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UNITED STATES ATOMIC ENERGY COMMISSION

**A SEMI-EMPIRICAL METHOD OF CALCULATING
THE ENERGY-ABSORPTION BUILD-UP FACTOR
WITH AN APPLICATION TO A UNIFORMLY
CONTAMINATED SPACE HAVING SPHERICAL
BOUNDARIES**

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A SEMI-EMPIRICAL METHOD OF CALCULATING THE
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ABSTRACT

A form for the point-source gamma-ray energy absorption build-up function valid for a material in the energy range where it is essentially a Compton scatterer is suggested as a modification of the asymptotic forms derived by U. Fano. Its parameters are evaluated by means of an energy equilibrium condition and a fit to experimental data. The results are compared with those obtained by other methods, and an application to the problem of uniformly and continuously distributed point-sources is discussed.

The point-source energy absorption build-up factor, as conventionally defined, is the ratio of the energy absorption rate from the total-radiation flux to that from the primary flux at some point in the material surrounding the point source. This material is assumed to be uniform and infinite in extent. The build-up function for a given source energy describes the variation of the build-up factor as a function of mean free path, and will be designated by $b(t)$.

If we assume a form of the build-up function consistent with the considerations of Fano (1), it is possible to compute build-up factors for Compton scatterers by using an energy-equilibrium condition (2,3,4,5,6) in combination with a fit to experimental data (7,8,9).

The dose-rate in a uniformly contaminated medium of infinite extent, when the contaminant is a monochromatic gamma emitter, is given by

$$I = \frac{E\mu_e}{\rho\mu_t} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \int_0^\infty \frac{dt b(t)e^{-t}}{4\pi}$$

where I is the dose-rate in Mev/sec-gram, E the energy emitted by the contaminant in Mev/sec-cm³, μ_e the energy absorption coefficient for the medium corresponding to the source energy in cm⁻¹, μ_t is the total attenuation coefficient for the medium corresponding to the source energy in cm⁻¹, ρ is the density of the medium in

grams/cm³, t the distance to a differential emitter dV in units of mean free path ($= \mu_t x$, where x is this distance in cm). The system is in spherical coordinates and ϕ and θ are the azimuthal and altitudinal angles respectively.

According to Fano (1) the asymptotic form of the build-up function for source-energies less than those corresponding to the minimum value of the attenuation coefficient is given by x^k . We propose to modify this and write

$$2 \quad b(t) = (1 + at)^\beta$$

If the use of a single form for the build-up function is to be justified it must be confined to a region where $d\mu_t/dE_0$ (E_0 being the source energy) has a regular behavior. For this reason and because this is the region for which our experimental data are applicable, we restrict Eq. 2 to the energy region where the material is a Compton scatterer.

Substituting Eq. 2 in Eq. 1 we get an answer in terms of the incomplete gamma function;

$$3 \quad I = \frac{E\mu_e}{\rho\mu_t} \int_0^\infty e^{-t}(1 + at)^\beta dt = \frac{E\mu_e}{\rho\mu_t} a^\beta e^{1/a} \Gamma(\beta + 1, 1/a)$$

Because the rate of energy absorption per gram equals the rate of energy production (the equilibrium condition), $I = E/\rho$ and therefore

$$4 \quad \int_0^{\infty} e^{-t}(1+at)^{\beta} dt = a^{\beta} e^{1/a} \Gamma(\beta+1, 1/a) = y$$

where $y = \mu_t/\mu_e$.

Because of the way that a and β appear in Eq. 4 it is difficult to perform numerical operations using it, and for convenience we have made use of two approximations. For $y < 2.8$ we use

$$5 \quad \beta = \frac{\ln[y(y-1)+1]}{\ln(1+ay)}$$

and for $y > 2.8$

$$6 \quad \beta = \frac{y-1}{\ln(1+ay)}$$

These approximations are discussed in the appendix.

