DOE-53250-006

DOE/ER/53250--6

DE92 010026

Annual Technical Report
Non-Local Kinetic Transport Studies
of a
Field Reversed Configuration*

for

July 1, 1990-June 30, 1991

by

Chan K. Choi
Principal Investigator

School of Nuclear Engineering
Purdue University
West Lafayette, IN 47907
(317) 494-6789

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

*Work supported by the U.S. Department of Energy under contract DE-FG02-87ER53250.A001.

MASTER

I. Introduction

A computer code was developed and tested, during this past period, to determine the global, linear kinetic stability of a one-dimensional Field-Reversed Configuration (FRC). A difficulty in verifying the code for fully electromagnetic perturbations caused the model to be simplified so that it would be easier to verify. The changes in the model were going from a fully electromagnetic field operator to an electrostatic field operator. The resultant simplifications allowed the code to be tested against well-known, analytic results from elementary plasma physics. This report will review the physical assumptions used to model the plasma and describe the extra assumptions inherent in using an electrostatic model. In addition, the rationale for simplifying the model and the results of the test of the model will be presented.

An FRC consists of an axisymmetric, toroidal plasma confined by a magnetic field that is nearly poloidal. FRC's belong to a class of experiments known as compact toroids (CT's). A CT is distinguished from other toroidal devices by having no mechanical structure in the center of the torus. An FRC is attractive as a potential energy source not only because of its simple mechanical structure, but also because it has a relatively high value of β , defined as the ratio of plasma pressure to magnetic field pressure at the wall ($\beta=8\pi p/B_w^2$). A schematic representation of an FRC is presented in Figure 1.

The physics of transport processes in an FRC are not well understood; in particular, the particle and energy loss times are greater than one would expect if cross-field diffusion were solely governed by Coulomb collisions. Also, recent experimental results [1] indicate that improved transport calculations [2] do not predict the correct scalings. This stability code is the next step towards understanding the processes that govern FRC transport.

Figure 1. Schematic of an FRC

II. The Physical Assumptions

In the past year considerable difficulty in verifying the code for the fully electromagnetic case was experienced. The difficulty was two-fold: first, there is a paucity of analytic solutions to the fully magnetized, electromagnetic stability problem. Most analytic work consisted of setting stability limits via Nyquist's method. Secondly, the model required a large amount of analytic results in addition to the numerical methods used, making it difficult to diagnose overall code performance. Since, it was uncertain whether the difficulty in verifying the code was due to an algorithmic error or whether the global method itself was inadequate, it was necessary to simplify the model by using an electrostatic approximation which made diagnostic evaluation easier. In this section the basic physical assumptions used to derive the equations that the code solves are discussed. Additionally, the requirements for the electrostatic approximation are discussed. This section presents both the strengths and shortcomings of the present code.

Models that have been used to explain anomalous transport in an FRC range from the local approximation [3] to kinetic simulation techniques [4,5]. The simulations suffer from drawbacks in the physical parameters used. By using a 2-species simulation code with a realistic electron-ion mass ratio, but unrealistic large drift parameters, Brackbill, et

al [4] found that Lower Hybrid Drift (LHD) turbulence developed but did not propagate to the interior of the plasma. A hybrid simulation using realistic drift parameters but unrealistic mass ratio was used to model the sheath region of an FRC and the LHD-like modes that were found were substantially different from those using the local approximation [5].

The local approximation requires that particle gyro-radii be much smaller than the gradient length scale. There are 2 regions in an FRC where this requirement fails. One is near the separatrix where the gradient scale lengths are typically 1 or 2 gyro-radii. The other is in the vicinity of the field null where the ion gyro-radius is ill-defined and particle excursions tend to be large, thus a particle sees a large part of the equilibrium.

A further assumption is made for the purpose of verifying the suitability of the global method. In the electrostatic assumption one assumes that the perturbed magnetic field vanishes. For this assumption to be valid, the ratio of plasma pressure to magnetic pressure ($\beta=8\pi p/B_w^2$) must be small. In modelling an FRC, this approximation is not valid since the beta may be as high as 90%. The purpose of approximation was to simplify the model to the extent that verification of the method of Lewis, Symon, and Seyler[8-10] was possible.

