are measured for a range of prescribed fluid mass fluxes, m_1 and m_2 . However, NMR, X-Ray, and ultrasonic imaging techniques can reveal internal core data on the saturation along the core length. Pressure transducers mounted along the core can measure the saturation weighted fluid pressure.

Then, the fluid 1 phase pressure can be estimated by:

$$p_1 = p - (1 - S_1)p_c \quad (9.31)$$

from which the following can be obtained:

$$\frac{dp_1}{dx} = \frac{dp}{dx} + \left[p_c - (1 - S_1) \frac{dp_c}{dS_1} \right] \frac{\partial S_1}{\partial x} \quad (9.32)$$

For history matching purposes two approaches can be taken. In the first approach, the relative permeability and capillary pressure data are represented by empirical functions and the parameters in them are determined by history matching.

For the purpose of an application consider the empirical relationship given as:

$$\eta_{r_l} = k_{r_l} = k_{r_l}^0 \left[\frac{S_l - S_{lr}}{1 - \sum_{l=1}^N S_{lr}} \right]^{n_l}, \quad l=1,2 \quad (9.33)$$

$k_{r_l}^0$ is the end-point relative permeability and S_{lr} denotes the irreducible fluid saturation.

According to Donaldson et al²⁹ capillary pressure curve can be represented by a three-constant hyperbolic function:

$$p_c = (a + bS_l)/(1 + cS_l) \quad (9.34)$$

In the second approach, the discrete values of the relative permeability and capillary pressure at prescribed saturation values are determined by history matching. This approach is more accurate because preassumed forms such as Eqs. 9.33 and 9.34 are not imposed.

In the present study the Marquardt-Levenberg⁵⁰ algorithm was used to determine the best values of k_{r_l} , $l = 1, 2$, and F_{12} so that the following objective function is minimized:

$$J(X) = [Y_m - Y_c]^T W [Y_m - Y_c] \quad (9.35)$$

in which X denotes the vector of the parameters to be estimated, i.e.

$$X = [k_{r_{ln}}, l = 1, 2, \text{ and } c_{pn} \text{ for } n = 1, 2, \dots, N, F_{12}]^T \quad (9.36)$$

where n denotes the discrete saturation values $S_{b_{ln}}$, selected for parameter estimation. Y denotes the vector of the matching values, i.e.

$$Y = [(\Delta p_j; \overline{S}_1 \text{ or } S_{1j}, j = 1, 2, \dots, M)^{t_1}, \\ (\Delta p_j; \overline{S}_1 \text{ or } S_{1j}, j = 1, 2, \dots, M)^{t_2}, \dots]^T \quad (9.37)$$

where j denotes the discrete points selected along the core, and $t_1, t_2, \dots$, denote the consecutive tests considered for a variety of u_{b1} and u_{b2} values, Y_m and Y_c are the measured and calculated values of the Y vectors.

As an initial guess for $k_{r_{ln}}$ and c_{pn} , $n = 1, 2, \dots, N$ and $l = 1, 2$ linearly distributed values are assigned at saturation values such as evenly spaced, and $F_{12} = 0$. Then an IMSL routine⁵⁰ implementing the Marquardt-Levenberg algorithm is used to determine the best estimates of these parameters.

9.3.2 Direct Method

In this method the internal core fluid data are facilitated to directly determine dp_i/dx and dS_l/dx using the actual p_i and S_l values at various locations along the core and Eqs. 9.31

and 9.32. These values are then substituted into the following equations. From Eq. 9.22:

$$k_{r_1} = \frac{v_1 m_1}{k} \left(1 + \frac{m_1 k}{\mu_1 \eta} \right) + \left[-\frac{dp_1}{dx} + \rho_1 g \sin \theta - \frac{F_{12}}{S_1} \right] \quad (9.38)$$

From Eqs. 9.18 and 9.20:

$$k_{r_2} = \frac{v_2 m_2}{k} \left(1 + \frac{m_2 k}{\mu_2 \eta} \right) + \left[-\frac{dp_1}{dx} - \frac{dp_c}{dS_1} \frac{dS_1}{dx} + \rho_2 g \sin \theta + \frac{F_{12}}{S_2} \right] \quad (9.39)$$

Alternatively, from Eq. 9.23:

$$k_{r_2} = \frac{v_2 m_2}{k} \left(1 + \frac{m_2 k}{\mu_2 \eta} \right) + \left[\frac{v_1 m_1}{k k_{r_1}} \left(1 + \frac{m_1 k}{\mu_1 \eta} \right) - \frac{dp_c}{dS_1} \frac{dS_1}{dx} + \frac{F_{12}}{S_1 S_2} - (\rho_1 - \rho_2) g \sin \theta \right] \quad (9.40)$$

The discrete values obtained from Eqs. 9.38 and 9.39 or 9.40 are plotted to construct the charts for k_r vs S_1 and F_{12} vs S_1 .

9.4 Application

A review of the literature on gas/liquid flow in sandstones revealed that there is not any reliable data available. Hence, for demonstration purposes, the data obtained from the literature will be utilized with reasonable assumptions on any missing data that the computational methods require.

Arastoopour and Semrau³ simulated the flow of water and nitrogen gas through a horizontal, tight sand core. The data from Arastoopour and Semrau³ with typical assumptions to make up for the missing data are presented in Table 9.1.

For determining the permeability and capillary pressure by the steady-state method the measured steady-state values of the pressure differential, Δp are the average saturation, $\bar{S}_{bi}$, are required for a set of values of injection fluxes of Phase 1 and 2. Due to the lack of reliable data, a set of simulated experimental data was derived and is shown in Table 9.2 by solving Eqs. 9.22 and 9.23 simultaneously over the core length by means of the Runge-Kutta-Fehlberg method. A few of the typical saturation profiles are shown in Fig. 9.4. The data given in Table 9.2 was then used as a substitute for experimental data for the steady-state method. As the initial guess for the values of the effective permeabilities and the capillary pressures, linearly varying discrete values were assumed over $N = 7$ sample points evenly distributed over the saturation range of $S_{1i} < S_1 < (1 - S_{2i})$ expressed at the base conditions. (See Fig. 9.5) These values were used in Eq. 9.37 to start the search for the optimal estimation of the discrete effective permeability and capillary pressure values in Eq. 9.36 using the IMSL routine⁵⁰ for the Marquardt-Levenberg algorithm based on the objective function defined by Eq. 9.35. During the computations intermediate values of the effective

permeabilities and the capillary pressures were obtained by interpolation of the discrete values at the sample points. As the convergence criteria an upper limit of 1.0×10^{-2} was considered for the objective function. The absolute relative errors between the estimated and the actual values of the effective permeabilities and the capillary pressures were in the order of 10^{-2} . The predicted discrete values and the actual curves are shown in Fig. 9.5.

9.5 Conclusion

The theoretical analysis of steady-state flow of compressible and variable viscosity immiscible phases through a porous core indicated that the internal and external fluid pressures are different and therefore the externally measured fluid pressure cannot be used as the boundary condition. The continuity equation is reduced to prescribed constant mass fluxes of the individual phases. The boundary values for the fluid saturations at the core inlet section depend on the mass fluxes of the fluid phases but cannot be specified rigorously. In the present study constant saturation near the injection end; i.e. $S_1 = \text{constant}$, $dS_1/dx = 0$, was assumed according to Richardson et al⁸² to determine the S_1 value at the inlet end of core. Hence, $dS_1/dx = 0$ is nothing more than an assumption. Based on this discussion it can be concluded that the saturation and pressure boundary conditions cannot be rigorously specified.

10. INDIRECT INTERPRETATION METHOD FOR DETERMINATION OF RELATIVE PERMEABILITY AND CAPILLARY PRESSURE FROM UNSTEADY-STATE NON-DARCY FLOW IN LABORATORY CORES

10.1 Introduction

Relative permeability and capillary pressure are the primary flow parameters required to model multiphase flow in porous media. They can be obtained by laboratory experiments by steady state or unsteady state methods. Both have some disadvantages. Steady state methods are time consuming. Unsteady state methods do not provide data over the full saturation range. Results from these experiments are valid for the macroscopic flow description by omitting the details of the physical process in the pore structure level. The usual assumptions include that each fluid remains continuous, all flow is in the same direction, there is no formation damage, the sample cores are representative of interwell reservoir rock and fluid properties, and the reservoir conditions are similar to the laboratory conditions.

Numerous investigations have been conducted to study these assumptions and estimate the capillary pressure and relative permeability simultaneously. Efforts have been made to gather internal core data such as saturation and pressure history profiles which can be measured along the core by gamma-ray attenuation technique, CT scanning, or NMR imaging in addition to the overall pressure drop and production data. Various functional representation of relative permeability and capillary pressure curves with a variety of interpretation methods and optimization techniques have also been investigated.

Kreig and Watson⁸⁸ used a regression based method to estimate relative permeability curves using a spline function representation. Richmond and Watson⁹³ extended this regression based method for simultaneous estimation of functional representation of capillary pressure and relative permeability curves. Chardaire et al¹² used piece wise linear functions to determine the relative permeabilities and capillary pressure simultaneously by using a multiscale representation of parameters. Chavent¹⁴ has proposed an automatic adjustment method to determine the capillary pressure and relative permeability by use of the optimal control theory. Ouenes et al⁸⁶ used a discrete representation of the relative permeability and capillary pressure using simulated annealing for Darcy flow of incompressible fluids.

For gas/liquid systems, non-Darcy flow and gas solubility effects are important to analyze their effects on the shape of relative permeability and capillary pressure curves and to represent the real phenomena occurring in reservoir condition. Capillary pressure curves are especially important to determine the fluid distribution in the reservoir and capillary trapping mechanisms in the reservoir. However, capillary pressure is usually neglected in the relative permeability measurements by conventional methods. Neglecting the capillary pressure make the relative permeability curves independent of the flow rate for Darcy flow and no Klinkenberg effect cases. Capillary pressure may be neglected for non-Darcy flow since viscous forces will dominate over capillary forces. But for special cases, such as during flow through tight gas reservoirs or naturally fractured reservoirs, capillary forces will be as important as other parameters.

It has been shown that saturation profiles and recovery curves from laboratory tests can be matched with different forms of relative permeability curves and capillary pressure curves. Frioozabadi et al⁴⁶ have shown that entirely different relative permeability models could match the same recovery performance. Thus, using only the recovery curves and pressure differential data does not guarantee a unique set of relative permeability and capillary pressure curves. But, the internal core data along with the conventional overall pressure and effluent fluid production data can help to achieve unique solutions.

