**Collaborative Research on Fluidization
Employing Computer-Aided Particle Tracking**

Final Report

Michael M. Chen

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Morgantown Energy Technology Center
P.O. Box 880
Morgantown, West Virginia 26507-0880**

**By
University of Illinois at Urbana-Champaign
Department of Mechanical and Industrial Engineering
1206 W. Green Street
Urbana, Illinois 61801**

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1. Introduction

Considerable excitement was generated when computer-aided particle tracking was introduced to fluidized bed measurements about ten years ago. The technique made possible intrusive measurements of detailed mean density and velocity distributions of the particles inside the fluidized bed, revealing circulation patterns which hitherto had only been inferred indirectly. Although much progress has been made in the technique, a comprehensive discussion of its basic methodology, instrumentation, capabilities and limitations does not exist. It is the purpose of this report to provide such a comprehensive discussion.

This report also serves as part I of the final report of Cooperative Agreement No. DE-FC21-88MC25048. Two major tasks of the final year of the contract were: 1. To explore methods of enhancing the spatial resolution of the particle tracking technique. 2. To write a comprehensive discussion of the technique, including the latest discoveries. Radioactive particle tracking is a technique for the study of the dynamics of suspension flows, including, in particular, fluidized bed. It makes use of one or more radioactive tracer particles which are dynamically identical to other particles in the suspension. The radiation emitted by the tracer particle(s) is monitored by a number of scintillation detectors located around the bed, permitting the determination of the particle position as a function of time. Time differentiation and other computations permit the determination of mean particle density and velocity distributions as well as distributions of other statistical quantities of importance to the dynamic description of the system, including Lagrangean integral time scale, cross correlations, and particles' kinetic contributions to the stress tensor. When a swarm of tracers are used, effective mixing rates can also be studied.

2. History

Kondukov [1964] was probably the first investigator to use a particle to study the motion of particles in fluidized beds. His short paper showed only a few dozen consecutive positions without any indication whether any useful data were obtained. Shirazi, Chao and Jones [1968] and Jones, Chao and Shirazi [1968] used a radioactive particle as a flow tracer in turbulent pipe flow of water. Three pairs of opposing detectors were used in analog circuits to obtain three coordinate measurements. The system made use of the current mode rather than the photon-counting mode of radiation detection, and relied on linear signal processing technology. These render the system less suitable to suspension flows, as will be explained below.

The method described here in is an adaptation of the above method, with three major improvements to render the technique more accurate and more suitable for suspension flow studies. The first improvement is to use energy-discriminated photon counting rather than photomultiplier current for gamma ray detection. This makes the measurements less subject to errors due to secondary gamma rays (i.e. gamma rays emitted by surrounding material due to gamma ray material interaction) from the surrounding matter of unknown density and geometry. The second improvement is the use of digital data acquisition and processing. This renders it possible to make use of the more sensitive, non-linear region of the gamma ray intensity/distance relationship, thus gaining in accuracy and spatial resolution. The third major improvement is the use of a large number of detectors. The resulting redundancy can be employed in the data reduction scheme to improve accuracy. A more recent development, not included in the above three but also a consequence of the photon counting - digital processing methodology is theoretical calibration, with automatic corrections of attenuations due to fixed structure, further improves the ease and accuracy of the method.

It should be mentioned that because of the intrinsic randomness of gamma rays and other physical limitation, spatial resolution of the radioactive particle tracking technique is limited. The improvements described above insures that the realized spatial resolution is as close to the theoretical maximum as possible. Of course, the value of radioactive particle tracking is that the data it is capable of obtaining cannot be obtained by any other method. Hence the modest spatial resolution, while regrettable, does not diminish its potential for making important contributions in the study of suspension flow dynamics.

3. Basic Methodology and Hardware Considerations

The hardware in the experiment is composed of radioactive tracer particle, the detectors for gamma ray detection, the amplifier for electrical signal amplification, the discriminator to eliminate background noise, the counter for signal counting and computer interface for data acquisition. Each of the components will be discussed in details for the following two systems. One is a large system developed earlier and the other is a improved system, designed for portability.

3.1 The Radioactive Tracer Particle

The radioactive tracer particle should be made of radioactive materials either inserted into a plastic spheres or coated with a layer of polyurethane to ensure that the specific gravity and size of the tracer match that of the particles in the bed. This, in ture, ensures that the dynamic behavior of the tracer resembles that of the particles in the bed.

In the CAPTF at University of Illinois at Urbana-Champaign, the tracer was made from scandium with specific gravity of 2.89, which is close to that of the glass microspheres used in the study (SG 2.45 to 2.49). It is coated with a layer of polyurethane which has a lower density, so that the finished particle to match the size and mass of the glass spheres used in this study. The coating also serves to prevent the erosive loss of radioactive material in the fluidized environment.

The coated tracer was irradiated in the nuclear reactor to obtain the scandium-46 isotope which has a half life of 84 days. In each disintegration of Sc-46, it emits two gamma photons at 0.889 MeV and 1.112 MeV. The use of the higher energy gamma rays is necessary to minimize attenuation and the dependence of the attenuation on the species of the medium. The tracer particles used in this study had diameters of 600 μm , 800 μm and 2.0 mm and contained 0.345 mg, 0.940 mg and 13.7 mg of scandium, respectively. They were irradiated each time to activities ranging from 400 to 600 μCi .

3.2 The Scintillation Detector Array

A. Original System

Sixteen Bicron (Newbury, OH) Model 2M2/2 scintillation detectors, comprised of 2"D x 2" (51 mm x 51 mm) NaI crystals with integral SRC B50B01 10 stage photomultiplier tubes (PMT) were strategically distributed around the perimeter of the bed as shown in Fig. 3.1 and Fig. 3.2.

A NaI crystal emits fluorescent radiation pulses in the ultraviolet or visible spectral regions when it is bombarded by gamma radiation. The photoelectrons generated from the cathode of the PMT are amplified by a series of dynodes and are finally collected at the anode of the PMT where they

Figure 3.1 Arrangement of the Scintillation Detectors

Figure 3.2 Arrangement of the 16 Detectors around a Cylindrical Bed

appear as electrical current pulses. Each successive electrode is at a higher potential than the previous one in order for the electrons to be attracted to the next dynode. The required potential difference of 1200 volts between the cathode and the anode was maintained by a Tennelec (Nashville, TN) Model TC952 high voltage power supply. The circuitry required to divide the potential difference among the dynodes and perform the necessary pulse shaping was constructed according to a design suggested by Bicron which has been replicated in Fig. 3.3

The rise time of the current pulses generated at the anode of the PMT is around 80 nano-seconds. The decay time of the pulses for NaI crystal is 230 nano-seconds. The amplification process inside the PMT tube takes about 200 nano-seconds. Therefore, the total time required for processing of each pulse is approximately 0.5 micro-seconds. Thus, the minimum time constant for differentiation of a current pulse at the anode of PMT is $R_4C_4 = 0.5 \mu s$ (Fig. 3.3), corresponding to absolute maximum count rate of 2 MHz. This count rate cannot be attained in practice for reasons to be explained later. A sample output signal of one of the detectors is shown in Fig. 3.4 in which the magnitude of each voltage pulse is directly proportional to the energy of the corresponding incident gamma ray.

B. Improved System

The circuits in the improved system is redesigned allowing an order of magnitude improvement of the maximum count rate.

In the improved system, the same number of detectors and power supplies are used. But the voltage supplied to the detectors was 1500V. With this high voltage, all the 16 detectors may be adjusted to generate output signals at 40 mV from the reading of the Tektronix 2235 oscilloscope (Tektronix Inc., Beaverton, Oregon). However, since the input resistance of the oscilloscope is the same as the detector output resistance ($1M\Omega$), the true signal height can not be determined. This can be demonstrated by measuring the count rate at various threshold settings as shown in Fig. 3.5. It is seen that the true signals should be at about 90 mV. Between 70 mV and 80 mV there is a sharp decrease of the count rates. This is likely due to the signals generated by the 0.889 MeV photons being cut off at that threshold range. From Fig. 3.5 we may set the threshold at 60 mV.

The detector count rate is approximately proportional to the inverse square of the distance between the detector and the point radioactive source. The relative error of the rate fluctuation from the mean due to the statistical nature of radiation is smaller when the absolute count rate is large. Therefore, the detectors should always be arranged as close to the bed as possible. But when the detector and the source is too close, the detector output pulses may overlap or the PMT current may saturate. Both will lead to a decrease of the apparent count rate. Figure 3.6 shows a typical behavior between the count rate and the distance, here the distance is

Figure 3.3 Voltage Divider String

Figure 3.4 Sample Detector Output Signal on an Oscilloscope for Sc-46

Figure 3.5 Change of Count Rate with Threshold Setting

Figure 3.6 Change of Count Rate with the Distance between Detector Center and Point Radioactive Source

measured from the detector crystal center (the center is on the detector axis and at 1 inch from the detector front surface) and the radioactive source. It is seen that the saturation starts when the distance is about 135 mm (the intensity of radioactive material is 0.5 Ci), and then the count rate decreases rapidly. This suggests that the minimum distance should be 135 mm between the detector centers and the inside wall of the bed column (or 110 mm (4.3 in) between the front surfaces of the detectors and the inside wall of the bed) in order that the detectors function properly.

3.3 Signal Processing Instruments

In the early development of the signal processing instruments [Lin 1981, 1985 and Lin, Chen & Chao 1985] The count rate was obtained by first converting the variable frequency digital signals (emitted gamma rays) into analog signals via a Frequency-to-Voltage (F/V) Converter and then re-digitized the analog signals via an Analog-to-Digital (A/D) Converter. The revised circuit made use of digital counters. In both cases, a digital signal was obtained, and sent to a digital computer for processing. The advantages are:

- (1) The present method makes use of the digital data reduction and thus can operate in nonlinear ranges of the gamma-ray intensity and distance calibration curve. The use of analog measurement often necessitates operation in relatively linear ranges of the calibration, which exist only when the distances between the tracer and detector are large. In fact the closer the tracer is to the detector, the more accurate is the position determination. Thus it is desirable to operate experiments in the nonlinear range of calibration.
- (2) The secondary Gamma rays must be filtered out. The effect of secondary emission and the hardware scheme to eliminate it will be described in detail later.
- (3) Practical limit of activity of a particle of 0.5-2 mm with a detector of reasonable size is 10^5 counts/sec. Coincidentally this is also comparable to the limit imposed by event overlap due to the scintillator time constant. Thus the physical nature of gamma ray emission sets an upper limit in the count rate, and one should be prudent in the use of all available gamma photons. Surrounding the bed with a large number of detectors is an obvious way to achieve this goal. It has also the advantage of having a nearby detector wherever the tracer is, providing a more accurate distance measurement.

Of the two schemes, the scheme using frequency-to-voltage conversion followed by A-to-D conversion is less desirable [Liljegren 1983; Moslemian 1987]. :

- (1) The A/D converter was a serial device: that is, it processed one signal after the other creating a time delay in processing the signals from the 16 detectors.

- (2) The F/V converter had a limited range (100 kHz) and drifted with time and temperature and its linearity was only 5 percent of full scale.
- (3) The sampling rate of the detection process was hardware dependent through the integration circuitry.
- (4) Due to the length of the transmission lines, any noise on these lines was mixed and digitized along with the relevant data.

