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**SCALE-UP OF CIRCULATING FLUIDIZED BED COAL COMBUSTORS
Final Report**

**By
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November 20, 1991

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Pittsburgh Energy Technology Center
Pittsburgh, Pennsylvania**

**By
Cornell University
Ithaca, New York**

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Final Report

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COMBUSTORS**

US Department of Energy Grant No. DE-FG22-88PC88929

submitted to

**U.S. Department of Energy
Pittsburgh Energy Technology Center
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November 20, 1991

Summary

Circulating fluidization is a promising technology for designing efficient coal combustors with high solid feed rates. Unfortunately, limited understanding of circulating fluidized beds (CFB) has rendered design extrapolations from pilot reactors to full-scale plants both empirical and expensive.

In CFBs, hydrodynamics and heat transfer are difficult to predict, and the behavior of the flow under scale-up is unclear. Thus the objectives of this research were to quantify the effect of scale-up on the hydrodynamics of CFB combustors, to carry out a rigorous analysis of the flow and heat transfer in vertical gas-solid risers, and to interact with industry at all stages of this work. The importance of this research resides in the great number of CFB coal-burning powerplants that may benefit from its results.

At the completion of this work, the results have exceeded the expectations outlined in the original proposal. The present report summarizes these accomplishments, which have led to five archival publications, two reviewed conference papers, twelve presentations, various quarterly reports, and the award of five graduate degrees.

1. Introduction and summary of accomplishments

In 1988, we proposed a program to elucidate the behavior of CFB coal combustors under scale-up. This program included three main objectives: (1) to perform experiments aimed at quantifying effects of scale-up in a new dedicated facility; (2) to model the hydrodynamics of vertical risers; and (3) to interact with industry in the context of these activities.

The results of this program have generally exceeded our original expectations in all aspect of this research. Each activity has led to publications in major research journals, and to significant progress of interest to industry.

(1) For the experiments, we have constructed a cold CFB facility with the ability to recirculate—rather than discard—fluidization gas mixtures of adjustable density and viscosity. Hydrodynamic analogy between the cold bed and a coal combustor was achieved by matching all relevant dimensionless parameters (*Glicksman, 1984; Louge, 1987*). Several choices of gas composition and particle properties made the cold flow analogous to that in combustors of diameters in the range 0.3 to 1m. Therefore, for the first time, scale-up effects could be quantified directly using a single cold flow facility.

During the facility's commissioning stage, we performed the first quantitative, time-dependent measurements of voidage in the wall region of the bed. The data established the presence of a dense curtain of solids falling near the wall (*Louge, Lischer & Chang, 1990*). In other experiments, we showed that, if unchecked, electrostatics has significant effects on the flow. In this regard, we have found a powerful anti-static additive that virtually eliminates the problem (*Chang & Louge, 1991a*).

By comparing data obtained in the scale-up experiments, we have verified the hydrodynamic analogy predicted by the dimensional analysis, determined scale-up rules for static pressure and its fluctuations, and investigated choking in risers of increasing sizes. In addition, by fluidizing two glass powders of identical characteristics but largely different surface friction coefficients, we have discovered, to our surprise, that particle friction affects the flow (*Chang & Louge, 1991b*).

(2) Because particles undergo frequent collisions in the CFB we have focused our efforts in modeling their interaction in gas-solid flows using elements of rapid granular theories. Because detailed measurements of velocity and velocity fluctuations are not yet available in dense flows, we have considered dilute, turbulent flows of relatively massive particles, for which Tsuji, *et al.* (1984) had published detailed data for vertical risers. A numerical solution of the resulting governing equations provided velocity and turbulent

energy profiles in good agreement with the measurements of Tsuji *et al.* (1984), and it showed that particle collisions can play an important role even under dilute conditions. The resulting paper (Louge, Mastorakos & Jenkins, 1991) has been published in the *Journal of Fluid Mechanics*. Encouraged by these results, we have exceeded the brief of our original proposal by considering heat transfer from a suspension of massive particles of low Biot number transported in a vertical pipe. The results, submitted to the *Int. J. of Heat and Mass Transfer*, have shed some light on the mechanism of energy exchange in vertical risers (Louge, Mohd. Yusof, & Jenkins, 1991).

(3) Our interactions with industry have taken many forms. During the course of this project, we have shared our results during several regular visits to ABB Combustion Engineering. In addition, with that company, students from our research group have adapted a new capacitance technique developed at Cornell to measure solid concentrations in the Chatham CFB coal combustor. After some promising trials of the new probe, this project is still underway at ABB Combustion Engineering.

(4) Other activities that exceed our original statement of work have also been made possible by this DOE grant. In particular, we have designed and constructed a miniature probe that measures simultaneously heat transfer rate and solid volume fraction at the wall of a CFB riser. Tests have recently demonstrated the feasibility and performance of this probe at the Cornell CFB facility.

(5) Five graduate students have been trained during the course of this project.

In this final report, rather than reproducing earlier quarterly reports, journal publications, or yearly summaries of activities, we have chosen to provide a list of all papers, reports and presentations resulting from this project and acknowledging DOE sponsorship. Through these, the reader will find a thorough description of the research and its results. The principal papers are reproduced by permission of the publishers in the Appendix.

2. List of publications

In this section we list all archival publications, conference proceedings, and presentations acknowledging the support of the DOE through this program. We also include abstracts of the archival publications.

2.1. Archival publications with abstracts

- Louge M., Lischer D.J. & Chang H.: "*Measurements of Voidage near the Wall of a Circulating Fluidized Bed Riser*", Powder Tech. 62, 267-74 (1990).

Quantitative, time-dependent, non-intrusive capacitance measurements of voidage in the wall region of a circulating fluidized bed of cracking catalyst are reported. The data confirms the presence of a denser annulus of solids near the wall, which exhibits a vertical voidage profile similar to that observed for the average cross-sectional voidage. A statistical analysis shows that the average wall voidage in the dense lower region of the bed is nearly independent of the total solid flux, that it expands linearly with superficial gas velocity, and that its fluctuations are limited to low frequencies.

- Louge, M.Y., Mastorakos, E. & Jenkins, J.T.: "*The Role of Particle Collisions in Pneumatic Transport*", J. of Fluid Mech. 231, 345-359 (1991).

A model for the dilute flow of relatively massive particles in a turbulent gas is developed for vertical pipes. The particle phase is treated as a dilute system of colliding inelastic grains, the balance equations for rapid granular flow are modified to incorporate the drag force from the gas, and boundary conditions, based on collisional exchanges of momentum and energy at the wall, are employed. The turbulence of the gas is treated using a one-equation closure. The model is developed in the context of steady, fully developed flow.

- Louge M. & Chang H.: "*Pressure and Voidage Gradients in Vertical Gas-Solid Risers*", Powder Tech. 60, 197-201 (1990).

The customary practice of inferring average voidage from measurements of vertical pressure gradients may lead to significant errors at the transition between the dense and the dilute regions of a circulating fluidized bed riser. In this context, a one-dimensional model is developed to account for rapid variations of vertical voidage in these calculations. In particular, the model explains the discrepancy observed by Arena, *et al.* between the voidage profiles inferred from pressure gradients and those measured by a quick-closing valve technique [*Circulating Fluidized Bed Technology*, Pergamon Press, NY (1985), p. 119]. The significance of this effect is discussed.

- Louge, M., Yusof J.M. & Jenkins J.T.: "*Heat Transfer in the Pneumatic Transport of Massive Particles*", Int. J. Heat & Mass Trans. (1991), under review.

We analyze the steady, fully-developed heat transfer to the walls of a vertical pipe from a dilute suspension of relatively massive particles of low Biot number in a turbulent gas. In this regime, the exchange of momentum in collisions between the particles and between the particles and the wall plays a significant role in the balance of forces of the dispersed phase. For this flow we adopt the hydrodynamic analysis of Louge, Mastorakos, and Jenkins [*J. Fluid Mech.* 1991, in press]. The energies of the particle and gas phases are balanced in separate equations, which are coupled through a term obtained by averaging a heat transfer correlation for a single particle. In the particle phase, conduction is calculated from the kinetic theory as a self-diffusive streaming flux and, assuming negligible transfer of heat during collisions, homogeneous boundary conditions are prescribed for the temperature. A numerical solution of the resulting balance laws reveals the parameters that govern heat transfer in this regime and provides heat transfer rates at the wall in agreement with the measurements of Jepson, Poll, and Smith [*Trans. Instn. Chem Engrs.* 41, 207-211 (1963)].

- Chang H. & Louge M.: "*Hydrodynamic scale-up of circulating fluidized beds*", Powder Tech. (1991), accepted, November 1991 (attached to the 12th quarterly progress report).

The effects of scale-up on the hydrodynamics of circulating fluidized beds (CFB) are investigated using a single cold laboratory facility with the ability to recycle fluidization gas mixtures of adjustable density and viscosity. By matching five dimensionless parameters, experiments employing plastic, glass and steel powders achieve hydrodynamic similarity with high-temperature CFB risers of 0.32, 0.46 and 1m diameter. Comparisons of results obtained with the plastic and glass powders indicate that static pressure and its fluctuations

scale with the riser and particle diameters, respectively. Experiments with the steel powder exhibit incipient choking behavior consistent with the greater analogous bed size that they simulate. The onset of choking with plastic and steel powders is well predicted by the correlation of Yang [*Powder Technology* 35 (1983) 143]. Experiments with coated glass beads suggest that the magnitude of the particle Coulomb friction coefficient affects CFB hydrodynamics in the limit where this coefficient is small.

2.2. Conference papers reviewed

- Mastorakos, E., Louge M., & Jenkins J.T.: "*The Role of Particle Collisions in Pneumatic Transport*", in Constitutive Relationships and Models in Continuum Theories of Multiphase Flows, R. Decker, ed, NASA Conference Publication 3047 (1989), pp. 127-37.
- Chang, H. & Louge, M.: "*Hydrodynamic scale-up of circulating fluidized beds*", in the Proceedings of the 1991 Int. Conf. on Fluidized Bed Combustion, Vol 3, E.J. Anthony, ed., ASME, NY (1991a), p.1215-18.

2.3. Contributed presentations

- Louge, M., Mohd. Yusof J., & Jenkins J.: "*A Model for Heat Transfer in the Pneumatic Transport of Large Particles*", Annual Meeting of the AIChE, San Francisco, CA, November 5-10, 1989.
- Louge, M., Mastorakos, E., & Jenkins J.: "*A Model for the Pneumatic Transport of Massive Particles*", Annual Meeting of the AIChE, San Francisco, November 6, 1989.
- Louge, M., Mastorakos, E., & Jenkins J.: "*The Role of Particle Collisions in Pneumatic Transport*", 1990 Annual Meeting of the AIChE, Chicago, IL, November 14, 1990.
- "*The Role of Particle Collisions in Pneumatic Transport*", 21st Annual Meeting of the Fine Particle Society, San Diego, CA, August 22, 1990.
- "*Voidage Measurements in the Wall Region of a Circulating Fluidized Bed*", Free Forum Poster Session, International Engineering Foundation Conference on Fluidization, Banff, Canada, May 9, 1989.
- "*Particle Collisions and Turbulent Kinetic Energy in Pneumatic Transport*", Free Forum Poster Session, International Engineering Foundation Conference on Fluidization, Banff, Canada, May 10, 1989.
- "*A Simplified Model of Gas-Solid Riser Hydrodynamics*", Annual Meeting of the AIChE, Washington, D.C., December 1, 1988.

2.4. Invited presentations

- Louge, M., Mohd. Yusof, J., and Jenkins, J.: *"Heat transfer in the pneumatic transport of massive particle"*, IUTAM Meeting on Fluidization, Stanford, Ca, July 2, 1991, organized by Bud Homsy and George Batchelor.
- *"The Role of Particle Collisions in Pneumatic Transport"*, invited seminar, The Ohio State University, Department of Chemical Engineering, June 26, 1990.
- *"Particulate Research at Cornell"*, invited lecture, JASON/MITRE Summer Study, San Diego, CA, June 26, 1989.
- *"Circulating Fluidization Research at Cornell"*, invited seminar, Department of Chemical Engineering, University of Cincinnati, OH, April 28, 1989.
- *"Particulate Research at Cornell"*, invited presentation, DOE Morgantown Energy Technology Center, October 22, 1990.

2.5. Reports

- Louge, M.: *"Scale-up of Circulating Fluidized Bed Coal Combustors"*, 12 consecutive Quarterly Technical Progress Reports for the period September 1, 1988 to August 31, 1991, DOE Grant DE-FG22-88PC88929.
- Louge, M.: *"Scale-up of Circulating Fluidized Bed Coal Combustors"*, DOE Research Assessment Report, DOE Grant DE-FG22-88PC88929, July 1990.
- Louge, M.: *"Scale-up of Circulating Fluidized Bed Coal Combustors"*, UCR/DOE Contractors Review Meeting, Pittsburgh, PA, three abstracts (1989, 1990, 1991), DOE Grant DE-FG22-88PC88929.

3. Graduate students

Five graduate students were trained during the course of this project. They are: Jamaludin Mohd. Yusof (Ph.D. candidate), Epaminondas Mastorakos (M.S., 1989), Catherine Acree Riley (M.S., 1989), D. Jeffrey Lischer (Ph.D., 1991), and Hongder Chang (Ph.D., 1991). Jamaludin Mohd. Yusof and Epaminondas Mastorakos developed the hydrodynamic and heat transfer models for vertical risers. Hongder Chang constructed the experimental facility and studied CFB scale-up. D. Jeffrey Lischer developed diagnostic instruments for this facility. After designing capacitance probes at Cornell, Catherine Acree Riley was dispatched to ABB Combustion Engineering to adapt this technique to real CFB coal combustors.

4. Next Research and Conclusions

During the course of this project, we have primarily investigated the behavior of CFB risers under scale-up. By matching the hydrodynamics of our facility analogous to those of a coal-burning CFB combustor, we have made the results of this study directly applicable to similar industrial facilities. In addition, our models have brought insight to the physical mechanisms that govern the hydrodynamics and heat transfer of vertical risers.

In the next phase of this research, we plan a research program integrating theory, computer simulation, and experiments that will focus on the dynamics of the wall layer and its effects on tube erosion and heat transfer. Experiments will be carried out in an existing circulating bed whose flow is analogous to that of a coal combustor. There, sensors recently developed at Cornell will measure particle volume fraction, collision frequency and heat transfer at the wall.

We wish to express our gratitude to the Department of Energy for making this research project possible.

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- M. Louge, E. Mastorakos & J.T. Jenkins: "The Role of Particle Collisions in Pneumatic Transport" *J. of Fluid Mech.* **231**, 345-359 (1991).
- M. Louge, J. Mohd. Yusof & J.T. Jenkins: "Heat Transfer in the Pneumatic Transport of Massive Particles", *Int. J. Heat and Mass Transfer* (1991), under review.
- Tsuji, Y., Morikawa, Y. & Shiomi, H. "LDV measurements of an air-solid two-phase flow in a vertical pipe", *J. Fluid Mech.* **139**, 417-34 (1984).

Hydrodynamic scale-up of circulating fluidized beds

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Summary

The effects of scale-up on the hydrodynamics of circulating fluidized beds (CFB) are investigated using a single cold laboratory facility with the ability to recycle fluidization gas mixtures of adjustable density and viscosity. By matching five dimensionless parameters, experiments employing plastic, glass and steel powders achieve hydrodynamic similarity with high-temperature CFB risers of 0.32, 0.46 and 1m diameter. Comparisons of results obtained with the plastic and glass powders indicate that static pressure and its fluctuations scale with the riser and particle diameters, respectively. Experiments with the steel powder exhibit incipient choking behavior consistent with the greater analogous bed size that they simulate. The onset of choking with plastic and steel powders is well predicted by the correlation of Yang [*Powder Technology* 35 (1983) 143]. Experiments with coated glass beads suggest that the magnitude of the particle Coulomb friction coefficient affects CFB hydrodynamics in the limit where this coefficient is small.

Introduction

Circulating fluidization is a promising technology for designing efficient reactors with high solid flow rates. Excellent contacting is achieved as solids are entrained in a vertical riser column by a gas stream at high velocity. Unfortunately, limited understanding of circulating fluidized beds (CFB) makes it difficult to extrapolate the design of a pilot reactor to that of a full scale plant.

Because basic measurements are challenging in the hostile environment of industrial CFB units, essential flow variables like overall solid flow rates are seldom recorded there. In contrast, cold facilities can produce detailed hydrodynamic data. However, because the density and viscosity of cold gases are markedly different than that of typical combustion products, the hydrodynamics of cold units are not directly relevant to CFB combustors. To avoid this problem, dimensional analysis has been used to match the hydrodynamics of bubbling and CFB combustors in cold laboratory facilities.

For bubbling beds, Glicksman [1] derived scaling relations from the dimensionless groups that arise in the balance Eqs. of Anderson and Jackson [2,3]. By inspecting the form of the drag force for different values of the particle Reynolds number, he distinguished three sets of these groups, depending on the relative importance of the viscous and inertial forces. Because by virtue of the Buckingham π -theorem, such groups may be constructed from a set of reference parameters regardless of the form adopted for the balance Eqs., other authors have come up with similar groups despite markedly different approaches. From an analysis of bubble dynamics, Horio, Nonaka, Sawa, and Muchi [4] derived numbers that are equivalent to Glicksman's groups in the viscous limit [5]. Employing a similar approach, Zhang and Yang [6] found scaling relations identical to Glicksman's [1], except that they incorporated particle sphericity in the other dimensionless groups instead of listing it as a separate parameter.

Fitzgerald, Bushnell, Crane, and Shieh [7] conducted experiments with bubbling beds in the viscous limit to verify a dimensional analysis comparable to Glicksman's. To this end, they pointed out similarities in the statistics of pressure fluctuations between a two-dimensional bed of limestone fluidized with helium and the analogous bed of copper fluidized with air. In addition, they simulated an industrial combustor using an air-copper suspension. Although they did not precisely satisfy the scaling relations in the latter tests,

the near similarity that they observed between the behavior of the combustor and its cold model was encouraging.

Other experiments further confirmed the dimensional analysis. Nicastro and Glicksman [8] demonstrated their scale-up rules by recording the dimensionless power spectrum and probability-density-function (PDF) of pressure fluctuations from a hot bubbling bed and its cold quarter-scale model of steel grit fluidized with air. Newby and Keairns [9] used high-speed movies to compare the bubbling frequency from two analogous cold beds. To match the appropriate dimensionless groups, they tuned the density of the gas by varying its composition and operating pressure. Zhang and Yang [6] tested their analysis by comparing dimensionless bed height, entrainment rate and flow patterns from analogous two-dimensional bubbling fluidized beds.

Roy and Davidson [10] performed experiments in bubbling beds at several temperatures and pressures to test scaling relations for different values of the Reynolds number Re_p based on particle diameter and superficial gas velocity. For analogous conditions, they found similar dimensionless frequency and amplitude of pressure fluctuations. In addition, they showed that the reduced set of dimensionless groups appropriate for the viscous limit is sufficient when $Re_p < 30$, but that Glicksman's complete set is necessary when $Re_p > 30$.

For circulating fluidization, Louge [11] added the ratio of the particle and gas mass flow rates (loading) to Glicksman's list of dimensionless groups. Horio, Ishii, Kobukai, and Yamanishi [12] proposed scaling relations that ignore gas and particle inertia. To test these, they constructed two scaled models (1/25 and 1/100) of a 175 MWe CFB combustor. There, they observed comparable voidage profiles and choking transitions for identical dimensionless operating conditions. Recently, Glicksman *et al.* [13] recorded similar vertical voidage profiles and spectrum of pressure fluctuations in a hot CFB and its geometrically similar cold model. Although these authors generally matched Louge's

complete set of dimensionless numbers, their experiments with two cold beds suggested that a reduced set of numbers for the viscous limit should be adequate for $Re_p < 30$. However, they found that this simplification may break down in the event of bed slugging. Finally, Ishii and Murakami [14] also reported similar conditions in two cold, geometrically similar CFBs matched according to the scaling laws of Horio, *et al.* [4].