In this study, relative permeability and capillary pressure curves are generated under different flow regimes, Darcy and non-Darcy, and for imbibition and drainage processes. The predictability and uniqueness of the flow functions are investigated by using the recovery curves and overall pressure differential data, the saturation history profiles data, and the saturation history profiles and pressure differential data. To the best of the authors' knowledge, relative permeability and capillary pressure have not been studied simultaneously under gas solubility and non-Darcy flow regime. Therefore, this study is aimed at developing numerical methods to estimate relative permeability and capillary pressure, simultaneously under these conditions. The Forchheimer equation has been modified for non-Darcy flow by a non-Darcy flow number for multiphase flow. The methodology has been illustrated by three applications involving unsteady-state core floods.

10.2 Formulation of Gas/Liquid Flow in Porous Media

The equations describing the flow of gas/water systems are derived based on the following assumptions: 1) no chemical reaction, 2) no adsorption, 3) only gas in the gaseous phase, water will not vaporize in gas phase, 4) only gaseous and aqueous phase exist and gas can dissolve in water, 5) isothermal flow, 6) compressible fluid and rock, 7) Newtonian fluids and Darcy law applicable, (Note that the correction for non-Darcy flow is given later) 8) one dimensional flow, 9) permeability and porosity distribution are known and there is no channeling due to permeability or porosity contrast.

The mass conservation equations for water and gas components are given respectively as:

$$\frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) + \frac{\partial}{\partial x} \left(\frac{u_w}{B_w} \right) + q_w = 0 \quad (10.1)$$

and

$$\frac{\partial}{\partial t} \left(\frac{\phi R_s S_w}{B_w} + \frac{\phi S_g}{B_g} \right) + \frac{\partial}{\partial x} \left(\frac{R_s u_w}{B_w} + \frac{R_s u_g}{B_g} \right) + q_g = 0 \quad (10.2)$$

S_w and S_g are the saturations of water and gas respectively; and U_w and U_g are the Darcy velocities of water and gas respectively. B_w , B_g and R_s are formation volume factor of gas and water, respectively. q_w and q_g denote the water and gas sources. The saturations add up to one:

$$S_w + S_g = 1 \quad (10.3)$$

The capillary pressure is given by

$$p_c = p_g - p_w \quad (10.4)$$

The formation volume factors for water and gas phase are given by

$$B_w = \frac{\rho_w^s}{\rho_w} \quad (10.5)$$

$$B_g = \frac{\rho_g^s}{\rho_g} \quad (10.6)$$

The gas solubility in water is expressed by

$$R_s = \frac{\omega_{gas,water} \rho_w^s}{\rho_g^s} \quad (10.7)$$

The Darcy equations for water and gas phases are given by

$$u_g = \frac{kk_{rg}}{\mu_g} \left(\frac{\partial p_g}{\partial x} - \gamma_g \frac{\partial z}{\partial x} \right) \quad (10.8)$$

$$u_w = \frac{kk_{rw}}{\mu_w} \left(\frac{\partial p_w}{\partial x} - \gamma_w \frac{\partial z}{\partial x} \right) \quad (10.9)$$

in Equations 10.8 and 10.9, μ_w and μ_g are the viscosities of water and gas; k_{rw} and k_{rg} are the relative permeabilities for the water and gas phases; and p_w and p_g are the pressures of water and oil phases; and γ_w and γ_g are the specific weights of water and gas, respectively. Combining the Darcy equations with the mass conservation equations for each phase yields the following pressure-saturation equations:

$$\frac{\partial}{\partial x} \left(K_w \frac{\partial p_w}{\partial x} + \gamma_w \frac{\partial z}{\partial x} \right) + q_w = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) \quad (10.10)$$

and

$$\frac{\partial}{\partial x} \left(K_g \frac{\partial p_g}{\partial x} + \gamma_g \frac{\partial z}{\partial x} \right) + \frac{\partial}{\partial x} \left(K_w R_s \frac{\partial p_w}{\partial x} + \gamma_w \frac{\partial z}{\partial x} \right) + q_g = \frac{\partial}{\partial t} \left(\frac{\phi S_g}{B_g} + \frac{\phi R_s S_w}{B_w} \right) \quad (10.11)$$

where K_w and K_g are conductivity of water and gas, respectively, and:

$$K_g = \frac{k k_{rg}}{B_g \mu_g} \quad (10.12)$$

$$K_w = \frac{k k_{rw}}{B_w \mu_w} \quad (10.13)$$

For numerical solution purposes the length of the core is divided into N equal blocks with the grid points located in the centers of these blocks. Thus, there are no points at the boundary. The boundary conditions can be specified as constant rate or pressure. The source and sink terms, q_w and q_g , are taken zero everywhere except at the inlet and outlet. A no flow boundary condition is used at the inlet, given by

$$\frac{\partial p_w}{\partial x} = 0, \quad x = 0, \quad t \geq 0 \quad (10.14)$$

The boundary condition at the outlet for pressure before breakthrough is given by

$$\frac{\partial p_w}{\partial x} = 0, \quad x = L, \quad t \geq 0 \quad (10.15)$$

and after breakthrough, by

$$p_w = p_{\text{specified}} \quad x = L, \quad t \geq 0 \quad (10.16)$$

The initial condition for the pressure and saturation equations are given by

$$p_w = p_{\text{initial}} \quad 0 \leq x \leq L, \quad t = 0 \quad (10.17)$$

$$S_w = S_{\text{initial}} \quad 0 \leq x \leq L, \quad t = 0 \quad (10.18)$$

For the numerical solution an implicit pressure and explicit saturation (IMPES) method is used. These pressure equations are discretized and matrix forms are obtained. The boundary conditions for each phase of water and gas can be either pressure specified or rate specified. The IMPES technique (Aziz and Settari⁴) is used to solve the pressure and saturation equations.

The additional pressure drop that occurs during high-velocity flow of fluid in porous media can be taken into account by a correction according to the Forchheimer Equation. For single phase flow, this equation reads as:

$$-\frac{\partial p}{\partial x} = \frac{\mu}{k}u + \beta \rho u^2 \quad (10.19)$$

To express the apparent pressure drop under Non-Darcy flow conditions, the non-Darcy flow number is used:

$$F_{nd} = \left(1 + \frac{\beta k \rho u}{\eta}\right)^{-1} \quad (10.20)$$

where β is the non-Darcy flow coefficient. If β or u approaches zero, then F_{nd} approaches 1, and the flow assumes Darcy flow. This approach can be extended for multiphase flow as

$$-\frac{\partial p_e}{\partial x} = \frac{\mu_e}{k k_{re}} u_e + \frac{1}{\eta \eta_{re}} \rho_e u_e^2; e = gas, water \quad (10.21)$$

In Equation 10.21, k_{re} is relative permeability for Darcy flow; η_{re} is relative non-Darcy flow coefficient for each phase. η is the inverse of β . Schulenberg and Muller⁹⁷ assumed that η accounts for the turbulent pressure drop of a single phase flow through porous media. They associated this term with the passability of porous media. Then, they defined relative passabilities η_{rg} and η_{rw} as functions of the effective saturation and particle shape. They approximated the relative passabilities of water and gas as:

$$\eta_{rw} = S^5 \quad (10.22)$$

$$\eta_{rg} = (1-S)^6 \quad (10.23)$$

where S is the water saturation.

The non-Darcy flow number for multiphase flow can be given as:

$$(F_{nd})_e = \left(1 + \frac{k k_{re} \rho_e u_e}{\eta \eta_{re} \mu_e}\right)^{-1} \quad e = gas, water \quad (10.24)$$

Chung and Catton¹⁶ considered that k_{re} and η_{re} are very close to each other and assumed they were equal. After this assumption the non-Darcy flow number for multiphase flow becomes:

$$(F_{nd})_e = \left(1 + \frac{k \rho_e u_e}{\eta \mu_e}\right)^{-1} \quad e = gas, water \quad (10.25)$$

in which

$$\eta = \frac{1}{\beta} \quad (10.26)$$

Eq. 10.25 for multiphase flow is similar to Eq. 10.20 for single phase flow. However, each phase has different viscosities, velocities and densities. Thus, to account for the non-Darcy effect in Eqs. 10.12 and 10.13, k_{rg} and k_{rw} must be replaced by $k_{rg}(F_{nd})_g$ and $k_{rw}(F_{nd})_w$.

10.3 Parameter Estimation Using the Simulated Annealing Method

The objective of the simulated annealing method is to transform an initial configuration of a stochastic field, into a configuration which allows a certain objective function to approach a global optimum (Farmer³⁹).

Simulated annealing is an optimization method developed by analogy to the physical annealing of solids undergoing a thermal process to attain lower energy states in a heat bath. The annealing process consists of two steps (Kirkpatrick et al.⁷¹, Aarts¹).

First, the temperature of the heat bath is increased to a value at which the solids melt, and then the temperature of the heat bath is carefully decreased until the particles rearrange themselves in the ground state of solid (Aarts, 1992).

In the liquid phase, the particles are distributed randomly, whereas in the ground state of solid, the particles are arranged in a highly structure lattice form. At this state the corresponding energy is minimal. Metropolis et al.⁸⁰ introduced an acceptance rule based on the probability of reaching lower energy states to search for a minimum similar to the rate of probability of reaching lower energy states to attain a thermal equilibrium given by

$$k = \exp\left(\frac{E_i - E_j}{k_b T}\right) \quad (10.27)$$

E_i is a current state of solid energy. E_j is a subsequent state of solid energy to be attained when the $(E_j - E_i)$ is less than or equal to zero. T is the temperature of the heat bath and k_b is physical constant. Temperature is lowered slowly to allow the solid to reach a thermal equilibrium.

The application begins with generating an initial random realization by using the cumulative distribution function and a perturbation is achieved by appropriate permutations on the randomly chosen node. Then, the new objective function is calculated and the change in the new objective function, ΔJ , is reduced according to the Metropolis acceptance rule. According to the acceptance rule, if $\Delta J < 0$, keep the new configuration. If $\Delta J > 0$, the new configuration is kept with a probability equal to

$$\exp\left(\frac{J_i - J_j}{T}\right) \quad (10.28)$$

Where T is the annealing temperature. The algorithm begins with an initial temperature, T_0 and T_0 is reduced by a chosen reduction factor which can be adjusted during the optimization procedure.

The objective function can be expressed in terms of saturation profiles, production or pressure history.

$$J(a) = W_q \sum_k (q_k^c - q_k^m)^2 + W_p \sum_k (\Delta p_k^c - \Delta p_k^m)^2 + W_s \sum_k (S_{k,i}^c - S_{k,i}^m)^2 \quad (10.29)$$

in which W_q , W_p and W_s are some appropriate weighting factors of production data, pressure drop and displaced phase saturation. The subscript c refers to the values computed by numerical method and m to the corresponding measured data.