The build-up function now has the form

$$7 \quad b(t) = \begin{cases} \exp \left\{ \frac{\ln[y(y-1)+1]}{\ln(1+ay)} \ln(1+at) \right\} & y < 2.8 \\ \exp \left\{ \frac{y-1}{\ln(1+ay)} \ln(1+at) \right\} & y > 2.8 \end{cases}$$

Fitting Eq. 4 to the experimental data of Refs. 7,8,9 we find that

$a \approx 1/y$.* Substituting this in Eq. 7 we have for the build-up function

$$b(t) = \begin{cases} \exp \left\{ \frac{\ln [y(y-1) + 1]}{\ln 2} \ln \left(1 + \frac{t}{y}\right) \right\} & y < 2.8 \\ \exp \left\{ \frac{y-1}{\ln 2} \ln \left(1 + \frac{t}{y}\right) \right\} & y > 2.8 \end{cases}$$

Figs. 1 - 3 show comparison of Eq. 8 with the calculations of Goldstein and Wilkins(4). The calculations for lead and for low energy photons on iron represent cases where departures from our assumptions become important. 500 Kev photons on iron have energies close to the photoelectric region, and lead is hardly a Compton scatterer even at 1 and 2 Mev. Eq. 8 agrees very poorly with the calculations of Goldstein and Wilkins for 0.255 Mev and water. However, experimental values for the build-up function, normalized to 4 mean free paths, are given for Hg^{203} (0.28 Mev) in Ref. 8 and are in reasonable agreement with Eq. 8. These build-up factors are exhibited with those of Goldstein and Wilkins for comparison in Figure 4.

* The work of Van Dilla and Hine(10) furnishes different results. For a discussion of this experiment see Ref. 4.

It is possible to modify Eq. 1 to include the dose-rate at a distance from a medium with a spherical boundary.

$$9 \quad I = \frac{E\mu_e}{\mu_t \rho} \int_0^{\phi_0} d\phi \int_0^{\theta_0} d\theta \sin \theta \int_{f(\phi, \theta, t)}^{g(\phi, \theta, t)} \frac{dt(1 + at)^\beta e^{-t}}{4\pi}$$

As this is rather difficult to solve exactly except for simple boundaries, we introduce another approximation. Let us expand Eq. 2 binomially, terminating it at n terms, where n is the largest integer less than $\beta + 2$. This approximation is, of course, exact when β is integral. A partial solution for Eq. 9 is then

$$10 \quad I = - \frac{E\mu_e}{\rho\mu_t} \int_0^{\phi_0} d\phi \int_0^{\theta_0} \frac{d\theta \sin \theta e^{-t}}{4\pi} \sum_{\nu=0}^n \frac{A_\nu t^\nu}{\nu!} \quad \left| \begin{array}{l} g \\ f \end{array} \right.$$

where

$$A_0 = y$$

$$A_1 = y - 1$$

$$A_2 = y - 1 - a\beta$$

$$A_3 = y - 1 - a\beta - a^2\beta(\beta - 1)$$

* * *

$$A_{n+1} = 0$$

The case of the dose-rate in air at some height t_h (mean free paths in air) over a uniformly contaminated half-space is of some interest. Here, $g = \infty$ and $f = t_h \sec \theta$, $\phi_0 = 2\pi$ and $\theta_0 = \pi/2$.

The solution is

$$11 \quad I = \frac{E\mu_e}{2\rho\mu_t} \left[t_h E_1(-t_h) + e^{-t_h} (B_0 + B_1 t_h + B_2 t_h^2 + \dots) \right]$$

where

$$B_0 = A_0$$

$$B_i = \sum_{\nu=i}^{n-1} \frac{A_{\nu+1}}{\nu(\nu+1)}, \quad i = 1, 2, \dots, n-1$$

and

$$-E_1(-t) \equiv \int_t^{\infty} \frac{e^{-x}}{x} dx$$

APPENDIX

An integral equal to Eq. 4 and containing an exponential
is

$$(*) \quad \int_0^{\infty} e^{-t/y} dt = y .$$

Clearly the integrands of Eqs. (*) and 4 intersect at some one point other than $t = 0$. A comparison of Eq. 4 and of the equations formed by setting the integrands equal indicates that this point is nearly $t = y$. If we assume it is exactly y , Eq. 6 follows immediately. For a wide range of values of α , β , and y Eqs. 4 and 6 are very close. All parameters are positive here, of course.

It is possible to better this approximation in the region about $\beta = 1$ by noting from Eq. 4 that $\beta = 1$ implies $\alpha = y - 1$. We substitute $t_0 = y + ky/(y - 1)$ and, only in the last stages letting k go to zero, Eq. 5 follows.

A comparison of Eqs. 4, 5, and 6 is shown in Fig. 5 for the particular case of $\alpha = 1/y$.

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Figure 1
Build-up Factors for H_2O

Figure 2
Build-up Factors for Fe

Figure 3
Build-up Factors for Pb

Figure 4
 Comparison of Theoretical and Experimental
 Build-up Factors for 0.28 Mev Photons on H_2O

Figure 5
Comparison of Various Approximations to the Incomplete Gamma Function