Conf-9305293--4

DOE/ER/40561-115-INT93-00-40

Solar Neutrino Oscillations

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*Invited talk presented at
the International Symposium on Nuclear Structure Physics Today
Taiwan, May 1993*

**PREPARED FOR THE U.S. DEPARTMENT OF ENERGY
UNDER GRANT DE-FG06-90ER40561**

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Solar Neutrino Oscillations

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I review the special properties of solar neutrinos that render this flux so uniquely important in searches for neutrino masses and flavor mixing. The effects of matter, including density fluctuations and turbulence, on solar neutrino oscillations are explained through analogies with more familiar atomic physics phenomena.

In the opening session of this workshop, Dr. S.C. Lee described his views of particle and nuclear physics, and his pessimism about the future of nuclear physics in Taiwan. His talk could be described accurately as an argument for channeling subatomic physics resources into user groups operating at facilities like CERN, Fermilab, and the SSC. Certainly much good physics is done or will be done at such facilities. But even if one is concerned only about particle physics, a strategy so narrow misses much that is vital in physics, and much that encourages creativity in young scientists. I am very happy that the topic I chose for this workshop, the solar neutrino problem and neutrino oscillations, illustrates this point clearly.

The solar neutrino problem has been an important physics problem for nearly two decades. In the U.S. the community most concerned with its solution has been the nuclear physicists: Ray Davis and his colleagues in the Brookhaven nuclear chemistry division mounted the ^{37}Cl experiment and demonstrated the feasibility of the ^{71}Ga experiment. Nuclear physics presently supports the U.S. participation in SAGE, GALLEX, and SNO, and funded the U.S. role in Kamioka II. The community's interest in this problem is both historical and technical. Historically the detection of solar neutrinos was viewed as an important test of our understanding of main sequence stellar evolution, a keystone of nuclear astrophysics. Can we predict the temperature and composition of the solar core and model the nuclear reaction chains with sufficient accuracy to predict the solar neutrino flux after five billion years of stellar burning? Technically, the experimental challenges were familiar to and attracted nuclear physicists: low-level counting, the radiochemistry of low-background materials, the calibration of Gamow-Teller strengths, etc.

The problem has also proved to have startling implications for particle physics, probing neutrino masses and mixing with unprecedented sensitivity. The full power of solar neutrino experiments in this regard was not appreciated until relatively recently, with the discovery of the Mikheyev-Smirnov-Wolfenstein mechanism [1]. We now know that such

experiments can probe neutrino properties associated with seesaw masses that are far beyond the reach of any contemplated accelerator. This talk is a review of this physics.

The earliest results from the ^{37}Cl experiment prompted speculations about vacuum neutrino oscillations, in analogy with the neutral kaon system. The requirement for vacuum oscillation to occur is that at least one neutrino is massive and that the neutrino emitted in β decay, the current eigenstate, is not a state of definite mass (i.e., it is not an eigenstate of the free Hamiltonian). In gauge theories such a scenario is highly plausible: we know of no fundamental argument requiring the neutrinos to be massless, and the occurrence of flavor mixing in the quark sector makes similar mixing among the leptons natural in unified theories. In the simple case of two mass eigenstates, the probability for observing a solar neutrino as an electron neutrino a distance L (in meters) from the sun is

$$P_{\nu_e} = 1 - \sin^2 2\theta_\nu \sin^2 \left[\frac{1.27(m_2^2 - m_1^2)L}{E} \right]$$

$$\xrightarrow{L \rightarrow \infty} 1 - \frac{1}{2} \sin^2 2\theta_\nu$$

where E is the neutrino energy (in MeV), θ_ν is the vacuum mixing angle, and m_1 and m_2 are the neutrino masses (in eV). The resulting extraordinary L/E ratio for solar neutrinos implies a sensitivity to extremely small masses, $m_2^2 - m_1^2 = \delta m^2 \gtrsim 10^{-12} \text{eV}^2$, provided the mixing angle is reasonably large. In many extended models a neutrino mass is generated of the form

$$m_\nu \sim M_D^2/M_R$$

where M_D is a quark or charged lepton mass (1 MeV - 1 GeV) and M_R is some large mass of the theory. Thus experiments sensitive to $\delta m^2 \sim 10^{-12} \text{eV}^2$ could uncover new physics governed by mass scales $M_R \sim (10^9 - 10^{15}) \text{GeV}$.