Consequently, an alternative scheme was adopted to eliminate the shortcomings of the technique. A digital pulse counting scheme was suggested to replace the F/V converter and the A/D converter. It has the following unique features:

- (1) The present method utilizes digital pulse counters to process the detector signals. The Transistor-Transistor Logic (TTL) devices employed in these counters are insensitive to time and temperature, require no calibration, are capable of handling noisy transmission lines, permits simultaneous measurement of the output of all the detectors (i.e. no time delay), and can be sampled and reset at desired rates under software control, thus eliminating the need for any hardware modification required as in the analog system.
- (2) The use of the NaI crystals in inorganic scintillation detectors sets an upper limit on the counting ability of the detectors due to the long decay time of the scintillation pulse. Thus, one should be prudent in the use of all the available gamma rays since due to the statistical nature of the rays, higher counting capability increases the signal-to-noise ratio of each detector which, in turn, improves the overall accuracy of the tracking system. Therefore, high speed timing/filter amplifier and leading edge discriminator modules are employed to achieve this goal. The nuclear physics instruments are built specifically for processing and conditioning of high frequency signals.

The electronic instrumentation for signal processing along with the supporting data acquisition system is schematically shown in Fig. 3.7. The design and purpose of the various components are described as below.

3.3.1 Amplifier

The pulse trains produced directly from scintillation detectors had very low magnitudes. Consequently they had to be properly amplified to retain their characteristics such as their rise and decay times. In segregating successive pulses, baseline stabilization must be accomplished for best amplifier performance. It normalizes the unpolar output pulses to a DC reference voltage (usually 0 Volt DC) to assure that successive pulses are precisely quantized in subsequent instruments. This process termed "baseline restoration" is necessary in high count rate applications.

Figure 3.7 Schematic Diagram of the Signal Processing Instruments along with the Data Acquisition System

A. Original System

The Canberra (Meriden, CT) Model 2110 Timing Filter Amplifiers were chosen for the above tasks in the original system in UIUC. These units are 50 MHz wideband DC coupled amplifiers which can accept positive or negative pulses and have adjustable gain (x1.5 to x100) with DC level setting. They deliver outputs of upto ± 5 volts into 50 ohm loads. The required power was provided by EG&G Ortec (Nashville, TN) Model 4001/4002 NIM/BIN power supplies.

B. Improved System

In the improved system, the amplifier is eliminated to reduce the weight and bulkiness of the system. The electric signal from the NaI crystal detectors are amplified by intrinsic PMTs and fed directly into discriminator.

3.3.2 Discriminators

The amplifier signals had a noisy background generated either by gamma rays which had only partially deposited their energy within the NaI crystal or previously interacted with the glass particles as well as pulses due to the spontaneous emission of the electrons from the cathode of the PMT. Since most of the secondary emissions consisted of gamma rays of fairly low energy, their contribution were effectively removed by employing leading edge discriminators.

A. Original System

The EG&G Ortec Model 584 discriminators were employed in UIUC to perform the discrimination against the noise. These units can process pulses of up to 100 MHz with amplitudes ranging from 0 to -5 Volts. The input impedances are 50 ohms. The discriminator threshold settings range from -5 MV to 1 V. The trigger threshold setting for each of the 16 channels corresponds to the base of the Compton edge in the spectrogram of the SC-46. It was determined from pulse-height analysis for each detector using a Canberra series 30 multichannel analyzer, Fig. 3.8 shows a typical spectrogram for Sc-46.

Pulses of greater magnitude than the threshold voltages were presumed to have resulted from gamma rays which had passed directly from the tracer to the detectors where they deposited their energy on the first interaction with the crystals. Subsequently, they were converted into logic pulses within the discriminator units, creating logic pulse trains at the output of each channel.

B. Improved System

In the improved system, the discriminator chosen is the LeCroy Model 4413 16-channel CAMAC discriminator. Signals enter directly from the detectors input to this device via a front panel Lemo connector. Using smaller size RG-174 cable the coaxial outputs of the detectors can be directly connected to the Lemo inputs of the discriminator. In the portable

Energy

Figure 3.8 Typical Spectrogram for Sc-46

system, the modified pulse enter directly into the discriminator. It houses 16 channels of discriminators in a single component. Each channel shares a common threshold which is adjustable between -15 mV and -100 mV. When the input signal reaches the threshold value the output logic pulse begins. As with the current system then discriminator's sole purpose is to convert the detector output signals to square waves readable by the counters. The main difference comes from the type of logic used. The current system uses TTL logic which employs positive voltages from 0 to 5 V, while the new version uses ECL signals which vary between -0.8 V and -1.7 V. While TTL has many advantages, very few of them apply in the new system. The counters currently being used were made using TTL chips and they therefore require TTL level signals. The advantages of ECL are many. First, the old discriminators have an output signal of 500 nsec which limits the count rate to 2 MHz by itself. The new discriminator can operate in several modes, one of which sets a backward threshold. When the input signal crosses the backward threshold the discriminator ends its output pulse. This allows for discriminator pulses to be the same width as the signal input, regardless of its duration. Another mode allow the operator to set the output pulse width anywhere between 3.5 nsec and 100 nsec. This allows a maximum count rate of 10-150 MHz, much greater than out 5 MHz maximum. Another advantage of the ECL level signal is the pulse height. Since the difference between a high and a low is less than a volt, rise time is much faster than TTL. This results in a cleaner output and better accuracy in the counting stage.

Signal output from this device is made from one of the two 34-pin connector through flat ribbon cable to the LeCroy CAMAC model 4434 32-channel, 24 bit scaler, which is served as a counter and buffer.

This is a latching scaler with an internal buffer for each channel. This allows the scaler to count while the previous periods data are being processed. This component is also quite fast, having a maximum rate of >20 MHz. The 4434 has an external bus connection which we will use.

3.3.3 Digital Pulse Counters

The pulse trains resulting from the interaction of gamma rays with NaI crystals had to be counted at this stage of signal processing.

A. Original System

Fig. 3.9 shows a schematic drawing of the digital counters for two of the 16 channels and the selector logic which were utilized to process the pulse trains in the original system in UIUC. Groups of four of SN74193 4-bit binary counters were cascaded to form 16-bit binary counters by feeding the CARRY outputs to the CLOCK inputs. The largest possible number of

Figure 3.9 Schematic Drawing of the Selector Logic and Two Channels of the 16-bit Counter Circuitry

counts for each of the counters was $2^{16}-1$ or 65,535. The limiting counting frequency of the counters was 32 MHz being much higher than the maximum possible frequency of the scintillation pulses (2 MHz), corresponding to the decay time constant of the scintillation events.

The counters were sampled through appropriate machine language software, implemented on a micro-computer, with utilization of programmable real-time clock (KW11-K0 and a general purpose interface module (DR11-C). The details of the operation of the counters and the selector logic are presented in Appendix A. Overflow of the real-time clock caused the interface module to send a signal (DATA READY) to the selector logic of the counters to indicate that it was time to sample the counters. Subsequently the output of all the counters was stored in the corresponding buffers. Then the output of the first counter was transmitted to the input registers of the PDP11/34A unibus for further analysis. Afterwards, a signal (DATA TRANSMITTED) was sent to the selector logic of the counters to receive the output of the next counter. This process was continued until the output of all the counters read into the computer for each sampling period.

B. Improved System

In the improved system, LeCroy Model 4434 32-channel, 24 bit scaler. This unit not only contains 32 channels for counting, but also the data bus. It is here that the computer attached to the system. Description of this segment is fairly simple. When the computer is ready to take a reading of the count for each of the channels it sends out a signal to the scaler's bus causing the counters to stop counting, dump their registers into a buffer to be read, reset, and begin counting again.

The buffers are a very important part of the counting segment. When it is time, the counters transmit their data (number of counts) into the buffers. The counters can therefore count while the computer is reading data from the buffers instead of waiting for the data to be read directly before counting can begin again. This means that counting can be virtually uninterrupted for the entire length of the experiment.

The counters themselves have a maximum count rate of 20 MHz, 30 MHz instantaneous which is still far above our current maximum count rate. The 24 bit buffer allows over 16 million pulses to be counted before rollover occurs. This is so far above what we need at present time that the system only employ 16 of the 24 bit which corresponds to over 65,000 counts, still well above our current load.

3.3.4 Computer Interface

The phenomena of gamma radiation emission is a random process causing the resulting pulse trains from the discriminators to have variable frequency of occurrence. This

causes instabilities at the outputs of the counters, over long transmission lines, at very high count rates if proper precautions are not taken. Simple termination resistors which are ordinarily used to eliminate standing waves (ringing) and to drive logic signals on the data and signal control lines are not adequate to resolve the problem. Pulse shaper and line drivers can be used to eliminate the problem.

A. Original System

Fig. 3.10 shows in detail the necessary circuitry to transmit logic signals from the counters to the interface module of the minicomputer and vice versa. The outputs of the counters were passed through SN7046 buffer gates to increase their stability. Then, they were run through a ten foot ribbon cable and were terminated by 916C181X2TR pull up/down resistors. Afterwards, the logic pulses were shaped using SN74LS240 octal line drivers which incorporated Schmitt Triggers. The terminators were put very close to the interface module to minimize creation of standing waves. These precautions resulted in very sharp square logic pulses throughout the transmission lines. Examination of the driving signals of the interface module (DATA READY and DATA TRANSMITTED pulses) revealed their poor quality but the quality was improved by taking similar steps in the opposite direction as shown in Fig. 3.10. The circuitry involved in improving the quality of the signals is referred to as TTL pulse shaper.

B. Improved System

In the improved system, the initial attempt to connect LeCroy 4434 scaler to a PC computer with a Burr-Brown data acquisition board was difficult. It becomes more apparent when we conclude that the LeCroy auxiliary bus is incompatible with the Burr-Brown interface card due to LeCroy's use of several pins on the auxiliary bus as both data and address bits. After many attempts to circumvent the problem, it was decided to use a LeCroy 8901A CAMAC crate controller in conjunction with a National Instruments IEEE-488 standard general purpose interface board (GPIB). This change avoids compatibility problems since the standardized GPIB interface is used for both electronics. Instead of connecting individual pins, one cable connects the entire interface.

The GPIB interface provides us with much in the way of system control. The databus is very easily accessed using simple commands. Using commands provided with the software (see next section), we can easily control the LeCroy 4434 scaler, loading buffers, resetting counters, reading data, etc. The LeCroy 4413 discriminator can also be accessed through the GPIB, allowing simplified threshold setting and other necessary functions that would otherwise have to be done manually.

Figure 3.10 TTL Pulse Shaper Circuitry

The crate controller has several front panel LEDs that allow the user to see whether commands are correctly executed, and to a small degree the results of those commands. Connection between the controller and the GPIB card within the computer consists of a Amphenol brand 24 pin GPIB cable.

4. Computation of Flow Variables

The software used in particle tracking facility is composed of: calibration raw data acquisition, calibration table generation, actual data acquisition, data conversion to position data, instantaneous velocity calculation, velocity distribution, fluctuating particle kinetic energy, particulate density calculation, solid translational stress calculation and Reynolds stress calculation. These will be discussed in detail in the following. The Accuracy and resolution problem will also be discussed.

For the improved particle tracking system, four programs has been written: RAWCAL for calibration data acquisition, PD for establishing calibration table, MAIN for actual data acquisition and PROCESS for position calculation. Other programs written in Fortran can easily ported to PC from DEC PDP-11 computer.

4.1 Data Acquisition

Data acquisition and storaction are controlled by an on-line computer with proper card for computer interface and proper software to control the data flow. To have a fast running speed for the software, it is usually written in C language.