Thus, previous studies suggest that dimensional analysis makes it possible to simulate the flow in an industrial CFB combustor using a cold model. However, because previous scaled models have only been made analogous to a single combustor, their results have not yet provided direct insight on the effects of scale-up. Therefore in this study, we have sought to quantify these effects directly using a single facility. To this end, we have constructed a cold CFB with the ability to recirculate, —rather than discard—, fluidization gas mixtures of adjustable density and viscosity. Using dimensional similitude, this cold CFB riser with diameter of 20cm, operating with different gas and solid systems, is made to simulate generic coal-burning CFB risers of 0.32, 0.46 and 1m diameter.

In this paper, we begin with a description of the dimensional analysis and the laboratory facility. We discuss the effects of electrostatics and total solid inventory on the flow. Then we present data collected with plastic, glass and steel powders to substantiate the hydrodynamic analogy and evaluate the role of scale-up in these flows. In this context, we discuss the onset of choking with increasing analogous riser size. Finally, we observe that, for spherical particles, the global flow behavior of the CFB riser is surprisingly affected by particle surface characteristics.

Dimensional similitude

In the absence of inter-particle forces or electrostatics, continuum Eqs. for suspensions of spheres in a gas derived, for example, by Anderson and Jackson [2,3] yield five dimensionless groups. From a dimensional analysis based on Buckingham's π -theorem, several possible sets of five groups may arise from combinations of the reference parameters that describe the CFB riser. One such set includes Froude number $Fr=u/\sqrt{gd}$ and solid loading $M=G/\rho u$, which determine the operational characteristics of the bed; Archimedes number $Ar=\rho_s \rho d^3 g/\mu^2$ and density ratio $R=\rho_s/\rho$, which combine gas and particle properties; and ratio of riser diameter to sphere diameter $L=D/d$. In this paper, u represents the superficial gas velocity; G is the average solid flux; ρ and ρ_s are the densities of the gas and the material of the particles, respectively; μ is the gas viscosity; and g is the acceleration of gravity. Implicitly, this analysis assumes that changes in gas density have negligible effects on the flow.

Thus, for spherical particles, risers of similar aspect ratios and identical values of Fr , L , Ar , R and L exhibit analogous hydrodynamic behavior [11]. For non-spherical particles, similitude requires that some dimensionless measure of particle shape should also be matched. In this context, Glicksman [1] proposed that the particle sphericity ϕ be made identical to achieve similitude. Because this requirement is inconvenient in tests that involve particles of different materials, we have sought to incorporate ϕ in other dimensionless groups, rather than treating it as a separate parameter.

In Ergun's correlation for a packed bed of particles, ϕ does not appear alone in the expression for the mean pressure gradient, but rather through its product with the particle diameter [15]. This observation suggests that, for suspensions, sphericity may arise from the balance Eqs. through a product of the form $d_s \phi^\alpha$, where α is an exponent to be determined and d_s is the diameter of a sphere having the same volume as that of the particle. To establish whether ϕ enters the problem in this way, we make the Eqs. of motion

dimensionless using $d_s \phi^\alpha$ and u as reference length and velocity scales, respectively. In these Eqs., the drag force per unit volume has the general form

$$F_D = C_D |g_s| g_s \frac{3\rho}{4d_s} (1-\epsilon) f(\epsilon), \quad (1)$$

where g_s is the average slip velocity vector between the gas and a particle, ϵ is the voidage and $f(\epsilon)$ is a correction of the drag coefficient C_D on a single particle that accounts for the presence of other particles [16]. By incorporating ϕ and d_s , this analysis leads to a modified set of five dimensionless groups. The balance Eqs. give rise to the density ratio R , a modified Froude number $u/\sqrt{gd_s \phi^\alpha}$ and a drag coefficient $C_D \phi^\alpha$. Boundary conditions produce the solid loading and a new ratio of length scales $D/d_s \phi^\alpha$ through the radial position of the wall. For isometric particles with $\phi \geq 0.67$, Haider and Levenspiel [17] propose the following correlation for single particle drag:

$$C_D = \frac{24}{Re_s} \{1 + [8.17 e^{-4.07\phi}] Re_s^{(0.096+0.056\phi)}\} + \frac{74 Re_s^{-5.07\phi}}{Re_s + 5.38 e^{6.21\phi}}, \quad (2)$$

where Re_s is based on g_s and d_s .

Therefore, under identical operating conditions, suspensions of spherical and non-spherical particles have analogous hydrodynamics if they have equal values of the products $d_s \phi^\alpha$ and $C_D \phi^\alpha$:

$$d_s \phi^\alpha = d, \quad (3)$$

$$C_D(Re_s, \phi) \phi^\alpha = C_D(Re, 1). \quad (4)$$

Here Re is based on the local slip g experienced by the spherical particle of diameter d , and the functional form of C_D is given by a correlation such as Eq. (2). For simplicity, we assume that (g_s/g) is approximately equal to the ratio of terminal velocities of the non-spherical and spherical particles. Then, from the balance of gravity and drag forces on a single particle, this ratio may be expressed through Eq. (2) as a known, —albeit

complicated—, function $f(Ar, \phi)$, where the Archimedes number Ar is based on d .

Combining eqs. (3) and (4) yields

$$C_D[Re \phi^{-\alpha} f(Ar, \phi), \phi] \phi^{\alpha} = C_D(Re, 1), \quad (5)$$

where $f(Ar, \phi)$ is kept in symbolic form for clarity.

Equation (5) yields α in terms of Re and ϕ . The resulting value of α is such that flows of non-spherical particles of equivalent diameter d_s and sphericity ϕ have the same hydrodynamic behavior as flows of spherical particles of diameter $\phi^{\alpha} d_s$ at the particle Reynolds number Re . As Fig. 1 indicates, α is remarkably independent of ϕ for $Re \leq 150$. For values of Re typically found in circulating beds ($Re \geq 3$), α lies between 0.5 and 1. Thus, for simplicity, we adopt $\alpha = 1$ in the present experiments. At worst, this approximation may introduce a relative mismatch of the drag coefficient less than 17% at the smallest value of $\phi = 0.69$ under consideration here.

From this analysis, suspensions in vertical risers with $\phi \geq 0.67$ have similar hydrodynamics if the following groups are matched:

$$Fr^* = u \sqrt{g \bar{d}_s \phi^{\alpha}}, \quad (6)$$

$$M = G/\rho u, \quad (7)$$

$$Ar^* = \rho_s \rho (\bar{d}_s \phi^{\alpha})^3 g / \mu^2, \quad (8)$$

$$R = \rho_s / \rho, \quad (9)$$

$$L^* = D / \bar{d}_s \phi^{\alpha}. \quad (10)$$

Here $\bar{d}_s \phi^{\alpha}$ is the modified mean Sauter diameter and the asterisk highlights modified groups involving ϕ .

The main objective of this study is to compare the flow behavior in risers of increasing size. Because our method accomplishes this end through direct examination of successive cold tests, our results need not be compared with actual, often uncertain data

from industrial units. Nevertheless, for the sake of relevance, we choose to make the flow in the cold CFB riser similar to that in a generic coal combustor. There a typical coal-limestone mixture has the PSD in Fig. 2 with a mean modified Sauter diameter $\phi_0 \bar{d}_{s0} = 256 \mu\text{m}$ and material density $\rho_{s0} = 1500 \text{ kg/m}^3$. Combustion gases at 1120°K have a density $\rho_0 = 0.3 \text{ kg/m}^3$ and viscosity $\mu_0 = 4.10^{-5} \text{ kg/m}\cdot\text{sec}$. Finally, the riser is commonly operated at superficial gas velocities and solid fluxes in the range $5 \leq u_0 \leq 9 \text{ m/sec}$ and $7 \leq G_0 \leq 100 \text{ kg/m}^2\cdot\text{sec}$.

Algebraic manipulations of Eqs. (6)-(10) relate the operating conditions and properties of the cold riser to those in the combustor that it models:

$$u/v^{1/3} = u_0/v_0^{1/3}, \quad (11)$$

$$G/\rho^{2/3}\mu^{1/3} = G_0/\rho_0^{2/3}\mu_0^{1/3}, \quad (12)$$

$$\phi \bar{d}_s/v^{2/3} = \phi_0 \bar{d}_{s0}/v_0^{2/3}, \quad (13)$$

$$\rho_s/\rho = \rho_{s0}/\rho_0, \quad (14)$$

$$D/v^{2/3} = D_0/v_0^{2/3}, \quad (15)$$

where $v = \mu/\rho$ is the kinematic viscosity. In this study, we employ plastic, glass and steel powders to simulate combustors of increasing sizes (Table 1). For similitude, each material must be fluidized with a gas of density specified by Eq. (14). This is achieved using appropriate mixtures of helium and carbon dioxide, which are two inert gases of widely different ρ and v . The viscosity of each mixture is evaluated through Wilke's semi-empirical formula [18]. Then the operating conditions of the cold model and the mean diameter of the test particles are specified by Eqs. (11)-(13). Through Eq. (15), each value of v associated with a different test powder corresponds to a new combustor diameter D_0 . In this way, a single facility is sufficient to investigate CFB scale-up.

Apparatus

The circulating fluidized bed facility has been described elsewhere [19,20]. Its unique feature is the ability to recycle fluidization gases and to monitor their contents using a thermal conductivity detector (Fig. 3). In addition, an oxygen analyzer constantly draws gas samples to detect possible leaks into the closed facility. The hot gases leaving the blower are cooled to the ambient temperature using a compact heat exchanger. The unit is made of aluminum to facilitate the discharge of electrostatic charges at the wall.

Static pressure is measured every 30 cm using 25 taps mounted flush along the height of the riser. Another 12 taps are located in the cyclone, downcomer and the solid return leg to complete the pressure profile along the entire circulation loop. The taps are read in sequence using a scanning valve connected to a single pressure transducer. The resulting signals are acquired by a computer system that also controls the position of the valve. The system samples each pressure tap for nine seconds at a sampling rate of 60Hz. After scanning the entire loop four times, the computer calculates the average pressure at each tap.

Instantaneous pressure fluctuations are recorded at two relative elevations $z/H = 0.12$ and 0.62 along the riser of height H using a differential transducer with one side connected to the ambient pressure. Dimensions of the line connecting the taps to the transducer are carefully chosen to minimize changes in the pressure fluctuations there [20,21]. The data is acquired in digital form at the rate of 50 Hz. Because pressure fluctuations are typically below 5Hz, this sampling frequency is fast enough to avoid aliasing. Signals are sampled over 82 sec periods to provide sufficient data for statistical analysis.

For each powder, sphericity is evaluated from photographs of at least 100 particles with random orientation suspended in an emulsion of clear epoxy. Images are digitized

using a graphics tablet to extract the projected area A and apparent circumference P of each particle. For isometric geometries, the square of the apparent circularity $C^2 = 4\pi A/P^2$ provides a robust estimate of ϕ . For example, for cubes with $\phi = (\pi/6)^{1/3}$, $\pi\sqrt{2}/(1+\sqrt{2})^2 \leq C^2 \leq \pi\sqrt{3}/6$; for cubes of random orientation, a rigorous statistical analysis yields the average $C^2 = 0.861$ with standard deviation 0.037. With this method, we find $\phi = C^2 = 0.69$ and 0.73 for the plastic and steel powders, respectively. For the round glass beads, $\phi = 1$. Then, through fastidious sieving and blending, the PSD of each of the three test powders relative to the mean Sauter diameter is matched to that in Fig. 2 to guarantee hydrodynamic similitude. Because the particles under study are isometric and nearly spherical, the PSD in Fig. 2 is obtained through a screen analysis, where the screen opening provides a convenient estimate of d_s [17]. Table 1 summarizes gas mixture properties, particle sizes and analogous combustor diameters associated with the three powders.

Effects of electrostatics

Static electrification is a serious problem in the transport of powders. It is usually the result of collisions among particles or between particles and the wall [22-24]. Here, we have noted significant levels of electrostatic charging with the plastic and glass powders, although the aluminum walls of the facility were carefully grounded in each experiment. A convenient measure of the severity of this effect is provided by capacitance probes, which are sensitive to the presence of free charges near their measurement volume [25]. With the plastic powder, we have found it nearly impossible to carry out stable capacitance measurements. Another evidence of electrostatics is the adhesion of particles on plexiglas windows located in the downcomer and the return leg to the riser. Clearly, the dimensional analogy outlined earlier would break down if electrostatics generated forces of magnitude comparable to the hydrodynamic forces on the particles. Because their effects on the flow

are difficult to reproduce, electrostatic charges must be eliminated. Unlike previous studies of CFB hydrodynamics, we cannot suppress them by humidifying the fluidization gases, because unacceptable changes in gas density, —and perhaps particle surface properties—, would result.

To solve this problem, we use a convenient powder additive available commercially under the brand name of Larostat 519 (Mazer Chemicals). It is a quaternary salt of ammonium ethosulfate of $\sim 20\mu\text{m}$ mean diameter that may be used in environments below 25% relative humidity. Because in our tests it is sufficient to introduce a small amount of Larostat to the bed material (0.5% and 0.2% by weight of the plastic and glass inventories, respectively), this additive is unlikely to affect CFB hydrodynamics. However, in the case of plastic particles, we observe reductions of the total pressure drop across the riser as high as 70% after introducing Larostat 519 (Fig. 4). Because these dramatic pressure reductions are also accompanied by stable capacitance signals and no particle adhesion, they clearly result from the elimination of static charges. For the glass and steel powders, electrostatic effects are considerably less pronounced, perhaps because, through the larger inertia and smaller static capacitance of these particles, electrostatics create forces of relatively smaller magnitude than those of hydrodynamics.

Effects of solid inventory

In vertical risers, two parameters generally determine the operating conditions *e.g.*, Froude number and solid loading. There, details of the entrance and exit conditions are quickly 'forgotten', even in the presence of a significant acceleration region. However, in certain facilities, vessels connected to the riser may alter the shape of entrance and exit flow profiles in such a way that their effects persist over significant distances into the riser. In

this case, the flow may depend on conditions that prevail in other sections of the facility, although superficial velocity and solid flux remain globally unaffected.

Weinstein, Graff, Meller, and Shao [26] and Li, Tung, and Kwauk [27] may have observed such persistence in their facilities. There they found conditions where the bottom dense region of the riser grows with larger downcomer inventory under fixed gas velocity and solid flux. Because these experiments suggest that, in some cases, conditions in the downcomer may affect the flow in the riser, we have undertaken to verify the effect of total solid inventory on flows of all three powders under study.

To this end, we have conducted successive experiments with identical values of superficial gas velocity and solid flux in the riser, but increasing values of the total powder inventory, so the bottom of the downcomer experiences a markedly higher static pressure. Despite the widely different downcomer conditions, we observe an identical vertical pressure profile in the riser (Fig. 5). This behavior is consistent with the analysis of Matsen [28] and the pressure model of Rhodes and Geldart [29,30]: at fixed gas velocity, any increase in the solid inventory coincides with a decrease in the opening area of the solid recirculation valve that counteracts the corresponding increase in pressure, so the solid flow rate remains constant.

Consequently, in this facility, fluidizing conditions in the downcomer hardly affect the flow in the riser. To achieve similitude, it is therefore superfluous to specify a dimensionless measure of total solid inventory *e.g.*, the relative bed elevation in the downcomer. Because it is difficult to produce a specific bed elevation there, this observation considerably simplifies experimental procedures.

Results and discussion

In these experiments, plastic, glass and steel powders are fluidized with suitable gas mixtures to simulate flows in vertical risers of increasing sizes (Table 1). With their relatively small analogous riser dimensions, suspensions of plastic and glass have similar flow behavior. In contrast, tests with the steel powder exhibit incipient choking consistent with the larger riser diameter that they model. In this section, we begin with a comparison between the plastic and glass tests that illustrates the behavior of risers of moderate diameter. Then, from results of the steel experiments, we study the effect of a large increase in riser size. Next, we correlate conditions at incipient choking with riser diameter. Finally, we examine the effect of particle surface friction on the flow.

Moderate riser diameter

First, experiments are conducted to compare vertical pressure profiles and pressure fluctuations obtained with the glass and plastic powders under identical dimensionless conditions. Here the Froude number varies between 102 and 174, and the solid loading between 5 and 34. Nine distinct sets of Fr^* and M are produced in these tests. The diameter ratio L^* is 1240 and 1830 for the plastic and glass powders, respectively. The Archimedes number $Ar^* = 46$ and density ratio $R = 5.10^3$ are typical of a generic coal combustor.

Because the two powders are fluidized with different gas mixtures, we must find the proper way to make their respective pressure profiles dimensionless for comparison. Riser height is an obvious scale for the elevation z . However, choosing the appropriate reference pressure is less straightforward. Figure 6 shows a typical pressure profile obtained with the two powders. Other conditions may be found in Chang [20]. For all values of Fr^* and M under consideration, the pressure profiles from the two distinct powders are virtually identical despite the different values of L^* , provided that they are

scaled with the product $\rho_s g D$. Other reference pressures such as $\rho_s g \bar{d}_s \phi^\alpha$ would fail to make the two profiles coincide. Thus for moderate values of riser diameter,

$$p^\dagger = \frac{p - p_{\text{top}}}{\rho_s g D} = \text{function} \left(\frac{z}{H}; Fr^*, M, Ar^*, R; \frac{H}{D} \right), \quad (16)$$

where p_{top} is the pressure at the top of the riser.

In addition, for moderate riser diameter, we find that pressure fluctuations scale with the product $\rho_s g \bar{d}_s$ for all conditions under study. These observations hold whether the fluctuations are recorded near the bottom of the riser ($z/H=0.12$), where these are rather vigorous, or near the top ($z/H=0.62$), where they are relatively weaker. As Fig. 7 illustrates, probability-density-functions (PDF) of dimensionless fluctuations $p' = (p - \bar{p})/\rho_s g \bar{d}_s$ are independent of relative riser size L^* ,

$$\text{PDF}(p') = \text{function} \left(p'; \frac{z}{H}; Fr^*, M, Ar^*, R; \frac{H}{D} \right), \quad (17)$$

where $\bar{p}$ is the time-average pressure and the PDF is normalized according to $\int \text{PDF}(p') dp' = 1$.

Therefore, for moderate riser diameter, scale-up predictions are relatively straightforward. In this case, the static pressure and its fluctuations scale with $\rho_s g D$ and $\rho_s g \bar{d}_s$, respectively. Through these scalings, it is remarkable that, for all values of Fr^* and M under study, the plastic and glass tests exhibit identical pressure profiles as well as identical PDFs, despite markedly different gas properties, and different particle size, shape and density between the two experiments. Clearly, the dimensional analysis produces the correct analogy, as long as electrostatics is eliminated from the riser.

Large riser diameter

Another series of experiments is conducted with steel powder to compare flows in a riser of larger diameter ($L^* = 4150$) than those simulated with the plastic and glass suspensions. Here, Ar^* and R are identical to their values in the previous experiments. However, because steel emulates a relatively large coal combustor, the suspension chokes more readily than the plastic and glass powders. Consequently, with steel powder, the riser has a narrower range of operating conditions: $M < 5$ for $Fr^* = 102$ and $M < 19$ for $Fr^* = 174$.

Figure 8 compares typical vertical profiles of pressure for the glass and steel experiments with identical values of Fr^* , M , Ar^* and R . There the larger bed shows higher pressure drops that indicate greater density near the bottom of the riser. However, the profiles have remarkable similarity in the more dilute region near the top. These observations suggest that, as the riser diameter increases, the suspension experiences a progressive collapse that develops first in the bottom region. As the solid flux increases or the fluidization velocity decreases, this denser region gradually moves up until violent choking eventually engulfs the entire riser.

In addition, as a likely consequence of the suspension's incipient collapse, pressure fluctuations have considerably greater amplitude in the larger riser (Fig. 9). Because pressure disturbances propagate throughout the riser at the appropriate wave speed [31], wider PDFs are observed at both $z/H = 0.12$ and 0.62 , although the suspension may not exhibit wide variations of particle volume fraction near the higher elevation.

Onset of choking

As the above results indicate, choking is an essential feature of the flow in risers of large sizes. Although it remains a relatively ill-understood phenomenon, its diagnostic is obvious. Satija, Young and Fan [32] characterize it as "a sharp transition [that occurs] as

gas velocity is reduced at a certain fine particle flow rate, the uniform suspension collapses and solids are then conveyed upwards in dense phase slugging flow." In our facility, it is unambiguously accompanied by loud banging noises and shaking of the riser, which result from the passage of slugs there.