For many years it was believed that oscillations were an unlikely explanation of the solar neutrino puzzle. As the ^{37}Cl discrepancy [2] is a factor of three, complete mixing of three neutrino species is required. This seems a priori quite improbable, and contrasts with the quark sector, where the mixing angles are small. [There is, however, the exception of "just so" oscillations [3], where the oscillation length $\sim$ the earth-sun separation, so that two species with $\theta_\nu = \pi/4$ can completely interfere at earth for some typical solar neutrino energy.]

This situation changed in 1985 when Mikheyev and Smirnov showed that the density dependence of the neutrino effective mass, a phenomenon first discussed by Wolfenstein, could greatly enhance flavor oscillations of solar neutrinos: a ν_e is adiabatically converted to a ν_μ as it traverses a level crossing at some critical density [1]. The result is a range of neutrino mass differences δm^2 and vacuum mixing angles $\sin^2 2\theta_\nu$ for which this phenomenon could account for the ^{37}Cl results.

Consider the case of two flavor eigenstates related to the mass eigenstates through a vacuum mixing angle θ_v .

$$\begin{aligned} |\nu_e\rangle &= \cos \theta_v |\nu_1\rangle + \sin \theta_v |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin \theta_v |\nu_1\rangle + \cos \theta_v |\nu_2\rangle \end{aligned}$$

If one writes the neutrino wave function in matter as

$$|\nu(x)\rangle = a_e(x)|\nu_e\rangle + a_\mu(x)|\nu_\mu\rangle$$

where x is the radial coordinate in the sun, one obtains (after a suitable linearization of the Dirac equation)

$$i \frac{d}{dx} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} 2\sqrt{2}G_F\rho(x) - \delta m^2 \cos 2\theta_v & \delta m^2 \sin 2\theta_v \\ \delta m^2 \sin 2\theta_v & -2\sqrt{2}G_F\rho(x) + \delta m^2 \cos 2\theta_v \end{pmatrix} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix} \quad (1)$$

where $\rho(x)$ is the solar density of electrons, the variable that determines the difference in the ν_e and ν_μ effective masses. It is convenient to rewrite Eq. (1) in a basis consisting of the local mass eigenstates (i.e., the states that diagonalized the right-hand side of Eq. (1))

$$|\nu_L(x)\rangle = \cos \theta(x)|\nu_e\rangle - \sin \theta(x)|\nu_\mu\rangle$$

$$|\nu_H(x)\rangle = \sin \theta(x)|\nu_e\rangle + \cos \theta(x)|\nu_\mu\rangle.$$

The local mixing angle is defined by

$$\begin{aligned} \sin 2\theta(x) &= \frac{\sin 2\theta_v}{\sqrt{X^2(x) + \sin^2 2\theta_v}} \\ \cos 2\theta(x) &= \frac{-X(x)}{\sqrt{X^2(x) + \sin^2 2\theta_v}} \end{aligned} \quad (2)$$

where $X(x) = 2\sqrt{2}G_F\rho(x)E/\delta m^2 - \cos 2\theta_v$. Thus $\theta(x)$ ranges from θ_v to $\pi/2$ as the density $\rho(x)$ goes from 0 to ∞ .