Following Civan and Evans²¹, the initial flow functions are taken as linear functions of saturation and a high annealing temperature is chosen to accept the high probability of non improving solutions. The objective function (J_i), Eq. 10.29, is then calculated by means of the numerical solution of the flow model (the forward problem). The analysis of the new objective function follows according to:

1. The initial flow functions are slightly perturbed randomly. The new objective function (J_{i+1}) is calculated.
2. If the new solution poses $J_{i+1} < J_i$, then the new perturbed flow function is accepted as the new configuration unconditionally.
3. If the new solution poses $J_{i+1} > J_i$, it may still be accepted if the metropolis acceptance rule is satisfied. Otherwise, the nonimproved new configuration is rejected and a new configuration is generated randomly.

This searching process is repeated at the same temperature until all the flow function properties are tested at each discrete saturation values selected to determine the discrete values of relative permeability and capillary pressure. When the convergence of the algorithm reaches an equilibrium state (thermal equilibrium), assuring the convergence of the algorithm at current temperature, the temperature is then decreased to reduce the probability of the acceptance of nonimproved solutions. This process is continued until the lowest defined temperature is attained indicating that the minimum of the objective function has been established.

The advantage of the simulated annealing compared to other optimization methods is the introduction of a slowly decreasing acceptance of the nonimproving solutions. Initially higher nonimproved solutions are accepted and the search ends up with a gradual fine local search which accepts the only improved solutions. One of the most important key steps is the method of generating the next candidate configuration which depends on the physical problem. In this study, it is created by randomly perturbing the configurations of the

previous flow function.

10.4 Applications

Three different data sets were selected from the literature and analyzed to test the computer code for forward solution for determination of relative permeability and capillary pressure curves simultaneously using the simulated annealing method. These applications can be classified in three categories:

Case 1- Imbibition: Displacement of oil by water in a water-wet core, using (1) the recovery curves and pressure differential data, (2) the saturation history profiles data, and (3) the saturation history profiles and pressure differential data.

Case 2- Drainage I: Displacement of oil by water in an oil-wet core at Darcy and non-Darcy flow regime.

Case 3- Drainage II: Displacement of water by nitrogen in a water-wet core.

Each of these applications and the results are described in the following.

Case 1-Imbibition: Displacement of oil by water in a water-wet core.

In order to test the numerical code, simulated experimental data obtained from Richmond and Watson⁹³ was used. The rock properties and operating conditions are given in the Table 10.1. The relative permeability and capillary pressure curves, Fig. 10.1, are representative of a strongly water-wet system. Results are compared with Richmond and Watson⁹³. As shown in the Fig. 10.2 and Fig. 10.3, the pressure drop history and cumulative production histories were matched successfully. The production rate history, saturation and pressure history profiles are given in Fig. 10.4, Fig. 10.5, and Fig. 10.6 respectively. As seen from Fig. 10.4, even though the arrival time of the front to the end of core is approximately 6 minutes, the breakthrough time is approximately 7 minutes. Due to the end-effect, breakthrough time is retarded. For imbibition processes, the capillary pressure should reach zero for the wetting phase pressure equal to the non-wetting phase pressure and then the production of the injected phase will begin.

After the computer code was tested, relative permeability and capillary pressure curves were estimated under three different constraints by using the same core properties and operating conditions. First, flow functions, Fig. 10.7, were estimated by using only recovery curves and pressure differential data. Then, pressure drop and cumulative production of oil and water recalculated by using the estimated relative permeability and capillary pressure. After that, the calculated pressure drop history (Fig. 10.8) and the cumulative production history (Fig. 10.9), and saturation history profiles (Fig. 10.10) from the estimated flow properties were compared with the initial history performance. As shown in Fig. 10.8 and Fig. 10.9, a good history match was obtained for pressure drop and cumulative production history with the estimated flow function properties. However, the saturation history profiles did not match well. Even though, the swept area of both case are close to each other.

Second, flow functions, Fig. 10.11, are estimated by using only saturation history profiles. In this case, the saturation history profiles (Fig. 10.14) are matched well. As a result, cumulative production history (Fig. 10.13) was also matched well. However, the pressure drop history (Fig. 10.12) did not match since there was no pressure constraint on the estimation of the flow functions.

Third, saturation history profiles and pressure drop history are used as constraints to generate the flow functions (Fig. 10.15). With these two constraints, the pressure drop history (Fig. 10.16), the cumulative production history (Fig. 10.17) and the saturation history profiles (Fig. 10.18) were matched well. From these three different runs, it can be concluded that recovery curves with the pressure drop history are not sufficient to determine the shape of the flow functions accurately. Saturation history profiles along with pressure drop history or pressure history profiles will give more realistic or a unique flow functions. By using porosity distribution along the core and saturation history profiles, the effect of core heterogeneity on the relative permeability and capillary pressure curves will be eliminated.

The estimated relative permeability and capillary pressure curves are not smooth functions like the original curves. In this study discrete values of relative permeability and capillary pressure curves were obtained in contrast to spline and other types of functional representations reported in the literature.

The simulation runs were conducted for the first 100 minutes, and larger grid size and time steps were chosen under stability condition to reduce the computation time during optimization. The initial temperature and the reduction factor for the temperature was obtained from the Fig. 10.19 which is based on the metropolis algorithm and average change (ΔJ) of objective function. The minimization of the objective function and the rate of convergence of the algorithm at each temperature are given Fig. 10.20. In order to increase the rate of convergence, the terms defined in the objective function such as cumulative production rate and the pressure drop had to be scaled to the same order of magnitude by appropriate weighting coefficients.

Case-2 Drainage I: Displacement of oil by water in an oil-wet core.

Experimental data were obtained from Richmond and Watson⁹³ to demonstrate that the core flood simulator could handle the end effect properly for drainage experiments under non-Darcy flow conditions. The rock properties and operating conditions are given in Table 10.2. The relative permeability and capillary pressure curves, Fig. 10.21, are representative of an oil-wet system. Results are compared with the Richmond and Watson's⁹³ data. A successful history matching was obtained. As shown in Fig. 10.22 and Fig. 10.23, pressure drop history and cumulative production history were matched very well. The saturation and pressure history profiles are given in Fig. 10.24, Fig. 10.25, and Fig. 10.26, respectively. The breakthrough time is not retarded in this case since the capillary pressure values are at the minimum value before the injected phase arrived at the end of the core. For illustration purposes, the previous drainage experiment was simulated assuming non-Darcy flow conditions for a range of beta coefficient values. As seen from Fig. 10.27 and Fig. 10.28, the pressure drop and cumulative production rate increased for increasing values of the β coefficient. But the saturation history profiles did not change as much under non-Darcy flow conditions, Fig. 10.29. The non-Darcy flow number for multiphase flow is greater than the

non-Darcy flow number for single phase flow. In other words, the non-Darcy multiphase flow properties are closer to the Darcy multiphase flow properties than to the non-Darcy single phase flow properties.

Case-3 Drainage II: Displacement of water by nitrogen in a water-wet core.

The gas/water data for this case were obtained from Ouenes and Fasanino⁶. The core properties and operating conditions are presented in Table 10.3. The history matching was not successful with their estimated relative permeability and capillary pressure curves, although in their paper a near perfect history match was reported. The estimated capillary pressure curve reported by Ouenes and Fasanino⁸⁸ is a typical imbibition capillary pressure curve rather than a primary drainage capillary pressure curve. The results obtained by the simulated annealing method in the present study indicate a successful match of pressure drop history and cumulative production histories, as shown in Fig. 10.30 and Fig. 10.31. The estimated capillary pressure and relative permeability curves are presented in Fig. 10.32. As can be seen, the present capillary pressure curve is representative of a primary drainage capillary pressure curve. The pressure drop before breakthrough did not match well compared to after breakthrough data since the cumulative injection rather than the injection rate information was available before breakthrough. The saturation and pressure history profiles are given in Fig. 10.33 and Fig. 10.34, respectively. Before breakthrough, there is not much difference between the two phase pressure profiles since the capillary pressure values are very small for average gas saturation values before breakthrough. When the water saturation approaches the residual water saturation, the two phase pressure profiles separate significantly (Fig. 10.35). The minimization of the objective function and the rate of convergence of the algorithm at each temperature for the number of simulations are given in Fig. 10.36. The same experimental data were rerun for non-Darcy flow conditions but there was not any difference between them using the non-Darcy flow coefficient estimated from the Katz et al⁴⁷ correlation for single phase flow. β was found to be 3.4×10^5 (1/cm) for this experimental data.

10.5 Conclusion

The present study has demonstrated that relative permeability and capillary pressure data from Darcy and non-Darcy fluid displacements in laboratory cores can be uniquely determined by the simulated annealing method provided that transient-state internal saturation profiles and overall pressure differentials are used for history matching. However, for heterogeneous cores, the information on the permeability and porosity distributions along the core are also required.

11. PORE SCALE MODELING

11.1 Introduction

Single and multiphase transport in porous medium is usually characterized by several macroscopic properties such as permeability, relative permeability and capillary pressure. These properties are traditionally obtained experimentally. Experimental results are analyzed in two different ways, forward (direct) and inverse (indirect) interpretation. Direct methods are based on integral formulations of Darcy equation while the indirect methods are based on history matching of experimental core data. Petrophysical properties obtained from these two methods are usually complex functions of morphology and topology of porous medium, phase distribution in porous medium and flow arrangements and regimes. The affect of all these properties on the petrophysical properties are lumped. Deconvolution of the affect of the individual properties and forces from the macroscopical petrophysical properties is near to impossible.

The key step to understanding the dynamics of multiphase fluid in porous media is to relate the physics at the microscopic scale to the macroscopic phenomena observed in laboratory. A theoretical model which can be treated mathematically can be developed from the Navier-Stokes equations and boundary conditions in the pore space.

Several pore level models have been proposed in the literature. These models are usually modified with respect to dimensionality and connectedness of pore space. The simplest models of porous medium is based on a bundle of straight, parallel capillary tubes with uniform diameter. Later, this model was extended considering different diameter tubes. In both cases the flow was assumed to be in one direction. The capillary tube models neglect the fact that the fluid path may branch and later on join together again. Fatt⁴¹ was the first to discuss a two dimensional model. The pore throats are represented by cylindrical tubes. Fatt⁴¹ assumed that there were non-dead end pores in porous medium, and no simultaneous two phase flow in the capillary channels. This original model was later extended by other researchers in terms of connectivity and shape of pore segments.