(A) Original System

In the original system in UIUC, data acquisition, reduction, and storage were controlled by a dedicated PDP11/34A (Digital Equipment Corporation, Maynard, MA) micro-computer with 256 kbytes of MOS main memory and two RL01 removable disk drives providing 5.2 Mbytes of storage each. Since the tracer location was solved for in real time, only the coordinates of the instantaneous positions needed to be stored. The storage capacity of the RL01 disk allowed continuous tracking for up to five hours at a sampling rate of 33 Hz. According to sampling theory, in order to accurately reproduce a signal of known frequency, it must be sampled at a rate equal to at least twice the known frequency. Measurements of the power spectra of bubbles in fluidized beds by a number of investigators such as Ho [1982] revealed a peak between 2 and 3 Hz and a decline to 1 percent of the peak value at about 12 Hz. Therefore, the sampling rate should be sufficient to observe relevant phenomena in gas fluidized beds. The five hours of sampling period assured significant sampling of the entire bed volume.

The software consisted of one main program (BMAINP) and three subroutines (BCOUNT, PROCESS, INVMAT). The main program, written in FORTRAN, read the test parameters from a file containing a description of the test and loaded the calibration data into a portion of memory shared with PROCESS. The subroutine BCOUNT controlled the data acquisition. This assembly language routine initialized the

real time clock of the computer and the general purpose interface module which linked the counters to the computer. The test began when the clock started and the sampling proceeded in an interrupt-driven fashion. That is, overflow of the clock preset buffer generated a high priority interrupt to the processor indicating that it was time to sample the 16 counters. Subsequently, the interface module transferred the outputs of the counters to the input register of the PDP11/34A until all the counters were read. The data were stored in two buffers. Once one buffer was filled, subroutine PROCESS was initiated to compute the positions of the tracer (using the mathematical formulation given in APPENDIX A) while the other buffer was being filled. When enough positions were determined to fill the output buffer, created by PROCESS, the routine initiated transfer of position data to RL01 disk. This scheme of filling one buffer while processing the data in the other and asynchronously writing the results to disk continued until the test was completed.

(B) Improved System

In the improved system, program MAIN.EXE takes count data during an experiment in the improved system and stores them in the computer. The basic process of this data acquisition program (MAIN.EXE) is the same as that of the calibration program (RAWCAL.EXE) to be discussed in the next sub-section. It is also written in C and uses the same subroutines and interrupt handler as those in program RAWCAL.EXE. Here, initialization is done and the interrupt is enabled. The counts are read into the computer and the interrupt routine ends. Two buffers of 1 kilobyte each are used to temporarily store the counts. When one buffer is full, its data are written on the hard disk of the computer while the other buffer is being filled. This process takes several hours in a typical experiment.

4.2 Establishment of Calibration Curves

A relationship between intensity (i.e. count rate) and distance between the tracer and each detector was established by calibration. The density dependence of gamma ray attenuation through the bed made it necessary to calibrate in situ because of the inhomogeneity of the gas fluidized bed. The process can be automated using the established data. In this section, we describe the manual calibration procedure.

(A) Original System

The procedure, similar to that employed by Lin [1981], involved attaching the tracer to the tip of a rod, which was held rigidly by a positioning device, and fixing it in a number of locations within the bed and then measuring the

number of gamma rays incident upon each detector at each location. The operating conditions of the fluidized bed during the calibration should be identical to those of the test under consideration.

It is desirable to express the calibration data in functional form using a curve fit for real time processing of the raw data. Polynomial fits of various orders may be used at different regions of the data to represent the relationship between intensity and distance of the tracer to each detector whose forms are

$$d = \sum_{n=0}^N a_n (1/I)^n$$

where d is the distance from the tracer of the crystal inside the detector, I is the intensity of gamma rays at the detector output expressed in number of counts over the sampling interval, a_i 's are the coefficients, and N is the order of polynomial fit between 3 and 7. The fifth-order polynomial adequately fitted most of the data. In order to minimize the execution time of the preprocessing routine, these polynomials were evaluated in advance and the results stored in a look-up table.

(B) Improved System

In the improved system, program PAWCAL.EXE is used to take the raw calibration data for the detectors which relate photon count rate to distance between the detector and the tracer. Basically, the user inputs a position in three coordinates which specifies the fixed position of the tracer particle. The computer then takes count data over several counting periods. The counts for each detector are averaged and the distance from the particle to the specific detector is calculated. The distance and its corresponding count are stored for each detector. This is done a number of times. Enough data must be taken here to fit a polynomial curve from which the final look-up tables will be constructed.

Specifically program PAWCAL.EXE is a C-language shell with two C-language subroutines and an assembly language interrupt handler. The main program takes care of initialization of the hardware as well as buffers and arrays into which data is read. The program prompts the user for the position of the tracer particle in three Cartesian coordinates (x,y,z) . When these are given and verified, the interrupt generator is enabled by setting the computer's internal clock with a user specified counting period which is set at the beginning of the program. At each interrupt, the program jumps into the interrupt handler. Here internal functions and parameters are set and the actual data taking procedures are called. The LeCroy 4434 scaler is the module accessed. A command register within the 4434 is programmed with the commands to be executed and the channels to be read. Once all sixteen channels have been read - the interrupt

routine returns control to the main program. This interrupt is preformed 32 times for each tracer location and the results are averaged to give statistical mean counts. This averaging is necessary because photon counting is purely statistical. The average count is then stored with its respective detector-to-source distance, for later work. When a significant amount of data has been taken, the user may exit the program.

Program PD.EXE is used to compile the look-up tables for the position-calculation program (PROCESS.EXE). Many count versus distance data have been stored for each detector during the calibration process. From these a characteristic polynomial curve fit can be found for each detector, and it is from this polynomial that the final table is generated. The degree of the polynomial is chosen from anywhere from 1 to 13 by the user and through a graphics routine the degree for the best curve fit may be deduced. In the table a distance is stored which corresponds to an integer count. This data is stored for each detector. The tracer position calculation program can then obtain a distance value for a detector from the location of the matching count in the table.

4.3 Other Factors affecting Calibration Curves

There are many other factors affecting a gamma radiation measurement. The most important ones are: the characteristics of the radioactive source; the interactions of gamma rays with matter; the relative position of the source to the detector; the detector efficiency; and the dead time of the whole measurement system. These phenomena are discussed in detail in many textbooks [Tsoulfanidis 1983, Knoll 1979, Tail 1980]. The corresponding formulations and empirical models can be found in these books. In the present study, these results were used with some justification and simplification due to the complexities of the situation for fluidized bed analysis. In the following sections, all of these phenomena will be discussed separately.

4.3.1 Source Effect

A radioactive source may affect a measurement by its geometrical configuration and physical properties (Fig. 4.1).

In the present study, the radioactive tracer particle is made dynamically identical to the solid particles to be studied, i.e. it is a sphere with the same diameter and density as the solid particle. Usually the diameters of the particles are less than 1 mm, which is very small compared with the attenuation length of gamma rays in the 0.1 - 10MeV range. Therefore, the tracer particle can be treated as an isotropic point radioactive source.

The tracer particle is made of Scandium-46. Its physical properties are listed In Table 4.1.

Table 4.1 Decay Data for Nuclide Sc-46

$T_{1/2}$ (day)	E(Kev)	$I^1(\%)$	$\Delta I/I(\%)$
83.7	889.2	99.98	0.0
	1120.5	99.99	0.0

For a radioactive material, the decay of its activity is described by

$$S = S_0 e^{-\frac{t}{T_m}} \quad (4.1)$$

where S_0 and S are activities at time zero and t , respectively, T_m is the mean decay time which equals 120.8 days for Sc-46. From the definition of activity, one Curie (Ci) is equivalent to 3.70×10^{10} disintegrations per second. Table 4.1 shows that each disintegration of Sc-46 produces two photons with energy levels of 0.8892 Mev and 1.1205 Mev, respectively.

4.3.2 Interactions of Gamma Rays with Matter

The substances being considered between the tracer and the detectors are the wall of the bed and the solid particles within the bed. The influence of the air is neglected because of its low density. The interactions of gamma rays with those materials can be simply treated as attenuation and scattering effects separately.

A. Attenuation

There are many possible interactions between photons and matter, but only the following three effects are the most important ones: photoelectric effect, Compton effect, and pair production effect. The Compton effect is the predominant interaction when the gamma ray energy is around 1 Mev, and the photoelectric effect and the pair production effect are important only in the lower or higher energy ranges, respectively. The total probability of interactions can be represented by the total linear attenuation coefficient which is composed of the three probabilities. Figure 4.2 shows the individual coefficients as well as the total linear attenuation coefficient for sodium iodide (NaI) as a function of photon energy. Other materials have similar relationships.

Let us denote the wall as substance 1 and the particles as substance 2. Then the attenuation of the gamma ray intensity I can be expressed as

$$I = I_0 e^{-(\alpha_1 \rho_1 R_1 + \alpha_2 \rho_2 R_2)} \quad (4.2)$$

1I : Gamma ray photon yield per disintegration.

$\Delta I/I$: Uncertainty in yielding figure.

Figure 4.1 Process of radiation detection in the fluidized bed

Figure 4.2 The photon attenuation coefficients for NaI(Tl)

where α_1 and α_2 , ρ_1 and ρ_2 , R_1 and R_2 are the total mass attenuation coefficients, densities, and gamma ray penetrating distances in substances 1 and 2, respectively. Mass attenuation coefficient is equal to the corresponding linear attenuation coefficient divided by the material density.

For the present calculation, the material of the particles is glass and the material of the wall is plexiglass. Then $\alpha_1\rho_1$ and α_2 are constants and were found as [Hubbell 1969], $\alpha_1\rho_1 = 0.0081/\text{mm}$, $\alpha_2 = 0.064 \text{ cm}^2/\text{g}$. ρ_2 is a function of position and can be obtained by fitting experimental data. R_1 and R_2 are evaluated along the line connecting the point source and the center of the NaI crystal as illustrated in Fig. 4.3. The formulas for determining R_1 and R_2 are given in APPENDIX B. Because of the variation of ρ_2 the attenuation term corresponding to the solid particles in Eq. (4.2) is integrated numerically as demonstrated in APPENDIX B.

B. Scattering

The entire gamma beam reaching the detector consists of two components: unscattered beam and scattered beam as shown in Fig. 4.3. However, it is convenient to express the entire beam in terms of a buildup function B , i.e.

$$I = B I_u \quad (4.3)$$

where I is the total beam and I_u the unscattered beam given by Eq. (4.2).

Compton scattering is the predominant component contributed to the buildup function B for the photon energy level of 1 Mev. It can be considered as a collision between a photon and a free electron [Tsoulfanidis 1983, Tait 1980]. The energy E of a photon scattered through angle θ with the incident direction is described by:

$$E = \frac{E_0}{1 + (1 - \cos \theta) E_0/mc^2} \quad (4.4)$$

where E_0 is the incident energy of the photon, and mc^2 is a constant which equals 511 Kev. Detailed analysis [Segre 1953, Segre 1964] shows that for an incident photon energy of about 1 Mev, the distribution of the scattered photons is in a small range of the angle change θ . This implies that the energy change of photons due to Compton scattering is very small (about the order of θ^2) and quite a few scattered photons are to the main beam. Therefore the buildup function may still reach higher values even though a threshold for gamma ray energy is set up.

For the case where an infinite shield is placed between the source and the detector, Berger's formula gives the buildup function B as [Tsoulfanidis 1983]

$$B = 1 + a\mu r e^{b\mu r} \quad (4.5)$$

where μ is the linear attenuation coefficient, r is the distance of gamma ray penetration, a and b are two constants which are dependent on both material and gamma ray energy.