Defining

$$|\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle,$$

eq. (1) becomes

$$i \frac{d}{dx} \begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} \lambda(x) & i\alpha(x) \\ -i\alpha(x) & -\lambda(x) \end{pmatrix} \begin{pmatrix} a_H \\ a_L \end{pmatrix} \quad (3)$$

with the local mass eigenstate splitting determined by

$$\lambda(x) = \frac{\delta m^2}{4E} \sqrt{X^2(x) + \sin^2 2\theta_v}$$

Figure 1. Schematic illustration of the MSW level crossing. The dashed lines correspond to the diagonal matrix elements H_{ee} and $H_{\mu\mu}$ on the right-hand side of Eq. (1). The solid lines are the corresponding heavy and light eigenvalues $\pm\lambda(x)$ of Eq. (3). If an electron neutrino is produced deep in the solar core and propagates adiabatically, it will follow the heavy-mass trajectory, emerging from the sun as a ν_μ .

while the mixing between the local mass eigenstates depends on the gradient of the density

$$\alpha(x) = \sqrt{2} \frac{E}{\delta m^2} \frac{\sin 2\theta_v}{(X^2(x) + \sin^2 2\theta_v)} G_F \frac{d}{dx} \rho(x).$$

The resulting physics is illustrated in Fig. 1. For a small mixing angle θ_v , one has $|\nu_e\rangle \sim |\nu_L(x)\rangle$ and $|\nu_\mu\rangle \sim |\nu_H(x)\rangle$ in vacuum, while at very high densities $|\nu_e\rangle \sim |\nu_H(x)\rangle$ and $|\nu_\mu\rangle \sim |\nu_L(x)\rangle$. Eq. (3) tells us that, if $|\alpha(x)| \ll 2\lambda(x)$ everywhere, an electron neutrino will remain on the heavy mass eigenstate trajectory if it is initially produced in a region of very high density. Thus the adiabatic [4] conversion $|\nu_e\rangle \rightarrow |\nu_\mu\rangle$ can be almost complete for a neutrino that travels from the center of the sun to the surface.

Of course, it may be that the adiabatic condition $|\alpha(x)| \ll \lambda(x)$ is not satisfied everywhere: this condition is clearly most stringent at the level crossing point, that value of x where the diagonal elements of Eq. (1) become degenerate and the splitting between the local mass eigenstates in Eq. (3) achieves its minimum value, $\delta m^2 \sin 2\theta_v / 2E$. As $\alpha(x)$ depends on the gradient of $\rho(x)$, one must then extend the adiabatic approximation described above, which depends only on the initial and final values of $\rho(x)$, to include the constraint that $\frac{d\rho(x)}{dx}$ not change too rapidly in the region of the crossing point.

Following Landau and Zener this can be done by solving Eq. (1) exactly for a linear density "wedge", as illustrated in Figure 2. The initial and final densities for the wedge are chosen to be the physical values (i.e., $\rho(x_i)$ is equated to the electron density in the solar core while $\rho(x_f) = 0$). This builds in the adiabatic approximation result in that

limit. The slope of the wedge is chosen to match the gradient of the solar density at the crossing point: this parameter should govern possible nonadiabatic behavior near the level crossing. The resulting solution of Eq. (1) can then be written in terms of parabolic cylinder functions, and is called the finite Landau-Zener approximation [5].

Figure 2. The top figure illustrates, for one choice of $\sin^2 2\theta$ and δm^2 , that the region of nonadiabatic propagation (solid line) is usually confined to a narrow region about the crossing point x_c . In the lower figure, the solid lines represent the solar density $\rho(x)$ and a linear approximation to $\rho(x)$ that has the correct initial and final densities and the correct slope at x_c . The MSW equation can be solved analytically on this finite wedge (Ref. 5). By extending the wedge to $\pm\infty$ (dotted lines and arrows) and assuming adiabatic propagation in the regions of unphysical density, one obtains the simple Landau-Zener result of Eq. (4).