Recent developments in modelling of the porous medium is the application of the effective medium theory (E.M.T.) to the network models. Koplik⁷³ applied E.M.T. to creeping incompressible fluid flow in two dimensional networks. Using the Stokes equation for the pressure difference between the centers of two neighboring pore bodies connected by a throat, it has been shown that E.M.T. provides a reasonable approximation to flow in two dimensional network.

Heiba⁵⁴ et al. used Bethe tree as an approximation to porous medium. They introduced the transport-path accessibility and transport-path conductivity functions to derive general formulae for the relative permeabilities of two phase systems.

Dias and Payatakes²⁸ developed a network model for the immiscible microdisplacement of two phase flow in porous medium. The porous medium was modelled as a network of randomly sized unit cells of the constricted tube. Under creeping flow conditions, they studied the effects of viscosity ratio and capillary number on the microdisplacement efficiency.

Mohanti et al⁸¹ conducted studies on a cubic network for given pore throat and pore body size distributions. Primary drainage and imbibition capillary pressure curves and relative permeabilities resulting from fluid distribution were determined. They also studied the dispersivity by network modelling and compared it with experimental results.

E.M.T. is also used to predict the permeability alteration by matrix damage in sandstone formation. Khatib and Vitthal⁶⁹ used 3-D network to calculate the absolute permeability and reduction of permeability by E.M.T. Rossen and Kumar⁹⁴ had used the E.M.T. to analyze the effect of fracture aperture distribution on the relative permeability curves for single phase and two-phase flow in natural fracture. They also extended the E.M.T. to the fluid flow non-Newtonian fluids in porous medium and pointed out that E.M.T. cannot be used for simultaneous multiphase flow in 2-D.

In the literature review, no study has been found related to the study of simultaneous rapid flow of two phases in a pore network model to obtain the non-Darcy flow properties by using E.M.T. Therefore, the work presented in this chapter is an effort to: 1) develop flow equations for rapid and simultaneous two phase flow in pore throat and pore body; 2) obtain the effective properties of 3-D network by E.M.T. under creeping and rapid flow; and, 3) compare the calculated effective flow properties of network with the macroscopic flow properties.

11.2 Flow Equations in Pore Scale

Single-phase flow through periodically constricted tubes was solved for small and intermediate values of the Reynolds number using a finite-difference method using a formulation of the stream function-vorticity type by Payatakes et al⁸⁸. For highly accelerated flow, Blick and Civan⁷ proposed capillary-orifice model to develop unsteady flow through porous media. But for multiphase flow, few experimental and theoretical studies in constricted tubes have been made. Due to the complexity of the subject, an exact solution of immiscible displacement in a periodically constricted tube is not available. The boundary condition at the interface of two phases and the pore throat-pore body boundaries are not well defined. In order to analyze the rapid flow in porous medium, first, single phase flow and then two phase simultaneous flow in pore throat and pore body are analyzed.

11.2.1 Single Phase Flow

11.2.1.1 Laminar Flow

Single phase flow for laminar flow in cylindrical pore described by Poiseuille's law. The pressure drop is given by:

$$\Delta p = \frac{(8\mu L)}{(\pi R^4)} q \quad (11.1)$$

In pore body which is spherical, pressure drop can be derived from the Navier-Stokes equation for the Fig. 11.1 and under the following assumptions:

Steady state, no end effects and no body forces. The radial and angular velocity

components are given by

$$V_r = V_\theta = 0 \quad (11.2)$$

$$V_\theta = V_\theta(r, \theta) \quad (11.3)$$

Invoking these assumptions, the continuity equation is given by:

$$V_\theta \sin \theta = C_1 = V_r \quad (11.4)$$

and the equation of motion (Bird et al.⁵) is:

$$\frac{1}{r} \frac{\partial p}{\partial \theta} = \mu \left[\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V_r}{\partial r} \right) \right] \quad (11.5)$$

The boundary conditions are given as follows:

$$V_r = 0 \quad r = R \quad (11.6)$$

$$p = p_1 \quad \theta = \epsilon \quad (11.7)$$

$$p = p_2 \quad \theta = \pi - \epsilon \quad (11.9)$$

The solution of Eq. 11.5 and the flow rate calculation is given in Appendix 11.A
The flow rate is given by:

$$q = \frac{\pi R^3 \Delta p}{6\mu E(\epsilon)} \quad (11.10)$$

where

$$E(\epsilon) = - \ln \left(\frac{1 - \cos \epsilon}{1 + \cos \epsilon} \right) \quad (11.11)$$

For incompressible flow, the flow rate in the pore throat is equal to the flow rate in pore body. By using Eq. 11.1 and Eq. 11.10, the ratio of pressure drop in pore neck to the Pore body is:

$$\frac{\Delta p_{throat}}{\Delta p_{body}} = \frac{R_b^3}{R_t^4} \frac{4L}{3E(\epsilon)} \quad (11.12)$$

where L is the length of the tube, $E(\epsilon)$ is given by Eq. 11.11. R_b and R_t are pore body and throat radius respectively. For example, if the length of the tube ($L \gg R_t$) is assumed as $L=10R_t$ and ϵ is 10^0 , then Eq. 11.11 simplifies to:

$$\frac{\Delta p_t}{\Delta p_b} = 2.7 \left(\frac{R_b}{R_t} \right)^3 \quad (11.12)$$

In porous medium, R_b is much larger than R_t . If the pressure drop in the pore throat is compared with the pore body, from Eq. 11.12, it can be concluded that pressure drop in pore throats and body segment is controlled by pore throats.

Conductivity ($q/\Delta p$) of the porous media is usually assigned to the pore throats and pore volume of the porous media is assigned to the pore bodies. Yuan and Swanson¹⁰⁹ developed an apparatus for pore examination (APEX) to resolve the pore space of porous medium into two interconnected parts. Pore bodies are considered as the low energy parts and pore throats as the high energy parts. If the pore bodies conductivity are not neglected in the pore throat-pore body, the average conductivity of the pore segment is the harmonic average of individual conductivity. Therefore, the conductivity of the pore bodies are negligible in the overall conductivity of the system. Multiphase flow in pore bodies require numerical solution requiring an increased computation time. By neglecting fluid flow in pore bodies, an analytical solution can be accomplished for pore throat under multiphase flow and turbulent flow regime. Analytical solutions can help to analyze different realization of pore throat and body distributions. For these reasons, fluid flow in pore bodies are neglected for the turbulent and multiphase flow; but saturation or storage of porous media is still controlled by pore bodies.

11.2.1.2 Turbulent Flow

It is assumed that rapid flow in porous medium can be expressed by a friction factor. Blasius's formula can be used to obtain pressure drop in cylindrical tube:

$$f = \frac{0.0791}{R_c^{1/4}} \quad (11.13)$$

Eq. 11.13 is valid for zero relative roughness (k/D) (Bird et al⁶) where, k is the height of protuberances, and for the Reynold number values in the range of $2.1 \cdot 10^3 < R_c < 10^5$. Under these conditions, the pressure drop in a cylindrical tube is given by:

$$\Delta p = 0.00897 \frac{\rho^{.75} \mu^{.25} L q^{1.75}}{R^{4.75}} \quad (11.14)$$

Derivation of Eq. 11.14 is given in Appendix 11.B. In porous medium, irregularity of pore

wall can be described by a relative roughness concept.

11.2.2. Multiphase Flow

Since, the pressure drop in pore body is negligible, multiphase fluid flow is only considered in pore throats. First, the analytical solution of the simultaneous segregated fluid flow in pore throat is presented. Second, the analytical solution of the dispersed bubble or spray flow of gas/liquid in pore throat is given.

It is assumed that the non-wetting phase flows in the inner part of the cylindrical tube ($0 < r < r_*$) and wetting phase flows in the outer part of the tube ($r_* < r < R$). Simultaneous flow of wetting and non-wetting phase velocity gradients can be expressed by Navier-Stokes equations for steady state, constant viscosity and density (Appendix 11.C). The velocity gradients are given, respectively, by:

$$V_w = \frac{\Delta p R^2}{4L\mu_{nw}} \left[1 - \left(\frac{r}{r_*} \right)^2 \right] \quad r_* < r < R \quad (11.15)$$

and

$$V_{nw} = \frac{\Delta p}{4L} \left[\frac{R^2 - r^2}{\mu_w} + \frac{(r_*)^2 - r^2}{\mu_{nw}} \right] \quad 0 < r < r_* \quad (11.16)$$

The flow rates of wetting and non-wetting phase are

$$q_w = \frac{\pi \Delta p}{8 \mu_w L} [R^2 - (r_*)^2]^2 \quad (11.17)$$

and

$$q_{nw} = \frac{\pi \Delta p}{4 L} \left[\frac{r_*^2 (R^2 - r_*^2)}{\mu_w} + \frac{r_*^4}{2 \mu_{nw}} \right] \quad (11.18)$$

where $0 < r_* < R$, Δp is the pressure drop across the tube in the direction of the flow, μ_w and μ_{nw} are viscosities of wetting and non-wetting phases, R is the radius of tube, and L is the length of tube.

The velocity difference between the wetting and non-wetting phases and difference in the viscosities will cause an interfacial drag on the interface surface between the wetting and non-wetting phases. The molecules of both phase at interference will cause a surface tension gradient. Previous derivation of two phase flow does not account for these two forces, the drag force and the surface tension forces. In Darcy flow which is assumed to be laminar flow, drag forces can be neglected for the liquid-liquid system. Since, both phase velocities and viscosities are close to each other compared to the gas-liquid system. However, for rapid flow, drag force becomes more important; but surface forces are less effective in rapid flow

compared to the Darcy flow. Surface forces are usually accounted indirectly by capillary pressure. As the flow rate increases, inertial forces will be more effective. For a gas/liquid system in rapid flow, as the gas flow rate increases, gas phase becomes irregular in shape and mixes with the liquid phase. Dispersed or spray flow of gas/liquid system can be treated as a mixture and each phase can be assigned different concentration. Drag and surface tension forces can be represented by interaction coefficient between two phases. Feizulbiev⁴⁵ proposed a model for mutually penetrating motion of two-phase flow. Here, this model is adopted to the rapid simultaneous flow of gas/liquid system. The proposed model for the wetting phase is given by:

$$f_w \mu_w \left(\frac{\partial^2 V_w}{\partial r^2} + \frac{1}{r} \frac{\partial V_w}{\partial r} \right) + K (V_{nw} - V_w) = f_w \frac{\partial p_w}{\partial z} \quad (11.19)$$

and for the non-wetting phase by:

$$f_{nw} \mu_{nw} \left(\frac{\partial^2 V_{nw}}{\partial r^2} + \frac{1}{r} \frac{\partial V_{nw}}{\partial r} \right) + K (V_w - V_{nw}) = f_{nw} \frac{\partial p_{nw}}{\partial z} \quad (11.20)$$

where, f_w and f_{nw} are the wetting and non-wetting phase concentration, V_w and V_{nw} are velocities of each phase, and p_w and p_{nw} are pressures of each phase. Due to the rapid flow, these two phase pressures can be assumed equal. μ_w and μ_{nw} are phase viscosities, and K is an interaction coefficient which has a unit of $[Kg / (m^3.s)]$.