The physical assumptions used in the code are strong enough to allow enough mathematical simplification to avoid the numerical difficulties of the kinetic simulations without losing important physics.

We specialize to the post-implosion phase of a hot FRC. Table 1 gives some physical quantities for a typical FRC plasma. First, we assume that the plasma can be well described by a collisionless equilibrium. This is justified by comparing the cyclotron and

Useful reference values	5 mTorr	20mTorr
electron gyrofrequency (rad/sec)	1.4×10^{11}	1.2×10^{11}
ion gyrofrequency (rad/sec)	3.8×10^7	3.4×10^7
electron plasma frequency (rad/sec)	2.4×10^{12}	4.0×10^{12}
electron gyroradius (cm)	3.9×10^{-3}	3.4×10^{-3}
ion gyroradius (cm)	4.5×10^{-1}	2.5×10^{-1}
electron collision rate (s^{-1})	2.4×10^7	1.5×10^8
ion collision rate (s^{-1})	5.8×10^4	1.3×10^6

Table 1. Physical quantities Characteristic of FRX-C

collision frequencies. In the 20 mTorr case the ratio of ion cyclotron to ion collision frequencies is not as large as in the 5 mTorr case, but clearly, the ion will undergo many oscillations before a collision. The assumption of a collisionless plasma greatly simplifies the problem mathematically. The collisionless equilibrium is computed by a code written by Bob Webster [6].

The next assumption is that the FRC can be modelled well by a one dimensional, cylindrically symmetric plasma. There are two motivations for making this assumption. First, the FRC is a long, thin plasma where the radial gradients are much greater than the axial gradients so that the mid-plane profile does not change very much over a significant portion of the configuration. The second reason for using a 1-D model is that in linear stability computations, the particle trajectories need to be computed. In a one dimensional plasma the particle orbits are bounded and periodic while in a general multi-dimensional plasma, the particle orbits can fill an entire region of phase space. What this means is that for a 1-D plasma the orbits only need to be computed over one period. (In a 2-D plasma the orbits would have to be computed over all phase space which is equivalent to integrating from $t=0$ to $t=\infty$.) This results in a tremendous mathematical and numerical simplification.

The final assumption is that the FRC is described well by linear stability. This means that the perturbations are small compared to equilibrium quantities. This code determines what instabilities are possible, but not their amplitudes.

The model used is that of a one-dimensional, cylindrically symmetric plasma, described by a collisionless distribution function [7]. The strengths of this model are the following:

- 1) The model is global so that the shortcomings of the local approximation will not mask the physics. The particle orbits are computed exactly.
- 2) The model allows enough mathematical simplification to be tractable numerically.

The shortcomings of this model are the following:

- 1) The model is one dimensional so effects due to magnetic field curvature and mirrors will not be seen.
- 2) The model does not determine any saturation mechanism for the instabilities and suffers from the limitations imposed by linear stability.

III. The Equations Solved

In this section the equations for the global kinetic stability are discussed. The basic goal is to characterize the solutions to the linearized Maxwell-Vlasov equations. First, there will be a short discussion about the equilibrium, then the equations the code is to solve will be presented.

III.1 The Collisionless Equilibrium

For a one dimensional, cylindrically symmetric equilibrium, we see that there exist three constants of the motion: H , the particle Hamiltonian; p_θ , the canonical angular

momentum; and p_z , the canonical axial momentum. If we assume that $T_{\parallel} = T_{\perp}$, we may eliminate one constant of the motion from the distribution function, i.e. we say that

$$f_{s0} = f_{s0}(H, p_{\theta}) \quad (1)$$

where f_{s0} is the Vlasov distribution function for species s . A computer code exists [6] that can compute the equilibrium distribution for realistic FRC profiles. The code assumes a parametric representation of f_{s0} given by

$$f_{s0}(H, p_{\theta}) = \exp \left\{ \frac{(-H + \beta(p_{\theta}))}{\alpha(p_{\theta})} \right\} \quad (2)$$

The functions α and β have parametric forms that were chosen so that the pressure and temperature are well modelled. The precise functional dependence of α and β are determined by requiring the moments of the distribution to satisfy the low frequency Maxwell equations and quasi charge neutrality [5]. The code we have written assumes a form for the distribution given by equation (2). The user is required to provide subroutines for α and β (in addition to subroutines for the equilibrium potentials). The formats and non-dimensionalizations are described in the comments of the main program.