Since the fluidized bed is confined in a cylindrical configuration, applying this equation directly will result in overestimations. Experiments showed that using R_3 , as shown in Fig. 4.3, instead of real penetrating distance ($R_1 + R_2$) gives better results. Constants a and b were determined from fitting experimental data.

4.3.3 Geometry Effect

The geometry effect concerns the size and shape of the radioactive source and detector and the distance between them. In this fluidized bed condition, these influences can be accounted for completely by the solid angle n of detector with respect to the point source. It represents the fraction of the photons emitted from the source reaching the detector.

4.3.4 Detector Effect

When a photon enters a detector, it may not produce a signal or it may produce a signal lower than the discriminator threshold, therefore it is not counted. This effect is accounted for by the detector efficiency defined as:

$$\epsilon = \frac{\text{number of photons recorded per unit time}}{\text{number of photons impinging upon the detector per unit time}}$$

In order to evaluate ϵ , the structure of the detector has to be discussed first.

A. Detector Assembly

Bicron model 2M2/2 scintillation detector is used for the detection of gamma rays emitted from the tracer. It is comprised of an NaI(Tl) cylindrical crystal and an RCA 4902 photomultiplier tube. The crystal is 2 inches in diameter and 2 inches long and is hermetically sealed by a chrome-plated aluminum container. The phototube is 2 inches in diameter and is housed within a chrome-plated Mu-metal magnetic light shield. The crystal and the phototube are optically connected as shown in Fig. 4.4. For this shape of the NaI crystal the radiation measurements would approximately depend upon only the distances between a given tracer particle and the detector. This has been experimentally demonstrated by Lin [1981].

There are two factors that may influence the measurements. One is that some photons reaching the detector may be reflected and absorbed by the aluminum container. Corrections for this influence are discussed in APPENDIX B. The other is that the conversion efficiency of the photo-

Figure 4.3 Relative position of the radioactive source with respect to the NaI crystal

tube window is not equal to unity over its entire area as shown in Fig. 4.5. This may also result in losses in the count rate. However, this effect can be included into other parameters and therefore is not treated individually.

B. Detector Efficiency [Tsoulfanidis 1983]

Statistically, the probability that a photon has at least one interaction in the NaI crystal is $1 - e^{-\mu r}$ where μ is the linear attenuation coefficient of sodium iodide and r is the distance that the photon penetrates in the crystal as shown in Fig. 4.3. By using the average energy of photons emitted from Sc-46, i.e. 1 Mev, it is found to be 0.0213 per millimeter [Tsoulfanidis 1983]. Then, for an isotropic radioactive point source, the detector efficiency can be expressed as

$$\epsilon = \frac{1}{\Omega_0} \int [1 - \exp(-\mu r)] d\Omega \quad (4.6)$$

where Ω is the solid angle. Using the spherical coordinate originating at the point source, $d\Omega$ is expressed as $\sin\theta d\theta d\phi$. Here r becomes a function of θ and ϕ .

This expression of ϵ is based on the assumption that each interaction of the incident photon with the crystal will produce a detectable pulse. It appears that the efficiency is overestimated. However, since some photons may produce multiple interactions in the crystal, the above expression can still give a good estimation.

The formulas and procedure to integrate Eq. (4.6) are given in APPENDIX B. The calculations showed that ϵ varies from 0.3 to 0.5 for the detectors in the fluidized bed condition.

4.3.5 Dead Time Effect

When several successive interactions occur too closely in a short period of time, the detection system may not be able to distinguish them resulting in some counts getting lost. The minimum time needed for a system to distinguish two successive events and record them as two counts is called the dead time of the counting system. The dead time losses may be particularly important in the case of high counting rates. The dead time may arise from a detector or from the associated electronics. However, since the dead time of the detector is usually much longer than the dead time of electronics, the latter can be ignored.

A. Dead Time Models

Two models of dead time behavior have been commonly used, paralyzable and nonparalyzable models [Knoll 1979]. These models represent idealized behaviors of two types of systems. The fundamental assumptions of the two models are illustrated in Fig. 4.6.

Figure 4.4 Configuration of Bicron model 2M2 scintillation detector

Figure 4.5 Relative sensitivities for a typical photomultiplier

Figure 4.6 Models of dead time behavior

The statistical relation of recorded count rate m , with the real interaction rate n , for the nonparalysable model is

$$m = \frac{n}{1+\tau n} \quad (4.7)$$

and for the paralysable model is

$$m = ne^{-\tau n} \quad (4.8)$$

where τ is the dead time of the system. These relations are plotted in Fig. 4.7. When the rates are relatively low, the two models give virtually the same result but the behaviors at high rates are quite different. Note that there is a maximum observed rate for the paralysable model above which the detector will be "saturated." This behavior restricts the maximum activity of a tracer that could be used in the fluidized bed.

B. Determination of the Dead Time Behavior

There are two ways of determining the dead time: the two-source method and the decaying source method. In order to check the behavior of the system and to obtain the dead time at the same time, the second method was used.

The decaying source method is based on the known decay behavior of radioactive materials, which gives the true rate as

$$n = n_0 e^{-\lambda t} + n_b \quad (4.9)$$

where n_0 is the true rate in the beginning of the measurement, λ is the decay constant (it equals $1/T_m$), and n_b is the background influence. Experiments showed that n_b is usually very small and therefore was neglected in the following analysis. Then, substituting Eq. (4.9) into Eq. (4.7), the nonparalysable model can be expressed as

$$me^{\lambda t} = -n_0\tau m + n_0 \quad (4.10)$$

where τ and n_0 are unknowns. If we take m as abscissa and $me^{-\lambda t}$ as ordinate, then Eq. (4.10) is a straight line equation with n_0 and $-n_0\tau$ as coefficients. From an experimentally established relationship between m and t , and n_0 can be obtained by least square fitting the relation.

For the paralysable model, substituting Eq. (4.9) into Eq. (4.8), we can get

$$\lambda t + \ln m = -n_0\tau e^{-\lambda t} + \ln n_0. \quad (4.11)$$

Taking $e^{-\lambda t}$ as the abscissa and $\lambda t + \ln m$ as the ordinate and using a procedure similar to the nonparalysable case, and n_0 can be determined.

Figure 4.7 Variation of the observed rate m as a function of the true rate n for two dead time losses

An experiment was conducted to determine the relation of m as a function of time t using a Sc-46 tracer particle as the radioactive source. To ensure the accuracy, the measurements were carried out over a period of 60 days which is about the half life of Sc-46. The results are plotted in Figs. 4.8a-d for the first two detectors. For other detectors, the results showed similar behaviors as those of the first two detectors. From these plots we conclude that the paralyzable model is more suitable for the present detection system.

The above measurements were performed in the medium and lower count rate range. In order to see the behaviors in the high count rate range, the two-source method was used to determine dead times in various ranges. It was found that the dead time is not a constant, it decreases as n increases. And it seems that a new value of dead time will be reached at very high values of n . This phenomenon may be explained as follows: when interaction rate n is very high, each output pulse from the detector will not be distinguished separately and all the pulses will pile up. As a result, the output signal is composed of a high average voltage with low amplitude fluctuations in high frequency. If the overall amplification of the signal is high enough, those fluctuations will be recorded according to the dead time behavior of the electronics. This suggests that there are two dead time limits for the detection system, one arises from the detector and the other arises from electronics. To simplify the calculation, the following dead time behavior was assumed:

$$m = (1 - \alpha) n e^{-\tau_1 n} + \alpha n e^{-\tau_2 n} \quad (4.12)$$

where τ_1 and τ_2 are two dead times and α is a weighting factor. These parameters will be determined later.

Equation (4.12) gives the dead time of the system as

$$\tau = \tau_1 - \frac{1}{n} \ln [1 - \alpha + \alpha e^{(\tau_1 - \tau_2)n}]. \quad (4.13)$$

This relation is plotted in Fig. 4.9a. The corresponding dead time behavior, Eq. (4.12), is plotted in Fig. 4.9b.

4.3.6 Electronics

The function of electronics is to amplify the input signals and then record the pulses as count rates. The existing electronics have a frequency-voltage-digital conversion process which usually changes the count rate m to a new rate m' . Assume that the actual converting factor is C , which may be a function of time, then m' equals Cm . Equation (2.8) can be rewritten as

$$m = n' e^{-\tau' n'} \quad (4.14)$$

Figure 4.8 a Application of the decaying source method to determine dead time

Figure 4.8b Application of the decaying source method to determine dead time

Figure 4.8c Application of the decaying source method to determine dead time

Figure 4.8d Application of the decaying source method to determine dead time

Figure 4.9a Typical relation between the dead time τ and the true rate n

Figure 4.91b Typical relation between the recorded rate m and the true rate n

where

$$n' = Cn \quad \text{and} \quad \tau' = \tau/C. \quad (4.15)$$

Equation (3.14) shows that the dead time behavior does not change by introducing the factor C. Also, it will be explained later that C can be included in another parameter. Therefore, the influence of the electronics is not considered separately.

4.4 Computation of Instantaneous position

The instantaneous position of the tracer is calculated from the count rate at that time using the calibration curve.

(A) Original System

In the original system in UIUC, the position of the tracer in the fluidized bed is determined from the count rate from all sixteen detectors. The system of equations is linearized and use is made of linear regression theory to find the solution in an overdetermined system. For details of this method, please refer to Appendix A.

(B) Improved System

In the improved system, program PROCESS.EXE converts the detector count data into tracer position data. From each count a distance is found from the look-up tables generated by program PD.EXE. From these distances and the detector locations the tracer particle position can be triangulated. Using sixteen detectors provides redundancy which is helpful in error reduction. Weighting factors are used in the calculation procedure so that those detectors which are closest to the tracer have the greatest emphasis in determining its location. This is done because the count versus distance data is more accurate and detailed in the short distance ranges.

4.5 Other Factors affecting position calculation

By combining all the correlation equations, the count rate can be expressed as a function of tracer position and other parameters. In this section, we will determine all these parameters from experimental data. Comparisons of this analysis with calibration data are also presented.

A. Formulations

The interaction rate n can be expressed as:

$$n = ABS \Omega e^{-(\alpha_1 \rho_1 R_1 + \alpha_2 \rho_2 R_2)} \quad (4.16)$$

where A is a constant and is assumed to be proportional to the converting factor C of the electronics, B is the buildup function, S is the activity of the tracer particle, n is the

solid angle, ϵ the detector efficiency, and the last term is due to the absorption of gamma rays by matter as expressed by Eq. (4.2). By substituting Eqs. (4.1) and (4.6) into Eq. (4.16), It becomes

$$n = ABS_0 e^{-(\alpha_1 \rho_1 R_1 + \alpha_2 \rho_2 R_2)} \int_{\Omega} (1 - e^{-\mu r}) \sin \theta d\theta d\phi \quad (4.17)$$

The recorded rate m , is

$$m = n e^{-\tau n} \quad (4.18)$$

where τ is given by relations (4.15) and (4.13).

Equations (4.17) and (4.18) are the basic equations to correlate the particle position and the count rate. A program computing these equations is given in APPENDIX B.

The parameters which need to be determined are: the properties of each detector (dead times τ_1 and τ_2 and weighting factor α), the properties of the material (constants a and b in the buildup factor B), and finally, the proportional constant A which is a property of the electronics of each channel.

B. Determination of Dead Times

An experiment with three tracers of total activity 335 Ci was made to establish the relationship between distance and count rate in the absence of a medium between the source and the detector. The measurements are plotted as points in Figs. 4.10a-c. The maximum count rate observed from these data further confirms that the paralyzable model is suitable to this system. These data also show the two dead time behaviors.