Whether or not a system chokes depends on particle characteristics (size, density, surface properties), gas property and vessel size. In dilute pneumatic transport at fixed solids flux, choking appears with decreasing gas velocity as particle weight overcomes gas shear in a global momentum balance of the flow [33]. From observations of denser pneumatic systems, Yang [34] proposed an empirical criterion for the transition to choking that is based on correlations of effective frictional losses attributed to the particle phase,

$$\left(\frac{u}{\epsilon_c} - u_t\right)^2 / 2gD(\epsilon_c^{-4.7} - 1) = 1.5 \cdot 10^{-6} \left(\frac{\rho_s}{\rho}\right)^{2.2},$$

where u_t is the terminal velocity of an individual particle of diameter equal to $\bar{d}_s$, and ϵ_c is the average voidage in the riser at incipient choking. Assuming further that, at choking, the suspension is uniform with average particle slip velocity equal to u_t , Yang wrote:

$$G = \rho_s \left(\frac{u}{\epsilon_c} - u_t\right) (1 - \epsilon_c).$$

Thus, the flux G and average voidage ϵ_c at incipient choking may be calculated from these Eqs. for any suspension at a given superficial gas velocity u .

Figure 10 compares Yang's correlation with our observations of the onset of choking for plastic ($L^* = 1240$) and steel ($L^* = 4150$) powders. For each value of L^* , the correlation produces a choking line that defines the relation between Fr^* and M where the transition occurs. To the left of the line are values of Fr^* and M where the riser chokes. From this Fig., it is clear that the major difference between risers of different sizes is the greater propensity for the suspension to collapse in larger risers. Note that the close agreement between our results and Yang's correlation further confirms the validity of the

dimensional analysis, at least through the range of riser size under consideration
 $1240 \leq L^* \leq 4150$.

Effects of particle friction

By fluidizing two glass powders of identical size distribution and density, but appreciably different surface friction coefficients, we have discovered, to our surprise, that particle surface quality affects the flow. One powder is the untreated spherical glass powder employed in the scale-up experiments (Potters Industries Spherglass®). The other is made of the same glass, but it is coated with a monolayer of moisture-resistant silicon that also serves as a lubricant (Potters CP00 coating), and consequently reduces surface friction. By changing microscopic asperities on the particle surface, the coating may also affect the coefficient of restitution of the particles [35].

Because a precise measurement of the Coulomb friction coefficient between a single particle and a flat wall is not straightforward [36], we have only recorded approximate magnitudes for the static and dynamic friction coefficients of the two powders. To measure static friction, particles bonded to a flat plate are made to rest on an inclined surface made of the same aluminum than that of the riser. The static friction coefficient f_s is found by raising the inclination β of the surface until the plate slides. At that point, $f_s = \tan(\beta)$. The dynamic friction coefficient f_d is obtained by monitoring the subsequent rate of acceleration of the sliding plate.

For the uncoated glass, we find $f_s \approx 0.33$ and $f_d \approx 0.29$. For the coated glass, friction is noticeably smaller. However, because our crude measurement technique is certain to damage the relatively fragile silicon monolayer, the coated particles should normally experience smaller friction coefficients than those recorded here i.e., $f_s \leq 0.28$ and $f_d \leq 0.18$. In addition, note that, because in our facility the fluidization gases come

directly from dry gas cylinders and circulate in a sealed environment, humidity is negligible during experiments. Therefore in this case, the friction coefficient is not affected by possible adsorption of moisture on the particle surface [37].

Figure 11 compares vertical profiles of pressure obtained with the two powders. The effect of surface quality is very pronounced in relatively dilute suspensions (Fig. 11a). Clearly, this observation has important consequences for modeling. In particular, it suggests that the frictional or collisional interaction of particles with the wall plays an important role, at least in dilute flows.

Nevertheless, the scale-up experiments have shown that the plastic and uncoated glass suspensions have analogous hydrodynamics over a wide range of conditions. Further, they have established a relation between the plastic and steel powders through a common dimensionless choking criterion. Because each of the three powders exhibits a different, but relatively large effective friction coefficient f_d (the plastic and steel are rough, the glass is frictional), variations in f_d appear therefore to have a minor effect on the hydrodynamics, as long as this coefficient is relatively large. In this case, it is superfluous to add surface properties to the groups (6)-(10) that arise in the dimensional analysis. Because many powders of industrial interest are rough, it may thus be reasonable to ignore their frictional properties. In contrast, the experiments with coated glass suggest that friction plays an increasingly noticeable role when f_d is small.

Recent theoretical developments may shed additional light on these observations. Louge, Mastorakos and Jenkins [33] have modeled the dilute flow of massive particles in a narrow vertical pipe. There they have shown that, because the coefficient of Coulomb friction governs the magnitude of the particle shear and the flux of particle fluctuating energy at the wall, it has a strong influence on the flow. In contrast, the flow is rather insensitive to the actual values of the coefficients of restitution. Further, in rapid granular flows of spheres interacting with a flat, frictional wall, Jenkins [38] showed that the ratio

S/N of shear to normal stresses exerted by the particles on the wall reaches an asymptotic limit for high friction that is independent of the magnitude of f_d ,

$$\frac{S}{N} = \frac{3}{7(1+e)} \frac{g}{\sqrt{3T}}, \quad (18a)$$

where g is the mean velocity of the contact point relative to the wall, T is the granular temperature of the particle phase and e is the coefficient of restitution for particle-wall collisions. In contrast, for low friction or high values of $g/\sqrt{3T}$, the stress ratio becomes

$$\frac{S}{N} = f_d. \quad (18b)$$

Louge, Jenkins and Hopkins [39] have recently verified these theoretical calculations using computer simulations. These results suggest that flows of particles with rough shape or high surface friction may be lying in a high friction limit. In that limit, it may not be necessary to add f_d to the list of dimensionless parameters (6)-(10), unless $g/\sqrt{3T}$ is very large. Our data suggest that the experiments with the suspensions in Table 1 belong to such a limit.

Because for risers with $L^* \gg 1$, the contribution of particle shear to the overall pressure gradient is dominated by particle weight, the larger gradient observed with greater friction cannot be the direct result of a change in the magnitude of the shear. Instead, an indirect mechanism may involve the curtain of solids falling near the wall [19], as follows. As the particle surface friction increases, so does the magnitude of the average particle shear. In turn, because it exerts a greater force on the falling solids, the wall may suspend a larger number of particles, so the downward solid flux may increase there. Consequently, in order to maintain the prescribed value of overall solid flux, the upward component of flux must increase in the core. Because at constant superficial gas velocity, any increase in solid flux typically leads to an increase in solid concentration, the average gas pressure gradient may thus increase. Further models and experiments focusing on the particle wall layer would be useful to test this argument.

Conclusions

In this work, we have employed dimensional similitude to study the hydrodynamics of geometrically similar CFB risers of increasing size. To this end, we have constructed a cold CFB with the ability to fluidize suspensions of arbitrary gas and particle properties. Particular care was taken to inhibit electrostatics using a powdered additive. For all conditions, the flow in the riser was independent of total solid inventory in the facility. Although this study has focused on recreating the flow in a generic coal combustor, a similar technique may be employed for other gas-solid suspensions.

Experiments have shown that, in risers of moderate diameter, vertical profiles of static pressure scale with riser diameter and particle material density, whereas pressure fluctuations scale with the product of particle mean sieve diameter, density and sphericity. Risers of larger diameter exhibited incipient choking characterized by a gradual collapse of the suspension originating from the base of the riser, and by considerably more intense pressure fluctuations. Observations of the onset of choking for different riser sizes agreed well with the empirical correlation of Yang [34]. Finally, experiments with glass beads of different surface properties suggested that particle surface characteristics play an important role in riser hydrodynamics, in the limit where the effective Coulomb friction coefficient is small.

Because typical industrial CFBs have very large dimensions, they are likely to exhibit the incipient choking behavior that we have observed with risers of large diameter. In fact, they often exhibit a nearly collapsed, dense bottom region, surmounted by a considerably more dilute region on top. The present study has shown that their hydrodynamics cannot be inferred directly from observations in pilot plants of moderate diameter. However, the experimental method that we have outlined may be used to mimic their behavior using cold units of relatively small dimensions.

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Table 1- Suspension properties

Fluidization gases				Solid powders				Analogous Diameter D ₀ m
He	CO ₂	ρ	$\mu \times 10^5$	type	ρ_s	$\bar{d}_s$	$\phi \bar{d}_s$	
%	%	kg/m ³	kg/m.sec		g/cm ³	μm	μm	
92	8	0.30	2.0	plastic grit	1.44	234	161	0.32
79	21	0.51	1.9	uncoated glass spheres	2.53	109	109	0.46
19	81	1.49	1.6	steel grit	7.40	67	49	1.00

Fig. 1. Numerical solution of Eq. (5) for typical values of the sphericity ϕ .

Fig. 2. Cumulative PSD relative to the mean Sauter diameter $\bar{d}_s$. The circles, squares and triangles represent the plastic, glass and steel powders, respectively. The solid line is a typical PSD in a generic coal-burning CFB riser.

Fig. 3. The circulating fluidized bed facility.

Fig. 4. Dimensionless pressure $p^\dagger \equiv (p - p_{top})/\rho_s g D$ in the riser versus relative elevation z/H for the conditions $Fr^* = 102$, $M = 5$. The material is plastic powder ($L^* = 1240$). The open circles represent the test with anti-static powder and the solid circles without it.

Fig. 5. Typical dimensionless pressure versus relative elevation in the CFB loop for the conditions $Fr^* = 131$, $M = 21$. The material is glass beads. Curve AB represents the riser, curve BC is the cyclone, curve CD is the downcomer, and curve DA is the solid flow rate control valve. The solid and dashed lines represent a total solid inventory of 98kg and 114kg in the facility, respectively. With higher solid inventory in the downcomer, the resulting increase of static pressure at the bottom of the downcomer is absorbed by the solid control valve, so the pressure profile remains unchanged in the riser.

Fig. 6. Dimensionless pressure $p^\dagger$ in the riser vs. relative elevation z/H for the conditions $Fr^* = 174$, $M = 33$. The open circles ($L^* = 1240$) and solid squares ($L^* = 1830$) represent the plastic and glass powders, respectively.

Fig. 7. PDF of dimensionless pressure fluctuations p' for the conditions $Fr^* = 131$, $M = 21$. The solid and dashed lines represent the plastic and glass powders, respectively. To evaluate the PDF with a normalized bias error below 1%, samples of p' are assigned to a number of 128 equally-spaced bins of width below one-fifth of the sample standard deviation of the data [40]. Relative elevations are (a) $z/H=0.12$ and (b) $z/H=0.62$.

Fig. 8. Vertical profiles of $p^\dagger$ for the conditions (a) $Fr^* = 102$, $M = 5$; and (b) $Fr^* = 131$, $M = 10$. The squares ($L^*=1830$) and triangles ($L^*=4150$) represent the glass and steel powders, respectively. In (a), the different abscissa scales in the intervals $0 \leq p^\dagger < 0.1$ and $p^\dagger \geq 0.1$ underscore the different flow behaviors in the bottom and top regions of the riser.

Fig. 9. PDF of dimensionless pressure fluctuations p' for the conditions $Fr^* = 131$, $M = 10$. The solid and dashed lines represent the glass beads and steel powders, respectively. Relative elevations are (a) $z/H=0.12$ and (b) $z/H=0.62$.

Fig. 10. Dimensionless choking lines. The circles and triangles represent conditions at incipient choking for the plastic and steel powders, respectively. The solid and dashed lines are the corresponding predictions of Yang's correlation [34] for $L^* = 1240$ and $L^* = 4150$.

Fig. 11. Vertical profiles of $p^\dagger$ for the conditions (a) $Fr^* = 102$, $M = 5$; and (b) $Fr^* = 131$, $M = 16$. The open and solid squares represent the coated and uncoated glass powders, respectively.

Heat Transfer in the Pneumatic Transport of Massive Particles

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ABSTRACT

We analyze the steady, fully-developed heat transfer to the walls of a vertical pipe from a dilute suspension of relatively massive particles of low Biot number in a turbulent gas. In this regime, the exchange of momentum in collisions between the particles and between the particles and the wall plays a significant role in the balance of forces of the dispersed phase. For this flow we adopt the hydrodynamic analysis of Louge, Mastorakos, and Jenkins [*J. Fluid Mech.* 1991, in press]. The energies of the particle and gas phases are balanced in separate equations, which are coupled through a term obtained by averaging a heat transfer correlation for a single particle. In the particle phase, conduction is calculated from the kinetic theory as a self-diffusive streaming flux and, assuming negligible transfer of heat during collisions, homogeneous boundary conditions are prescribed for the temperature. A numerical solution of the resulting balance laws reveals the parameters that govern heat transfer in this regime and provides heat transfer rates at the wall in agreement with the measurements of Jepson, Poll, and Smith [*Trans. Instn. Chem Engrs.* 41, 207-211 (1963)].

1. INTRODUCTION

Because accurate predictions of heat transfer are essential for the design of pneumatic transport lines, measurements of effective heat transfer rates through the wall have been reported in gas-solid suspensions at high velocity (e.g., Jepson, Poll, and Smith [1], Soo [2], Boothroyd and Haque [3]) and phenomenological models have been proposed to correlate the resulting data with local flow parameters [4]. However, because detailed studies of turbulent suspensions addressed only the dilute flow of small, non-

interacting particles (e.g., Elghobashi and Abou-Arab [5], Pourahmadi and Humphrey [6]), physical analyses of the energy transfer were limited to regimes where particle collisions play a minor role in the flow (e.g., Derevich, Yeroshenko, and Zaichik [7]).

In a recent paper, Louge, Mastorakos, and Jenkins [8] consider dilute suspensions of massive particles in a gas and show that collisions are essential to predict the transfer of momentum and fluctuating energy in the particle phase. In their analysis, the particles exchange momentum through collisions with the wall and between themselves, while the mean drag exerted by the gas suspends the particles. However, the particles are assumed to be massive enough to be unaffected by the turbulent velocity fluctuations. The turbulence is modeled using a one-equation closure that incorporates contributions from the particle phase.

In the present study, we analyze fully-developed heat transfer in the same regime. To this end, separate energy equations are employed for the particle and the gas phase. These are coupled through a source term calculated by averaging a heat transfer correlation for a single particle. In this flow the heat transferred to particles in and between collisions is negligible. Consequently, conduction in the dilute particle phase is treated as a self-diffusive streaming flux derived from the kinetic theory and homogeneous boundary conditions are prescribed for the particle phase at the wall. The equations are solved numerically for constant wall heat flux or constant wall temperature to predict heat transfer rates at the wall and temperature profiles across the flow. The resulting predictions are compared with the measurements of Jepson, Poll, and Smith [1].

We begin with a summary of the hydrodynamic treatment for the fully developed flow of massive particles in a pipe. Then, through an analysis of the energy balance of the two phases, we isolate the parameters that govern the steady, fully-developed heat transfer in the same regime. Because the presence of particles tends to reduce temperature variations across the pipe, we assume that all material and fluid properties are constant, so that the earlier hydrodynamic predictions [8] may be employed directly in the heat transfer analysis.

2. HYDRODYNAMIC ANALYSIS

Louge, Mastorakos, and Jenkins [8] consider the dilute, fully-developed, steady flow of particles in a vertical pipe. They focus on particles massive enough to be unaffected by the velocity fluctuations in the turbulent gas. For such particles, the hydrodynamic relaxation time of the particle velocity fluctuations is much greater than a typical roll-over time of the turbulent eddies that is based on their integral length scale l and their root-mean-

square (rms) turbulent velocity u' . Because the turbulent eddy size is of the order of the pipe diameter, such a flow would typically be observed in pipes of relatively small dimensions. In this case, the particles are suspended by the drag force exerted by the mean gas flow, but their velocity fluctuations are the result of collisions with other particles or with the wall.

For the particle phase, these authors adopt the assumption of molecular chaos central to the treatment of collisions in rapid granular flows [9] and assume that interparticle collisions are nearly elastic and frictionless. They also focus on flows where a particle undergoes at least several collisions during its hydrodynamic relaxation time. Consequently, a particle loses only a small fraction of its fluctuation energy in and between collisions and the velocity distribution function for the particles is nearly Maxwellian. Finally, they focus on fully-developed, steady flows, which permits them to carry out averages for both the gas and particle properties on long vertical strips. Then, no matter how dilute the flow, the control volumes can always include a large enough number of particles to define the appropriate averages, so the dispersed particles may be regarded as a continuum. For the particles, they assume that the averages on the strips are equivalent to averages based on a velocity distribution function.

With these assumptions, well-established results from the kinetic theory of gases lead to constitutive relations for the pressure and shear stress in the particle phase [10]. The resulting momentum equation for the particles is then modified to incorporate the drag force from the gas and the gravitational force. For a dilute, fully-developed, axisymmetric flow, the vertical component of this momentum balance is

$$0 = \frac{1}{r} \frac{d}{dr}(rS) + \frac{\rho_p}{T} (1-\epsilon)(u-v) - \rho_p(1-\epsilon)g, \quad (1)$$

where u and v are, respectively, the vertical components of the mean gas and particle velocities, r is the radial coordinate, g is the gravitational acceleration, ϵ is the voidage, ρ_p is the material density of the particles, T is the hydrodynamic relaxation time of the mean relative (slip) velocity between the phases, and S is the particle shear stress on surfaces at constant radius.

The shear stress is given by

$$S = \frac{5\sqrt{\pi}}{96} \rho_p d \sqrt{\Theta} \frac{dv}{dr} \frac{1}{1+\lambda/R}. \quad (2)$$

Here, Θ is the 'granular temperature', expressed in terms of the rms particle velocity fluctuations $\underline{v}$ as $(3/2)\Theta = (1/2) \underline{v}^2$. Note that this 'temperature' is a hydrodynamic measure of the agitation of the grains that bears no relation to the conventional thermal temperature of the particles. Also, the factor λ/R in the denominator is a heuristic correction for the very dilute flow of relatively large particles that is important when the length of the mean free path between collisions $\lambda = d/[6\sqrt{2}(1-\epsilon)]$ is comparable to the radius R of the pipe.

The hydrodynamic relaxation time is defined in terms of the drag force on one particle by

$$\frac{\rho_p}{T} = C_d |u - v| \frac{3\rho}{4d}, \quad (3)$$

where ρ is the gas density and d is the particle diameter. For the drag coefficient C_d , the empirical expression

$$C_d = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) \quad (4)$$

is valid in the range $0 < Re_p < 800$, where $Re_p = |u - v| \rho d / \mu$ is the Reynolds number based on the average slip velocity between the phases and μ is the viscosity of the gas [11].

In these dilute flows, the particle momentum balance in the radial direction becomes

$$dN/dr = 0, \quad (5)$$

where N is the particle pressure, which is related to the particle volume fraction $(1-\epsilon)$ by an expression analogous to an equation of state in a molecular gas:

$$N = \rho_p (1-\epsilon) \Theta. \quad (6)$$

In order to close the equations describing the particle phase, a balance of fluctuating energy is written to determine the granular temperature Θ . The energy equation derived by Jenkins and Savage [9] is, for fully developed flow,

$$0 = \frac{1}{r} \frac{d}{dr}(rq) + S \frac{dv}{dr} - D_1 - D_2. \quad (7)$$

Here q is the diffusive flux of particle fluctuation energy and D_1 and D_2 are rates of dissipation of the fluctuating energy per unit volume due, respectively, to the inelasticity of the particles and to their interaction with the gas. The energy flux is given by the kinetic theory as

$$q = \frac{25\sqrt{\pi}}{128} \rho_p d \sqrt{\Theta} \frac{d\Theta}{dr} \frac{1}{1+\lambda/R}, \quad (8)$$

where the correction for long mean free path is analogous to that in (2). In the dilute limit, for nearly elastic particles

$$D_1 = \frac{24(1-e)\rho_p}{\sqrt{\pi} d} \Theta^{3/2}(1-e)^2, \quad (9)$$

where e is the coefficient of restitution for a particle-particle collision [9]. The rate of energy dissipation per unit volume D_2 results from the working of the fluctuating force exerted by the gas through the fluctuating velocity of the particles. It may be written as

$$D_2 = -(\rho_p/T)(1-e) \overline{v_i'(u_i' - v_i')} = -(\rho_p/T)(1-e) (\overline{u_i'v_i'} - 3\Theta), \quad (10)$$

where primes indicate fluctuating velocities and the overbar denotes the average. The term $\overline{u_i'v_i'}$ is the correlation between the velocity fluctuations of the gas and those of the particles. For this correlation, the expression of Koch [12] is extended to other than low particle Reynolds numbers using a relaxation time T_1 based on the rms fluctuating slip velocity between the two phases:

$$\overline{u_i'v_i'} = \frac{4}{\sqrt{\pi}} \frac{d(u-v)^2}{T_1 \sqrt{\Theta}}. \quad (11)$$

For the boundary conditions of the particle phase at the wall, we adopt expressions calculated by Jenkins [13] in the limit where the coefficient of friction μ_f is so small that the point of contact of a particle always slips during a collision:

$$S = -\mu_f N \quad (12a)$$

and

$$q = \frac{3}{8} N \sqrt{3\Theta} \left[\frac{7}{2}(1+e_w)\mu_f^2 - (1-e_w) \right], \quad (12b)$$

where e_w is the coefficient of restitution for a particle colliding with the wall.