III.2 The Maxwell-Vlasov System

In this section the dispersion relation for the linear stability of the computed equilibria will be discussed. The basic methodology was developed by Lewis, Symon, Seyler, and Schwarzmeier [8-10]. We begin with the nonlinear, time-dependent, Maxwell-Vlasov equations given by

$$\frac{\partial f_s}{\partial t}(q, p, t) + \{f_s(q, p, t), H\} = 0 \quad (3)$$

$$\text{and} \quad \sum_{n=1}^4 \Lambda_{mn} \phi_{nm} = \int dp J_m f_s(q, p, t) \quad (4)$$

where $\{f, g\}$ is the classical Poisson bracket given by

$$\{f, g\} = \sum_{i=1}^3 \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right). \quad (5)$$

f_s is the single particle distribution function for species s ; q and p are the canonical variables, Λ_{mn} is a field operator representing Maxwell's equations, ϕ_n is a component of an array containing the electric and magnetic potentials and J_m is a component of the charge-current kernel given by

$$\mathbf{J} = q \left(1, \frac{\mathbf{v}}{c} \right). \quad (6)$$

We follow general procedures by assuming that $f_s(q, p, t)$ can be split into two parts: a time-independent equilibrium distribution, $f_{s0}(q, p)$ and a smaller time-dependent perturbation, $f_{s1}(q, p, t)$

$$f_s(q, p, t) = f_{s0}(q, p) + f_{s1}(q, p, t). \quad (7)$$

Further, we assume that the Hamiltonian and potentials also split into two parts:

$$H(q, p, t) = H_0(q, p) + H_1(q, p, t), \quad (8)$$

$$\phi_n(q, t) = \phi_{n0}(q) + \phi_{n1}(q, t). \quad (9)$$

These assumptions lead to a dispersion relation as was discussed in the previous annual report. Briefly, we have

$$\det |D_{mn}(\omega)| = 0, \text{ where} \quad (10)$$

$$D_{nn'}(\omega) \alpha = 0$$

$$D_{nn'}(\omega) = \Lambda_{nn'}(\omega) - \sum_r \frac{\kappa_r \Phi_{nr}^* \Phi_{n'r}}{\mu_r - \omega}$$

$$\Lambda_{nn'}(\omega) = \int dq \eta_n^* \Lambda \cdot \eta_{n'}$$

$$\Phi_{nr}^* = \int dq dp w_r j \cdot \eta_n$$

$$\Phi_{n'r}^* = \int dp dq w_r^* j \cdot \eta_{n'} \quad (11a-e)$$

and where the vector α is an array of coefficients for the basis functions, w_r are the Liouville eigenfunctions and η_n are the basis functions in the Galerkin discretization. Further,

$$j = q_s \left(1, \frac{v_0}{c} \right), \text{ where} \quad (12a)$$

$$v_0 = \left(p_i - \frac{q}{c} A_0 \right) \quad (12b)$$

A_0 is the equilibrium magnetic vector potential and

$$\kappa_r = \left[\mu_r \frac{\partial f_0}{\partial H_0} + m \frac{\partial f_0}{\partial p_\theta} + k \frac{\partial f_0}{\partial p_z} \right] \quad (13)$$

The matrix $D_{nn'}(\omega)$ contains all the information about the linear stability properties of the equilibrium. In particular, it contains all the information about the particle motion as it affects the field perturbations. The eigenfunctions w_r have been found in one dimension and depend on the solution to the particle equations of motion, which are easily solved for a time-independent Hamiltonian with 2 ignorable coordinates.