When there is no material between the source and the detector, the buildup factor B and the absorption term in Eq. (4.17) are dropped. Therefore, Eqs. (4.17) and (4.18) can be used to determine the dead time behavior. By choosing the constant A and τ_1 and τ_2 and α , which must be agreeable to the dead time measured by using the decaying source method and the two-source method, the experimental data were fitted as the solid lines in the figures. In the high count rate region, those curves do not fit the data well but usually this region is not used in experiments because of the low accuracy.

The dead times τ_1 and τ_2 may not be constants due to the change of the converting factor C of the electronics. The influence of C to the dead times is determined by relation (4.15). However, since A is proportional to C , $A\tau_1$ and $A\tau_2$ are constants.

The actual values of the dead time cannot be obtained exactly because of the unknown parameter C . If C were known, with the sampling time of each measurement and the disintegration data

Figure 4.10a Experimental data and curve fit of count rate m versus the distance between the tracer and the detector crystal center

Figure 4.10b Experimental data and curve fit of count rate m versus the distance between the tracer and the detector crystal center

Figure 4.10c Experimental data and curve fit of count rate m versus the distance between the tracer and the detector crystal center

of the source given earlier, A can be computed and dead times can be determined.

C. Buildup Function

A calibration was performed with the bed filled with glass particles at static condition. In this state, the particle density is a constant, and the gamma ray absorption and scattering effects reach maximum. The calibration data are plotted in Fig. 4.11 for the first detector. The parameters a and b in the buildup relation (4.5) were determined by fitting these calibration curves for all the detectors. This curve fit gives $a = 0.9$ and $b = 0.03$. The maximum value of the buildup function B observed in this condition reaches as high as 2.61. This means that the scattered gamma beam is even more important than the unscattered gamma beam.

D. Comparison of the Analysis with Experiments

The parameters in Eqs. (4.17) and (4.18) have been determined above. Now these equations are used to calculate for different tracer activities and fluidized bed situations. Figures 4.11a-e show some predicted calibration curves compared with those from real experiments for the first detector. Similar results were obtained for other detectors. These experiments were performed at different times using different tracer particles.

E. The dependence of count rate on the polar angle from the centerline of the detectors

Fig. 4.12 shows the relationship between the count rate and the polar angle of the tracer from the center line of the detectors. It can be seen that in general, the count rate is smallest in the center of the detector and largest on the side of the detectors. This is expected because when the tracer is in the center of the detector, the solid angle projected from the the detector to the tracer is smallest. That is the area of the detector exposed to the tracer is smallest, resulting a relative smaller count rate measurement from the detector. It should be pointed out that the vertical axis does not start from 0 to show that the count rate varies with the polar angle. If it were start from 0, the curve would be relatively smooth, indicating that the variance of the count rate with the polar angle is not large.

4.6 Automatic Calibration Procedure and a Test Case

The calibration to establish the relationship between the intensity (i.e. count rate) and the distance between the tracer and the detectors is an important step for the particle tracking measurement. The accuracy of the calibration data determines the accuracy of the particle tracking data. Since the radioactive intensity of the tracer varies and the experiment conditions are different, a calibration has to be made for each experiment. At UIUC, a manual calibration

Figure 4.11a Comparison of experimental calibration data with analytical predictions

Figure 4.11b Comparison of experimental calibration data with analytical predictions

Figure 4.11c Comparison of experimental calibration data with analytical predictions

Figure 4.11d Comparison of experimental calibration data with analytical predictions

Figure 4.11e Comparison of experimental calibration data with analytical predictions

Fig. 4.12 Count rate vs. the polar angle from the centerline of the detector

procedure has been employed. The procedure involved attaching the tracer to the tip of a rod, which was held rigidly by a positioning device, and fixing it in a number of locations within the bed and then measuring the counts for all detectors at each location. Then, the calibration curves were obtained for all detectors, as described in detail earlier. However, this technique is not convenient to be used. Therefore, it was decided to explore a theoretical procedure to establish the calibration curves.

There are two methods to automatically generate the calibration curves:

- (1) using a complete theoretical description of the radiation detection by accounting for the attenuation, build up, and detector efficiency, as discussed in section 3.3.3, and
- (2) from empirical iterations based on successive substitution and improvement of calibration curve iterates. In this case, use can be made of the relationship between the count rate, distance, polar angle and attenuation distance and material itself as discussed in earlier.

Since the latter has the appeal of easy programming, decision was made to try it first. With this method, a pre-calibration is not required before an experiment. The tracer particle is thrown into the bed which is running at a specified condition. Its subsequent motion is monitored by the detectors in terms of the counts at a sampling rate of, e.g., 20 ms. The detector counts are continuously stored on the computer hard disk for a total time period of a few hours. These count data can be used first to determine the calibration curves and then the tracer positions. In this section, the procedure for the automatic calibration will be discussed.

In order to find the calibration curves, an initial estimation of the curves for all the detectors is needed. From these curves, the optimum tracer positions may be predicted from the detector count data by linear regression. Then the new calibration curves can be established from the predicted tracer positions and the existing detector count data. This iteration process continues until the curves converge. Theoretically, since the linear regression scheme usually gives the optimum solution for a non-bias system, it is hoped that final converged curves can serve as the calibration curves. The procedure can be completely performed by a computer. It should be pointed out, due to time constraint, this procedure was not used in our experiment. But the idea is successfully tested using the actual experiment data obtained from METC 1-foot fluidized bed. The detail is in the following sub-section.

4.7 Computation of Instantaneous velocity and mean velocity

The instantaneous Lagrangian velocity of the tracer was obtained by differentiating the location of the tracer with respect to time in Lagrangian coordinates:

$$v(a, t) = \frac{ds(a, t)}{dt}$$

where s denoted the distance the tracer traveled for time t since its release at position a . In finite difference form, the above equation became

$$v(a, t) = \frac{s(a, t+\Delta t) - s(a, t)}{\Delta t}$$

in which Δt was the sampling interval for position measurements.

The mean velocity distribution was determined by dividing the bed into imaginary sampling compartments in a cylindrical coordinate system whose origin was at the center of the air distribution surface as shown in Fig. 4.13. Ten radial, sixteen circumferential, and fifty axial subdivisions were chosen giving a total of 8,000 compartments. By running the experiments for sufficiently long times, the repeated appearance of the tracer in a given compartment enabled the evaluation of ensemble averages of the Lagrangian velocity calculated from the above equation. Therefore, at a given position η' , mean velocity was given by

$$v(\eta') = \frac{1}{N_{\eta'}} \sum_{i=1}^{N_{\eta'}} v(\eta_i)$$

where $N_{\eta'}$ was the total number of occurrences of the tracer in the sampling compartment $\delta v(\eta')$ centered at η' and

$$\eta_i = \frac{s(a, t+\Delta t) + s(a, t)}{2} \in \delta v(\eta')$$

Thus, the components of the mean velocity in each imaginary compartment were computed as the ensemble averages of the corresponding components of the instantaneous velocity. The resulting mean velocities were averaged circumferentially due to the near axisymmetry that existed for the cylindrical bed to give the distribution of mean velocities in radial and axial directions.

4.8 Determination of solid velocity distribution

Once the instantaneous velocity at each point in the fluidized bed is obtained, it is possible to find out the velocity distribution of the solid particles at a given point in space. This distribution function is very important in the granular flow theory and is only possible when the measurement of the velocity in the fluidized bed is feasible.

Figure 4.13 Configuration of Data Reduction Compartments in Cylindrical Coordinates

The velocity distribution can also be obtained using the swarm-particle tracking technique.

4.9 Computation of fluctuating particle kinetic energy

From the particle velocity distribution data, it is easy to find out the fluctuation particle kinetic energy defined as

$$\sum_{i=1}^N \frac{1}{2} m_p (v_i - u)^2$$

where m_p is the mass of the particle, and v and u are the instantaneous velocity and ensemble average velocity of the particle.

4.10 Stream function and Streamline plots

The average movement of the particles in the fluidized bed is simplified to a steady, compressible, axisymmetrical two-dimensional flow. The velocity distributions, and the density distribution which is assumed to be proportional to the total tracer occurrence distribution, were obtained from averaging a long time movement of a single tracer particle inside the bed. During the measurements and the data processing, some errors were inevitably introduced into the resulting data. This can be easily seen from a typical experimental circulation pattern as shown in Fig. 4.14a. The lines in the plot are particle trajectories which represent the streamlines of the flowfield. The problem is that some lines end on the boundaries and some cross each other. This contradicts the definition of streamlines. Improvement for the velocity data therefore becomes necessary. Nevertheless, it is assumed that the main structure of the velocity distributions is correct and the errors are small and random. Otherwise, the corrections will be worthless.

A. Calculation of Stream Functions

In steady, two-dimensional, axisymmetrical flow, the continuity equation is

$$\frac{\partial(r\rho u)}{\partial r} + \frac{\partial(r\rho v)}{\partial z} = 0 \quad (4.19)$$

where u and v are the velocity components in the r and z directions, respectively; ρ is the density; z is measured from the bottom of the bed, and r , from the center line of the bed as shown in Fig. 4.15.

The stream function Ψ is defined by

$$\frac{\partial \Psi}{\partial z} = r\rho u \quad (4.20)$$

$$\frac{\partial \Psi}{\partial r} = -r\rho v \quad (4.21)$$

Figure 4.14a Streamlines in fluidized bed plotted by trajectory method

Figure4.14b Streamlines in fluidized bed plotted by stream function method (without smoothing)

Figure 4.14c Effect of stream function smoothing (smoothing once)

Figure4.14d Effect of stream function smoothing (smoothing twice)

Figure 4.14e Effect of stream function smoothing (smoothing three times)

Figure 4.14f Effect of stream function smoothing (smoothing four times)

Figure 4.15 Coordinates and boundary conditions of the stream functions for the fluidized bed

subjected to the boundary conditions

$$\Psi(r, 0) = 0 \quad (4.22)$$

$$\Psi(0, z) = 0 \quad (4.23)$$

$$\Psi(r, z_0(r)) = 0 \quad (4.24)$$

$$\Psi(r_0, z) = 0 \quad (4.25)$$

where r_0 is the radius of the bed and $z_0(r)$ is a postulated upper boundary line at which the velocities become zero as shown in Fig. 4.15. This upper limit can be determined when the zero velocities are found within the domain of the measured flowfield. When the velocities are non-zero at the top boundary of the domain, the upper boundary condition (4.24) does not apply. This case will be treated in APPENDIX C when deriving the formulas for programming.

Integrations of Eqs. (4.20) and (4.21) subjected to the boundary conditions (4.12) and (4.13) give

$$\Psi_1(r, z) = r \int_0^z \rho u dz \quad (4.26)$$

$$\Psi_2(r, z) = - \int_0^r \rho v dr. \quad (4.27)$$

These two stream functions, Ψ_1 and Ψ_2 are usually not equal and do not satisfy boundary conditions (4.24) and (4.25) due to the errors in the density and velocity data. The deviations can be considered measures of the errors. It was found that the deviations are usually less than 5 percent of the maximum values of Ψ_1 and Ψ_2 along the respective integration paths in the calculation of the fluidized bed data.

To modify the stream function Ψ_1 we assume that for each fixed r , the error in the product ρu is constant throughout the integration (4.26), i.e. we assume

$$\overline{\rho u} = \rho u + \delta_1 \quad (4.28)$$

where $\overline{\rho u}$ is the corrected value and δ_1 is the constant error.