However, because the nonadiabatic behavior is generally confined to the region around the crossing point, one can derive a simpler, but still very accurate result. The density wedge is extended, as illustrated by the dotted lines in Fig. (2), to $\pm\infty$. Under the

assumption that the evolution in the regions of unphysical density is adiabatic, the exact solution in the physical region can be recovered by choosing the boundary conditions

$$\begin{aligned} a_L(-\infty) &= -a_\mu(-\infty) = \cos \theta(x_i) e^{-i \int_{-\infty}^{x_i} \lambda(x') dx'} \\ a_H(-\infty) &= a_e(-\infty) = \sin \theta(x_i) e^{-i \int_{-\infty}^{x_i} \lambda(x') dx'} \end{aligned}$$

This allows one to evaluate the parabolic cylinder functions in their asymptotic limit. The result is the Landau-Zener formula for the electron survival probability a long distance from the sun,

$$P_{\nu_e}^{LZ} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_\nu \cos 2\theta(x_i) (1 - 2e^{-\pi\gamma_c/2}) \quad (4)$$

where the adiabatic parameter γ_c is

$$\gamma_c = \frac{\lambda(x_c)}{|\alpha(x_c)|} = \left(\frac{\delta m^2}{2E} \right)^2 \frac{\sin^2 2\theta}{|\sqrt{2} G_F \frac{d}{dx} \rho(x_c)|} \quad (5)$$

where x_c is the radial coordinate at the crossing point. In the limit $\gamma_c \rightarrow \infty$, Eq. (4) reduces to the adiabatic result [4]. The factor $2e^{-\pi\gamma_c/2}$ thus describes a hopping probability [6] from the heavy mass trajectory in Fig. 2 to the light mass trajectory.

Strong conversion $\nu_e \rightarrow \nu_\mu$ requires both that the neutrino passes through a "crossing point" at a critical density ρ_c

$$2\sqrt{2} G_F \rho_c E / \delta m^2 = \cos 2\theta$$

and that the crossing be at least somewhat adiabatic

$$\gamma_c \gtrsim 1.$$

The combination of these constraints, illustrated in Fig. 3, defines a triangle of interesting parameters in the $\frac{\delta m^2}{E} - \sin^2 2\theta_\nu$ plane, as Mikheyev and Smirnov first found numerically.

One can envision superimposing the spectrum of solar neutrinos, plotted as a function of $1/E$ and weighted by the response of the ^{37}Cl detector, on Fig. 3. Since Davis sees some solar neutrinos, the solutions must correspond to the boundaries of the triangle in Fig. 3. The horizontal boundary indicates the maximum $\frac{\delta m^2}{E}$ for which the sun's central density is sufficient to cause a level crossing. If a spectrum properly straddles this boundary, we obtain a result consistent with the Homestake experiment in which low energy neutrinos (large $1/E$) lie above the level-crossing boundary (and thus remain ν_e 's), but the high-energy neutrinos (small $1/E$) fall within the unshaded region where strong conversion takes place. Thus such a solution would mimic nonstandard solar models in that only the ^8B neutrino flux would be strongly suppressed. The diagonal boundary separates the adiabatic and nonadiabatic regions. If the spectrum straddles this boundary, we obtain a second solution in which low energy neutrinos lie within the conversion region, but the high-energy neutrinos (small $1/E$) lie below the conversion region and are characterized by $\gamma \ll 1$ at the crossing density. (Of course, the boundary is not a sharp one, but

is characterized by the exponential of Eq. (4)). Such a nonadiabatic solution is quite distinctive since the flux of pp neutrinos, which is strongly constrained in the standard solar model and in any steady-state nonstandard model by the solar luminosity, would now be sharply reduced. An experimental confirmation of such a depletion would then demand new particle physics of the type under discussion here.

Figure 3. The shaded areas define those values of $\delta m^2/E$ and $\sin^2 2\theta_\nu$ that, for a neutrino produced at the sun's center, fail to satisfy the level-crossing condition (above the horizontal boundary) or the adiabatic condition ($\gamma_c \lesssim 1$, below the diagonal boundary). The conversion $\nu_e \rightarrow \nu_\mu$ is efficient in the unshaded region.