III.3 Final Form Of The Dispersion Relation

Attempts have been made to compute $D_{nn'}(\omega)$ directly and have failed because of the enormous computation involved. For every ω considered, the particle motion must be solved and used to compute the orbit integrals over all the accessible phase space. An attempt to pose equation (11) in a more tractable form was made. In essence, the ω dependence in the sum over r was removed by a simple identity.

$$D_{nn'}(\omega) = \Lambda_{nn'}(\omega) - \int_{-\infty}^{\infty} d\mu \frac{S_{nn'}(\mu)}{\mu - \omega}, \quad (14)$$

$$\text{where } S_{nn'}(\mu) = \sum_r \delta(\mu - \mu_r) \kappa_r \Phi_{nr}^* \Phi_{n'r}, \quad (15)$$

The matrix S is called the dynamic spectral matrix. What makes it so useful is that much of the computation can be done analytically. In this section we will present the form of the dynamic spectral matrix computed by the computer code.

The label r hides much of what makes computing the dynamic spectral matrix difficult. If we call the integrand of equation (15) $G_{nn'}$, then writing the sum over r out completely gives

$$\sum_a \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} dp_{\theta} \int_{U_m(p_{\theta})}^{\infty} dH \int_{-p_{z0}(H,p_{\theta})}^{p_{z0}(H,p_{\theta})} dp_z G_{nn'}(p_{\theta}, H, p_z; l, a) \quad (16)$$

The l is an integer used to label Liouville eigenfunctions and a is an index used to distinguish degenerate eigenfunctions (i.e. when there are two possible orbits for the same H , p_{θ} and p_z). Using the dynamic spectral matrix allows the integration over p_z and the infinite summation to be done analytically.

The matrix S is defined by equations (14) and (15). To reduce the amount of calculation required, a change of variables is made:

$$H_{\perp} = H - \frac{p_z^2}{2m} \quad (17)$$

The matrix then becomes

$$S_{nn'} = \sum_a \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} dp_{\theta} \int_{U_m(p_{\theta})}^{\infty} dH_{\perp} \int_{-\infty}^{\infty} dp_z \delta(\mu - \mu_r) \kappa_r \Phi_{nr}^* \Phi_{n'r} \quad (18)$$

Where,

$$\Phi_{nr} = \int dp dq w_r j \cdot \eta_n \quad (19)$$

$$\eta_n = (\eta_{n1}, \eta_{n2}, \eta_{n3}, \eta_{n4}) \quad (20)$$

$$w_r = d_{sr} \delta(H - H') \delta(p_{\theta} - p'_{\theta}) \delta(p_z - p'_z) \exp(i[m\theta + kz]) u(\tau) \quad (21)$$

$$u(\tau) = \exp(i[l\Omega_r + m\Theta(\tau) + kZ(\tau)]) \quad (22)$$

$$\Theta(\tau) = \int_0^{\tau} \frac{\partial H_0}{\partial p_{\theta}} d\tau' - \Omega_{\theta} \tau \quad (23)$$

$$Z(\tau) = \int_0^{\tau} \frac{\partial H_0}{\partial p_z} d\tau' - p_z \tau = 0 \quad (\text{for } 1-D), \quad (24)$$

$$\begin{aligned}\Omega_r &= \frac{2\pi}{T_{sr}} \\ T_{sr} &= \int_{r1}^{r2} \frac{dr}{|v_r|} \\ v_r &= \frac{p_r}{M}\end{aligned}\tag{25a-c}$$

$r1$ and $r2$ are the inner and outer turning points defined by $p_r = 0$ and μ_r is the eigenvalue associated with the Liouville eigenfunction w_r

$$\mu_r = l\Omega_r + m\Omega_\theta + k\frac{p_z}{M} \tag{26}$$

The form of the Liouville eigenfunctions w_r is derived in Seyler and Lewis [10]. The η_{ni} are the basis functions used in the discretization of equation (10). The quantity $\mathbf{j}$ is the charge-current kernel defined in equation (12). To finish defining the terms in the preceding equations we have

$$\begin{aligned}H_\perp &= \frac{p_r^2}{2M} + U(r; p_\theta) \\ U(r; p_\theta) &= \frac{\left[p_\theta - \frac{q}{c}\psi(r)\right]^2}{2Mr^2} + q\phi(r) \\ \psi(r) &= rA_\theta(r)\end{aligned}\tag{27a-c}$$