The boundary conditions for two phase flow are given as follows:

$$V_w = 0 \quad r = R \quad (11.21)$$

$$V_{nw} = 0 \quad r = R \quad (11.22)$$

$$V_w, V_{nw} = \text{finite} \quad r = 0 \quad (11.23)$$

It is clear from Eq. 11.19 and Eq. 11.20 that if the interaction coefficient (K) is equal to zero, Eq. 11.20 and Eq. 11.21 become the equations of parabolic velocity profiles of single phase in cylindrical tube. The analytical solution of Eq. 11.20 and Eq. 11.21 is also given by Feizulaev⁴⁵. The velocity profiles for the wetting and non-wetting phases are given, respectively, by:

$$V_w = - \frac{1}{m^2} \frac{\partial p}{\partial z} \left[\left(\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} - \frac{1}{\mu_w} \right) \frac{I_0(mr)}{I_0(mR)} + \frac{K}{4 f_w \mu_w f_{nw} \mu_{nw}} \right]$$

$$(R^2 - r^2) - \left(\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} - \frac{1}{\mu_w} \right) \quad (11.24)$$

$$V_{nw} = - \frac{1}{m^2} \frac{\partial p}{\partial x} \left[\left(\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} - \frac{1}{\mu_{nw}} \right) \frac{I_0(mr)}{I_0(mR)} + \frac{K}{4 f_w \mu_w f_{nw} \mu_{nw}} \right. \\ \left. (R^2 - r^2) - \left(\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} - \frac{1}{\mu_{nw}} \right) \right] \quad (11.25)$$

where,

$$m = K^{\frac{1}{2}} \left[\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} \right]^{\frac{1}{2}} \quad (11.26)$$

and I_0 and I_1 are the modified Bessel functions of the first kind of zero and first order. If Eqs. 11.25 and 11.26 are integrated from 0 to R, the flow rate of each phase will be obtained. The flow rate of the wetting phase is given by:

$$q_w = - \frac{2 \pi R f_w}{m^2} \frac{\partial p}{\partial x} \left[\left(\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} - \frac{1}{\mu_w} \right) \frac{I_1(mR)}{m I_0(mR)} + \right. \\ \left. \left(\frac{K R^2}{8 f_w \mu_w f_{nw} \mu_{nw}} + \frac{1}{\mu_w} - \frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} \right) \frac{R}{2} \right] \quad (11.27)$$

and the non-wetting phase by:

$$q_{nw} = - \frac{2 \pi R f_{nw}}{m^2} \frac{\partial p}{\partial x} \left[\left(\frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} - \frac{1}{\mu_{nw}} \right) \frac{I_1(mR)}{m I_0(mR)} + \right. \\ \left. \left(\frac{K R^2}{8 f_w \mu_w f_{nw} \mu_{nw}} + \frac{1}{\mu_{nw}} - \frac{1}{f_w \mu_w + f_{nw} \mu_{nw}} \right) \frac{R}{2} \right] \quad (11.28)$$

The flow rate is directly proportional to the pressure drop per unit length. In case of high flow rate, in turbulent flow, the friction factor and flow rates of each phase can also be derived in the following manner:

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$$f = \frac{f_w \tau_{w,max} + f_{nw} \tau_{nw,max}}{f_w \frac{1}{2} \rho_w < V_w^2 > + f_{nw} \frac{1}{2} \rho_{nw} < V_{nw}^2 >} \quad (11.29)$$

where ρ_w and ρ_{nw} are densities of each phase, and $\tau_{w,max}$ and $\tau_{nw,max}$ are the maximum values of the viscous stress, and $< V_w >$ and $< V_{nw} >$ are the mean velocities.

As seen from Fig. 11.2 and Fig. 11.3, as the interaction coefficient increases, the two phase velocity profiles approach to each other. It is also observed that as the viscosity ratio (μ_{nw}/μ_w) decreases, the effect of interaction coefficient on the velocity profiles increases. As shown in Figs. 11.4 and 11.5, when the interaction coefficient is close to one or zero, the velocity profiles for each phase become single phase flow velocity profiles for dispersed bubble or spray flow arrangements, respectively. In segregated flow arrangements, the velocity profiles are shifting at the interface of the two phases (Figs. 11.6 and 11.7). Comparisons of flow rate for the different flow arrangements are presented in Table 11.1. Total flow rate of the wetting and non-wetting phase is decreasing with the increase of the interaction coefficient for constant pressure drop. As the viscosity ratio (μ_{nw}/μ_w) is decreasing, the effect of the interaction coefficient on the total flow rate is increasing. It can be concluded that interfacial drag and surface tension forces cause for increased dissipation. These forces should be accounted for in the prediction of fluid flow in pore throat and porous medium.

11.3 Representation of Porous Media by Network Model

Naturally occurring porous media is highly complex. Majority of the porous media consists of an interconnected three dimensional network of pore bodies and pore throat. Porous medium can be represented in a computationally efficient way by a network of pore throats (bonds, capillary tube, pore neck) and pore bodies (nodes, sites). The network is characterized by the coordination number of Z , the number of pore throats connected to a pore body. Z defines the structure and connectivity of the medium (Dullien³⁰). As Z becomes larger, the connectivity of the medium increases. For example, a bundle of capillary tube model has one direction and, therefore, Z is infinite. In two-dimensional networks only one phase can be continuous; but it is possible to have two continuous phases present simultaneously for three-dimensional networks. Koplik⁷⁴ carried out some calculations for various network models for permeability.

Pore throats and pore bodies can converge or diverge like sinusoidal shaped constricted capillary tubes. If the geometry of the bonds and nodes are complex, numerical solution is required to calculate the fluid conductivity of each pore segment. In the present study a representative of bonds and sphericals as a representative of nodes are used as approximately. The relation between throat radius r_t and pore length L is defined by Fatt⁴¹ as:

$$L = Cr_t^\alpha \quad (11.30)$$

where C and α are constants to define the length of tube in terms of pore throat radius.

Each pore body is joined by Z throats. The volume of each segment, V_p is the sum of the pore throat volume V_t , and the $2/Z$ pore body volume V_b .

Pore volumes of a bond and a node are :

$$V_t = \pi r_t^2 L \quad (11.31)$$

$$V_b = \frac{4}{3} \pi r_b^3 \quad (11.32)$$

The volume of a pore segment is:

$$V_p = \pi \left(C r_t^{2+\alpha} + \frac{4}{3} r_b^3 \right) \quad (11.33)$$

Pore size distribution is commonly measured by mercury intrusion porosimetry. For modeling purposes, pore size distributions are usually represented by normal, lognormal or Rayleigh distributions. The statistical variation of pore sizes can be described as normalized Gaussian distribution, that is:

$$n(r) = \frac{\sqrt{2}}{\sqrt{\pi} \sigma} \frac{1}{\left[1 + \operatorname{erf}\left(\frac{\mu}{\sigma \sqrt{2}}\right) \right]} \exp \left[-\frac{1}{2} \left(\frac{r - d_t}{\sigma} \right)^2 \right] \quad (11.34)$$

where, σ is the standard deviation and d_t is the mean pore throat radius. All statistical distribution functions suffer from having infinite size range since real pore size distributions have lower and upper limits (Sharma and Yortsos⁹⁸). If the capillary pressure curves are given, the pore size distribution can be obtained by Purcell's equation (Khatib and Vitthal⁶⁸), that is:

$$n(r) = \frac{r_p^{1/4} \frac{\partial S_{nw}}{\partial P_c}}{\int_0^\infty r_p^{1/4} \frac{\partial S_{nw}}{\partial P_c} \cdot dr} \quad (11.35)$$

Yuan and Swanson¹⁰⁹ had proposed rate controlled mercury injection porosimetry to resolve the pore space of a porous medium into pore bodies and pore throats. Pore throats have higher capillary pressure (rison) at low saturation. However, pore bodies have much higher saturation (subison) to previously attained capillary pressure levels. The apparatus for Pore EXamination (APEX) provides that pore throat size and pore body size distribution can be obtained separately compared to the conventional capillary pressure curve.

Topological features of porous medium such as interconnectedness of the pore structure is defined by z coordination number. The geometry of pore structure such as dimensions and orientations of pore and morphological features of pore structure such as roughness and irregularity of pore wall and converging - diverging cross-sections of pore segments can be chosen arbitrarily or based on information of thin section images.

11.4. Effective Medium Theory (E.M.T.)

E.M.T has been used in transport properties of disordered materials, such as porous media, as a mathematical model to study properties of disordered medium. Kirkpatrick⁷⁰ used the E.M.T. to derive the electrical conductance of a mixture of materials. The theory used in petroleum engineering as a hydraulic conductivity is based on the work of Kirkpatrick⁷⁰. The basic idea of E.M.T. is to replace random microscopic conductances with a certain mean value so that the mean field produced by the random conductances is the same as that produced when all parameters have this mean value (Koplik⁷³).

The network with hydraulic conductance variation is replaced by one which has all elements of identical conductances. There is a fluctuation in pressure field due to replacement of a random conductance, g , by effective medium conductance, g_m , but the average of all the fluctuations over a large scale is still assumed zero (Koplik¹³, Heiba⁵⁴, Levine⁷⁶).

Kirkpatrick¹⁶ showed that the effective medium conductance or average conductance of a network, g_m can be obtained by solving the following integral equation.

$$\int_0^{\infty} \frac{g - g_m}{g + \left(\frac{Z}{2} + 1\right) g_m} n(r) dr = 0 \quad (11.36)$$

where $n(r)$ is a pore size distribution function, g is conductance ($q/\Delta p$) of tube radius r , and Z is the coordination number which defines the connectivity.

An analytical expression for g_m cannot be obtained for normal or lognormal distribution. However, a solution can be obtained numerically. It is the solution of conductivity for infinitive network. As stated by Rossen⁹⁴, E.M.T. approximates a physical heterogeneous network with a hypothetical homogeneous network of bonds of uniform conductivity. E.M.T. has also some shortcomings, Kirkpatrick⁷⁰ and Levine et al⁷⁶ showed that E.M.T. has poor approximation close to the percolation threshold and E.M.T. is based on bond percolation theory not on the site percolation problem.