This assumption is applied to ρv , i.e.

$$\overline{\rho v} = \rho v + \delta_2. \quad (4.29)$$

Then the corrected stream functions Ψ_1 and Ψ_2 are

$$\overline{\Psi}_1(r, z) = \Psi_1(r, z) + \delta_1 \rho z \quad (4.30)$$

$$\overline{\Psi}_2(r, z) = \Psi_2(r, z) - \frac{1}{2} \delta_2 r^2. \quad (4.31)$$

By applying boundary conditions (4.24) and (4.25), δ_1 and δ_2 are found to be

$$\delta_1 = -\frac{\Psi_1(r, z_0(r))}{rz_0(r)} \quad (4.32)$$

$$\delta_2 = \frac{2\Psi_2(r_0, z)}{r_0^2} \quad (4.33)$$

The stream functions Ψ_1 and Ψ_2 defined by Eqs. (4.30) and (4.31), errors in the data still exist. This can be illustrated by plotting the stream functions Ψ_1 and Ψ_2 in Fig. 4.14b.

B. Reconciliation of the Two Stream Functions

By simply averaging the values of the stream functions at one point with those of the surrounding four points, the stream functions can be combined into one single function and, at the same time, the random errors will be eliminated. In order that the two stream functions have the same weighting in the averaging process, a factor of 4 is multiplied to the middle point stream function because the surrounding four points belong to the other stream function. Figures 4.14c-f show the effects of different times of stream function averagings.

The averaging scheme, as discussed above, can smooth the stream function very effectively. But, at the same time, the relative values of the stream function will be reduced. Sometimes the boundaries dividing different vortices will be shifted slightly if the values of the stream function at different regions are widely different. This is the case for the fluidized bed calculations. Therefore, the process should not be carried out more than a few times--usually two or three averagings are enough for streamline plottings.

C. Program Development

Program SMOOTH was developed for two-dimensional, compressible, Cartesian, or cylindrical coordinate stream function calculations. It can also handle the calculations for the case where a solid ball is placed on the axis of the bed. The formulas and the program are given in APPENDIX C. In this program, the density distribution is smoothed first since this will improve the modified velocity distributions. Program STRMLN is also listed. It gives the streamline plottings.

For the case of a solid ball inside the bed, the stream function calculation and smoothing effects are illustrated in Figs. 4.16a-c. For the Cartesian coordinate case, the effects are shown in Figs. 4.17a-c.

4.11 Computation of Particulates Density

The tracer position and velocity data obtained represent a Lagrangian description of the motion of a single particle in the bed. However, it is usually desired to present the data in Eulerian form. To this end, the Lagrangian data are used to evaluate local means of dynamic variables as functions of position in the fluidized bed. This is

Figure 4.16a Streamlines in fluidized bed with a sphere plotted by trajectory method

Figure4.16b Streamlines in fluidized bed with a sphere plotted by stream function method (without smoothing)

Figure 4.16c Effect of stream function smoothing (smoothing twice)

Figur 4.17a Streamlines in the fluidized bed for Cartesian case
plotted by trajectory method

Fig. 4.17b Streamlines in the fluidized bed for Cartesian case plotted by stream function method (without smoothing)

Figur 4.17c Effect of stream function smoothing (smoothing twice)

accomplished by dividing the cylindrical bed into imaginary sampling compartments in a cylindrical coordinate system, with origin at the bottom center of the bed as shown in Fig. 4.13. In this study, ten radial, sixteen circumferential, and fifty axial subdivisions were chosen to give a total of 8000 compartments. By running the experiments for sufficiently long times, usually 5 hours, the tracer particle typically appears many times in each sampling compartment. Ensemble averages of the Lagrangian quantities of the tracer when it appears in a sampling compartment give the 40 values of the corresponding Eulerian quantities for that compartment. The resulting data are then averaged circumferentially because of the near axisymmetry of the data.

If the tracer particle is a true representation of all the particles in the bed, and the solids fluctuation in the bed is a stationary stochastic process, the solids mean density and velocity may be evaluated in the following manner. Let us denote V_c as the volume of a compartment, n as the local particle number density which is unknown, and N as the total number of particles in the bed. The particles in the bed are assumed to have the same size δp and the same mass m . Then, nV_c/N is the probability to find a single tracer particle in the compartment at a time instant. It is also the fractional time duration when the tracer is found in the compartment. Therefore, if the total duration of an experiment is Δt , we have

$$\frac{nV_c}{N} = \frac{\sum_k \delta t_k}{\Delta t} \quad (4.38)$$

where δt_k is the duration of the k th residency of the tracer in the compartment. If we multiply the particle mass m to N and n in Eq. (4.38), noticing that $Nm=M$ is the total mass of the bed, the mean solids density $\rho = nm$ is then determined from

$$\rho = \frac{M}{V_c \Delta t} \sum_k \delta t_k \quad (4.39)$$

Correspondingly, the local mean solids velocity can be computed from

$$v = \frac{\sum_k \delta t_k v_k}{\sum_k \delta t_k} = \frac{M}{\rho V_c \Delta t} \sum_k \delta t_k v_k \quad (4.40)$$

The above computations can be viewed as a form of conditional time average. Unfortunately, for a general compartment in cylindrical coordinates, the computation of

the residence duration for a particle in straight line motion is quite complex and time consuming. Instead, we count only the number of occurrences of the tracer in the compartments. One occurrence is defined to be equivalent to one sampling duration δt_s , and is assigned on the center location of the tracer trajectory in δt_s . Let us denote $N_0 (= \Delta t / \delta t_s)$ as the total number of occurrences of the tracer in the bed, and n_0 as the number of occurrences of the tracer in the compartment V_c , the mean solids density and velocity may be computed from the conditional ensemble averages

$$\rho = \frac{M n_0}{V_c N_0} \quad (4.41)$$

$$v = \frac{M}{\rho V_c N_0} \sum_k v_k \quad (4.42)$$

where the summation is performed only when the tracer occurrence is in the compartment. With the tracer occurrences we simplified the continuous tracer trajectory into discrete point tracer trajectory. Therefore, the probability of finding the tracer in a compartment, which is $n V_c / N$ as discussed above, should also equal to the fractional occurrence of the tracer in the compartment, n_0 / N_0 . This shows that when $\Delta t \rightarrow \infty$, which is equivalent to $N_0 \rightarrow \infty$. Eqs. (4.41) and (4.42) are identical to Eqs. (4.39) and (4.40) respectively. But when Δt is finite, Eqs. (4.41) and (4.42) would have lower accuracy since they are derived from simplified discrete point motion of the tracer particle.

The above consideration for large Δt , of course, presupposes the existence of intrinsic randomness of the system. In the event the system is not random, the mean quantities defined in Eqs. (4.41) and (4.42) become meaningless and values computed from them may attain physically unacceptable values. For example, in a portion of the bed where the particles are stagnant for a significant fraction of the time, unrealistically small or large computed densities, including those greater than the material density of the solid, may be observed. Whether there can be any statistical tests capable to detecting such pathological behaviors is unclear at this point. Until such tests prove feasible, one must be vigilant in examining the consistency of the data before accepting them as bona fide representations of the flow field.

From the brief discussion in the above paragraph it is seen that there are three sources affecting the accuracy of the data: (1) the randomness of the system, (2) the duration of the experiment Δt , and (3) the measurement error.

4.12 Computation of the Reynolds Stresses

4.12.1 The Concept of Reynolds Stresses

When working with the time-averaged equations for fluctuating and/or turbulent flows, some of the contributions of random momentum fluxes represented by products of the fluctuating components, known in statistical language as correlations, appear as average stress terms in the equations. These contributions of the random momentum fluxes are called the Reynolds stresses, or sometimes turbulent stresses. For single phase constant density flows, the Reynolds stress τ_{ij} is the familiar form $\rho \langle u_i' u_j' \rangle$. For multiphase flows or flows with variable densities, the situation is more complex, and greater scrutiny is necessary.

For single phase variable density flows, $\rho \langle u_i' u_j' \rangle$ represents only part of the contributions of the Reynolds stress. Other correlations involving the fluctuating components of ρ must also be included.

For multiphase flows there are at least two possible types of stochastic motion. In the continuum level, there may be turbulence-like fluctuations of a scale much greater than the average diameter or distance of the particles. On the other hand, in the scale of the dimensions of the particles, there may also be stochastic motion analogous to the motion of molecules in gases. This analogy, however, is not complete. As will be explained below.

In a molecular gas, the contribution of random momentum fluxes due to the thermal motion of the molecules is in fact, essentially equal to the viscous stress and the pressure. This is because the gas density is usually so small that the molecules are almost never caught in the act of actually colliding with each other.

In a dense multiphase system, the fraction of time the particles are engaged in colliding with each other is not nearly as negligible. Since the internal stresses of particles under collision are quite high, they account for a significant fraction of the total stress. This fraction is comparable or frequently greater than the contribution due to random momentum fluxes.

In the following, two methods of determining the Reynolds stresses are described. The first is a measurement of $\rho \langle u_i' u_j' \rangle$. The second is a determination of the total Reynolds stress by actually measuring the average momentum flux. It is interesting to note that the two methods yielded essentially identical results. On the other hand, it is important to remember that even the total Reynolds stress does not represent the total turbulent stress, since the contributions of collisional stresses was not included.

4.12.2 Reynolds Shear Stress component $-\rho \langle u' r u' z \rangle$

The evaluation of the Reynolds shear stress component $-\rho \langle u' r u' z \rangle$, requires the determination of the local particulates density, and the cross-correlation of the solids

fluctuating velocities in the radial and axial directions, $\langle u'_r u'_z \rangle$. The local particulates density was evaluated using the method described earlier. The cross correlation of solids fluctuating velocities, $\langle u'_r u'_z \rangle$, was computed by multiplying the radial and axial fluctuating velocities, u'_r and u'_z , by each other followed by ensemble averaging. The ensemble average is denoted by $\langle \cdot \rangle$. The fluctuating velocities were obtained by subtracting the mean, $U(\underline{n}')$, from their respective instantaneous velocities, $v(\underline{n}_i)$, for each position $\underline{n}'$, i.e.

$$u'_r(\underline{n}_i) = v_r(\underline{n}_i) - U_r(\underline{n}') \quad (4.43)$$

$$u'_z(\underline{n}_i) = v_z(\underline{n}_i) - U_z(\underline{n}'). \quad (4.44)$$

Therefore, the cross correlation at position $\underline{n}'$ was given by

$$\langle u'_r(\underline{n}') u'_z(\underline{n}') \rangle = \frac{1}{N_{n'}} \sum_{i=1}^{N_{n'}} u'_r(\underline{n}_i) u'_z(\underline{n}_i) \quad (4.45)$$

where $\underline{n}_i \in \delta V(\underline{n}')$ and $N_{n'}$ denotes the number of occurrences of the tracer at position $\underline{n}'$.