The simplifying effect of the electrostatic approximation is easily seen at this point. The approximation is essentially to ignore Ampere's law in the field operator. Thus, the system of three (a gauge transformation is used to eliminate one equation) coupled differential equations become one. Further, the four dimensional vectors η and $\mathbf{j}$ reduce to

$$\eta = (\eta_1, 0, 0, 0), \tag{28}$$

$$\text{and } \mathbf{j} = (q, 0, 0, 0). \tag{29}$$

This reduces the number of terms in the dynamic spectral matrix by $2/3$.

IV. Numerical Analysis

The results of applying the code to a test problem are presented in this section. The problem was the drifting bi-Maxwellian distribution. In the electrostatic approximation, propagation along the magnetic field in a uniform plasma is equivalent to parallel propagation in the field-free case. The ion acoustic instability arises when a drifting Maxwellian distribution of electrons and a stationary ion distribution are subject to a perturbation. The distribution is unstable if $T_e \gg T_i$ and if the drift velocity, $u_0 > (T_i/m_i)^{1/2}$. The computer code gave rough agreement with analytical results, e.g., the computed real frequency and growth rate corresponded to the analytical approximation. Also, the criterion $u_0 > (T_i/m_i)^{1/2}$ did provide a boundary for stability, although the code was not very sensitive to the boundary. That is, if u_0 was slightly greater than the minimum, the code could not find a root.

The rest of this section is organized as follows. In section IV.1, the basic structure of the code and input requirements are discussed, including the physical constants for the drifting bi-Maxwellian and the code results. In section IV.2 the regularity and boundary conditions are discussed, particularly, how they differ in the electrostatic case from the fully electromagnetic case. Finally in section IV.3 the question of convergence is discussed. This question is important because the code did not converge well to the answer (in the sense that increasing the number of basis function did not increase the accuracy). Possible reasons for this behavior will be discussed.

IV.1 Code Structure and Test Problem

The computer code, renamed SCOVS (Stability Code for One-dimensional Vlasov Stability) is an eigenvalue code. It determines the eigenvalue (frequency) and eigenfunction

(mode structure) defined by equations (11-25). There are two types of user input. One is an ordinary data file containing the mode numbers (m and k), particle masses, particle charges, tolerance requirements and initial guesses for the root finding algorithm. The other type of input consists of a set of user-defined subroutines and functions. These routines provide all the equilibrium information for the code. They include routines for evaluating the distribution function and the appropriate moments of the distribution, like density. The user-defined subroutines are summarized in table 2.

Name	Type	Input	Return
GETPOT	SUBROUTINE	RADIAL POSITION	EQ. FLUX AND POTENTIAL
GETDPOT	SUBROUTINE	RADIAL POSITION	DERIVATIVES OF FLUX AND POTENTIAL
GETA	FUNCTION	RADIAL POSITION	THETA COMP. OF MAG. POT.
GETDA	FUNCTION	RADIAL POSITION	DERIVATIVE OF MAG. POT.
GETNI	FUNCTION	RADIAL POSITION	ION DENSITY
GETNE	FUNCTION	RADIAL POSITION	ELECTRON DENSITY
ALPHAI	FUNCTION	PTHETA	ANSATZ FUNC. FOR IONS
ALPHAE	FUNCTION	PTHETA	ANSATZ FUNC. FOR ELECTRONS
BETAI	FUNCTION	PTHETA	ANSATZ FUNC. FOR IONS
BETAE	FUNCTION	PTHETA	ANSATZ FUNC. FOR ELECTRONS
DALPHI	FUNCTION	PTHETA	DERIVATIVE WRT PTHETA
DALPHE	FUNCTION	PTHETA	DERIVATIVE WRT PTHETA
DBETAI	FUNCTION	PTHETA	DERIVATIVE WRT PTHETA
DBETAE	FUNCTION	PTHETA	DERIVATIVE WRT PTHETA

Table 2. User defined subroutines and functions for SCOVS

Figure 2 is a flow diagram that illustrates the basic code structure. The program consists of essentially two parts. The first part incorporates all the information relating the the equilibrium particles to the linear stability into the dynamic spectral matrix. The second part of the code uses the dynamic spectral matrix to solve the dispersion relation (10).