In the gas phase the momentum equation for fully-developed, axisymmetric flow is

$$0 = \frac{1}{r} \frac{d}{dr}(r \varepsilon \tau) - \frac{\rho_p}{T} (1-e)(u-v) - \frac{\partial(\varepsilon p)}{\partial z}, \quad (13)$$

where $\tau = (\mu + \mu_t) du/dr$ is the gas shear stress on vertical surfaces at constant radius and the eddy viscosity μ_t is assumed to be given by the one-equation closure described, for example, by Reynolds [14]:

$$\mu_t = C_\mu \rho \sqrt{k} l. \quad (14)$$

Here l is the turbulent mixing length, $C_\mu = 0.49$, and $k = \overline{u'_i u'_i} / 2$ is the turbulent kinetic energy per unit mass of the gas. Following common practice in pipe flow, the analysis in [8] assumes that near the wall the mixing length is proportional to the distance from the wall and that near the center of the pipe it is constant:

$$l/R = \begin{cases} \kappa (1-r/R), & r/R \geq 0.7; \\ 0.3 \kappa, & r/R \leq 0.7, \end{cases} \quad (15)$$

where $\kappa = 0.41$ is von Kármán's constant.

In addition, the balance of turbulent kinetic energy k is obtained from that of Elghobashi and Abou-Arab [5] by ignoring the voidage fluctuations ϵ' , which are zero for the steady velocity distribution function, and by suppressing other terms on the basis of an order-of-magnitude analysis [8]. For fully-developed flow, the result is

$$0 = \frac{1}{r} \frac{d}{dr} \left(r \epsilon \left(\frac{\mu_t}{\sigma_k} + \mu \right) \frac{dk}{dr} \right) + \mu_t \epsilon \left(\frac{du}{dr} \right)^2 - \rho \epsilon E - \frac{\rho p}{T} (1-\epsilon) (2k - \overline{u'_i v'_i}). \quad (16)$$

The isotropic dissipation rate E is modeled after Reynolds [14] using $E = C_2 k^{3/2}/l$, where $C_2 = C_\mu^3$.

Boundary conditions for the gas phase are enforced at a dimensionless distance $y^+ = \rho(R-r)u^*/\mu = 30$ away from the wall, where $u^* = \sqrt{\epsilon \tau_0/\rho}$ is the shear velocity and τ_0 is the gas shear stress at the wall. In dilute flow, the gas velocity at small values of y^+ is approximately given by the universal 'law of the wall':

$$u/u^* = \begin{cases} 5 \ln y^+ - 3, & 5 \leq y^+ \leq 30; \\ (1/\kappa) \ln y^+ + 5.7, & y^+ \geq 30. \end{cases} \quad (17)$$

The shear velocity is calculated from the global momentum balance:

$$\partial(\epsilon p)/\partial z = -2\rho u^{*2}/R + 2S_0/R - g \rho_p (1-\bar{\epsilon}), \quad (18)$$

where S_0 is the particle shear stress evaluated at the wall and $\bar{\epsilon}$ is the average voidage across the pipe. Because the production and dissipation of k are nearly equal near the wall, and because the molecular viscosity is much smaller than the eddy viscosity at $y^+ = 30$, there is no flux of turbulent kinetic energy there, so

$$\partial k / \partial r = 0. \quad (19)$$

Finally, radial symmetry implies that the radial derivative of the velocities and the temperatures vanish at the centerline of the pipe.

The resulting non-linear, coupled, two-point boundary value problem of equations (1), (5)-(7), (13), (16) subject to the boundary conditions (17), (19) are solved using the quasi-linearization method of Bellman and Kalaba [15]. In Figs. 1 and 2, the results are compared with velocity and turbulent kinetic energy profiles measured by Tsuji, Morikawa, and Shiomi [16] for typical conditions. In these the loading m is defined as the ratio of the particle to gas mass flow rates. Because a treatment that ignores shear stress in the balance of forces for the particle phase (dashed line in Fig. 1) clearly fails to reproduce the observed velocity profile, we conclude that particle collisions play an essential role in these flows. In particular, the momentum transport in the particle phase provides the mechanism that permits the average particle velocity to be larger than that of the gas near the wall. Significantly, the hydrodynamic analysis also predicts the reduction in the turbulent velocity fluctuations associated with the presence of particles and the trend toward larger fluctuations for larger particles (Fig. 2).

3. THERMAL ANALYSIS

In this section, we write balance laws for the thermal temperature of the gas and particle phases. We assume that particles are massive enough for the flow to lie within the regime studied by Louge, Mastorakos, and Jenkins [8] but have Biot numbers small enough to ignore temperature variations in their interior.

3.1 Gas Phase

In a dilute turbulent flow, the thermal energy balance for the gas phase has nearly the same form as the corresponding equation for the pure gas. It is obtained by multiplying appropriate terms by the voidage. Because in fully-developed flow radial velocities vanish, it becomes

$$\epsilon \rho c u \frac{\partial T_g}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r Q_r) + \frac{\partial}{\partial z} (Q_z) + H, \quad (20)$$

where c is the heat capacity per unit mass of the gas, Q_r and Q_z are heat fluxes through the gas in the radial and axial direction, respectively, and H is the average rate of energy per unit volume supplied by the particle phase to the gas. In this formulation, T_g is the temperature of the gas averaged over time. In keeping with the simple turbulence closure in (14)-(15), we employ an eddy diffusivity ϵ_H to relate the correlation between the

instantaneous fluctuations of temperature and velocity of the gas to the temperature gradient. In this case, heat fluxes take the form:

$$Q_r = (k_g + \rho c \epsilon_H) \epsilon \frac{\partial T_g}{\partial r}, \quad (21a)$$

and

$$Q_z = (k_g + \rho c \epsilon_H) \epsilon \frac{\partial T_g}{\partial z}, \quad (21b)$$

where k_g is the gas conductivity. Following common practice, we assume that the turbulent Prandtl number $Pr_t \equiv \epsilon_M/\epsilon_H$ is 0.9, where $\epsilon_M \equiv \mu/\rho$ is the eddy diffusivity of momentum.

3.2 Particle Phase

For nearly elastic particles, a simple analysis permits us to ignore the heat exchanged by thermal conduction during a collision between a particle and the wall. Using the elastic collision model of Timoshenko and Goodier [17], we compare estimates of the collision time τ_c with the time τ_e to equilibrate the particle temperature by conduction through the area of contact. By approximating the relative velocity of the particles before collision as the square root of the granular temperature, these estimates are written in terms of average flow parameters,

$$\tau_c \sim \frac{\rho_p^{2/5} \Delta^{2/5} d}{\Theta^{1/10}},$$

and

$$\tau_e \sim \frac{\rho_p^{3/5} d^2 c_p}{\Delta^{2/5} \Theta^{2/5} k_p},$$

where c_p and k_p are the particle's heat capacity per unit mass and thermal conductivity, respectively, and $\Delta = 3(1-\sigma^2)/2E$ combines Young's modulus E and Poisson's ratio σ for the solid particle. Here, the heat conducted during a collision may be ignored as long as the following ratio is large:

$$Rc \equiv \frac{\tau_e}{\tau_c} \sim \frac{\rho_p^{1/5} d c_p}{\Delta^{4/5} \Theta^{3/10} k_p} \gg 1. \quad (22)$$

Because for the flows under consideration, Rc is of the order 10^6 to 10^9 , the solid thermal conductivity plays no role in the mechanism of heat transfer, and a particle exchanges virtually no heat during its nearly instantaneous collisions with the wall. These observations are consistent with the more detailed theoretical analysis of Sun and Chen [18]. This argument implies that the flux of thermal energy from the wall to the particles is

zero. Then, because the granular temperature does not vanish in the neighborhood of the wall, the gradient of the thermal temperature in the particle phase must be zero there.

In a wider context, because experience shows that heat transfer rates are often a weak function of the thermal conductivity of the solid material, the transfer of heat by conduction through particles in direct contact is generally negligible in gas-particle suspensions at high velocities [19].

The establishment of a balance equation for the thermal energy of the particle phase requires the definition of an average particle temperature T_p . Because unlike other particle flow parameters, the thermal energy of the particle phase varies along the pipe, we cannot adopt a definition of T_p based on long vertical strips. Instead we assume that time or ensemble averages of the particle thermal temperature carried out at a point are equivalent to the strip averages employed in the hydrodynamic analysis. In this case the balance equation for the thermal energy of the particle phase is

$$(1-\epsilon) \rho_p c_p v \frac{\partial T_p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r Q_r^p) + \frac{\partial}{\partial z} (Q_z^p) - H. \quad (23)$$

In such dilute systems, the radial and longitudinal fluxes Q_r^p and Q_z^p arise from the streaming transport of thermal energy carried by the fluctuating particles, and the average transfer of heat between colliding particles is small compared to the streaming flux. In general, because particles and gas have different thermal temperatures, they may exchange energy during this transport. The product of the particle collision frequency and a characteristic time for this transfer of heat is of order $[\Theta^{1/2} (1-\epsilon) (1+\lambda/R)] c_p \rho_p d / k_g$. Because for the flows under consideration this product varies between 4 and 10^2 , it is large enough to neglect the energy exchange between collisions and, in these dilute, hardly dissipative flows, the streaming conduction fluxes Q_r^p and Q_z^p may be derived from the self-diffusion of a hard-sphere gas [10],

$$Q_r^p = \frac{\sqrt{\pi}}{16} \rho_p c_p d \sqrt{\Theta} \frac{\partial T_p}{\partial r} \frac{1}{1+\lambda/R}, \quad (24a)$$

and

$$Q_z^p = \frac{\sqrt{\pi}}{16} \rho_p c_p d \sqrt{\Theta} \frac{\partial T_p}{\partial z} \frac{1}{1+\lambda/R}, \quad (24b)$$

where the correction for long mean free path is analogous to that in (2) and (8). Note that, as a consequence of this correction, the fluxes approach zero as $(1-\epsilon)$ vanishes, so that equation (23) remains valid in this limit.

In more concentrated systems of particles the streaming contribution to the energy flux is diminished and collisional contributions to the particle energy flux due to conduction through contacts become, potentially, more important. However when the ratio Re in (22) is much greater than one, the collisional contribution to the thermal energy flux in the particle phase is negligible even in concentrated systems. In this case, the net result of increasing the concentration is to decrease the total thermal energy flux in the particle phase.

3.3 Source Term

The source term H in equations (20) and (23) averages the contribution from all particles to the thermal energy of the gas. Because the fluctuating velocities of the gas and the particles are much smaller than their average values [8], the flow around each particle may be regarded as nearly steady. Consequently, to evaluate the heat convected away from each particle in dilute flows, we adopt a correlation appropriate for a single sphere in a steady, uniform, infinite gas stream [20]:

$$Nu = 2 + (0.4 Re^{1/2} + 0.06 Re^{2/3}) Pr^{0.4}, \quad (25)$$

where Nu is the Nusselt number based on the particle diameter, and the particle Reynolds number depends on the instantaneous magnitude of the slip velocity $\underline{S}$ between the gas and the particle.

Because Nu depends explicitly on the instantaneous slip, it is conveniently averaged using a velocity distribution function $f(\underline{S})$. This function is normalized so that its integral over the entire velocity space equals the number density of particles n at the point under consideration,

$$\iiint f(\underline{S}) d\underline{S} = n = (1-\epsilon)/(\pi/6)d^3. \quad (26)$$

Using this distribution, the average $\langle \psi \rangle$ of any function of the slip $\psi(\underline{S})$ is

$$\langle \psi \rangle = \frac{1}{n} \iiint \psi(\underline{S}) f(\underline{S}) d\underline{S}. \quad (27)$$

In the present treatment we assume that this average is equivalent to ensemble or time averages employed for the gas and particle phases. To evaluate the source term, we multiply the rate of energy transferred from each particle to the gas by the total number of particles per unit volume. Upon averaging the result, we obtain

$$H = (1 - \epsilon) \frac{6k_g}{d^2} (T_p - T_g) \langle Nu \rangle. \quad (28)$$

To average the fractional powers of Re arising in (25), we write the magnitude of the instantaneous slip Reynolds number in the form

$$Re = \left(\frac{\rho d \sqrt{s^2 + 2k + 3\Theta}}{\mu} \right) [1 + \xi(s'; k, \Theta, s)]^{1/2}, \quad (29)$$

where $s = (u - v)$ is the average slip along the pipe axis, s' is the fluctuating slip, and

$$\xi(s'; k, \Theta, s) = \frac{(2s \cdot s' + s'^2 - 2k - 3\Theta)}{s^2 + 2k + 3\Theta}. \quad (30)$$

Upon integrating the binomial expansion of $(1+\xi)^{a/2}$ through the velocity distribution function, we find

$$\langle Re^a \rangle = \left(\frac{\rho d \sqrt{s^2 + 2k + 3\Theta}}{\mu} \right)^a \left[1 + \frac{a}{2} \langle \xi \rangle + \frac{a}{4} \left(\frac{a}{2} - 1 \right) \langle \xi^2 \rangle + o(\langle \xi^3 \rangle) \right],$$

so, upon ignoring terms of order higher than the first,

$$\langle Re^a \rangle = \left(\frac{\rho d \sqrt{s^2 + 2k + 3\Theta}}{\mu} \right)^a \left[1 - a \frac{\langle u_i' v_i' \rangle}{s^2 + 2k + 3\Theta} \right]. \quad (31)$$

Finally, with $a=1/2$ or $2/3$, respectively, we have

$$\begin{aligned} \langle Nu \rangle = & [2 + 0.4 Pr^{0.4} \left(\frac{\rho d \sqrt{s^2 + 2k + 3\Theta}}{\mu} \right)^{1/2} \left(1 - \frac{1}{2} \frac{\langle u_i' v_i' \rangle}{s^2 + 2k + 3\Theta} \right) + \\ & 0.06 Pr^{0.4} \left(\frac{\rho d \sqrt{s^2 + 2k + 3\Theta}}{\mu} \right)^{2/3} \left(1 - \frac{2}{3} \frac{\langle u_i' v_i' \rangle}{s^2 + 2k + 3\Theta} \right)], \end{aligned} \quad (32)$$

where an estimate of $\langle u_i' v_i' \rangle$ is provided by (11). Because the contribution of this term is relatively small, the rate of transfer is influenced primarily by the mean slip.

3.4 Fully-developed heat transfer

To discuss the mechanisms of heat transfer in this regime, we focus on two separate examples. In the first, a constant heat flux q_w is imposed at the wall. In the second, a constant temperature T_w is maintained there. In either case, the thermal energy flux carried by the particles vanishes at the wall. Then, because the streaming conductivity of the particles is finite, the thermal temperature of the particle phase satisfies the homogeneous boundary condition at the wall

$$\frac{\partial T_p}{\partial r} = 0. \quad (33a)$$

In the gas phase, because the turbulent diffusivity vanishes at the wall,

$$\epsilon k_g \frac{\partial T_g}{\partial r} = q_w, \quad (33b)$$

where q_w remains an unknown obtained by iteration when a constant T_w is imposed. At the centerline, radial symmetry requires $\partial T_p / \partial r = \partial T_g / \partial r = 0$.

In order to define what is meant by fully developed thermal flows in these two cases, we first introduce the total heat capacity rate M and the average bulk temperature T^b ,

$$M \equiv \int_0^R [\rho c u \epsilon + \rho_p c_p v (1-\epsilon)] 2\pi r dr = c Q (1 + m \frac{c_p}{c}), \quad (34)$$

and

$$T^b \equiv \frac{1}{M} \int_0^R [\rho c u \epsilon T_g + \rho_p c_p v (1-\epsilon) T_p] 2\pi r dr, \quad (35)$$

where Q is the average mass flow rate of gas, and m is the average loading. In steady turbulent flow, a cross-sectional heat balance shows that the rate of change of the instantaneous bulk temperature along the pipe is related to the heat flux across the wall, so on average

$$\frac{dT^b}{dz} = \frac{2\pi R q_w}{M}. \quad (36)$$

This balance is nearly exact; because instantaneous velocity and temperature fluctuations are much smaller than their time-averaged values, we ignore in this analysis temporal moments of order higher than the first in the expressions for T^b and M .

We define the average heat transfer coefficient h at the wall as

$$h (T_w - T^b) = q_w. \quad (37)$$

Fully developed thermal flows are defined as flows in which the temperature profiles are similar i.e., the average normalized temperatures

$$\theta_g \equiv (T_g - T_w) / (T^b - T_w), \quad (38a)$$

$$\theta_p \equiv (T_p - T_w) / (T^b - T_w), \quad (38b)$$

are independent of z . Because from (33b), (37) and (38a)

$$h = -\epsilon k_g \left(\frac{\partial \theta_g}{\partial r} \right)_{r=R},$$

it follows that h is also independent of z .

When the heat flux q_w is constant at the wall,

$$\frac{\partial T_g}{\partial z} = \frac{\partial T_p}{\partial z} = \frac{dT^b}{dz} = \frac{2\pi R q_w}{M},$$

so

$$\frac{\partial^2 T_g}{\partial z^2} = \frac{\partial^2 T_p}{\partial z^2} = 0,$$

and the balance laws (20) and (23) become ordinary differential equations (ODEs) in r :

$$\epsilon \rho c u \frac{dT_w}{dz} = \frac{1}{r} \frac{d}{dr} (r Q_r) + H, \quad (39)$$

$$(1 - \epsilon) \rho_p c_p v \frac{dT_w}{dz} = \frac{1}{r} \frac{d}{dr} (r Q_r^p) - H, \quad (40)$$

where $dT_w/dz = dT^b/dz$ is a constant and Q_r and Q_r^p are given by (21a) and (24a).

For constant wall temperature, through differentiation of (36) and (38) the balance laws (20) and (23) may be rearranged as two coupled ODEs:

$$\epsilon \rho c u \theta_g \frac{2\pi R}{M} q_w = \frac{1}{r} \frac{d}{dr} (r Q_r) + \frac{\partial}{\partial z} (Q_z) + H, \quad (41a)$$

with

$$\frac{\partial}{\partial z} (Q_z) = - (k_g + \rho c \epsilon H) \epsilon \theta_g \left(\frac{2\pi R}{M} \right)^2 \frac{q_w^2}{T_w - T^b} \quad (41b)$$

and Q_r given by (21a); and

$$(1 - \epsilon) \rho_p c_p v \theta_p \frac{2\pi R}{M} q_w = \frac{1}{r} \frac{d}{dr} (r Q_r^p) + \frac{\partial}{\partial z} (Q_z^p) - H, \quad (42a)$$

with

$$\frac{\partial}{\partial z} (Q_z^p) = - \frac{\sqrt{\pi}}{16} \rho_p c_p d \sqrt{\Theta} \theta_p \left(\frac{2\pi R}{M} \right)^2 \frac{q_w^2}{T_w - T^b} \frac{1}{1 + \lambda/R} \quad (42b)$$

and Q_r^p given by (24a). Because q_w is unknown *a priori*, the solution of (41) and (42) must be found by iteration.