11.4.1 Single Phase Flow Properties

According to the Poiseuille's law the resistivity of the single pore throat for creeping flow is given by:

$$R = \frac{8 \mu L}{\pi r^4} \quad (11.37)$$

The conductance of a single pore throat is the inverse of resistivity given by:

$$g(r) = \frac{\pi r^4}{8 \mu L} \quad (11.38)$$

Using Eq. 11.36, the effective conductance of the medium (g_m) can be calculated numerically for the given pore size distribution function $n(r)$ and coordination number (Z). The permeability of the medium is obtained from the effective conductance of the medium by using Darcy's law, that is:

$$k = \frac{g_m \mu L}{A} \quad (11.39)$$

where L is the length of pore network parallel to the flow direction, A is the cross sectional area perpendicular to the flow direction and μ viscosity of the fluid.

For rapid flow, Eq. 11.14 will be used to calculate the conductance of a single pore throat instead of using Poiseuille's law. The conductance of single pore ($q/\Delta p$) is

$$g(r) = \frac{r^{4.75}}{0.00897 \rho^{.75} \mu^{.25} L q^{.75}} \quad (11.40)$$

After the effective conductance of the medium for rapid flow is calculated similar to the creeping flow for the same network, the non-Darcy flow coefficient can be calculated by comparing the effective conductivity of the network to the conductivity of macroscopic flow which is obtained from the Forchheimer equation. Since the permeability of the medium is known from the creeping flow. The conductivity of the macroscopic rapid flow is

$$\frac{q}{\Delta p} = \frac{1}{\frac{\mu}{k} + \beta \rho \frac{q}{A}} \quad (11.41)$$

where β is the non-Darcy flow condition, ρ is the density of the fluid and ($q/\Delta p$) is equal to the g_m which is obtained from Eq. 11.36. For illustration purposes, a simple network is constructed, where permeability and the non-Darcy flow coefficient is calculated.

11.4.2 Multiphase Flow Properties

In order to calculate the multiphase flow properties, flow arrangement (fluid distribution) should be decided upon. Most of the investigators studied the channel flow concept. This assumption is good and valid for the capillary dominant matrix flow. In this assumption, small pore throats are assigned to flow of only the wetting phase and large pore throats are assigned to flow of only the non-wetting phase. Simultaneous two phase flow cannot take place in any pore throat. In funicular flow concept, wetting and non-wetting

phases flow simultaneously in all capillary tubes; the wetting phase is on the outside surrounding the non-wetting phase. Funicular flow concept seems a good approximation for the viscous dominant liquid/liquid flow system. But for the rapid flow of gas/liquid system, the bubble dispersed or spray flow arrangement will be a good approximation. Fluid flow of three flow arrangements were given in pore scale modelling for the cylindrical tube.

The effective conductivity of the (g_m) corresponds to the single phase fully saturated network. For partially saturated network, the E.M.T., Eq. 11.36 should be rewritten for the partially saturated network. In this study, pore size distribution of pore throats are divided into three regions. The wetting phase conductivity of pore throat should be calculated in the following way.

$$g_w(r) = \begin{cases} \text{wetting phase flow conductivity for } 0 < r < r_1 \\ \text{two phase flow conductivity for } r_1 < r < r_2 \\ 0 \text{ for } r_2 < r \end{cases} \quad (11.42)$$

Similarly for the non-wetting phase:

$$g_{nw}(r) = \begin{cases} 0 \text{ for } 0 < r < r_1 \\ \text{two phase flow conductivity for } r_1 < r < r_2 \\ \text{non-wetting phase flow conductivity for } r_2 < r \end{cases} \quad (11.43)$$

The flow equations given in pore scale modelling are used to calculate the conductivity of pore throats for creeping flow and rapid flow. The effective conductivity of pore network for wetting phase is given by:

$$\int_0^{r_1} \frac{g_w(r) - g_{mw}}{g_w(r) + \left(\frac{Z}{2} + 1\right)g_{mw}} n(r) dr + \int_{r_1}^{r_2} \frac{g_w(r) - g_{mw}}{g_w(r) + \left(\frac{Z}{2} + 1\right)g_{mw}} n(r) dr + \int_{r_2}^{\infty} \frac{n(r)}{\left(\frac{Z}{2} + 1\right)} dr = 0 \quad (11.44)$$

and for non-wetting phase by:

$$\int_0^{r_1} \frac{n(r)}{\left(\frac{Z}{2} + 1\right)} dr + \int_{r_1}^{r_2} \frac{g_{nw}(r) - g_{mnw}}{g_{nw}(r) + \left(\frac{Z}{2} + 1\right)g_{mnw}} n(r) dr + \int_{r_2}^{\infty} \frac{g_{nw}(r) - g_{mnw}}{g_{nw}(r) + \left(\frac{Z}{2} + 1\right)g_{mnw}} n(r) dr = 0 \quad (11.45)$$

where g_{mw} is the effective conductivity of the network for wetting phase, g_{mnw} is the effective conductivity of network for non-wetting phase, respectively. The relative permeability of wetting phase is

$$k_{rw} = \frac{g_{mw}}{g_m} \quad (11.46)$$

and non-wetting phase is

$$k_{rnw} = \frac{g_{mnw}}{g_m} \quad (11.47)$$

where g_m is the conductivity of the fully saturated network. After the single phase flow properties such as permeability and the non-Darcy flow coefficient and two phase flow relative permeability for creeping flow are calculated, the methodology of calculating the rapid flow properties in single phase can be used in a similar way to calculate the rapid flow of multiphase systems.

The macroscopic momentum balance equations for rapid flow are given by Schulenberg and Miller⁹⁷ as:

$$-\frac{\partial p_i}{\partial x} = \frac{\mu_i}{k k_{ri}} u_i + \frac{\rho_i}{\eta \eta_{ri}} u_i^2 + \frac{F_{i,j}}{S_i} \quad i=w,nw \quad j=nw,w \quad (11.48)$$

where, η is the reciprocal of the inertial flow coefficient, β . $F_{w,nw}$ and $F_{nw,w}$ are the interfacial drag forces between wetting and non-wetting phase.

If the k , η and k_{ri} are known for given network, the rapid flow of relative permeability (η_{ri}) can be estimated by running the same network under rapid flow and with/without the interaction coefficient.

11.5 Application

A simple and small size network is chosen to illustrate calculation of the permeability and inertial flow coefficient of the network. The geometrical structure and dimensions of the network are given in Fig. 11.8 and Table 11.2.

In order to calculate the conductivity of the each throat, first the pressure at the pore body must be calculated. The mass balance of compressible fluid at the pore body is given by:

$$\sum_{i=1}^4 q_i = 0 \quad (11.49)$$

Using the Poiseuille's equation, Eq. 11.40 leads to:

$$(r_1^4 + r_2^4)(p_1 - p_A) + (r_3^4 + r_4^4)(p_2 - p_A) = 0 \quad (11.50)$$

If the over all pressure drop is chosen to be 1 Atm, the pressure at the pore body is calculated as 0.598 Atm. After knowing individual throat flow rate, total flow rate of network can be

calculated. The flow rate of the network should be equal to the Darcy flow rate. The permeability of the given network yields the following equation:

$$k = \frac{\pi}{8} \frac{p_1 - p_A}{p_1 - p_2} \frac{L_{network}}{L_{tube}} \frac{1}{A} (r_1^4 + r_2^4) \quad (11.51)$$

By using Eq. 11.51, the permeability of the network is 17.2 md.

For rapid flow, instead of Poiseuille's law, Eq. 11.14 is used for mass balance at the pore body. Then, Eq. 11.49 yields to:

$$\left(\frac{4.75}{r_1^{1.75}} + \frac{4.75}{r_2^{1.75}} \right) (p_1 - p_A)^{\frac{1}{1.75}} + \left(\frac{4.75}{r_3^{1.75}} + \frac{4.75}{r_4^{1.75}} \right) (p_2 - p_A)^{\frac{1}{1.75}} = 0 \quad (11.52)$$

If the over all pressure drop is chosen to be 4 Atm, the pressure at the pore body is 2.19 Atm. Same as the creeping flow, the flow of network is equal to the macroscopic flow rate of rapid flow. The Forchheimer equation is used to calculate the macroscopic flow rate (Eq. 11.41). The only unknown is the inertial flow coefficient since the permeability (k) is known from the creeping flow regime. The non-Darcy flow coefficient is found to be 0.024 cm⁻¹.

This methodology can be extended for multiphase flow. In order to calculate the multiphase flow properties, network must contain more pore throats and pore bodies. As the number of pore throats and pore body increase, the computation time increases exponentially (Koplik⁷⁴).

11.6 Discussion and Conclusions

Darcy's Law fails to describe flow through porous media at high rates due to the increased contribution of inertial forces. This deviation does not necessarily imply a turbulence flow regime. Although, the flow regime is assumed to be turbulence flow regime in this study. The pressure drop is increasing with the reverse flow and with the heterogeneity of the medium such as roughness and irregularity at the pore walls.

The effect of flow rate and the viscosity ratio of phases on the relative permeability curves have long been questionable subjects. The effects of so many parameters on the relative permeability curves are lumped. It is very hard to differentiate the affect of flow rates and the viscosity ratio on relative permeabilities. By using the interaction coefficient which is a function of interfacial drag and surface tension forces, the affect of flow rate on the relative permeability can be differentiable by setting different values of interaction coefficient in pore network modeling.

The mid size of pore throats ($r_1 < r < r_2$) are assigned to flow in simultaneous phases. The phase concentration in any mid size pore throat can be assigned linear function of pore throat size. Capillary pressure curves usually determine the fluid distribution in reservoir. In the piston like displacement of two immiscible fluid flow, there is a meniscus present in pore throat. The capillary pressure jump which is function of pore throat size can be added to the pressure drop of the single phase flow in pore throat (Koplik⁷⁴)

The most widely used method for determining pore size distribution is mercury porosimetry that determines the only accessible pore throat and pore body from the outside of the rock specimen. Accessibility functions must be introduced when a statistical pore size distribution function is used. Even though pores are allowed to be accessible, some fraction of the pores will still not be accessible (Heiba and Sahimi⁶⁴).

The pore body (nodes) are assumed to allow the simultaneous passage of any number of phases. When a wetting phase displaces a non-wetting phase, the wetting phase will form thin films on the surfaces of the pore throat. This thin film can be neglected in the fluid flow conductivity.

Applicability of the E.M.T. to the simultaneous fluid flow in pore throat should be reviewed and validated based on the percolation theory in the 3-D network modelling.