4.12.3 Turbulent Normal Stresses components $-\rho \langle u'^2_r \rangle$ and $-\rho \langle u'^2_z \rangle$

The Reynolds normal stresses, $-\rho \langle u'^2_r \rangle$ and $-\rho \langle u'^2_z \rangle$, were determined by evaluating the respective mean square velocities using their Lagrangian velocity autocorrelations. The correlation between the values of the fluctuating velocity at two different times is referred to as the autocorrelation. For a stationary flowfield (i.e. statistically steady), it is called the Lagrangian autocorrelation. The Lagrangian autocorrelation coefficient $R_{\alpha\alpha}(\underline{n}', \tau)$ at a given position $\underline{n}'$ is defined as (Tennekes and Lumley 1972):

$$R_{\alpha\alpha}(\underline{n}', \tau) = \frac{\langle v'_\alpha(a, t) v'_\alpha(a, t + \tau) | a \in \delta V(\underline{n}') \rangle}{\langle v'^2_\alpha(a, t) | a \in \delta V(\underline{n}') \rangle} \quad (4.46)$$

where $v'_\alpha(\underline{n}, t)$ is the fluctuating velocity in the Lagrangian field in either the radial ($\alpha = r$) or the axial ($\alpha = z$) direction and a is the initial position of the tracer. The Lagrangian fluctuating velocity was computed as

$$v'_\alpha(a, t) = v_\alpha(a, t) - \langle v_\alpha(a, t) \rangle \quad (4.47)$$

where $v_\alpha(a, t)$ is the instantaneous Lagrangian velocity and $\langle v_\alpha(a, t) \rangle$ is the ensemble average of this velocity. In order to evaluate $\langle v_\alpha(a, t) \rangle$, it was assumed that an "experiment" was initiated each time the tracer was found in the compartment under consideration, $\delta V(\underline{n}')$, provided the time

interval between two successive "experiments" was above a prescribed limit. This was necessary to insure that the two successive experiments were statistically independent. This time limit is equal to or greater than the time lag for which the Lagrangian autocorrelations become negligible.

The mean Lagrangian velocity of the tracer at a later time t^* after its release from $\delta V(n')$ was computed as

$$\langle v_{\alpha}(a, t^*) | a \in \delta V(n') \rangle = \frac{1}{N_{n'}} \sum_{i=1}^{N_{n'}} v_{\alpha}(n_i, t^*) \quad (4.48)$$

where $v'_{\alpha}(n, t^*)$ is the instantaneous Lagrangian velocity at t^* for the "experiment" initiated at $n_i \in \delta V(n')$, and $N_{n'}$ is the total number of experiments in which the tracer was released from $\delta V(n')$. Therefore, using Eqs. (4.46) and (4.47), the fluctuating velocities in the Lagrangian frame of reference were obtained at time $t = 0$ and the next successive time steps. Thus the Lagrangian velocity autocorrelation was then computed using

$$\langle v'_{\alpha}(a, 0) v'_{\alpha}(a, t) | a \in \delta V(n') \rangle = \frac{1}{N_{n'}} \sum_{i=1}^{N_{n'}} v'_{\alpha}(n_i, 0) v'_{\alpha}(n_i, \tau) \quad (4.49)$$

where the $N_{n'}$ experiments were initiated at $t = 0$ at position a . The flowchart for the calculation of the autocorrelation as well as the cross correlation functions of solids fluctuating velocities is shown in Fig. 4.18.

The emission of gamma rays has a Gaussian distribution at high count rate applications. This is mostly the cause for spurious noise in the velocity measurements. Therefore, the Mean Square velocities were determined indirectly in order to achieve better accuracy. They were assessed from the Lagrangian autocorrelations by extrapolation based on polynomial curve fitting of the autocorrelation function. Since the autocorrelation function is expected to be symmetric with respect $\tau = 0$, the following polynomial was postulated:

$$P = A\tau^4 + B\tau^2 + C \quad (4.50)$$

where coefficients A , B , and C were determined from three measured velocity correlations: $\langle v'_{\alpha}(n_i, 0) v'_{\alpha}(n_i, \tau) \rangle$ with $\tau = \delta t$, $2\delta t$, and $3\delta t$, δt being the sampling interval between velocity measurements. The desired Mean Square velocity was obtained from

$$\langle v'^2_{\alpha}(a, 0) \rangle = C. \quad (4.51)$$

The Reynolds normal stresses were assessed afterwards by multiplying the particulates density by the Mean Square velocities in the respective directions. The RootMeanSquare (RMS) velocity was also determined using the above expression. The Mean Square velocity was employed in Eq. (4.46) to evaluate the autocorrelation coefficient, $R_{\alpha\alpha}(n', \tau)$. This coefficient decreases as the time interval,

Figure 4.18 Solids Mean Particulate Stresses in a Cylindrical Fluidized Bed for 500 μm Glass Particles at $u_0/u_{mf} = 1.5$ ($u_{mf} = 21.9$ cm/s)

increases if the velocities $v'_\alpha(a,t)$ and $v'_\alpha(a,t+\tau)$ become less correlated with in creasing τ .

The area under the curve representing the correlation coefficient is given by

$$T_a = \int_0^{\infty} R_{\alpha\alpha}(n',\tau) d\tau. \quad (4.52)$$

The time T_a is called the Lagrangian integral time scale which is the correlation time of the solids fluctuating velocity. This quantity was computed for both radial and axial directions using the above expression. The dispersion coefficient of the solid particles, was also obtained using the Mean Square velocity from Eq. (4.51) as follows (Tennekes and Lumley, 1972):

$$D_a = \langle v'_\alpha(a,0) \rangle F_a \quad (4.53)$$

Even though Eqs (4.52) and (4.53) are only valid for homogeneous isotropic turbulent flows, they were used for the fluidized bed system which is non-homogeneous and anisotropic. Therefore, the results are approximations of the true values of F_a and D_a .

4.13 Computation of Total Translational Reynolds Stress

The particulate stress τ_{ij} is simply defined as the fluctuating momentum flux. It can be easily shown

$$\tau_{ij} = \frac{M}{\Delta A_i \Delta t} \sum \text{sign}(v'_i) v'_j \quad (4.54)$$

where M is the total mass of the bed and the summation accounts for only the test particle passing through the sampling area ΔA_i in time period Δt . In the following calculation, Δt is taken to be very long, τ_{ij} therefore the mean particulate stress tensor.

4.14 Comparison of $\rho \langle u'_i u'_j \rangle$ and τ_{ij}

A comparison of $\rho \langle u'_i u'_j \rangle$ and τ_{ij} is made of figures 4-18 to 4.23. It is interesting to note that the two are nearly identical, within the expected accuracy of the data. This could seem to indicate that of the terms in the Reynolds stress expression, those correlations containing density fluctuations do not play an important role. The physical reason for this is not understood.

Figure 4.19 Solids Reynolds Stresses in a Cylindrical Fluidized Bed for 500 μ m Glass Particles at $u_0/u_{mf} = 1.5$ ($u_{mf} = 21.9$ cm/s)

Figure 4.20 Solids Mean Particulate Stresses in a
Cylindrical Fluidized Bed for 500 μ m Glass
Particles at $u_0/u_{mf} = 2$ ($u_{mf} = 21.9$ cm/s)

Figure 4.21 Solids Reynolds Stresses in a Cylindrical Fluidized Bed for 500 μ m Glass Particles at $u_0/u_{mf} = 2$ ($u_{mf} = 21.9$ cm/s)

Figure 4.22 Solids Mean Particulate Stresses in a
Cylindrical Fluidized Bed for 500 μ m Glass
Particles at $u_0/u_{mf} = 4$ ($u_{mf} = 21.9$ cm/s)

Figure 4.23 Solids Reynolds Stresses in a Cylindrical Fluidized Bed for 500 μ m Glass Particles at $u_0/u_{mf} = 4$ ($u_{mf} = 21.9$ cm/s)

5. Accuracy and Resolution

5.1 Limits of Accuracy - First Principle Considerations

As a bench mark against which future analysis can be measured, it is useful to make approximate estimate of the accuracy attainable from considerations of first principles. Consider a gamma source at a distance r from the detector, which consists of a sodium iodide (NaI) scintillation crystal optically coupled to a photomultiplier (PMT). For the approximate analysis to be attempted at this point, we will assume the crystal to be sufficiently small so that we do not need to be concerned with the finite size of the crystal. The distance r will be assumed to be measured from the geometrical center of the crystal. The relationship between the source activity A and the rate of scintillation S at the crystal is approximated by the relationship

$$S = \frac{d^2}{16r^2} \eta_c A \quad (5.1)$$

Here the first factor represents the fraction of gamma photons intercepted by the scintillation crystal. It is equal to the solid angle extended by the detector crystal to the source, with d denoting the crystal diameter. η_c is the efficiency of the crystal, depending on both the material and the size of the crystal, and A is the activity of the source.

In the radioactive particle tracking methodology, the distance is inferred by measuring the scintillation rate S . The approximate relationship between the accuracy of the distance measurement and the scintillation rate from Eq. (5.1) is thus

$$\frac{|\delta r|}{r} = \frac{1}{2} \frac{|\delta S|}{S} \quad (5.2)$$

The scintillation process in the crystal converts each gamma interaction into a burst of light detectable by the photomultiplier tube (PMT), which converts the light to a current pulse, typically with a pulse length on the order of 1 μ s. The scintillation rate S can be measured in two ways. The simplest procedure is to simply to measure the average current of the PMT as an analog signal. This procedure has two disadvantages. The first disadvantage is that it cannot distinguish between the contributions of the primary gamma photon emitted by the source from the secondary gamma photons which are the results of the interaction of the primary photon with the bed material. The second disadvantage is the fact that the gain of PMT's are usually unstable, in that the current generated for a given count rate tends to vary with time. To overcome these difficulties, more precise measurements tends to be based on photon counting principles. By this method, the pulses of current are first processed by a energy discriminator circuit which discards pulses caused by gamma photons with energies outside a selected range. The

accepted pulses are then counted for a given time interval to determine the total count rate. Because of the finite length of the pulses (on the order of 1 μ s), this approach is difficult to implement when the count rate are too high, when pulses may overlap and cause miscounts. For NaI-PMT scintillators and gamma photon energies in the vicinity of 1 MeV, the maximum practical count rate $S_{\max}$ is about 100 kcounts/s.

The duration of the counting period depends on application. Recent measurements at the UIUC CAPTF have been based on counting period δt in the range of 10 to 30 ms. For a stochastic process, the accuracy with which one can determine the average value is on the order of $1/N^{1/2}$. Hence the maximum accuracy for count rate is about $(\delta t S_{\max})^{-1/2}$:

$$\frac{\delta S}{S} = \frac{1}{(\Delta t S_{\max})^{1/2}} \quad (5.3)$$

This maximum accuracy is attained only when the tracer is at the minimum distance from the NaI crystal. For particle tracking measurements, the system must be able to measure distance accurately for a range of distances. Let the closest working distance be denoted r_0 and the mean working distance be denoted r_m . It is clear that optimally the activity A should be selected such that the maximum counting rate $S_{\max}$ would occur at r_0

$$S_{\max} = \frac{d^2}{16r_0^2} \eta_c A \quad (5.4)$$

Hence the count rate at $r = r_m$ is

$$S_m = S_{\max} \frac{r_0^2}{r_m^2} \quad (5.5)$$

Combining the above results, the expected accuracy for distance for a mean working distance r_m is

$$|\delta r| = \frac{1}{2} \frac{r_m^2}{r_0} \frac{1}{(\Delta t S_{\max})^{1/2}} \quad (5.6)$$

It will now be convenient to express $r_m - r_0$ as the range R :

$$|\delta r| = \frac{1}{2} \frac{(r_0 + R)^2}{r_0} \frac{1}{(\Delta t S_{\max})^{1/2}} \quad (5.7)$$

According to the above estimates, the mean error for distance measurement for R of about 50 mm, δt of 20 ms, and

$S_{\max}$ of 10^5 count/s would be about 2 mm. These are conditions typical of CAPTF at UIUC. As a general rule, one may estimate R to be about $1/4$ or $1/3$ of the diameter of the bed, since the redundancy of a multi-detector arrangement reduces the importance of the most distal detector. For the METC 12" bed, R should be about 100 mm, leading to an error estimate of about 4 mm.

The above simple analysis suggests that previous measurement had not been performed at optimum detector placement.