In the hydrodynamic analysis the governing equations for the flow give rise to eight dimensionless numbers. The loading m and the Reynolds number Re_D based on the superficial gas velocity and the pipe diameter D define the flow conditions. Combinations of the particle and gas properties produce the Archimedes number $Ar = \rho_p \rho_g d^3 / \mu^2$ and the density ratio ρ_p / ρ . The size of the particles relative to that of the pipe is D/d . Finally, the coefficients e , e_w and μ_f are dimensionless collision properties. In addition, the Prandtl number of the gas and the ratio c_p / c arise from the thermal equations (20) and (23). Because of the boundary condition (33b), it is convenient to make the temperatures of both phases dimensionless through the product $(q_w R / k_g)$. In this case, the slope of the dimensionless gas temperature profile versus relative radius r/R is always $(1/\epsilon) \sim 1$ at the wall. Finally, because heat transfer at the wall occurs through the gas phase alone, the Nusselt number $Nu_D = hD/k_g$ is a natural measure of h .

4. RESULTS AND DISCUSSION

The balance equations for fully-developed heat transfer are first written as a set of four coupled, linear, first-order ODEs. These equations are integrated from the centerline of the pipe to the wall using a fourth-order Runge-Kutta algorithm, combined with an iterative Newton-Raphson scheme to satisfy the boundary conditions at the wall.

Because the closure model (14) and (15) is only valid in the turbulent core, in the thermal analysis we use values of the flow parameters that are extrapolated through the relatively thin buffer and viscous sublayers. In particular, because the particle velocity and granular temperature have finite values at the wall, these are extrapolated assuming a constant gradient through that region. This assumption is rather inconsequential, because for small friction, the predicted heat transfer rates are relatively insensitive to the coefficients of particle friction and restitution and to large variations in the streaming conduction fluxes (24a) and (24b) of the particle phase. In addition, for the flows under consideration, variations in the form of the turbulent kinetic energy profile near the wall do not affect the trends predicted by the thermal analysis. As a result, we simply assume that k varies linearly with the radius between its predicted value at $y^+ = 30$ and zero at the wall. In the future, a refined thermal analysis could invoke near-wall turbulence models (see for example Patel, Rodi, and Scheurer [21]). However, to be meaningful, such a refinement should also treat the effect of massive particles on the near-wall turbulence.

To highlight the parameters that govern the heat transfer in this regime, we focus on the case where a constant heat flux is imposed at the wall. Figure 3 shows typical temperature profiles across the pipe. In general, the particles affect heat transfer in two opposite ways.

First, they generally reduce the turbulent kinetic energy [8], unless they are so very massive that turbulence is augmented, perhaps by their own wakes or by deflections of the mean flow streamlines induced by the particles [16]. Because at relatively small loading the source term is dominated by the turbulent conduction term in equation (39), the gas heat flux Q_g remains nearly unchanged for small m , so lower turbulent transport coefficients result in higher gradients of gas temperature in the pipe. Consequently, as Fig. 3 indicates, the gas temperature predicted by the complete analysis (solid line) is lower than if turbulence were not reduced (dashed line). In addition, because of its coupling through the source term, the particle temperature also decreases with decreasing turbulence intensity.

Second, particles promote the radial transfer of thermal energy through streaming conduction. Consequently, through the source term H , they increase the temperatures of both phases with increasing loading. To illustrate this effect alone, Fig. 3 shows hypothetical temperature profiles for two loadings (dashed and dotted lines), where the turbulence intensity is artificially kept identical to that of clear gas. In the absence of turbulence reduction, as the dotted line in Fig. 4 indicates, the heat transfer coefficient would rapidly increase with m .

Therefore, particles have competing effects on the temperature profiles. On the one hand, they lower these profiles through turbulence reduction, and on the other, they increase them through streaming conduction and the source term. The resulting changes in the difference $(T_w - T^b)$ between the wall and bulk temperatures affect the heat transfer coefficient in (37). For loadings small enough that $(m c_p/c) \ll 1$, the total heat capacity rate M in (34) is approximately equal to $c Q$. In this case, through (35), the difference between the wall and bulk temperatures is approximately

$$(T_w - T^b) \approx (T_w - T_g^b) + m \frac{c_p}{c} (T_w - T_p^b), \quad (43)$$

where T_g^b and T_p^b represent the cross-sectional average temperatures for each of the two phases:

$$T_g^b \equiv \frac{1}{Q} \int_0^R \rho u \varepsilon T_g 2\pi r dr, \quad (44a)$$

$$T_p^b = \frac{1}{mQ} \int_0^R \rho_p v (1-\epsilon) T_p 2\pi r dr . \quad (44b)$$

In most instances at small loading, the effects of turbulence reduction dominate or balance the effects of streaming conduction, so $(T_w - T_g^b)$ and $(T_w - T_p^b)$ increase with m , or at least become independent of it. Therefore, the heat transfer coefficient generally decreases with m at small loading.

This trend would persist at higher values of m in the absence of a particle streaming conduction flux. In this hypothetical case, equation (40) yields the following algebraic expression for the dimensionless temperature difference between the two phases:

$$\frac{T_g - T_p}{q_w R / k_g} = \frac{\rho_p c_p}{\rho c} \frac{v}{\epsilon u} \left(\frac{d}{R}\right)^2 \frac{1}{3\langle Nu \rangle (1 + c_p m / c)} , \quad (45)$$

where the overbar denotes the cross-sectional average and $\langle Nu \rangle$ is given by (32). For typical conditions, this difference is small at any radial position. Therefore, if we assume for simplicity that $[\rho c \epsilon u + \rho_p c_p (1-\epsilon)v]$ is constant across the pipe, the sum of equations (39) and (40) would integrate to

$$(T_w - T_g) = \frac{q_w}{\rho c} \int_{r/R}^1 \frac{\zeta d\zeta}{\epsilon(\epsilon_H + \alpha)} , \quad (46)$$

and, because in this case $(T_w - T_g) = (T_w - T_p) = (T_w - T^b)$, any reduction in ϵ_H would increase $(T_w - T^b)$ and reduce h . Therefore, in the absence of streaming conduction, the heat transfer coefficient would decrease as long as turbulence intensity decreased with loading (thin dashed line in Fig. 4).

In contrast, with streaming conduction, the effects of reduction in turbulence intensity become less significant as loading increases. The direct integration of either equations (39) or (40) yields

$$Nu_D = 6\left(\frac{R}{d}\right)^2 \overline{\langle Nu \rangle} \frac{(1-\epsilon)(T_g - T_p)}{T_w - T^b} \left(1 + \frac{c}{c_p m}\right) , \quad (47)$$

where the overbar denotes the cross-sectional average. Assuming uniform profiles of ϵ , u and v , $(1-\bar{\epsilon}) \ll 1$ and $m c_p / c \gg 1$, this equation simplifies to

$$Nu_D = m \langle Nu \rangle \approx 6 \left(\frac{R}{d} \right)^2 \frac{\rho}{\rho_p} \frac{\bar{u}}{\bar{v}} \left(\frac{T_g^b - T_p^b}{T_w - T_b^b} \right). \quad (48)$$

Our experience in solving equations (39) and (40) numerically is that the temperature ratio in (48) is a weak function of m for values of m large enough that relative changes in turbulent intensity are insignificant. As equation (48) shows, in this limit the heat transfer coefficient grows linearly with m , and the growth is more pronounced for smaller particles or larger pipes (Fig. 4, heavy line). By inspection, it is clear that the source term is responsible for this through the factors $(R/d)^2$ and $\langle Nu \rangle$ in (48). Therefore, in the absence of a source term, the transport of a conserved scalar would not be enhanced by the presence of particles at high loading. Ebert, Glicksman and Lints [22] made a similar observation in the different regime of circulating fluidization. There, they showed that, although heat transfer is greatly enhanced by particles, the rate of mass transfer from a solid block of naphthalene mounted flush with the bed wall is no greater than in the absence of particles.

The present thermal analysis provides explanations for all trends summarized by Boothroyd and Haque [3] in the regime under consideration. In particular, these authors reported a sharper drop of heat transfer coefficient at low loading for higher values of Re_D . The relatively greater reduction of turbulence associated with increasing Re_D provides the mechanism responsible for this. At higher loading, Boothroyd and Haque [3] also observed that heat transfer coefficients are increased by the solids far more in larger pipes than smaller ones. This trend is confirmed by equation (48).

Figure 5 compares predictions of this analysis with the experimental data of Jepson, Poll, and Smith [1]. In those experiments, particle size ranges from $422\mu\text{m}$ to $599\mu\text{m}$. For these particles, the Biot number $Bi = h d / k_p$ and Re are of order 10^{-2} and 10^9 , respectively, and the ratio of particle hydrodynamic relaxation time to a typical roll-over time of turbulent eddies is approximately 20. Because heat transfer is a strong function of particle diameter through the source term H , we bracket the data with predictions corresponding to the limits of the experimental size distribution. As Fig. 5 indicates, the predictions are satisfactory, and they exhibit the minimum heat transfer coefficient observed at low values of loading. The calculations also demonstrate the importance of measuring the correct particle size in the experiments.

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FIG. 1. Calculated profiles of gas and particle velocities normalized by the centerline air velocity $u_{cl} = 9.65$ m/s for relatively dilute flows of $500\mu\text{m}$ polystyrene particles ($\rho_p = 1.02$ g/cc) at a loading $m = 1.1$. The solid and open circles represent the data of Tsuji, Morikawa, and Shiomi [16] for gas and particle velocities, respectively. The dashed lines represent particle velocities predicted by an analysis that would ignore particle shear. In all calculations we adopt a coefficient of restitution $e = 0.9$ for particle-particle collisions, $e_w = 0.7$ for particle-wall collisions, and a coefficient of dynamic friction $\mu_f = 0.2$ between a particle and the wall.

FIG. 2. Calculated profiles of normalized rms gas velocity fluctuations u'/u_{cl} for relatively dilute flows. The solid circles represent the data of Tsuji, Morikawa, and Shiomi [16] for $200\mu\text{m}$ particles, $u_{cl} = 12.8$ m/sec, $m = 1.3$; the open circles are $500\mu\text{m}$ particles, $u_{cl} = 13.3$ m/sec, $m = 1.3$. The solid and dashed lines are the respective predictions of the analysis, which also yields $(1-\bar{e}) = 0.16\%$, $\Theta^{1/2}/u_{cl} \approx 1.8\%$ for $200\mu\text{m}$ particles, and $(1-\bar{e}) = 0.18\%$, $\Theta^{1/2}/u_{cl} \approx 2\%$ for $500\mu\text{m}$.

FIG. 3. Typical average gas (g) and particle (p) temperature profiles with constant q_w for the flow conditions $Re_D = 15000$, $\rho_p/\rho = 2100$, $Ar = 814$, and $R/d = 90$. Temperature is made dimensionless through the product $(q_w R/k_g)$. The thermal parameters are typical of suspensions of glass particles in air with $c_p/c = 0.8$ and $Pr = 0.7$. The solid lines are predictions of the complete analysis for $m = 0.4$. In this case, $h/h_0 = 1.01$, where h_0 is the heat transfer coefficient for the same flow of clear gas. The dashed and dotted lines correspond to $m = 0.4$ and 3.0 , respectively. These lines are hypothetical predictions from a hydrodynamic analysis that artificially ignores turbulence reduction associated with the introduction of particles.

FIG. 4. Predictions of the analysis for Nusselt number Nu_D versus loading m for the conditions of Fig. 3. The solid and heavy dashed lines are predictions of the thermal analysis with constant q_w and constant T_w , respectively. The dotted and thin dashed lines are hypothetical predictions that ignore turbulence reduction and streaming conduction, respectively. The heavy line corresponds to smaller particles with $R/d = 250$.

FIG. 5. Predictions of the analysis compared with the data of Jepson, Poll, and Smith [1] for sand particles of $422\mu\text{m} \leq d \leq 599\mu\text{m}$ suspended by air in a pipe of 38mm in diameter. Here, $Re_D = 45000$, $\rho_p/\rho = 2100$, $6300 \leq Ar \leq 18000$, $c_p/c = 0.8$ and $Pr = 0.7$. The solid and dashed lines are predictions for $422\mu\text{m}$ and $599\mu\text{m}$, respectively. For these conditions, $\Theta^{1/2}$ is approximately 1% of the gas velocity at the centerline.

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Pressure and Voidage Gradients in Vertical Gas-Solid Risers

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SUMMARY

The customary practice of inferring average voidage from measurements of vertical pressure gradients may lead to significant errors at the transition between the dense and the dilute regions of a circulating fluidized bed riser. In this context, a one-dimensional model is developed to account for rapid variations of vertical voidage in these calculations. In particular, the model explains the discrepancy observed by Arena et al. between the voidage profiles inferred from pressure gradients and those measured by quick-closing valve technique. The significance of this effect is discussed.

INTRODUCTION

In circulating fluidization and pneumatic transport, measurements of time-averaged vertical pressure profiles are conveniently used to infer average voidage in a section of the riser. To this end, the static pressure drop $\Delta p/\Delta z$ is equated to the bulk weight in the riser section [1-4]:

$$\frac{\Delta p}{\Delta z} \approx [\rho_s(1 - \epsilon) + \rho_g \epsilon]g \quad (1)$$

where ρ_s and ρ_g are the densities of the solid and the gas and ϵ is the average voidage. However, this calculation is erroneous when momentum losses due to wall friction or solid acceleration cannot be neglected as compared with the static head of solids. In particular, Stermerding [5] showed that wall friction becomes significant at the higher velocities of pneumatic transport.

Acceleration effects, which are common at the bottom of pneumatic transport lines and circulating fluidized beds (CFB), also require substantial corrections to the simple expression above. For example, Weinstein and Li

[6] recently proposed a method to evaluate the actual bed voidage in the acceleration zone of a CFB, provided that the length of the acceleration section, the solid terminal velocity and the pressure profile are known.

In this context, the quick-closing valve technique provides a direct measurement of voidage that may be compared with the values inferred from vertical pressure gradients. In this technique, gate valves located at several vertical locations are actuated at precisely the same instant to trap solids in successive sections of the riser. Using this technique in a CFB, Arena et al. [7] observed unexpected discrepancies between the measured and inferred values of voidage at positions located well above the bottom acceleration zone of the riser. These authors assumed that the discrepancies were related to frictional losses at the wall.

In the present paper, a simple model offers a different explanation for Arena's observations. This model attributes the voidage discrepancies to the rapid vertical variations of voidage that are present at the transition between the dense and the dilute regions of a CFB riser. Hence, this discussion does not focus on the bottom acceleration region of the CFB. There, details of the solid injection system often make the flow pattern three-dimensional, thus difficult to predict accurately. Instead, this paper concerns the portion of the riser well above the bottom acceleration zone.

MODEL

The proposed model assumes that the radial profiles of voidage and velocity observed in CFB risers do not contribute significantly to the transfer of momentum in the vertical direction. Thus, the shear in the gas and solid phases are neglected, and the

flow is considered one-dimensional. As a result, the momentum equations for the gas and solid phases are simplified to read

$$\rho_s \epsilon U \frac{dU}{dz} \approx -\epsilon \frac{dP}{dz} - F_d$$

$$\rho_s(1-\epsilon)U_s \frac{dU_s}{dz} \approx -\rho_s(1-\epsilon)g - (1-\epsilon) \times \frac{dP}{dz} + F_d$$

where U and U_s are the average gas and solid velocities. F_d is the drag force per unit volume exerted by the gas on the solid phase. The term $(1-\epsilon) dP/dz$ arises from the effect of the gas pressure gradient on the particles, which is similar to the buoyancy force in a fluid. Since the gas convection term is much smaller than the corresponding term in the solid eqn., $\rho_s \epsilon U(dU/dz) \ll \rho_s(1-\epsilon)U_s(dU_s/dz)$, the two eqns. may be combined to give

$$-\frac{dP}{dz} \approx \rho_s(1-\epsilon)U_s \frac{dU_s}{dz} + \rho_s(1-\epsilon)g \quad (2)$$

For one-dimensional flow, the continuity equation for the solid phase reduces to

$$\frac{d}{dz}[\rho_s(1-\epsilon)U_s] = 0 \quad (3)$$

Differentiating eqn. (3) yields

$$\frac{dU_s}{dz} = -\frac{U_s}{1-\epsilon} \frac{d(1-\epsilon)}{dz} \quad (4)$$

Using the definition of solid flux, $G_s = \rho_s(1-\epsilon)U_s$, eqn. (4) can be substituted into eqn. (2) to obtain the following ODE governing the actual voidage profile:

$$\frac{d(1-\epsilon)}{dz} = \frac{\rho_s^2 g}{G_s^2}(1-\epsilon)^3 + \frac{\rho_s}{G_s^2} \frac{dP}{dz}(1-\epsilon)^2 \quad (5)$$

The solution of this first-order ODE requires the knowledge of the vertical pressure profile and an initial voidage condition specified at any one location in the riser. For typical CFB operations, the voidage profile near the top of the riser is often dilute and nearly fully developed. As a result, acceleration effects are negligible there. Provided that friction plays a minor role in the momentum balance, eqn. (1) provides an estimate for the voidage near the top of the riser, which can serve as the initial condition required to solve eqn. (5).

The integration of eqn. (5) is then carried out backwards for decreasing values of height.

However, this integration should be interrupted well before it reaches the solid injection port at the bottom of the riser. There, unless the riser includes a smooth solid injection design such as that of Weinstein and Li [6], the assumption of a one-dimensional flow is clearly wrong, since the solid flux is primarily pointed in the radial direction, and the particles are accelerating. An estimate of the height of this acceleration region can be obtained by calculating the height at which particles, initially held motionless at the distributor and then suddenly released in a riser of constant voidage, reach their terminal velocity under constant gas velocity. Under these assumptions, the solid momentum eqn. can be written

$$\frac{d}{dz}[\rho_s(1-\epsilon)U_s^2] \approx -\rho_s(1-\epsilon)g + F_d \quad (6)$$

where F_d , the drag force per unit volume, is obtained by multiplying the drag force on a single particle by the number of particles per unit volume [8]:

$$F_d = C_d \left| \frac{U_0}{\epsilon} - U_s \right| \left(\frac{U_0}{\epsilon} - U_s \right) \frac{3\rho_s}{4d_p} (1-\epsilon)\epsilon^{-1.8} \quad (7)$$

$$C_d = \frac{24}{Re_p} [1 + 0.15 Re_p^{0.687}] \quad (8)$$

$$Re_p = \frac{\left| \frac{U_0}{\epsilon} - U_s \right| \rho_s d_p}{\mu} \quad (9)$$

In the simple formulation of eqn. (6), note that the buoyancy term $-(1-\epsilon) dP/dz$ has been ignored, since it is of order $(1-\epsilon)^2 \rho_s g < (1-\epsilon) \rho_s g$. In eqn. (7), the coefficient $\epsilon^{-1.8}$ is a correction for the drag on a single particle that accounts for the presence of other particles in the flow [9]. Given specific values of ϵ , G_s and U_0 , the numerical integration of eqn. (6) provides an estimate for the height of the acceleration region, which is excluded from our calculations.

RESULTS AND DISCUSSION

In this section, the voidage predictions of the one-dimensional model are tested against

the data of Arena *et al.* [7], who fluidized glass beads of mean diameter 88 μm and density 2.6 g/cm^3 in humidified air. These authors had the foresight to report vertical pressure profiles for three different operating conditions, as well as the corresponding voidage profiles measured using quick-closing valves.

In our approach, the only input to the model is the measured pressure profile. The pressure gradient is assumed constant between two successive pressure taps, located at intervals of 80 cm along the height of Arena's 6.4-m riser column. (A more precise account of the vertical pressure gradient is not possible, because Arena's riser includes only 8 pressure taps). The ODE (5) is then solved numerically for decreasing values of height using a fourth-order Runge-Kutta algorithm, with the initial voidage condition provided by eqn. (1) at the top of the riser.

Figure 1 shows excellent agreement between the voidage predicted by the one-dimensional model and Arena's voidage measurements using quick-closing valves. From this figure, it is also clear that the voidage directly inferred from eqn. (1) is greatly in error at the transition between the dense and the dilute regions of the CFB. There, substantial vertical gradients of voidage make the pressure drop significantly different from the static head of the gas-solid mixture. Similar results are obtained for the conditions of Fig. 2.

Fig. 1. Comparison of axial profiles of solid volume fractions in a CFB riser 41 mm ID and 6.4 m high, with the superficial gas velocity $U_0 = 7$ m/s and the solid flux $G_s = 600$ $\text{kg}/(\text{m}^2 \text{ s})$ (Arena *et al.* [7]). The open squares are inferred directly from the pressure profile and eqn. (1). The solid squares are average solid volume fractions measured using quick-closing valves. The solid line is the prediction of the one-dimensional model.