Figure 2. Flow Diagram for SCOVs

The integration routines used are Gaussian quadrature and Simpson's rule. Their convergence is discussed in Papanikolaou and Choi [11] as well as the previous annual report [12]. The determinant of the dispersion matrix is found via LU decomposition of the matrix followed by multiplication of the diagonal elements. The root finder used is Mueller's algorithm with a modification to prevent guesses with $\text{Im}(\omega) < 0$.

For the drifting bi-Maxwellian equilibrium, the following physical parameters were used:

$$\begin{aligned} T_e &= 100 \text{ keV} \\ T_i &= 1 \text{ keV} \\ N_e &= N_i = 1 \times 10^{13} \text{ cm}^{-3} \\ u_0 &= 10^8 \text{ cm/sec} \\ m &= 0 \\ k &= 1.0 \end{aligned}$$

The analytical results are that $\omega_r = 2.8 \times 10^9 \text{ sec}^{-1}$ and $\omega_i = 3.4 \times 10^7 \text{ sec}^{-1}$. The code results were $2.9 \times 10^9 \text{ sec}^{-1}$ and $3.2 \times 10^7 \text{ sec}^{-1}$, respectively. Some of the possible reasons for the differences are discussed in the following sections.

IV.2 Regularity and Boundary Conditions

Since Maxwell's equations are being applied to a cylindrical geometry, one must be concerned about the behavior of the computation near the origin. Lewis and Schwarzmeier (unpublished) approached this difficulty several years ago and developed two methods to attack this problem. The first was to determine the asymptotic form of the distribution function near $r=0$. The second was to find basic functions that guarantee that the computed potentials would be infinitely continuous near the origin.

The latter method is used in our code. The basic requirements differ for basis functions representing scalar and vector potentials, but since the code is being tested for the

electrostatic case, only the scalar potential needs to be defined. For the electric potential, the requirement is

$$\phi(r) = r^m \psi(r^2), \quad (30)$$

where m is the poloidal mode number. To test whether these conditions work, we used them to solve the wave-guide problem in a cylindrically symmetric geometry. This was discussed in our previous report [12] and in [11]. Since the regularity conditions worked well in the fully electromagnetic case, it was not necessary to retest them for the electrostatic. Clearly the regularity conditions did not introduce any error.

The gauge choice made in determining the form of $\Lambda_{mn}(\omega)$ for the fully electromagnetic case is that the radial component of the perturbed magnetic potential, $A_{1r} = 0$. While this simplifies the problem by reducing the number of functions that must be found, it results in frequency dependent boundary conditions. This required the redefinition of the basis functions representing the potentials near the boundary. To cope with the frequency dependent boundary conditions, it was necessary to redefine the basis functions in such a way that the basis conditions were satisfied. For the electrostatic case, the boundary conditions were very much simplified. The only boundary condition was that the tangential component of the electric field be continuous. Along with the gauge choice, this resulted in the boundary condition that the electric potential vanish at the wall. This is the most probable reason for the slight difference between the analytical and the computed result.

IV.3 Integration Convergence

As figure 3 illustrates, the convergence of the computation to a specific eigenfrequency was not very good. First, the tolerance requirement on the root finder

Figure 3. Convergence of the eigenfrequency. (The eigenfrequency here is defined as the absolute value of the complex frequency.)

could not be increased beyond 0.01, or roots would not be found. This indicates that the computation of the dispersion equation contains a lot of "noise". In addition, increasing the number of basis functions did not increase the accuracy, meaning that as the number of basis functions increased, the eigenfrequency did not get closer and closer to a specific answer. At this time we are not able to say why this is so, although we are able to make some speculation.