The physics of the MSW effect has a simple and familiar atomic physics analog, illustrated in Fig. 4a. In such a spin-filter, the two directions along the fixed z-axis (up and down) correspond to the two neutrino flavors (electron and muon). The two directions along the applied magnetic field $\vec{B}(t)$ (parallel and antiparallel) correspond to the

instantaneous neutrino mass eigenstates (light and heavy). $\vec{B}(t)$ evolves in time as

$$\vec{B}(t) = B_x \hat{x} - B_z t \hat{z},$$

changing from parallel to antiparallel to $\hat{z}$ as t goes from $-\infty$ to $+\infty$, with $|\vec{B}(t)|$ reaching a minimum at the "crossing point" $t = 0$. The analog with MSW neutrino oscillations is clear: A spin-up state will become a spin-down state provided $\vec{B}(t)$ does not change too rapidly at the crossing point, where the splitting between the instantaneous eigenstates in the magnetic field achieves its minimum value.

Figure 4. Atomic physics analogs of the MSW mechanism (a) and of periodic density perturbations or turbulence effects (b). The z-axis is the analog of flavor (up (down) corresponds to electron (muon) type), and similarly the two projections along the axis defined by the applied field $\vec{B}$ are the analog of the instantaneous mass eigenstates. In (a), reversal in $\vec{B}$ transforms a spin-up state into a spin-down state: the spin remains pointed along $\vec{B}$ if the change is made adiabatically. In (b), a rotating perturbation in the x-y plane of angular frequency ω will drive a spin reversal provided $\hbar\omega \sim 2\mu B_z$.

Numerical studies by Schäfer and Koonin [7] and by Krastev and Smirnov [8] showed that additional phenomena are contained in the MSW equations. These authors explored the effects of periodic density fluctuations on flavor oscillations. Schäfer and Koonin found changes in the electron neutrino survival probability $P_{\nu_e}(E_\nu)$ that were linear in the fluctuation amplitude h and sizeable for $h \sim 0.01$. However, as the effect on $P_{\nu_e}(E_\nu)$ was primarily an E_ν -dependent oscillation about the unperturbed value, little net change in P_{ν_e} would remain when integrated over a spectrum of solar neutrinos. In contrast, Krastev and Smirnov found a general enhancement of P_{ν_e} , though for a very large fluctuation amplitude ($h \sim 0.1 - 0.2$).

Haxton and Zhang [9] offered a simple, analytic description of this phenomena, and showed that the physics again has a simple atomic physics analog, illustrated in Fig. 4b. A strong magnetic field again points in the $\hat{z}$ axis. To this fixed field, a small, rotating perturbation is added in the x - y plane

$$\vec{B}_{\text{pert}}(t) = \delta B(\hat{x} \cos \omega t + \hat{y} \sin \omega t)$$

where ω is a frequency that can be tuned to the difference between spin-up and -down unperturbed eigenstates. The resulting reversal of the spin direction is the phenomenon of adiabatic fast passage. It also illustrates the well-known exception to the adiabatic condition: if the perturbation is harmonic and in resonance, its effects grow linearly with time even though $\hbar\omega \gg \delta B$. The obvious neutrino oscillation analog suggests that a (nearly) harmonic density fluctuation (or turbulence pattern [9]) can have large effects provided it is resonant with the local oscillation frequency.

Figure 5. The local neutrino oscillation length in units of the solar radius $R_{\odot}$ as a function of the radial coordinate r , for $\delta m^2/E = 10^{-5} \text{ eV}^2/\text{MeV}$ and $\sin^2 2\theta_v = 0.01$. The intersection with the dashed line illustrates two points where a perturbation of fixed oscillation length could lead to resonance.