Fig. 2. Same caption as in Fig. 1, with the conditions $U_0 = 7$ m/s and $G_s = 382$ $\text{kg}/(\text{m}^2 \text{ s})$.

Further insight into this effect can be gained by rearranging eqn. (5):

$$\frac{dP}{dz} = -\rho_s g(1 - \epsilon) - \frac{G_s^2}{\rho_s} \frac{d}{dz} \left(\frac{1}{1 - \epsilon} \right) \quad (10)$$

In this form, the pressure gradient appears to be composed of two terms. The first one is the static head of the mixture, and the second represents the contribution of the voidage gradient to the overall pressure gradient. In the limit of no voidage gradient (e.g., for fully developed flow) or for a small solid flux, the second term becomes negligible. This case is illustrated in Fig. 3. There, the discrepancy between the two measurement methods nearly disappears.

Fig. 3. Same caption as in Fig. 1, with the conditions $U_0 = 7$ m/s and $G_s = 199$ $\text{kg}/(\text{m}^2 \text{ s})$.

Figure 4 is a voidage profile inferred from data collected in our own CFB facility. The bed consists of Sigma 200 Fluid Cracking Catalyst (FCC) fluidized with air. The mean particle diameter is 72 μm and the density is 1.3 g/cm^3 . Under the conditions shown, the solid flux is small enough to render the second term of eqn. (10) negligible. In this

Fig. 4. Comparison of axial profiles of solid volume fractions in a CFB riser 203 mm ID and 7 m high, with the superficial gas velocity $U_0 = 2$ m/s and the solid flux $G_s = 40$ kg/(m² s). The open squares are inferred directly from the pressure profile and eqn. (1). The solid line is the prediction of the one-dimensional model.

Fig. 5. Least-squares fit of the vertical time-averaged pressure profile for the experiment of Fig. 4. Equation (11) was taken as a basis for the least-squares fit. Each data point was time-averaged for 5 consecutive intervals of 2 s, separated by approximately 30 min.

case, the solid line of the model agrees well with the local voidage values inferred from eqn. (1). The scatter of the local voidage data (open squares) results from the inevitable noise amplification associated with the calculation of the derivative dP/dz . Therefore, the first step in the data reduction is to fit the experimental pressure profile using the following expression (Fig. 5):

$$P(z) = \rho_s g z_0 \left\{ (\epsilon_a - 1) \frac{z_1 - H + z}{z_0} - (\epsilon_a - \epsilon^*) \ln \left[1 + \exp \left(\frac{z_1 - H + z}{z_0} \right) \right] \right\} + \text{const.} \quad (11)$$

This pressure profile is the integral of the convenient fit proposed by Li and Kwauk [10] for vertical voidage profiles in a CFB. The parameter z_1 is the location of the inflection point of the voidage profile, z_0 is a characteristic length and H is the riser height. ϵ_a and ϵ^* are two asymptotic voidages characteristic of the voidage profile. Using the corresponding function for dP/dz , eqn. (5) is then integrated between the top of the riser and the edge of the acceleration zone.

CONCLUSIONS

This paper has discussed the effect of vertical voidage gradients on the measured pressure drop in circulating fluidized bed risers. In particular, a one-dimensional model has explained the discrepancies observed by Arena *et al.* [7] between the voidage measured using quick-closing valves and that inferred from the static head of the mixture. Such discrepancies are significant under high solid fluxes and are primarily confined to regions of large vertical voidage gradients, such as the transition between the dense and the dilute regions in a CFB riser. Finally, this paper has outlined a data reduction procedure that may be used to infer vertical voidage profiles from time-averaged pressure measurements when these effects are significant.

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LIST OF SYMBOLS

C_d	drag coefficient on a single particle
d_p	particle diameter
F_d	drag force per unit volume

G_s	solid flux in riser
H	height of riser
P	time-averaged pressure
Re_p	Reynolds number based on the average slip velocity ($U_0/\epsilon - U_s$)
U	gas velocity
U_0	superficial gas velocity
U_s	solid velocity
z	vertical co-ordinate with origin at the bottom of the bed
z_0	characteristic length (eqn. (11))
z_i	location of the inflection point of the voidage profile from top of riser

Greek symbols

ϵ	voidage (gas volume fraction)
ϵ_a	asymptotic voidage at the bottom of the riser (eqn. (11))
ϵ^*	asymptotic voidage at the top of the riser (eqn. (11))
ρ_g	gas density
ρ_s	solid density
μ	gas viscosity

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The role of particle collisions in pneumatic transport

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We analyse the dilute, steady, fully developed flow of relatively massive particles in a turbulent gas in the context of a vertical pipe. The idea is that the exchange of momentum in collisions between the grains and between the grains and the wall plays a significant role in the balance of forces in the particle phase. Consequently, the particle phase is considered to be a dilute system of colliding grains, in which the velocity fluctuations are produced by collisions rather than by the gas turbulence. The balance equations for rapid granular flow are modified to incorporate the drag force from the gas, and boundary conditions, based on collisional exchanges of momentum and energy at the wall, are employed. The turbulence of the gas is treated using a one-equation closure. A numerical solution of the resulting governing equations provides velocity and turbulent energy profiles in agreement with the measurements of Tsuji *et al.* (1984).

1. Introduction

Gas-solid flows satisfy the principles of mass, momentum and energy balance. While the Navier-Stokes equations govern the motion of the gas phase, there is not yet universal agreement upon the way of incorporating the particle phase. One approach is to treat this phase as a continuum and use some form of averaging to obtain the balance laws. Then, depending on the situation, the fluxes and sources of momentum and energy for the particle phase are modelled in order to close the system of equations.

For small particles, dilute in a turbulent gas, momentum transfer in the particle phase is due to turbulent diffusion of the particles. In this regime, Elghobashi & Abou-Arab (1983) have derived a two-fluid $k-\epsilon$ closure that relates the Reynolds stress of the particles to gradients of the mean particle velocity using an eddy viscosity that is a fraction of the eddy viscosity of the gas. This analysis predicts the reduction of turbulent energy observed in the presence of small particles (e.g. Modares, Tan & Elghobashi 1984). Other works in this regime include those of Pourahmadi & Humphrey (1983), Chen & Wood (1985) and Berker & Tulig (1986). However, these analyses do not apply to flows with massive particles.

For massive particles, the experiments of Soo, Ihrig & El Kouch (1960) show that the intensity of the particle velocity fluctuations may exceed that of the fluid, an observation that cannot be explained if the particles respond only to the turbulence. Min (1967) attributes this high particle 'turbulence' to particle-wall collisions. Lourenco, Riethmuller & Essers (1983) also focus attention on collisions when

predicting profiles of density and gas and particle velocities in a 10 cm wide horizontal duct loaded with 500 μm glass particles. By analogy with molecular dynamics, these authors introduce a particle velocity distribution function that, they assume, is determined as a solution of a Boltzmann transport equation by particle collisions, rather than by the gas turbulence. In other words, they assume that the gas influences the mean velocity of the particles, but that it has no effect on their random motion. In their work, this assumption is justified, because the ratio of particle relaxation time to a typical large-eddy turbulent timescale is large (Hinze 1972). Sinclair & Jackson (1989) have recently presented a model for vertical gas-solid flow in a pipe that also treats the particle phase as a rapid granular flow. They produce a surprising variety of flow regimes in the context of the model. However, they ignore the gas turbulence and, consequently, do not distinguish the regimes in which collisions dominate the interactions of particles with the turbulent eddies. Neither do they attempt a quantitative comparison of their results with experimental data.

Here, a more detailed analysis of dilute gas-solid flow in a vertical pipe is undertaken. Particles are assumed to be sufficiently massive to be unaffected by the turbulent velocity fluctuations of the gas. The particle phase is treated as a rapidly flowing granular material in which momentum and energy are transferred by the velocity fluctuations of the particles. The momentum balance for rapid granular flow given, for example, by Jenkins & Savage (1983) is modified to include a drag term that provides the force necessary to suspend the particles and the corresponding energy balance includes rates of production and dissipation associated with interactions between the particles and the gas. Finally, the turbulence of the gas phase is treated using a one-equation closure.

Here, we focus on fully developed, steady flows. This assumption is crucial to the analysis for it permits us to carry out averages for both the gas and particle properties on long vertical strips. For the gas, we equate these with ensemble averages or with the time averages determined in the experiments. For the particles, we assume that they are equivalent to averages based on a velocity distribution function. Then, no matter how dilute the flow, the control volumes can always include a large enough number of particles to define the appropriate averages, so the dispersed particles may be regarded as a continuum.

We obtain numerical solutions of the resulting balance laws, constitutive relations and boundary conditions and compare these with the experimental profiles of particle velocity, gas velocity, and turbulent gas fluctuations reported by Tsuji, Morikawa & Shiomi (1984). The agreement of this data with the results of the analysis suggests that particle collisions play an important role in flows involving relatively massive particles, even when the particle phase is rather dilute.

2. Hydrodynamics

In this section, we outline the hydrodynamics for dilute, fully developed, steady flow in a vertical pipe of relatively massive particles of uniform diameter d . We focus on particles massive enough to be unaffected by the velocity fluctuations in the turbulent gas. In other words, the hydrodynamic relaxation time of the particle velocity fluctuations is much greater than a typical roll-over time of the turbulent eddies that is based on their integral lengthscale l and their root-mean-square (r.m.s.) turbulent velocity u' .

2.1. Particle phase

In the regime of interest, particles are suspended by the drag force exerted by the mean gas flow, but their velocity fluctuations are the result of collisions with other particles or with the wall. For the particle phase we adopt the assumption of molecular chaos central to the treatment of collisions in rapid granular flows (Jenkins & Savage 1983) and we assume that interparticle collisions are nearly elastic and frictionless. For the conditions of Tsuji *et al.* (1984), the product of the collision frequency and the hydrodynamic relaxation time of the velocity fluctuations varies between 3 and 230. Consequently, a particle loses only a small fraction of its fluctuation energy in and between collisions, so the velocity distribution function for the particles is nearly Maxwellian.

2.1.1. Momentum

With these assumptions, well-established results from the kinetic theory of gases lead to constitutive relations for the pressure and shear stress in the particle phase (Chapman & Cowling 1970). The resulting momentum equation for the rapidly flowing particles is then modified to incorporate the drag force from the gas and the gravitational force:

$$\rho_p \frac{\partial[(1-\epsilon)v_i]}{\partial t} + \rho_p \frac{[(1-\epsilon)v_i v_j]}{\partial x_j} = \frac{\partial S_{ij}}{\partial x_j} + \frac{\rho_p}{T} (1-\epsilon)(u_i - v_i) - (1-\epsilon)g_i \rho_p, \quad (1)$$

where u_i and v_i are the average gas and particle velocities, p is the gas pressure, g_i is the gravitational acceleration, ϵ is the voidage, ρ_p is the material density of the particles, T is the hydrodynamic relaxation time of the mean relative velocity (slip velocity) between the phases, and

$$S_{ij} = -N\delta_{ij} + \frac{5\pi^{\frac{1}{2}}}{96} \rho_p d\Theta^{\frac{1}{2}} \left[\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial v_k}{\partial x_k} \delta_{ij} \right] \quad (2)$$

is the total stress transmitted through the particle phase. The particle pressure N is related to the particle volume fraction $(1-\epsilon)$ by an expression analogous to an equation of state in a molecular gas:

$$N = \rho_p (1-\epsilon) \Theta, \quad (3)$$

where Θ is the 'granular temperature', expressed in terms of the r.m.s. particle velocity fluctuations v' as $\frac{1}{2}\Theta = \frac{1}{2}v'^2$. In these dilute flows, terms in the momentum exchange between the phases that are proportional to the gradient of the voidage are neglected.

For fully developed, axisymmetric flow, the momentum balance in the vertical direction reduces to

$$0 = \frac{1}{r} \frac{d}{dr} (rS) + \frac{\rho_p}{T} (1-\epsilon)(u-v) - \rho_p (1-\epsilon)g, \quad (4)$$

where u and v are the vertical components of the mean gas and particle velocities, r is the radial coordinate and S , the particle shear stress on surfaces at constant radius, is given through (2) by

$$S = \frac{5\pi^{\frac{1}{2}}}{96} \rho_p d\Theta^{\frac{1}{2}} \frac{dv}{dr}. \quad (5)$$

In this simple flow, the particle momentum balance in the radial direction becomes

$$dN/dr = 0, \quad (6)$$

so that N is constant across the pipe.

For the very dilute flow of relatively large particles, the length λ of the mean free path between collisions may be comparable to the radius R of the pipe. In this case, the transport of particle momentum is similar to that in the free-molecule regime of gas dynamics and, by analogy with the heuristic treatment of shear stress in Couette flow there (e.g. Vincenti & Kruger 1977), so we adopt the following correction for S :

$$S = \frac{5\pi^{\frac{1}{2}}}{96} \rho_p d \Theta^{\frac{1}{2}} \frac{dv}{dr} \frac{1}{1 + \lambda/R}, \quad (7)$$

where $\lambda = d/[6\sqrt{2}(1-\epsilon)]$. Unlike (5), this expression for shear stress approaches zero as $(1-\epsilon)$ vanishes, so that (4) remains valid in this limit.

The hydrodynamic relaxation time T of a particle is defined in terms of the drag force on one particle by

$$\frac{\rho_p}{T} = C_d |u - v| \frac{3\rho}{4d}, \quad (8)$$

where ρ is the gas density. For the drag coefficient C_d we adopt the empirical expression

$$C_d = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}), \quad (9)$$

which is valid in the range $0 < Re_p < 800$, where $Re_p = |u - v| \rho d / \mu$ is the Reynolds number based on the average slip velocity between the phases and μ is the viscosity of the gas (Clift, Grace & Weber 1978).

2.1.2. Energy

In order to close the equations describing the particle phase, an energy balance is written for the determination of the granular temperature Θ . Following Jenkins & Savage (1983), it is

$$\frac{1}{2} \rho_p (1-\epsilon) \frac{\partial \Theta}{\partial t} + \frac{1}{2} \rho_p (1-\epsilon) v_i \frac{\partial \Theta}{\partial x_i} = - \frac{\partial q_i}{\partial x_i} + S_{ij} \frac{\partial v_i}{\partial x_j} - D_1 - D_2, \quad (10)$$

where q_i is the diffusive flux of particle fluctuation energy, given by kinetic theory as

$$q_i = - \frac{25\pi^{\frac{1}{2}}}{128} \rho_p d \Theta^{\frac{1}{2}} \frac{\partial \Theta}{\partial x_i},$$

and D_1 and D_2 are rates of dissipation per unit volume due, respectively, to the inelasticity of the particles and to their interaction with the gas.

The second term of the right-hand side of (10) is the working of the stress through the mean particle velocity gradient. In the dilute limit, D_1 is given for nearly elastic particles as

$$D_1 = \frac{24(1-\epsilon)\rho_p}{\pi^{\frac{1}{2}}d} \Theta^{\frac{1}{2}} (1-\epsilon)^2, \quad (11)$$

where e is the coefficient of restitution for a particle-particle collision (Jenkins & Savage 1983). The expression for D_1 is obtained by considering the kinetic energy lost in each collision, then averaging over all possible collisions.

The rate of energy dissipation per unit volume D_2 results from the working of the fluctuating force exerted by the gas through the fluctuating velocity of the particles. This fluctuating force includes a drag component associated with the slip velocity and a Basset component that arises from the rapid variation of the slip velocity with time. A calculation of the Basset component indicates that in the experiments of Tsuji *et al.* (1984) it is at most 20% of the drag component. Upon ignoring its contribution, D_2 may be written as

$$D_2 = -(\rho_p/T)(1-\epsilon)\overline{v'_i(u'_i - v'_i)} = -(\rho_p/T)(1-\epsilon)(\overline{u'_i v'_i} - 3\Theta), \quad (12)$$

where primes indicate fluctuating velocities and the overbar denotes the average.

The term $\overline{u'_i v'_i}$ is the correlation between the velocity fluctuations of the gas and those of the particles. Koch (1990) has recently calculated this term for a dilute gas-solid suspension at very low particle Reynolds number in the limit where solid-body collisions determine the particle velocity distribution function. He finds that

$$\overline{u'_i v'_i} = \frac{4}{\pi^{\frac{1}{2}}} \frac{d(u_i - v_i)(u_i - v_i)}{T_0 \Theta^{\frac{1}{2}}}, \quad (13)$$

where T_0 is the Stokes relaxation time. In the experiments of Tsuji *et al.* (1984), the Reynolds number based on the r.m.s. fluctuating slip velocity lies well above one. In this regime there are no theoretical prediction for $\overline{u'_i v'_i}$. In this case we adopt the functional form given by (13) but use the relaxation time T_1 given by the analogue of (8) based on the r.m.s. fluctuating slip velocity between the two phases. This extension of Koch's expression is analogous to the extension of the Stokes drag coefficient to other than low particle Reynolds number. In the situations considered here its contribution is typically dominated by other terms.

For fully developed, axisymmetric flow, (10) becomes

$$0 = -\frac{1}{r} \frac{d}{dr}(rq) + S \frac{dr}{dr} - D_1 - D_2, \quad (14)$$

where

$$q = -\frac{25\pi^{\frac{1}{2}}}{128} \rho_p d\Theta^{\frac{1}{2}} \frac{d\Theta}{dr}.$$

To treat the very dilute flow of large particles, we modify this expression using a correction similar to that in (7):

$$q = -\frac{25\pi^{\frac{1}{2}}}{128} \rho_p d\Theta^{\frac{1}{2}} \frac{d\Theta}{dr} \frac{1}{1 + \lambda/R}.$$

2.1.3. Boundary conditions

Boundary conditions for the particle phase have been calculated by Jenkins (1991) who considers collisions of inelastic frictional spheres with a frictional wall. For relatively small values of the coefficient of sliding friction μ_f , the resulting tangential momentum balance and the balance of energy are, respectively,

$$S = -\mu_f N \quad (15a)$$

$$\text{and} \quad q = -\frac{1}{2} N (3\Theta)^{\frac{1}{2}} [\frac{1}{2}(1 + e_w)\mu_f^2 - (1 - e_w)]. \quad (15b)$$

where e_w is the coefficient of restitution for a particle colliding with the wall. These relationships have been derived based on the assumption that the coefficient of friction is so small that the point of contact of a particle always slips during a collision.

Finally, because the flow is axisymmetric, the boundary conditions at the centreline of the pipe are that S and q vanish there.

2.2. Gas phase

2.2.1. Momentum

In our treatment of the fully developed, steady particle flow, we considered a control volume of infinitesimal width but arbitrary extent in the vertical direction and assumed that the velocity distribution function depended only upon the radial position of the control volume. Further, we restricted attention to particles that are too massive to be affected by the turbulent velocity fluctuations; their velocity fluctuations are governed by a velocity distribution function $f(v, x, t)$. The particle volume fraction $(1 - \epsilon)$ is related to this velocity distribution function by

$$\iiint f(v, x, t) dv \equiv (1 - \epsilon) / (\frac{1}{2}\pi) d^3.$$

In the flow under consideration f and, consequently, ϵ are independent of time, so $\epsilon' = 0$. Thus, a Reynolds decomposition of the momentum equation leads to

$$\rho \frac{\partial(\epsilon u_i)}{\partial t} + \rho \frac{\partial(\epsilon u_i u_j)}{\partial x_j} = -\frac{\partial(\epsilon p)}{\partial x_i} + \frac{\partial(\epsilon \tau_{ij})}{\partial x_j} - \frac{\rho_p}{T} (1 - \epsilon)(u_i - v_i) \quad (16)$$

and

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \delta_{ij} - \rho \overline{u'_i u'_j}, \quad (17)$$

respectively.