There are two places where "noise" can accumulate. One is in the computation of the dynamic spectral matrix. Each term in the matrix is a sum of very many terms. Errors may propagate when the matrix elements are small. This, however, is a small source of error since our previous work has indicated that the dynamic spectral matrix converges well to its computed value. A more likely source of noise is in evaluating the dispersion relation. The method used is LU decomposition, followed by multiplication of the diagonal elements (with the appropriate sign). As the computation nears an eigenvalue, the matrix tends to become singular. This results in the LU decomposition not being as accurate. Although the code takes steps to prevent the LU decomposition from failing, it may result in an inaccurate evaluation of the dispersion relation.

To improve the evaluation, a more robust method of evaluating the determinant of a matrix will have to be implemented. We believe that using LU decomposition until the vicinity of a root has been reached and then go to a direct evaluation of the determinant, i.e., expansion by minors, may alleviate this problem. The reason this method is not used currently is that it is prohibitive computationally. LU decomposition is an $O(N^3)$ method while expansion by minors is $O(N!)$.

V. Summary

The computer code SCOVS has been simplified by the electrostatic approximation and applied to a well known stability problem. The results show that the global model in SCOVS is correct and reasonable computationally (6.2 minutes were required on a Gould

NP2 to run the problem). There were some difficulties numerically in that increasing the number of basis functions did not increase the accuracy of the computation (in the sense of asymptotically approaching a number). The difference between the computed and analytic answers are attributable to the affects of the boundary conditions. The success in applying the electrostatic approximation to the code aids us in diagnosing where difficulties occur in the fully electromagnetic problem. The difficulties can occur in only two places: the parts of the model that describe electromagnetic perturbations, or in the evaluation of the dispersion relation. The evaluation of the dispersion relation is more difficult for the electromagnetic case since the dispersion is $3N \times 3N$ instead of $N \times N$.

REFERENCES

- [1] D. J. Rej et al. Flux Confinement Measurements in Large Field-Reversed Configuration Equilibria, *Phys Fluids B*, 2 (7) (1990) 1706.
- [2] N. A. Krall. The Effect of Low-Frequency Turbulence on Flux, Particle, and Energy Confinement in a Field-Reversed Configuration, *Phys. Fluids B*, 1 (9) (1989) 1811.
- [3] R.C. Davidson and N.A. Krall. Anomalous Transport in High Temperature Plasmas with Applications to Solenoidal Fusion Systems, *Nucl Fusion*, 17 (6) (1977) 1313.
- [4] J. U. Brackbill, D. W. Forslund, K.B. Quest, and D. Winske. Nonlinear Evolution of the Lower Hybrid Drift Instability, *Phys. Fluids*, 27 (11) (1984) 2682.
- [5] N. T. Gladd, A. G. Sgro and D. W. Hewett, Microstability Properties of the Sheath Region of the Field-Reversed Configuration, *Phys. Fluids*, 28 (7) (1985) 2222.
- [6] Robert B. Webster, A Study of the Equilibrium Phase of the Field Reversed Configuration, PhD Thesis, Purdue University, 1988.
- [7] C. K. Choi. Non-local Kinetic Transport Studies of a Field-Reversed Configuration. Annual Report 1989. DOE-53250-003
- [8] H. R. Lewis and K.R. Symon, Stability of Inhomogeneous Vlasov Equilibria, *J. Math. Phys.*, 20 (1979) 413.
- [9] K.R. Symon, C. E. Seyler and H.R. Lewis, Stability of Vlasov Equilibria. Part 1 General Theory, *J. Plasma Phys.*, 27 (1) (1982) 13.
- [10] H.R. Lewis and C. E. Seyler, Stability of Vlasov Equilibria. Part 2 One non-ignorable coordinate, *J. Plasma Phys.*, 27 (1) (1982) 25.
- [11] P.G. Papanikolaou and C.K. Choi, Plasma Stability Study for the FRC Compact Torus. *Fusion Tech.* May (1991) 1317.
- [12] C.K. Choi, Non-Local Kinetic Transport Studies of a Field-Reversed Configuration, Annual Report 1990. DOE-53250-005.

END

**DATE
FILMED**

4 / 23 / 92