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Commercialization of Atom Interferometers for Borehole Gravity Gradiometry.

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The majority of the time in this quarter was spent completing the work remaining on Task # 1. Since many of the tasks contained in the statement of work are strongly interrelated and affect the progress in each other, work in quarter # 2 has been distributed among various tasks in parallel, and this work is still in progress. Notably, we anticipate that a significant portion of the work on Tasks # 2, 3, 4, 5, 7 and 7 will be complete within the next quarter.

Work on Task # 1:

Perform a mathematical analysis of the tomography inversion problem for various configurations of a borehole gravity gradiometer.

This task is the most important one of the work and had to be done first so as to specify the basic sensitivity requirements for the device, upon which subsequent work is based. Much of the work was described in the first quarter's report, and this task is now essentially complete. Within the limits of our presently available computer power (IBM 486 clones running at 50 MHz) we have analyzed the one - hole and two - hole tomography problems. An extension of the two - hole problem to a full instrument limited range and resolution will require a more powerful computer (and/or an array processor), and is considered beyond the scope of the present work.

For instrument requirements, we have successfully attempted to work within the limits imposed by a device that does not employ axial magnetic field gradients, as these necessarily complicate the construction, *vis à vis* borehole diameter limitations. Within this constraint, we have found that an instrument (such as ours) that can measure two of the five independent gravity gradient tensor components

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$$\Gamma_{xz} \equiv \frac{\partial g_x}{\partial z}, \text{ and } \Gamma_{yz} \equiv \frac{\partial g_y}{\partial z},$$

where g_x and g_y are the x and y components of the local gravity force field, and where z represents the borehole axis, with an instrumental error in the range 0.1 - 1.E will yield particularly useful well-logging data. Here we use the standard unit of gravity gradient measurement, the Eötvös, which is $1 \text{ E} = 10^{-9} \text{ sec}^{-2}$. Naturally, the lower the error, the greater the anticipated range and resolution. Given the present state of the work on other tasks, it is believed that our first instrument can be built too reach within this accuracy range.

Our work on Task # 1 has proceeded in various steps. These are delineated as follows:

1. We formulated and created computer programs to numerically solve the mass-tomography inversion problems. The problem was solved as an "over-determined" one, i.e. we specify sampling of more gradient points along the well-bore than are necessary to perform the inversion (sampling too few points yielded a singular inversion). Assuming that all measurements would contain some error, we include a free parameter γ that allows us to balance matrix near-singularity and its associated computational accuracy against measurement (instrumental) error.
2. We considered various test mass distributions and calculated the gravity gradient signals produced at the borehole by each of these distributions.
3. We applied the inversion procedure to the test signals in order to reconstruct the parent mass distribution. Simultaneously, we simulated instrumental random noise (with various specified statistical distributions), added it to the gradient signals, and noted how the reconstruction accuracy deteriorated with increasing instrumental noise.
4. For a given coordinate geometry we then repeated the above steps while simultaneously varying the simulated instrument noise magnitude and the parameter γ . We thus optimized our calculation by balancing the computational error

(reconstruction accuracy) for a given instrumental error against the instrumental error and determined the optimum γ for each case.

5. We again repeated all of the above while systematically varying the coordinate geometry (thereby varying the anticipated range and resolution). So doing, we determined how the accuracy of the (optimized) reconstruction varies with respect to specified range and/or resolution.
6. Finally, all of the above methodology was applied both to measurements taken from one borehole, and also to correlated measurements taken from two adjacent boreholes (for increased azimuthal resolution).

Conclusions for Task # 1:

1. Naturally, as higher resolution and/or range is specified the reconstruction accuracy deteriorates. With too fine a resolution and/or at too great a range, the reconstructed density acquires sufficiently large error as to be meaningless. For a specified instrument accuracy we thereby determined the range and resolution limits of our instrument. Not surprisingly, as the instrumental accuracy improves, greater range and/or resolution is fostered.
2. In a given ring surrounding a single borehole, by using data from that hole alone, one can determine only the lowest Fourier coefficients of its azimuthal mass distribution. If a single mass anomaly is localized somewhere on that ring, then its range, bearing, elevation and mass can be determined. However, an ambiguity remains as to whether it is a positive mass in the specified direction or a negative one in the antipodal direction. If measurements of the same localized mass anomaly are made from two adjacent boreholes then the ambiguity is resolved.
3. With ideal instruments and correlated measurements taken from two adjacent holes it is possible to have very high azimuthal resolution. However, minuscule instrumental error quickly

reduces this enhancement to essentially a determination of no more than information regarding the second Fourier component of the azimuthal resolution. Thus with data from two holes, two independent localized mass anomalies can be located. Presumably, the use of correlated data from N holes can, in turn locate N localized mass anomalies, although a direct calculation to prove this conjecture was beyond the scope of the current project and computer power. By limiting the range and number of points considered for the one hole problem, the accuracy and resolution of the one and two hole solutions was found comparable with that of the two hole solution, but with the latter exhibiting twice the azimuthal accuracy w.r.t the former.

4. Depth and range resolution deteriorate with increasing range from the hole, and simultaneously with increasing instrumental error. For optimal radial resolution, ring size increases quadratically with range.
5. Regarding depth resolution, there is a tradeoff available. Naturally, it is possible to determine more accurately the average mass of a larger volume. Correspondingly, there is a tradeoff between the accuracy with which density is determined and the resolution to which it must be determined. This trade off is not, however, uniform with range from the hole. Thus, one is content with low resolution at short range from the hole then the accuracy improves for determining the density at high range with constant resolution at this high range. Correspondingly, if one demands high short range resolution, then the accuracy is reduced at large range.

The attached Figure demonstrates this interdependence. Here we simulate the use of a gravity gradiometer with a sensitivity of $0.25E$ for reconstruction of the mass density surrounding a hole in a cylindrical volume 3km tall. This volume is divided into six nested cylinders with centroid radii shown on the Figure. The outermost cylinder is divided vertically into seven rings, each 428m tall. One may then grade

the vertical resolution on successively inner cylinders. If one divides the heights of the inner cylinders all the same (ratio = 1) as that of the outermost cylinder, then typically the mass density in all such created rings is determined to about the same accuracy i.e. $2\% \times 0.1\text{g/cm}^3 = 0.002\text{g/cm}^3$. If, on the other hand, if one specifies successively finer resolution as one moves inward, say 7 rings in the outermost cylinder, 14 rings in the next inner, 28 rings in the next, etc. (ratio = 2), so that now the number of density values thus extracted is significantly more, then the accuracy in the various annular rings decreases, with a worst case accuracy of about $70\% \times 0.1\text{g/cm}^3 = .07\text{g/cm}^3$. Intermediate ratios are depicted on the Figure.

6. In addition to an ability to locate isolated mass anomalies, another particularly important feature of our instrument is its ability to determine simultaneously local layering dip inclination and orientation from data taken from a single hole. We can readily extract from the data the quantity φ and $\rho \sin 2\vartheta$, where ρ is the inclined slab density, ϑ is the inclination angle of a given inclined slab pierced by hole, and φ is the azimuthal orientation of the inclination. Assuming that the density ρ is available from coring samples, then ϑ is also uniquely determined. The algorithm for doing this was incorporated into the one - hole program and was found to work well.

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Total Error (Mea&Cal) vs Radius

$z \rightarrow [-2000, 2000]$, $H \rightarrow [-1500, 1500]$, $NZ=2001$, $Dev=.25E$, $Den=.1g/cm^3$
 $dr(i)=6*i^2$, $outmost \# = 7$, $gamma=7$. ($i=1, \dots, 6$)

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