We adopt a closure of the Reynolds stress based on the eddy viscosity μ_t :

$$\rho \overline{u'_i u'_j} = -\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{2}{3} \delta_{ij} \left(\rho k + \mu_t \frac{\partial u_k}{\partial x_k} \right). \quad (18)$$

where $k = \frac{1}{2} \overline{u'_k u'_k}$ is the turbulent kinetic energy per unit mass of the gas. The eddy viscosity is assumed to be given by the one-equation closure described, for example, by Reynolds (1976):

$$\mu_t = C_\mu \rho k l. \quad (19)$$

where l is the turbulent mixing length and $C_\mu = 0.49$. Following common practice in pipe flow, we assume that near the wall the mixing length is proportional to the distance from the wall and that near the centre of the pipe it is constant:

$$\frac{l}{R} = \begin{cases} \kappa(1 - r/R), & r/R \geq 0.7; \\ 0.3\kappa, & r/R \leq 0.7. \end{cases} \quad (20)$$

where $\kappa = 0.41$ is von Kármán's constant.

For axisymmetric, steady, fully developed flow, the vertical component of (16) becomes

$$0 = -\frac{\partial(\epsilon p)}{\partial z} - \frac{\rho_p}{T} (1 - \epsilon)(u - v) + \frac{1}{r} \frac{d}{dr} (r \epsilon \tau_{rz}). \quad (21)$$

where $\tau_{rz} = (\mu + \mu_t) du/dr$ and z is the upward vertical coordinate. The corresponding radial component is

$$0 = -\frac{\partial(\epsilon p)}{\partial r} + \frac{1}{r} \frac{d}{dr} (r \epsilon \tau_{rr}) - \frac{\epsilon \tau_{\theta\theta}}{r},$$

where $\tau_{rr} = -\rho \overline{u_r'^2}$ and $\tau_{\theta\theta} = -\rho \overline{u_\theta'^2}$. Using (18), we find that

$$\rho \overline{u_r'^2} = \rho \overline{u_\theta'^2} = \frac{2}{3} \rho k.$$

Consequently, the radial component is

$$\frac{\partial(\epsilon p)}{\partial r} = \frac{2}{3} \rho \frac{d(\epsilon k)}{dr}. \quad (22)$$

We do not require a solution to (22): from it, it follows that $\partial(\epsilon p)/\partial z$ is a constant independent of r and z in fully developed flow.

2.2.2. Kinetic energy

The balance of turbulent kinetic energy is obtained by multiplying the instantaneous gas momentum equation by u_i' , taking the time average of all terms, and subtracting the energy equation for the mean flow. For steady flows involving massive particles the resulting equation reduces to eleven terms:

$$\begin{aligned} \frac{\partial(\rho \epsilon u_i k)}{\partial x_j} = & -\overline{u_i' u_j'} \frac{\partial(\rho \epsilon u_i)}{\partial x_j} - \frac{\partial}{\partial x_i} (\overline{\epsilon p' u_i'}) - \overline{p' u_i'} \frac{\partial \epsilon}{\partial x_i} - \rho \epsilon \overline{u_i' u_j'} \frac{\partial u_i'}{\partial x_j} \\ & - \rho \epsilon \overline{u_i' u_j'} \frac{\partial u_j'}{\partial x_i} + \frac{\rho_p}{T} (1 - \epsilon) \overline{u_i' (v_i' - u_i')} + \mu \epsilon \overline{u_i' \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)} \\ & + \mu \frac{\partial \epsilon}{\partial x_j} \overline{u_i' \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)} - \frac{2}{3} \mu \epsilon \overline{u_i' \frac{\partial^2 u_j'}{\partial x_i \partial x_j}} - \frac{2}{3} \mu \frac{\partial \epsilon}{\partial x_i} \overline{u_i' \frac{\partial u_j'}{\partial x_j}}. \end{aligned} \quad (23)$$

We adopt the closures proposed by Elghobashi & Abou-Arab (1983) for the velocity and pressure correlations. In particular, we use (18) for the Reynolds stress and a gradient diffusion model for the turbulent transport of k :

$$\rho \epsilon \overline{u_i' u_j'} = \epsilon \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j}, \quad (24)$$

where σ_k is a turbulent Prandtl number appropriate for the diffusion of k . Following common practice we assume that $\sigma_k = 1$. We ignore a counter-gradient diffusive flux.

In fully developed, axisymmetric flow, (23) simplifies to

$$\begin{aligned} 0 = & \frac{1}{r} \frac{d}{dr} \left\{ r \epsilon \left(\frac{\mu_t}{\sigma_k} + \mu \right) \frac{dk}{dr} \right\} + \mu_t \epsilon \left(\frac{du}{dr} \right)^2 - \rho \epsilon E - \frac{\rho_p}{T} (1 - \epsilon) \overline{u_i' (u_i' - v_i')} \\ & - \frac{1}{r} \frac{d}{dr} (\overline{\epsilon r p' u_r'}) - \overline{p' u_r'} \frac{d\epsilon}{dr} - \frac{2}{3} \mu \frac{k}{r} \frac{d}{dr} \left(r \frac{d\epsilon}{dr} \right) + \frac{2}{3} \mu \frac{d\epsilon}{dr} \frac{dk}{dr} + \frac{10}{9} \mu \frac{k}{\epsilon} \left(\frac{d\epsilon}{dr} \right)^2, \end{aligned} \quad (25)$$

where $E = (\mu/\rho) \overline{\partial u_i' \partial u_j' / \partial x_j \partial x_i}$ is the isotropic turbulent dissipation rate.

The first term of (25) represents the turbulent and viscous diffusion of k . The second term is the rate of production of k by the working of the mean shear. Because the particles modify the mean gas velocity profile through the drag term in (21), they affect the gas velocity gradient and, through the second term in (25), the production

of k . The third term is the isotropic dissipation of k . For dilute conditions, it is closed as described by Reynolds (1976): $E = C_2 k^3/l$, where $C_2 = C_\mu^3$. The fifth and sixth terms are the work of turbulent pressure fluctuations. The next three terms are viscous dissipation terms. According to Hinze (1975), the relative pressure fluctuations p'/p are at least an order of magnitude smaller than the relative velocity fluctuations u'/u . Because the fifth term is much less than order $\epsilon \rho u'^3/R$, which is itself smaller than the turbulent diffusion term of order $\epsilon \rho u'^3/l$, it is negligible. Because the last four terms involve the average voidage gradient $\partial \epsilon / \partial r$, which is of order $(1-\epsilon)/R$, in dilute flow they may be ignored. With these simplifications, (25) becomes

$$0 = \frac{1}{r} \frac{d}{dr} \left\{ r \epsilon \left(\frac{\mu_t}{\sigma_k} + \mu \right) \frac{dk}{dr} \right\} + \mu_t \epsilon \left(\frac{du}{dr} \right)^2 - \rho \epsilon E - \frac{\rho_p}{T} (1-\epsilon) (2k - \overline{u_i' v_i'}). \quad (26)$$

Our experience in analysing the experiments of Tsuji *et al.* (1984) is that the production of turbulent kinetic energy is dominated by the working of the mean shear, and that its dissipation is primarily governed by the isotropic dissipation rate E .

2.2.3. Boundary conditions

Because the closure outlined above is not valid in the viscous and buffer layers, boundary conditions for the gas phase at the wall are enforced at a dimensionless distance $y^+ = \rho(R-r)u^*/\mu \approx 30$ from the wall. The parameter $u^* = (\epsilon \tau_0/\rho)^{1/2}$ is the shear velocity, where τ_0 is the gas shear stress at the wall. In the dilute flow, we assume that the gas velocity profile near the wall is not greatly affected by the presence of the particles. In this case, the gas velocity at small values of y^+ is given by the universal 'law of the wall':

$$\frac{u}{u^*} = \begin{cases} 5 \ln y^+ - 3, & 5 \leq y^+ \leq 30; \\ (1/\kappa) \ln y^+ + 5.7, & y^+ \geq 30. \end{cases} \quad (27)$$

The shear velocity is calculated from a global momentum balance obtained by adding the momentum balances of the gas and particle phases and integrating between $r = 0$ and $r = R$:

$$\partial(\epsilon p)/\partial z = -2\rho u^{*2}/R + 2S_0/R - g\rho_p(1-\epsilon), \quad (28)$$

where S_0 is the particle shear stress evaluated at the wall and ϵ is the average voidage across the pipe.

For the turbulent kinetic energy, we take the boundary condition at the wall to be

$$\partial k / \partial r = 0. \quad (29)$$

Provided that the molecular viscosity μ is much smaller than the eddy viscosity μ_t (a condition typically met for $y^+ \approx 30$), this condition is equivalent to equating the production and dissipation of turbulent kinetic energy. In particle-laden flows, unlike pure gas flows, equating the production and the dissipation of k at the boundary does not yield a straightforward algebraic expression for k because of the last term of (26). In this context, (29) is as exact and more convenient than specifying a value for k at the wall in terms of u^{*2} .

The system of equations (21) and (26) was first solved for pure gas ($\epsilon = 1$) subject to the boundary conditions (27)–(29) at the wall and the requirement that the radial

FIGURE 1. Calculated profiles of (a) gas velocity u and (b) r.m.s. gas velocity fluctuations u' for clear gas, normalized by the centreline gas velocity $u_{cl} = 13.4$ m/s. The solid circles represent the data of Tsuji *et al.* (1984).

derivative of u and k vanish at the centreline. As indicated in figure 1, the predicted profiles of gas velocity and turbulent kinetic energy are in satisfactory agreement with the measurements of Tsuji *et al.* (1984) despite the rather rudimentary turbulence closure used. In addition, the predictions of the skin friction coefficient at the wall, $f \equiv 2\tau_w/\rho\bar{u}^2$, are within 10% of the Blasius correlation for a smooth pipe, $f = 0.079Re^{-1/2}$, where Re is based on the pipe diameter and the average gas velocity $\bar{u}$ over the cross-section.

3. Results and discussion

In this section, we compare the detailed measurements of Tsuji *et al.* (1984) with the predictions of our analysis. Using a laser-Doppler anemometer, these authors measured the profiles of mean gas velocity, mean particle velocity, and gas velocity fluctuations in a vertical pipe of 30.5 mm diameter under fully developed, steady

FIGURE 2. Calculated profiles of normalized gas velocity u/u_{cl} and particle velocity v/u_{cl} for relatively dilute flows of (a) 200 μm and (b) 500 μm particles. The solid and open circles represent the data of Tsuji *et al.* (1984) for gas and particle velocities, respectively. The dashed lines represent particle velocities predicted by an analysis that would ignore particle shear. The conditions are: (a) $u_{cl} = 18.9 \text{ m/s}$, $m = 1.0$; and (b) $u_{cl} = 9.65 \text{ m/s}$, $m = 1.1$.

flow. In these experiments, polystyrene spheres with density $\rho_p = 1020 \text{ kg/m}^3$ and diameters in the range 200 μm to 3 mm were suspended in air. Ratios of particle-to-gas mass flow rates m (loading) were as high as 3.6. For these conditions, the hydrodynamic relaxation time of the particles is between 60 ms and 850 ms, while the slowest turbulent timescale is of order $R/u \sim 2 \text{ ms}$. Clearly, the particles cannot follow the gas turbulence.

The system of ordinary differential equations (3), (4), (6), (14), (21), (26) and boundary conditions constitutes a nonlinear, coupled, two-point boundary-value problem. We solve this system numerically using the quasi-linearization method of Bellman & Kalaba (1965). In the calculation we assume a coefficient of restitution $e = 0.9$ for particle-particle collisions, $e_w = 0.7$ for particle-wall collisions, and a coefficient of dynamic friction $\mu_f = 0.2$ between a particle and the wall. The

FIGURE 3. Calculated profiles of normalized r.m.s. gas velocity fluctuations u'/u_{cl} for relatively dilute flows. The solid circles represent the data of Tsuji *et al.* (1984) for 200 μm particles. $u_{cl} = 12.8$ m/s. $m = 1.3$; the open circles are 500 μm particles. $u_{cl} = 13.3$ m/s. $m = 1.3$. The solid and dashed lines are the respective predictions of the analysis. We predict $(1-\epsilon) = 0.16\%$ and $\Theta^1/u_{cl} \approx 1.8\%$ for 200 μm particles and $(1-\epsilon) = 0.18\%$ and $\Theta^1/u_{cl} \approx 2\%$ for 500 μm .

coefficients of restitution are consistent with the observations of Govan, Hewitt & Ngan (1989), who filmed the trajectories of glass spheres transported in a small pipe under conditions similar to those of Tsuji *et al.* (1984). For particle-wall collisions, these authors estimated that $0.52 < e_w < 0.95$.

In particle-laden flows, two parameters determine the operating conditions, i.e. gas velocity and solid loading. Rather than specifying these values at the onset of the computations, it was more convenient to input to the program the overall pressure gradient and the particle volume fraction at the wall. These were iterated until the gas velocity at the centreline and the loading were within 1% of the measured values.

Figures 2 to 5 compare the measurements of Tsuji *et al.* and the predictions of our analysis for a range of particle diameters and loadings. Note that the predicted particle velocity at the wall is positive. Comparable observations were made by Lee & Durst (1982) in a similar situation. At first sight, these observations might be surprising. Because the gas velocity is zero at the wall, one might expect the particles to fall. In fact, there is a region near the wall where the particles can acquire a velocity higher than that of the gas. This effect is due to the shear stress in the particle phase. Particles further from the wall are lifted by the gas and, through collisions, transfer momentum to particles closer to the wall. In figures 2 and 5(a), the dashed line represents the prediction of an analysis that ignores this shear stress in the balance of forces for the particle phase. Such a treatment clearly fails to reproduce the observed particle velocity profile. Therefore, stresses in the particle phase are essential for an accurate description of the flow. In addition, because the loading is proportional to the average product of the particle velocity and volume fraction across the pipe, the agreement of the calculated particle velocity with the measured values suggests that equations (3) and (6) adequately predict the particle volume fraction, at least with respect to the mean.

Figure 3 compares the measured and predicted r.m.s. velocity fluctuations for 200 μm and 500 μm particles at relatively low particle loadings. The present analysis predicts correctly the magnitude of the r.m.s. velocity fluctuations and the trend

FIGURE 4. Calculated profiles of (a) u/u_{cl} , v/u_{cl} , and (b) u'/u_{cl} for relatively dense suspensions of 200 μm particles. In (a), the solid circles represent the data of Tsuji *et al.* (1984) for u/u_{cl} and the open circles are the data for v/u_{cl} . In (b), the solid circles are the data for u'/u_{cl} . The solid lines are the prediction of the analysis. The conditions are: (a) $u_{cl} = 14.8$ m/s, $m = 4.2$; (b) $u_{cl} = 10.8$ m/s, $m = 3.2$. For (b), we predict $(1-\varepsilon) = 0.43\%$ and $\Theta^2/u_{cl} \approx 1.8\%$.

towards larger fluctuations for larger particles. Figures 4 and 5 concern the largest particle loadings considered by Tsuji *et al.* (1984) for 200 μm and 500 μm particles. Here, the mean velocities are well predicted by the analysis, but the r.m.s. velocities are underpredicted near the centreline of the pipe. In this case it may be desirable to refine the turbulence closure of equations (18)–(20) to obtain a better agreement with the measured gas velocity fluctuations at higher particle loadings.

Figure 6 compares our predictions of the total pressure gradient $-\partial(\varepsilon p)/\partial z$ with the measurements of Tsuji *et al.* (1984). Despite the crudeness of the turbulence closure, the predictions of equation (28) are within 15% of the measured values. For these conditions, the contribution of the particle weight $g\rho_p(1-\varepsilon)$ to the total pressure gradient is relatively small ($\approx 20\%$), and that of the particle shear stress $-2S/R$ is even smaller ($\approx 8\%$).

FIGURE 5. Calculated profiles of (a) u/u_{cl} , v/u_{cl} , and (b) u'/u_{cl} for relatively dense suspensions of 500 μm particles. Symbols have the same meaning as in figure 4. The conditions are: (a) $u_{cl} = 8.07$ m/s, $m = 3.6$; and (b) $u_{cl} = 10.7$ m/s, $m = 3.4$. For (b), we predict $(1-\varepsilon) = 0.5\%$ and $\Theta^1/u_{cl} \approx 2.2\%$.

Unfortunately, our predictions of the profiles of granular temperature Θ cannot be compared with the experiments of Tsuji *et al.* (1984), who did not measure the particle velocity fluctuations or the concentration of the particle phase. Such measurements would add considerably to the understanding of this flow regime. In particular, they would help clarify the form of the particle velocity distribution function, as well as the nature of the correlation $u'_i v'_i$. If such experiments are undertaken, we recommend that the coefficients of restitution e and e_w and the coefficient of sliding friction μ_s be reported, because these values affect the magnitude of the granular temperature Θ .

Our experience in analysing the experiments of Tsuji *et al.* (1984) is that the mean velocity profiles and the gas velocity fluctuations are rather insensitive to the actual values of the coefficients of restitution. In contrast, because the coefficient of friction governs the magnitude of the particle shear at the wall and the flux of granular

FIGURE 6. Calculated versus measured fully developed pressure gradient $-\partial(\epsilon p)/\partial z \approx -\partial p/\partial z$ for 500 μm particles. The circles are the total pressure gradient predicted by equation (28). The triangles represent the predicted contribution of the particle weight. The squares are the sum of the contributions of the gas shear and the particle weight. The total particle mass flow rate is 0.03 kg/s. The mean gas velocities are, from left to right, $u = 11.6, 13.0, 14.8$, and 17.2 m/s. For these conditions, we predict, respectively, $(1 - \epsilon) = 0.39, 0.35, 0.30$, and 0.25% and $\Theta^1/u_{c1} \approx 1.8, 1.7, 1.6$, and 1.4% .

energy, it has a strong influence on the flow over a range of particle diameters. For example, for typical conditions (500 μm polystyrene particles, $u = 10$ m/s, $m = 1$), the total predicted pressure gradient rises sharply above $\mu_f \approx 0.33$. These observations suggest that the boundary conditions of the particle phase play an important role in the flow.

Finally, our analysis predicts the behaviour of the vertical pressure gradient observed in pneumatic transport lines. At constant loading, the predicted fully developed pressure gradient first decreases with gas velocity. Here the gas shear $2\rho u^{*2}/R$ is the dominant term in equation (28). However, as the gas velocity decreases, the particle volume fraction must increase to maintain the constant loading, so that the weight of the particle phase becomes increasingly important. Eventually, the particle weight dominates equation (28) and the total pressure gradient increases with decreasing gas velocity. Eventually the suspension can no longer be sustained and the 'choking' occurs. It is interesting to note that, for the dilute flows under consideration, the particle shear predicted by the analysis plays a relatively minor role in the total pressure gradient, despite its importance in the predictions of the particle mean velocity profiles.

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Measurements of Voidage Near the Wall of a Circulating Fluidized Bed Riser

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SUMMARY

Quantitative, time-dependent, non-intrusive capacitance measurements of voidage in the wall region of a circulating fluidized bed of cracking catalyst are reported. The data confirm the presence of a denser annulus of solids near the wall, which exhibits a vertical voidage profile similar to that observed for the average cross-sectional voidage. A statistical analysis shows that the average wall voidage in the dense lower region of the bed is nearly independent of the total solid flux, that it expands linearly with superficial gas velocity, and that its fluctuations are limited to low frequencies.

INTRODUCTION

The behavior of circulating fluidized beds (CFB) is a direct consequence of their local hydrodynamic structure. Therefore, quantitative, time-dependent measurements of voidage are essential to understand the details of the flow. In this context, several measurements of vertical and radial voidage profiles have been reported for CFB risers. Horio *et al.* [1], Hartge *et al.* [2, 3] and Tung *et al.* [4] have correlated the amount of light back-scattered onto an optical fiber system with local voidage. Hartge *et al.* [2] traversed a needle probe through a CFB riser and correlated the resulting capacitance with voidage. Brereton and Stromberg [5] also measured radial voidage profiles using a needle capacitance probe. Weinstein *et al.* [6] used an X-ray system to visualize the flow. Other measurements of local solid flux by Monceaux *et al.* [7], Rhodes *et al.* [8] and Bader *et al.* [9] were achieved using suction probes.