In rapid flow of gas/liquid system, interfacial drag force is not negligible. Non-Darcy flow properties are not only functions of petrophysical properties but also functions of flow arrangements and flow regimes. However, Darcy flow properties are not function of flow regimes. A theoretical model such as 3D pore network modeling may be a good approach to differentiate the effect of flow regimes from the petrophysical properties and is recommended for future work.

APPENDIX 11.A

Solution of the single phase flow in sphere

Separating Eq. 11.5 into two parts leads to:

$$\sin \theta \frac{\partial p}{\partial \theta} = B \quad (11.A1)$$

$$\frac{\mu}{r} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) = B \quad (11.A2)$$

where B is the separation constant.

Eq. 11.A1 can be solved for the following boundary condition

$$p = p_1 \quad \theta = \pi - \epsilon \quad (11.A3)$$

$$p = p_2 \quad \theta = \epsilon \quad (11.A4)$$

By integrating Eq. 11.A1 , the following equation is obtained:

$$\Delta p = B \left| \ln (\operatorname{cosec} \theta - \cot \theta) \right|_e^{\pi - \epsilon} \quad (11.A5)$$

and taking limits, Eq. 11.A5 yields:

$$\Delta p = -B \ln \left(\frac{1 + \cos \epsilon}{1 - \cos \epsilon} \right) \quad (11.A6)$$

or

$$\Delta p = -B E(\epsilon) \quad (11.A7)$$

The solution of Eq. 11.A2 is

$$V = \frac{\Delta p r}{2 \mu E(\epsilon)} - \frac{C_1}{r} + C_2 \quad (11.A8)$$

where C_1 and C_2 are integral constants. The boundary conditions are given by:

$$V = 0 \quad r = R \quad (11.A9)$$

$$V = \text{finite} \quad r = 0 \quad (11.A10)$$

Invoking the boundary conditions, Eq. 11.A8 yields:

$$V = \frac{R \Delta p}{2 \mu E(\epsilon)} \left(1 - \frac{r}{R} \right) \quad (11.A11)$$

Thus, the volumetric flow rate is given by:

$$q = \int_0^R \int_{\theta=0}^{\theta=2\pi} V r \, d\theta \, dr \quad (11.A12)$$

Taking the integral and limits, Eq. 11.A12 yields:

$$q = \frac{\pi R^3 \Delta p}{6 \mu E(\epsilon)} \quad (11.A13)$$

APPENDIX 11.B

According to the definition of the friction factor in the cylindrical tube (Bird et al⁵)

$$f = \frac{1}{2} \frac{D}{L} \frac{\Delta p}{\frac{1}{2} \rho \langle v \rangle^2} \quad (11.B1)$$

where, $\langle v \rangle$ is average velocity and D is the diameter of the cylindrical tube, ΔP is pressure drop along the tube.

The Blasius formula for approximation of friction factor

$$f' = \frac{0.0791}{R_e^{1/4}} \quad (11.B2)$$

where, R_e is the Reynold number which is:

$$R_e = \frac{\rho \langle v \rangle D}{\mu} \quad (11.B3)$$

The flow rate of the pipe is:

$$q = \frac{\pi D^2}{4} \langle v \rangle \quad (11.B4)$$

By using Eqs. 11.B1, 11.B2, 11.B3 and 11.B4 the following flow rate equation is obtained.

$$\Delta p = 0.00897 \frac{\rho^{.75} \mu^{.25} L q^{1.75}}{R^{4.75}} \quad (11.B5)$$

APPENDIX 11.C

The equation of motion in cylindrical tube for Newtonian fluid with constant viscosity and density is given by Bird et al⁶. In terms of shear stress:

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) \quad (11.C1)$$

In terms of velocity gradient:

$$-\frac{\partial p}{\partial z} = \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V_z}{\partial r} \right) \right] \quad (11.C2)$$

The solution of Eqs. 11.C1 and 11.C2

$$\tau_{rz} = \frac{\Delta p}{2L} r + \frac{C_1}{r} \quad (11.C3)$$

$$V_z = -\frac{r^2}{4\mu} \frac{\Delta p}{L} - \frac{C_1}{r} + C_2 \quad (11.C4)$$

The boundary conditions are:

$$B.C.1 \quad V_{s,w} = 0 \quad r = R \quad (11.C5)$$

$$B.C.2 \quad V_{s,w} = V_{s,nw} \quad r = r_* \quad (11.C6)$$

$$B.C.3 \quad \tau_{rz,w} = \tau_{rz,nw} \quad r = r_* \quad (11.C7)$$

$$B.C.4 \quad V_{s,nw} = \text{finite} \quad r = 0 \quad (11.C8)$$

$$\tau_{rz,nw} = \text{finite} \quad r = 0 \quad (11.C8)$$

By using the 11.B.C.4 and 11.B.C.3 the following constants are obtained:

$$C_{1,nw} = 0 \quad (11.C9)$$

and

$$C_{1,w} = 0 \quad (11.C10)$$

Then, by using the B.C.1:

$$C_{2,w} = \frac{R^2}{4\mu_w} \frac{\Delta p}{L} \quad (C11)$$

The velocity profile for wetting phase is given by:

$$V_{s,w} = \frac{R^2}{4\mu_w} \frac{\Delta p}{L} \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (11.C12)$$

After that, use the 11.B.C.2:

$$C_{2,nw} = \frac{\Delta p}{L} \left[\frac{R^2}{4\mu_w} - \frac{r_*^2}{4\mu_w} + \frac{r_*^2}{4\mu_{nw}} \right] \quad (11.C13)$$

The velocity profile for non-wetting phase is given by:

$$V_{s,nw} = \frac{\Delta p}{L} \left[\frac{(R^2 - r^2)}{\mu_w} + \frac{(r_*^2 - r^2)}{\mu_{nw}} \right] \quad (11.C14)$$

The volumetric flow rate can then be obtained by using the following equation:

$$q = \int_0^r V_r r \partial r \quad (11.C15)$$

12. PUBLICATIONS RESULTING FROM THE PROJECT

1. Civan, F. and Evans, R.D., "Non-Darcy Flow Coefficients and Relative Permeabilities for Gas/Brine Systems", SPE 21516 paper, Proceedings of SPE Gas Technology Symposium, Houston, TX, Jan. 23-25, 1991, pp. 341-352.
2. Civan, F. and Evans, R.D., "New Insights in the Formulation of Porous Media Mass and Momentum Equations", SPE 26149 paper, Proceedings of the 1993 SPE Gas Technology Symposium, Calgary, Canada, June 28-30, 1993, pp. 119-128.
3. Civan, F. and Evans, R.D., "Relative Permeability and Capillary Pressure Data from Non-Darcy Flow of Gas/Brine Systems in Laboratory Cores", SPE 26151 paper, Proceedings of the 1993 SPE Gas Technology Symposium, Calgary, Canada, June 28-30, 1993, pp. 139-153.
4. Ucan, S., Civan, F. and Evans, R.D., "Simulated Annealing for Relative Permeability and Capillary Pressure from Unsteady State Non-Darcy Displacement", Proceedings of the 1993 SPE Annual Technical Conference and Exhibition, Houston, TX, October 3-6, 1993, pp. 673-687.

NOMENCLATURE

A	cross-sectional area, cm^2 ; constant in the correlation of non-Darcy flow coefficient; or solid-fluid contact surface
a_i, b_i	coefficients of polynomials
B	constant in the correlation of non-Darcy flow coefficient
B_g	formation volume factor of gas, $(\text{cm}^3)_{\text{RC}}/(\text{cm}^3)_{\text{STC}}$
B_w	formation volume factor of water, $(\text{cm}^3)_{\text{RC}}/(\text{cm}^3)_{\text{STC}}$
c	pore volume compressibility (psi^{-1}), or isothermal compressibility, atm^{-1}
C	an integration constant, $\text{atm}\cdot\text{cm}^3$; an empirical constant; or constant in the correlation of non-Darcy flow coefficient
c_k	Kozeny constant
d	diameter of pore, grain or pipe (cm)
d_t	mean pore throat diameter (μm)
D	constant in the correlation of non-Darcy flow coefficient
E	energy at the state of solid
f	fractional flow, fraction; friction factor
F	a lumped function or formation resistivity factor
F_i, F_{12}	interfacial drag force, dynes
F_i	interfacial drag
f_{nd}	non-Darcy factor
g	gravitational constant, 981 cm/s^2 ; conductance of circular tube (μm^4)
I	unit tensor
J	objective function
k	absolute permeability, darcies; apparent permeability, darcies; or thermal equilibrium constant
k_b	physical constant
k_o	intrinsic permeability, darcies
k_r	relative permeability, fraction
K	interaction coefficient, $\text{kg}/(\text{m}^3\text{s})$
L	core length, cm; or pore throat length
m	exponent
M	molecular weight, g/mole
n	an exponent, or unit normal vector
$n(r)$	pore throat size distribution
p	pressure, atm
p_c	capillary pressure, atm
q	volumetric flow rate, cm^3 / s
Q	cumulative volume, cm^3
r_b	pore body radius (L)
r_t	pore throat radius (L)
R	sample correlation coefficient; or ideal gas constant, $82 \text{ atm}\cdot\text{cm}^3/\text{mole}\cdot\text{K}$
R_1, R_2	some lumped functions
Re	porous media Reynolds number, unitless
R_s	Gas solubility in water $(\text{cm}^3)_{\text{STC}}/(\text{cm}^3)_{\text{STC}}$
S	saturation, fraction, or specific surface area (cm^{-1})
t	time, seconds
T	temperature, K

u	volume flux, $\text{cm}^3 / \text{cm}^2 \cdot \text{s}$
v	fluid velocity, cm/s
V	volume (cm^3)
V_b	pore body volume (L^3)
V_p	pore segment volume (L^3)
V_t	pore throat volume (L^3)
x	distance in flow direction, cm
y	distance in normal direction to x , cm
z	vertical distance, cm or real gas deviation factor; coordination number
Z	coordination number

Greek Symbols

α	constant in compressibility equation; constant in pore length equation
β	non-Darcy flow (or inertial flow) coefficient (cm^{-1} or ft^{-1})
η	reciprocal inertial flow coefficient, cm
η_r	relative reciprocal inertial flow coefficient, fraction
γ	surface tension (dynes/cm); specific weights
θ	angle of inclination, degrees
μ	viscosity (cp)
λ	empirical coefficient, cp
ν	kinematic viscosity, cp/g/cm^3
ω	mass fraction
π	dimensionless group
ρ	fluid density (gm/cm^3)
σ	overburden stress, dynes/cm^2 ; or effective stress (psi), or standard deviation
τ	tortuosity factor; viscous stress
ϕ	porosity, fraction
Ψ	flow potential, $\text{atm-cm}^3/\text{g}$