The local oscillation length for one choice of δm^2 and $\sin^2 2\theta_v$ is shown in Fig. 5. The pattern is typical: the oscillation length varies greatly as the neutrino propagates, passing through a well-defined maximum at the crossing point. Therefore if there is a solar density/current perturbation of some characteristic frequency, the prospects for matching the local oscillation frequency at some density need not be small.

Figure 6. Neutrino survival probabilities for a highly adiabatic transition ($\gamma_c \sim 24$) with $\sin^2 2\theta = 0.01$, $\delta m^2/E = 10^{-5} \text{ eV}^2/\text{MeV}$, and $h = 0.01$, with h the amplitude of an oscillatory perturbation (eddy currents or density fluctuations). Results are given as a function of the perturbation scale L^{per} , in units of $R_\odot$, for an exact integration (a) and for the approximate expression derived in Ref. 9. The band in the approximate calculation represents the region within which P_{ν_e} will oscillate, a feature apparent in (a). The oscillation length at crossing is $3.56 \cdot 10^{-3} R_\odot$.

Furthermore the quantum mechanics in the case of solar neutrinos is quite rich. In general there will be two points of "stationary phase", which the neutrino encounters on entering and on leaving the resonance region. There will be phase interference between these separated regions of strong perturbation. If the stationary phase points are well separated, each region can be treated by a standard quadratic expansion. But if they approach one another, which occurs if the resonant frequency is close to the frequency characterizing the crossing point, a more complicated expansion is required. Finally, the perturbation may be of some importance even if the length scale exceeds (somewhat) that characterizing the crossing point: that is, one must treat the "classically forbidden" region.

Haxton and Zhang showed that this problem could be attacked by using the uniform approximation, and that a very accurate solution exists which contains both the adiabatic and Landau-Zener results, in the appropriate limits, when the perturbation is turned off [9].

Rather than describing the detailed derivation, I will show an illustrative numerical example in Fig. 6. The analytic and exact results are in impressive agreement. Furthermore dramatic effects persist over a range of perturbation frequencies (and, it can be shown, a range of neutrino energies). Thus this example is similar to that found by Krastev and Smirnov [8], though the necessary fluctuation amplitude ($h = 0.01$) is much smaller than they found necessary.

Several years ago there was some indication that the signal in the ^{37}Cl experiment was anticorrelated with the solar cycle. Recent Homestake data make such an anticorrelation less plausible [10]. As density fluctuations or turbulence in the sun could be plausibly tied to the solar cycle, the mechanism just described seemed a possible explanation, one that required no new particle physics (e.g., neutrino magnetic moments [11,12] other than that already implicit in the MSW mechanism. Now the most plausible site for the phenomena is in a core-collapse supernova, where convective velocities ($v/c \sim h$) and density perturbations much larger than those of Fig. 6 are generated.

There is much interesting physics in the responses of various solar neutrino detectors to the neutrino oscillation physics described here. I will not attempt a discussion of this aspect of the solar neutrino problem today (see, however, ref. 13). However I hope I have made the point that even simple phenomena, like the MSW mechanism and its generalization to include density fluctuations, can be both unanticipated and extremely significant to physics generally. This stands as a counterargument to those who believe significant subatomic physics can only come from a certain kind of activity, such as experimentation at the highest energy accelerators. I hope the audience has also noted that fundamental physics of this sort can still be done on a rather human scale. It is physics that Taiwan could participate fully in, and that I believe your young scientists would find appealing. Finally, I note that the solar neutrino problem is by no means unique as an example of nuclear physics that addresses very deep issues of the standard model and its possible extensions. There are enthusiastic and talented nuclear physicists studying lepton number violation in $\beta\beta$ decay, right-handed currents and other new interactions in β decay, hadronic weak neutral currents in nuclear parity violation, CP-violation in atomic electric

dipole moment measurements, etc. I believe Taiwan will choose to pursue such physics opportunities.

This work was supported in part by the U.S. Department of Energy and by NASA under grant # NAGW 2523.

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