These studies have shown that the CFB riser includes a relatively dilute core near the

center surrounded by a denser annular region near the wall. In addition, the suspension is denser near the bottom of the column than at the top. The solids in the denser annulus recirculate downward unless the gas velocity is high enough for the suspension to lie in the pneumatic transport regime. In this context, Yang [10] proposed a hydrodynamic model where the lower region of the bed is held at the minimum fluidization voidage near the wall and operates under choking conditions in the core, while the more dilute top region is modeled using pneumatic transport correlations. In a somewhat different description, Bolton and Davidson [11] postulated that "the lower part of the riser is occupied by a violently bubbling or slugging fluidized bed", while the upper region exhibits a core-annulus structure. This description suggests that the core-annulus structure may be absent from the lower, dense bed region. Similarly, Rhodes and Geldart [12] distinguish a uniform fluidized region at the bottom of the CFB with an entrainment zone above, a description which may nevertheless be "inappropriate for higher gas velocities". As our observations of a CFB of small particles will show, the core-annulus structure is present throughout the entire height of the riser for all the experimental conditions summarized in the next section.

In the practical design of a CFB unit, the annular wall region plays a particularly important role, because it is where both erosion and heat transfer to the water wall take place. Because the magnitude of the local voidage has a substantial effect on these phenomena, we have carried out quantitative, time-dependent measurements of voidage near the wall of a circulating bed of fluid cracking catalyst (FCC) fluidized with ambient air. The present paper focuses on wall voidage

measurements at several elevations along the CFB riser. We begin by describing the CFB facility and the conditions of our experiments. A description of the capacitance probe follows. Then, the voidage measurements are reported and discussed.

EXPERIMENTAL

The circulating fluidized bed facility is sketched in Fig. 1. Its riser is 19.7 cm in diameter and 7 m high. It includes five tubular sections assembled with flanges. A 90° disengagement elbow connects the top of riser to a primary cyclone, followed by a smaller secondary cyclone. The collected particles fall in a cylindrical downcomer 14.6 cm in diameter. In an effort to minimize the effects of electrostatics, the riser and downcomer are made of aluminum, and the cyclones are made of steel. Plexiglas windows are located in the downcomer to monitor the circulation of solids visually. A gate valve controls the solid flow rate from the downcomer to the riser. The gases leaving the secondary cyclone are cleaned by a dust collector before being recycled to a tank. A positive displacement 'roots' blower driven by a 40-hp variable-speed dc motor recirculates these gases to the facility. The hot gases leaving the blower are cooled to the ambient temperature using a compact heat exchanger. A vortex flowmeter provides a direct measurement of the volume flow rate supplied to the

riser. The downcomer is fluidized using a smaller gas manifold monitored by a rotameter. The desired superficial gas velocities are achieved by setting the angular velocity of the blower and by recycling a fraction of the gas flow to the tank.

Pressure readings are taken every 30 cm using 25 taps mounted along the height of the riser. Another 12 taps are located in the downcomer and the solid return leg to complete the pressure profile along the entire circulation loop. Stainless steel screens of 200 mesh are soldered onto the tip of all pressure taps to avoid particle contamination of the manometer system. The pressure taps are read in sequence using a scanning valve connected to a single pressure transducer. The solid recirculation flux G_s is measured in the downcomer section using a sintered butterfly valve. The flux is determined by closing the valve and measuring the time rate of change of the pressure gradient created by the accumulation of particles fluidized above the valve.

In these experiments, a fluid cracking catalyst (FCC) of 61 μm Sauter mean diameter was fluidized using ambient air. The particle size distribution of this powder is approximately Gaussian, with a mass-weighted average diameter of 77 μm and a standard deviation of 28 μm . In a first series of tests, vertical pressure profiles were recorded in the riser. To this end, the readings from all pressure taps were time-averaged for five consecutive intervals of 2 s, separated by approximately 30 min. Figure 2 summarizes the conditions of these experiments with a plot of the total pressure drop across the riser *vs.* solid flux for different superficial velocities in the riser. As expected, the total pressure drop increases with solid flux at a fixed superficial gas velocity. The vertical pressure profiles were fitted in the least-

Fig. 1. The circulating fluidized bed facility.

Fig. 2. Summary of experimental conditions where vertical pressure profiles were recorded. Δ , $u_0 = 2$ m/s; $\circ$, 3 m/s; $\square$, 4 m/s; $\blacklozenge$, 5 m/s.

squares sense using an integral of the empirical fit proposed by Li and Kwauk [13]:

$$P(z) = \rho_s g z_0 \left\{ (\epsilon_a - 1) \frac{z_1 - z}{z_0} - (\epsilon_a - \epsilon^*) \right. \\ \left. \times \ln \left[1 + \exp \left(\frac{z_1 - z}{z_0} \right) \right] \right\} + \text{constant} \quad (1)$$

where the parameter z_1 is the location of the inflection point of the voidage profile, z_0 is a characteristic length, H is the riser height, z is the vertical co-ordinate (positive downwards) and ϵ_a and ϵ^* are two asymptotic voidages characteristic of the voidage profile. The vertical profiles of voidage were derived from the pressure fit using the one-dimensional model proposed by Louge and Chang [14]. The values of ϵ_a and ϵ^* were correlated with the dimensionless group $Kw = (18Re_s + 2.7Re_s^{1.687})/Ar$ that arises from the one-dimensional momentum analysis of Li *et al.* [15]. Ar is the Archimedes number and Re_s is the Reynolds number based on the average particle diameter and the average relative velocity at the relevant riser elevation. In the dimensionless group Kw , the numerical coefficients arise from a correction to Stokes' law of drag on a single particle.

The trend in the present data of ϵ_a is in reasonable agreement with the earlier study of Li *et al.*, although our values of ϵ_a appear to decrease above $Re_s \approx 50$ (Fig. 3). However, the sharp decrease of ϵ^* shown in Fig. 4 for increasing values of Kw contradicts the observations of Li *et al.*, who presented a correlation where ϵ^* increases with Kw . Clearly, further experiments are needed to resolve this issue, and the correlations of ϵ^* proposed by Li *et al.* should be treated with caution.

In other tests, we have measured the voidage near the wall of the CFB riser using

Fig. 3. Correlation of ϵ_a with $Kw = (18Re_s + 2.7Re_s^{1.687})/Ar$.

Fig. 4. Correlation of ϵ^* with $Kw = (18Re_s + 2.7Re_s^{1.687})/Ar$.

the capacitance technique of Acree Riley and Louge [16]. This technique is based on a guard circuit that, unlike previous designs, eliminates virtually all stray and cable capacitances and permits quantitative measurements of voidage with fluctuations as rapid as 2 kHz. We have carried out the measurements using a non-intrusive probe mounted flush with the wall of the CFB riser (see [16], p. 56). In order to eliminate the effects of stray capacitances from other conductors present in the vicinity, the riser section that holds the probe was driven at the probe's guard voltage [16]. In this configuration, the electric field lines are circular arcs confined to a narrow semi-toroidal measurement volume near the wall. For the probe used in this study, the measurement volume penetrates approximately 2.1 mm into the bed and has a maximum diameter of 11 mm.

The rectified output of the capacitance measuring system is a voltage V between 0 and 10 V that is related to the probe capacitance C by $V = Q_s/g_s C$, where $Q_s = 3.44 \times 10^{-12}$ C is a constant charge amplitude supplied by the amplifier and g_s is a gain that may be adjusted between 1 and 7. Local changes in the particle volume fraction modify the effective dielectric constant K_{eff} of the suspension and, in turn, the probe's capacitance according to $C = C_0 K_{eff}$, where C_0 is the probe's capacitance in the absence of particles. For powders of negligible conductivity, K_{eff} is primarily a function of geometrical parameters, because the frequency of the applied electric field is relatively high (16 kHz):

$$K_{eff}/K_h = f(\epsilon, \phi; K_h, K_s).$$

In this expression, K_h is the dielectric constant of the host fluid and K_s is that of the material of the particles. Among the geometrical parameters, the voidage ϵ and the

sphericity ϕ play an important role, while the particle size and its distribution have little influence on K_{eff} unless the particles have a diameter comparable to the dimensions of the capacitance probe [17]. Thus, to obtain a quantitative measurement of voidage without *in situ* calibration, the dependence of K_{eff} on ϵ must be established independently. In this context, Louge and Opie [17] recently developed a technique to measure the effective dielectric constant of powders suspended in petroleum jelly at any arbitrary voidage. For the FCC powder used in this study, they showed that a model proposed by Böttcher predicts K_{eff} accurately:

$$\frac{K_{eff} - K_h}{3K_{eff}} = \frac{(1 - \epsilon)(K_s - K_h)}{K_s + 2K_{eff}} \quad (2)$$

However, they noted that the value of K_s for humid FCC particles is considerably larger than that for dry FCC. They recommended that the value of K_s be inferred on a regular basis by immersing the capacitance probe in bulk FCC powder and by recording the corresponding value of K_{eff} . In this case, they showed that the Böttcher model can be used as an accurate interpolation function between the dielectric constant for pure fluid ($\epsilon = 1$, $K_{eff} = K_{gas} = 1$) and the effective dielectric constant of the bulk powder ($\epsilon = \epsilon_{bulk}$, $K_{eff} = K_{bulk}$).

In this work, three capacitance probes were mounted at any of four elevations along the riser: $h = 1.1, 1.4, 2.9$, and 4.4 m above the distributor. In order to maximize the sensitivity of the system, the probes were uncovered before each test and the gain g_s was set to produce a voltage output $V_0 = 10$ V in air. After each test, K_s was calculated by immersing the lowest capacitance probe in the bulk FCC powder of voidage $\epsilon_{bulk} \approx 39\%$. For the experiments of Fig. 2, the values of K_s ranged between 15.4 and 16.5. During the experiments, K_{eff} was calculated from the ratio of V_0 and the output voltage V :

$$K_{eff} = V_0/V = \left(\frac{Q_s}{g_s C_0} \right) / \left(\frac{Q_s}{g_s C} \right) = C/C_0.$$

Finally, the Böttcher model was used to infer the voidage from K_{eff} and K_s . Because of the excellent sensitivity of this system, we estimate that the absolute voidage uncertainty

does not exceed $\Delta\epsilon \leq \pm 2\%$ for $\epsilon = 40\%$, $\Delta\epsilon \leq \pm 1.0\%$ for $\epsilon = 60\%$, or $\Delta\epsilon \leq \pm 0.3\%$ for $\epsilon = 80\%$.

RESULTS

In this work, the capacitance probe was placed at various heights along the riser. When the probe is immersed in the dense region of the bed, the wall voidage is relatively low, but occasional spikes of higher voidage are recorded by the probe. In contrast, the voidage is generally high in the upper region of the riser, except for the brief passage of denser aggregates. In the transition region, the voidage fluctuates between the extreme values of the dense and the dilute regions.

Figure 5 compares the time-average wall probe voidages with the vertical voidage profile inferred from the average pressure gradient for typical CFB conditions. In all cases, the average voidage across the riser is greater than the voidage at the wall. This result confirms the presence of a denser annular region near the wall, which becomes progressively more dilute at increasing elevations.

The voidage probability density function (PDF) also behaves differently at different elevations. The PDFs shown in Figs. 6-8 were evaluated by assigning voidage samples to a number of adjacent, equally-spaced 'bins'. In order to keep the normalized bias error below 1%, the bin spacing was kept below one-fifth of σ , the sample standard deviation of the voidage data [18]. Here the PDF is normalized according to

Fig. 5. Vertical voidage profiles in the riser for the conditions: $u_0 = 2$ m/s and $G_s = 17$ kg/(m²·s). □, Average voidages inferred from the vertical pressure gradients; —, a fit using the exponential expression of Li and Kwauk [14]; ■, time-averaged voidages measured by the capacitance probe for a total sampling time of 120 s.

Fig. 6. Voidage PDF at different riser elevations for the conditions of Fig. 5. Each PDF is constructed with 4 096 samples at a sampling rate of 100 Hz. Relative elevations h_1/H are shown. The inflection point of the vertical voidage profile is located at $h_1/H = 0.17$ above the distributor. At $h_1/H = 0.17$, the average voidage is $\bar{\epsilon} = 0.52$, its standard deviation is $\sigma = 0.07$ and its skewness is $S = 2.4$; at $h_1/H = 0.20$, $\bar{\epsilon} = 0.61$, $\sigma = 0.11$, $S = 0.60$; at $h_1/H = 0.65$, $\bar{\epsilon} = 0.84$, $\sigma = 0.08$, $S = -0.73$.

Fig. 7. Voidage PDFs in the dilute region for the conditions of Fig. 5 and 6. Relative elevations are shown. At $h_1/H = 0.44$, $\bar{\epsilon} = 0.81$, $\sigma = 0.09$ and $S = -0.63$.

$$\int_0^1 \text{PDF}(\epsilon) d\epsilon = 1.$$

The typical voidage PDFs plotted in Fig. 6 clearly shift towards higher voidages at increasing elevations. Further, the PDFs are rather narrow in the dense region of the bed, then widen in the transition region, and finally become narrow again in the dilute, upper region of the riser. This evolution of the width of the PDF is also reflected in the sample standard deviation σ .

A dimensionless measure of the asymmetry of the PDF is provided by the skewness:

$$S = (1/N) \sum_{i=1}^N [(\epsilon_i - \bar{\epsilon})/\sigma]^3$$

where N is the total number of voidage samples. For typical conditions, S is positive at the bottom of the riser, where the tail of

Fig. 8. Comparison of the voidage PDFs at the elevation $h_1/H = 0.17$. (a), A 'turbulent bed' with $u_0 = 0.7$ m/s, $G_s = 0$ kg/(m²·s) and a dense CFB region with inflection point h_1/H above 0.38, $u_0 = 2$ m/s, $G_s = 35$ kg/(m²·s); (b), the 2 m/s condition in (a) and a dense CFB region with h_1/H above 0.38, $u_0 = 5$ m/s, $G_s = 125$ kg/(m²·s).

the PDF points to higher voidages. In contrast, it is negative at the top, where the tail points to lower voidages. This observation suggests that the sign of the skewness can serve to identify whether the probe is immersed in a region above or below the inflection point of the vertical voidage profile. Other means of identifying the bed region were also proposed by Drahoš *et al.* [19]. The values of skewness, however, must be treated with caution. Because it is a third-order moment, skewness is a less robust indicator than the average or the standard deviation. In this context, Press *et al.* [20], p. 457, suggest that reliable absolute values of skewness should greatly exceed $(6/N)^{1/2}$, the standard deviation of the skewness of a normally distributed random variable. In our experiments, $N = 4 096$ samples, so that the criterion $|S| > (6/N)^{1/2}$ is met in both the upper and lower regions of the riser.

Our results agree well with the data of Hartge *et al.* [3], who measured similar voidage PDFs near the wall at various vertical locations in the riser using an optical fiber probe. Our voidage PDFs at the wall do not exhibit the bimodal distributions that are normally present farther from the wall, in

regions alternately swept by dense aggregates falling near the wall and by a more dilute suspension from the core. This absence of a bimodal distribution apparently contradicts the recent observations of Li [21], who has suggested that a bimodal distribution may also be obtained at the wall if an optical probe of small enough measurement volume is used. Because the penetration of our measurement volume (2.1 mm) is much smaller than its height (11 mm), Li's observations suggest that the length scale of the vertical voidage fluctuations near the wall may be smaller than 11 mm under our experimental conditions. In an attempt to resolve this issue, we will re-examine the voidage PDF near the wall using a smaller capacitance probe of similar geometry.

Figure 7 compares the voidage PDFs at two locations in the dilute region of the riser, where the cross-sectional average voidage is 97%. The similarity between the two curves suggests that the wall voidage approaches a fully developed distribution. This observation agrees with the data of Monceaux *et al.* [7], who reported fully developed mass flux profiles in dilute flow when the average cross-sectional voidage exceeds approximately 95%.

In the dense lower region of the bed, the wall voidage PDF appears independent of the average solid flux G_s : identical PDFs were obtained for three different values of the flux but identical values of the superficial gas velocity ($u_0 = 2$ m/s, $G_s = 17, 27$, and 40 kg/(m²·s)). A similar observation was made for another gas velocity that left the probe clearly immersed in the dense region of the bed while the flux was varied ($u_0 = 3$ m/s, $G_s = 60, 69$, and 80 kg/(m²·s)). In contrast, the voidage PDF in the dense region is a strong function of superficial gas velocity. Figure 8 shows PDFs for a 'turbulent' bed and for the dense region of a CFB with two distinct values of u_0 . Note the leading edges of the PDFs, which represent a measure of the voidage in the emulsion phase. In Fig. 8(a), the emulsion phase of the CFB ($u_0 = 2$ m/s, $G_s = 35$ kg/(m²·s)) appears to be significantly more expanded than that of the 'turbulent' bed ($u_0 = 0.7$ m/s, $G_s = 0$). However, the probability of observing a voidage above 0.5 is nearly identical for the two cases. In Fig. 8(b), the emulsion phase voidage in the dense region of the CFB appears unchanged despite

Fig. 9. Time-averaged wall voidage in the dense lower region of the CFB. In each experiment, the inflection point of the voidage profile was located well above the capacitance probe at $h/H = 0.17$.

a substantial increase in u_0 , while the tail of the PDF extends towards higher voidages at higher u_0 . On average, the voidage expansion near the wall is primarily a function of the superficial gas velocity in the dense lower region of the CFB. As Fig. 9 indicates, the time-averaged wall voidage increases linearly with u_0 from the 'turbulent' bed through the CFB regime. These observations suggest a review of the model of Yang [10], who assumed that the voidage in the wall region is constant and nearly equal to the voidage at minimum fluidization.

The power spectral density (PSD) of the voidage fluctuations in the dense, transition, and dilute regions of the bed was also examined. The PSD is calculated and normalized using the maximum entropy algorithm of Press *et al.* [20], pp. 430 - 435, with 50 poles. The results show larger peak values of the PSD in the transition region where the voidage fluctuations are largest. In addition, the frequency of the largest fluctuations appears independent of the riser elevation, and no significant fluctuations are observed above 20 Hz. This limitation of the signal band-width agrees well with the frequency content of typical pressure fluctuations [19]. However, it is unclear whether this upper frequency bound is affected by the vertical size of our measurement volume. Finally, Fig. 10 indicates that the shape of the PSD and the position of its peak frequency are rather insensitive to the superficial gas velocity. These observations suggest that, near the wall, the voidage fluctuations are not primarily governed by the gas turbulence, but perhaps by local instabilities of the granular motion in this denser annular region.

Fig. 10. PSD of voidage fluctuations in the dense region of the bed ($h/H = 0.17$) for the conditions $u_0 = 0.7$ m/s, $G_s = 0$ kg/(m²·s); $u_0 = 2$ m/s, $G_s = 35$ kg/(m²·s); $u_0 = 5$ m/s, $G_s = 125$ kg/(m²·s). The sampling rate is 50 Hz with 256 points per trace. To obtain these curves, the PSDs of 64 traces were averaged.

CONCLUSIONS

The present paper has reported measurements of voidage in the wall region of a CFB riser using a non-intrusive, quantitative capacitance probe. In these experiments, a cracking catalyst was fluidized with air. A comparison of these measurements and the average cross-sectional voidage has confirmed the presence of a denser annulus of solids near the wall. The voidage in this annular region exhibits a vertical profile similar to that observed for the average cross-sectional voidage. A statistical analysis indicates that the skewness of the voidage PDF can be used to identify the relative elevation of the probe and the inflection of the vertical voidage profile. In the relatively dense region located below the inflection point, the voidage PDF appears independent of the overall solid flux. However, the average wall voidage in this region grows linearly with the superficial gas velocity. Finally, for the conditions reported in this study, the frequencies of the voidage fluctuations are limited to approximately 20 Hz, and their distribution appears to be nearly independent of superficial gas velocity.

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LIST OF SYMBOLS

C	probe capacitance
d_p	particle diameter
g_s	amplifier gain
G_s	solid mass flux, kg/(m ² ·s)
h	vertical co-ordinate with origin at bottom of bed
H	riser height
K	dielectric constant
P	gas pressure
Q_s	system constant of amplifier, C
Re_p	average particle Reynolds number = $[u_0/\epsilon - G_s/\rho_s(1 - \epsilon)]d_p/\nu$
S	skewness
u_0	superficial gas velocity, m/s
V	amplifier output voltage, V
z	vertical co-ordinate with origin at top of bed
z_i	location of inflection point of voidage profile
z_0	characteristic length in eqn. (1)
<i>Greek symbols</i>	
ϵ	voidage
ϵ_a	asymptotic voidage at bottom of riser (eqn. (1))
ϵ^*	asymptotic voidage at top of riser (eqn. (1))
ν	kinematic viscosity of gas
ρ_s	solid material density
σ	sample standard deviation

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