Operational Symbols

∇	gradient operator
$\langle \rangle$	representative volume average
$\cdot$	dot product
Δ	difference operator

Subscripts

1	wetting phase
2	nonwetting phase
A	actual
b	body
c	calculated data
f	fluid material
g	gas
i	inertial

l	liquid
m	measured data; effective data
nw	non-wetting
o	original, oil or base conditions
r	relative, residual or bulk material
s	standard condition or solid matrix
t	throat
v	viscous
w	water; wetting

Superscripts

'	spatial deviation
-	average
*	inlet conditions
s	standard conditions

SI Metric Conversion

1 atm	= 1.01325×10^5 Pa
1 darcy	= $0.9869233 \mu\text{m}^2$
1 cp	= 1.0×10^{-3} pa.s

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APPENDIX A

TABLES

Table 5.1: Results of Non-Darcy Flow Factor Length Analysis		
Core Length [in]	β [1/cm ²]	Permeability, K[md]
2	7.766E5	138.0
3	8.685E5	142.0
4	8.530E5	150.0

Table 5.2: Sensitivity of the Non-Darcy Flow Factor to Core Length Investigation Data				
(in) Core Length	Run #	(1/cm) β	(1/cm ²) Intercept	(md) Permeability
4.0	1	7.38 E05	7.00 E08	147.4
	2	8.10 E05	6.85 E08	150.6
	3	7.80 E05	7.00 E08	147.4
3.0	1	7.65 E08	7.15 E08	141.8
	2	7.80 E05	7.00 E08	147.4
	3	8.05 E05	6.90 E08	146.8
2.0	1	8.10 E05	7.40 E08	136.9
	2	8.36 E05	7.38 E08	137.3
	3	8.04 E05	7.48 E08	135.5
1.0	1	9.80 E05	7.65 E08	132.5
	2	9.58 E08	8.10 E08	125.1
	3	10.16 E08	7.82 E08	129.6

Table 7.1. Core, Fluid, and Displacement Data by Ouenes⁸⁵ and the Relative Permeability and Capillary Pressure vs. Saturation Calculated in the Present Study

Core Data:		Displacement Data:			Calculated Values:			
		Q_1, cm^3	Q_2, cm^3	$\partial p/\partial x, \text{atm/cm}^\diamond$	S_1	kr_1	kr_2	P_c, atm
L	= 37.8 cm	8.9	8.9	-0.008	0.	0.	1.330	0.497
A	= 11.7 cm ²	15.9	15.9	-0.010	0.000	0.	1.047	0.264
θ	= 90° = $\pi/2$	19.5	19.5	-0.012	0.026	0.003	0.757	0.213
ϕ	= 0.301	23.5	22.0	-0.010	0.076	0.010	0.517	0.174
k	= 0.311 darcy	25.8	23.0	-0.010	0.095	0.013	0.429	0.158
		28.6	24.0	-0.009	0.106	0.015	0.401	0.142
	Fluid Data:	34.8	26.0	-0.008	0.118	0.019	0.404	0.115
		48.4	29.2	-0.005	0.142	0.030	0.386	0.081
1	= Nitrogen	68.4	32.8	-0.004	0.176	0.042	0.312	0.056
2	= Brine	90.9	34.8	-0.004	0.212	0.051	0.185	0.041
ρ_1	= 0.00875 g/cm ³	123.0	36.4	-0.003	0.233	0.055	0.119	0.030
ρ_2	= 1.003 g/cm ³	172.7	38.1	-0.003	0.247	0.059	0.088	0.020
μ_1	= 0.018 cp	231.0	39.6	-0.003	0.255	0.063	0.076	0.014
μ_2	= 1.02 cp	580.0	45.1	-0.002	0.289	0.077	0.047	0.004
S_{1r}	= 0	890.0	47.4	-0.002	0.319	0.078	0.026	0.001
S_{2r}	= 0.633	1150.0	48.4	-0.002	0.340	0.080	0.015	0.000
q	= 0.0417 cm ³ /sec	1450.0	48.8	-0.001	0.367	0.076	0.000	0.000
σ	= 72 dynes/cm [†]							
p_c	= 0.003 [†]							
C	= 4. [†]							
W	= 55. [†]							

† data assumed

♦ values estimated from the pressure drop data given by Ouenes⁸⁵

Table 9.1: Data for Plug Number 6686³

$$\theta = 0^\circ$$

$$L = 3.88 \text{ cm}$$

$$A = 5.07 \text{ cm}^2$$

Nonwetting fluid: Nitrogen, $I = 2$

Wetting fluid: Water, $I = 1$

Base Conditions: $P_b = 1 \text{ atm}$ and $T_b = 306.5\text{K}$

Effective Permeability, Darcies:

$$k_{a1} = kk_1 = 10.02 \times 10^{-6} [(S_1 - S_{1i}) / (1 - S_{1i})]^4$$

$$k_{a2} = kk_2 = 31.9 \times 10^{-6} (1.0 + 2.92 / p_2)$$

$$(-0.0995 - 0.472 \ln S_1)$$

Capillary Pressure, atm:

$$p_c = 0.701 + 3.02 / S_1^2 - p_b$$

$$dp_c/dS_1 = -6.04 / S_1^3$$

Porosity: $\phi = 0.1367$

Immobile Wetting Fluid Saturation: $S_{1i} = 0.4065$

Immobile Gas Saturation: $S_{2i} = 0.0$

Temperature: $T = 306.5\text{K}$

Liquid Density: $\rho_{p1} = 0.9947 \text{ g/cm}^3$

Liquid Viscosity: $\mu_{b1} = 0.744 \text{ cp}$

Gas Viscosity: Empirical correlation

Isothermal Compressibility for Water: $c_T \approx 0 \text{ atm}^{-1}$

Table 9.2: Experimental data obtained by numerical simulation.
(Note that only nine of the data points are listed here.)

Run Number	Assumed		Calculated	
	Fluxes for Phase 1 and Phase 2, $\text{gr/cm}^2\text{-s}$		Average Saturation, fraction	Pressure Differential, atm
	m_1	m_2	$\bar{S}_1$	Δp
1	5.00×10^{-6}	5×10^{-4}	0.604	0.00334
2	1.00×10^{-6}	5×10^{-4}	0.621	0.000618
3	1.00×10^{-7}	5×10^{-4}	0.555	0.000113
4	1.00×10^{-7}	1×10^{-4}	0.576	0.0000871
5	1.00×10^{-6}	1×10^{-4}	0.592	0.00074
6	1.00×10^{-5}	1×10^{-4}	0.707	0.0031
7	4.00×10^{-8}	1×10^{-5}	0.576	0.0000348
8	1.00×10^{-8}	1×10^{-5}	0.570	0.928×10^{-5}
9	1.00×10^{-7}	1×10^{-5}	0.591	0.748×10^{-4}

Table 10.1. Core Properties and Operating Conditions, Simulated Imbibition Waterflood (Richmond and Watson ⁹³)	
Core Properties	
k, md	1,270
ϕ	0.249
L, cm	7.62
A, cm ²	5.06
S_{rw}	0.2882
S_m	0.2373
Operating Conditions	
Q, cm ³ /min	0.36
μ_w , cp	1
μ_n , cp	10
Total fluid injected, PV	14.8

Table 10.2. Core Properties and Operating Conditions, Drainage Waterflood Experiment (Richmond and Watson ⁹³)	
Core Properties	
k, md	9.6
ϕ	0.262
L, cm	7.13
A, cm ²	11.3
Swi	0.10
Operating Conditions	
Q, cm ³ /min	2.0
μ_w , cp	0.262
μ_o , cp	0.725
Total fluid injected, PV	17.8

Table 10.3. Core Properties and Operating Conditions, Actual Gas/Water Draining (Ouenes ⁸⁵)	
Core Properties	
Core length L	37.8 cm
Core cross section A	11.7 cm ² 311
Permeability k	md
Porosity ϕ	30.1
Initial gas saturation S_{1ni}	0%
Residual water saturation S_{2res}	63.3%
Operating Conditions	
Injection flow rate Q	150 cm ³ /hr
Gas density ρ_1	0.00875 g/cm ³
Brine density ρ_2	1.003 g/cm ³
Gas viscosity μ_1	0.018 cp
Brine viscosity μ_2	1.02 cp

Table 11.1 : Comparison of flow rates in cylindrical tube for the different flow arrangements

<u>Two Phase Flow</u>			<u>Single Phase Flow</u>
Dispersed Bubble or Spray Flow K=1.Kg/(m ³ s) K=100. K=500.			Segregated Flow
<u>$\mu_{nw}/\mu_w = .01$</u>			
Q_w	.00067	.00148	.00152
Q_{nw}	.08923	.00848	.00466
Q_r	.08991	.00996	.00618
			.00014
			.07763
			.07777
			.00157
			.15708
<u>$\mu_{nw}/\mu_w = .3$</u>			
Q_w	.00047	.00072	.00086
Q_{nw}	.00364	.00281	.00236
Q_r	.00412	.00353	.00322
			.00014
			.00323
			.00337
			.00157
			.00523

$\Delta P = 4. \text{ (Atm)}$, $L = 1. \text{ (mm)}$, $R = .1 \text{ (mm)}$, $\mu_w = 1. \text{ (cP)}$, $S_w = .3$, $S_{nw} = .7$

APPENDIX B
FIGURES

Fig. 3.1 Non-Darcy flow apparatus

Fig. 3.3 Schematic flow paths through the API proppant pack conductivity test

Courtesy of the Western Company

Fig. 3.4 Exploded schematic diagram of API proppant pack conductivity test unit

Courtesy of the Western Company
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Fig. 4.1 — Correlation of β Including Effect of Tortuosity

Fig. 4.2 — Correlation of β in A Dimensionally Consistent Form

Fig. 4.3 — Effect of Immobile Liquid on The β -Coefficient

Fig. 4.4 — Correlation of β in Terms of K , ϕ and $(1-s_i)$

Fig. 4.5 — A General Correlation of β vs. K , ϕ for Various Porous Media Including Effects of Immobile Liquid and Overburden Stress

Fig. 5.1 Straight-line plot for the different core length

Figure 5.2 - Core length average permeability and non-Darcy flow factor vs core length.

Figure 7.1 Comparison of Relative Permeabilities

Figure 7.2 Comparison of Capillary Pressures

Fig. 9.1 : Schematic of A Steady- State, Multiphase Flow Experiment

Fig. 9.2 : Comparison of the External and Internal Fluid Pressures

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