5.2 Enhancement of Resolution by Deconvolution

If a set of data representing a function of position is "smeared" by poor position resolution, the original function can sometimes be partially restored by de-convolution. To accomplish this, the nature of "smearing", represented by a weighting function in the convolution integral, must be either known a priori, or experimentally measurable, or determinable by parameter estimation schemes. It also follows that if a set of data has random position errors, then the average of many sets of such data would have a statistically predictable smoothing effect, which can be partially undone by de-convolution. Also discussed is convolution-deconvolution problems for velocity distributions and methods of determining the weighting function.

5.2.1. The Basic Concept of Deconvolution

In many experimental observations the observed function is an integral, with an appropriate weighting function, of the function of interest. Examples include radiation (x-ray and visible light) measurements and data degraded by poor resolution and/or certain types of random noise. Another example is the remote sensing of temperature or heat flux moderated by intermediate heat conduction. The ready availability of modern computers renders it quite straightforward to obtain the original function by computation, although the accuracy of the process depends strongly on the quality of the data and the precision with which the weighting function is known. This class of problems is related to the inverse problems of which much has been written. The following discussion has been written specifically to illustrate the recovery of data due to poor spatial resolution, and hence may not conform to the terminology of the general field.

We will introduce the discussion by considering two simple 1-D examples in discrete form for which the deconvolution process can be easily explained.

Example 1.

Consider a 1-D function $f(x)$ represented by a discrete sequence of numbers corresponding to positions. It will be assumed that because of poor resolution, the observed value

at is not equal to $F(I)$, but is contaminated by the two neighboring values $F(I-1)$, $F(I+1)$:

$$\bar{F}(I) = G_- F(I-1) + G_0 F(I) + G_+ F(I+1), \quad I = 2, 3, \dots, N-1 \quad (5.8)$$

This is system of $N-2$ linear equations. $\bar{F}(0), \bar{F}(N)$ are not defined. If $\bar{F}(I)$ and the three coefficients G_- , G_0 and G_+ are knowns, the matrix can be inverted to yield $N-2$ values of $F(I)$ in a straightforward manner. Two values of $F(I)$, typically the end values $F(0)$ and $F(N)$, must also be known.

Example 2.

We will now extend the above example by considering a measuring system with even poorer resolution, so that the measured function $\bar{F}(I)$ represents a weighted average of $(2M+1)$ values of $F(I)$ centered around I . Here M is proportional to, and perhaps equal in magnitude to two or three times the ratio of the spatial resolution and Δx . The dependence of $\bar{F}(I)$ on $F(I)$ is described by the equation

$$\bar{F}(I) = \sum_{J=I-M}^{I+M} G(I, J) F(J) \quad I = M+1, M+2, \dots, N-M \quad (5.9)$$

The weighting factors $G(I, J)$ are typically functions of $(J-I)$, with decreasing magnitudes for large values of $|J-I|$. One example which might apply for many problems with random errors is related to the Gaussian distribution:

$$G(I, J) = \frac{e^{-|(J-I)\Delta x/R|^2}}{\sum_{J=I-M}^{I+M} e^{-|(J-I)\Delta x/R|^2}} \quad (5.10)$$

Here R denotes the length scale of the spatial resolution. It is quite clear that even though there are more terms in Eq. (5.9) than in Eq. (5.8), the mechanics of convolution and deconvolution is basically the same. The only difference is that there is now a deficit of $2M$ equations. Hence $2M$ values of $\bar{F}$, presumably near both ends of the sequence, must be known in order to carry out the deconvolution.

Note also the denominator in Eq. (5.9) in effect normalizes the weighting function. This is not necessary in the general formulation of the problem but is convenient and necessary if the two functions F and $\bar{F}$ are not defined to differ by a proportionality constant. In the normalized form the two functions approach each other in the limit that the scale of its spatial variations is large relative to the resolution.

Determination of the Weighting Function

From the above, it is quite clear that an accurate knowledge of the weighting function is needed for the deconvolution process. Note that for the problem of recovery of data degraded by poor resolution, the weighting function $G(J,I)$ actually represents the distribution function of $\bar{F}(J)$ due to a unit value of F at I . This can be determined in one of three ways:

Direct Experimental Observation

Experimental observation of G can be made by observing a simple known function, such as a delta function or a step function, centered at different I 's. In the context of radioactive particle tracking, this can be best accomplished by placing the radioactive particle at known positions, and then observe the distribution of the observed positions.

Theoretical Computation

In some cases the causes of the poor resolution are either known or can be modeled with adequate accuracy. For example, in optical measurements poor resolution is usually dominated by the accuracy of focus or diffraction. The former can be computed by ray tracing optics and the latter by wave optics. For gamma ray particle tracking, much of the position uncertainty is due to the randomness of the stream of discrete gamma photons. This is predictable. However, the random attenuation/scattering due to bubbly or turbulent bed materials is probably much more difficult to predict.

By Inverse Parameter Estimation

Under some conditions, the function G can also be determined by parameter estimation techniques based on experimental data. This is especially true when the general shape of G is known. For example, the Gaussian form of Eq. (5.10) might be a good first approximation in many resolution-degraded data. It has essentially only one parameter, R . The value of M is not too significant as long as it is large enough. If there is an *independent measure of the "quality"* of the recovered data, then different values of R can be tried, with the value producing the best data being accepted as the best value. If the accuracy or statistical quality of the data warrants, the basic Gaussian shape of the function can be replaced by more complex functions with more parameters, which can then be varied in the same way to achieve the "best" recovery.

At least in some aspects of radioactive particle tracking, there is indeed an *independent measure of the "quality"* of data. For example, in particle density distribution measurements, the density distribution should be zero outside the bed walls. In reality, the poor resolution of the technique often leads to a significant fraction of the data being outside the walls. Hence the percent of data outside the bed proper can be viewed as a negative measure of the overall quality of the data. Until there is more precise information, the Gaussian distribution of Eq. (5.10) may vary

well be a suitable first approximation to try out this technique.

5.2.2 Other Numerical Schemes

In the following analysis, the true value of an experiment is $f(x)$, where x is an independent variable range from 0 to L . Due to the error intrinsic to the experiment setup, the actual measured value is $F(x)$. The two values are related by

$$F(x) = \int_0^L f(\xi) N(x-\xi) d\xi \quad (5.11)$$

where the noise function $N(x-x)$ is inherent to the system itself. It can be determined by a calibration process in which the function $f(x)$ is known and sometimes taken to be delta function. Then the noise function of the system can be determined by solving (5.11) by collocation method. If $f(x)$ is a delta function at x_i , from the definition of delta function, then $F(x) = N(x-x_i)$. Therefore, the noise function can be constructed from the calibration data $F(x)$.

A. Collocation

If equation (5.11) is discretized on a uniform set point $0 \leq x_i \leq L$ and $0 \leq x_j \leq L$, where $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$, it can be written as:

$$F(x_i) = \sum_{j=1}^m f(\xi_j) N(x_i - \xi_j) \quad i = 1, 2, \dots, n \quad (5.12)$$

or in a vector and tensor form:

$$\underline{F} = \underline{N} \underline{f} \quad (5.13)$$

where vectors

$$\underline{F} = \begin{pmatrix} F(x_1) \\ F(x_2) \\ \dots \\ F(x_n) \end{pmatrix} \quad \text{and} \quad \underline{f} = \begin{pmatrix} f(\xi_1) \\ f(\xi_2) \\ \dots \\ f(\xi_m) \end{pmatrix} \quad (5.14)$$

and the tensor

$$\underline{N} = \begin{pmatrix} N(x_1 - \xi_1) & N(x_1 - \xi_2) & \dots & N(x_1 - \xi_m) \\ N(x_2 - \xi_1) & N(x_2 - \xi_2) & \dots & N(x_2 - \xi_m) \\ \dots & \dots & \dots & \dots \\ N(x_n - \xi_1) & N(x_n - \xi_2) & \dots & N(x_n - \xi_m) \end{pmatrix} \quad (5.15)$$

which is an $n \times m$ second rank tensor. When $n = m$, the equation (5.13) can be solved for $f(x)$ by an linear equation solver. When $n < m$, $f(x)$ can be obtained by a minimization process.

B. Spectral Method

If the value of the function $f(x)$ is not solved at the location ξ_j , it is rather solved for the magnitude of the modes in the Fourier Series

$$f(x) = \sum_{j=0}^{\infty} A_j \cos(2\pi x/L) \quad (5.16)$$

Substituting into (5.11), we have

$$F(x) = \sum_{j=0}^{\infty} A_j \int_0^L \cos\left(\frac{2\pi\xi}{L}\right) N(x-\xi) d\xi \quad (5.17)$$

Solve the equation for $x = x_1, x_2, \dots, x_n$ for mode upto $j = m$. After discretization of (5.17) we have:

$$F(x_i) = \sum_{j=0}^{\infty} A_j \int_0^L \cos\left(\frac{2\pi\xi}{L}\right) N(x_i-\xi) d\xi \quad (5.18)$$

which can be written as:

$$\underline{F} = \underline{B} \underline{f} \quad (5.19)$$

where tensors

$$\underline{N} = \begin{pmatrix} N(x_1 - \xi_1) & N(x_1 - \xi_2) & \dots & N(x_1 - \xi_m) \\ N(x_2 - \xi_1) & N(x_2 - \xi_2) & \dots & N(x_2 - \xi_m) \\ \dots & \dots & \dots & \dots \\ N(x_n - \xi_1) & N(x_n - \xi_2) & \dots & N(x_n - \xi_m) \end{pmatrix} \quad (5.20)$$

and

$$B(x_i - \xi_j) = \int_0^L \cos\left(\frac{2\pi\xi_j}{L}\right) N(x_i - \xi_j) d\xi \quad (5.21)$$

When $n=m$, the system of linear equations can be solved using Gaussian elimination. Form $m < n$, the following method can be used to solve the A_i 's which gives a least square fit of the experimental curve:

$$\underline{B}^T \underline{F} = \underline{B}^T \underline{B} \underline{f} \quad (5.22)$$

5.3 Sample Results

Fig. 5.1-5.4 showed the experimental data for the density distribution of a Couette flow. The fitted curve using three, four, five and six terms are plotted. It can be seen that with

Fig. 5.1 Data fitting using Fourier cosine series truncated at third term.

Fig. 5.2 Data fitting using Fourier cosine series truncated at fourth term.

Fig. 5.3 Data fitting using Fourier cosine series truncated at the fifth term.

Fig. 5.4 Data fitting using Fourier cosine series truncated at sixth term.

the increase of the number of terms in the fit, the fitted curve is more close to the experimental curve. The resolved solution can be obtained from the approximation.

6. Concluding Remarks

Discussions presented in this report highlight all the important considerations for making particle dynamics measurements using the radioactive active particle tracking methodology. It is seen that the technique makes it possible to obtain data which are not accessible by any other means. Among the advantages of the methods are:

1. It is completely uninvasive, and hence cannot disturb the flow.
2. It can penetrate opaque suspensions.
3. It yields detailed, spatially resolved (subject to resolution limitations) data on mean velocity and density distributions as well as other statistical quantities.

Its disadvantages are:

1. It can only provide time averaged data. It cannot provide instantaneous "snap shots" of the flow field.
2. It cannot yield information of flows around bubbles, which are of great interest in fluidization theories.
3. It has only modest resolution - of the order of a two to four percent of the flow field. However, the resolution can be improved by almost an order of magnitude, if long test runs can be used to acquire data of high statistical quality.
4. For data of adequate resolution, long test runs - several hours and up - are needed.

Clearly if used with a clear understanding of its capabilities as well as limitations, the radioactive particle tracking technique can make possible measurements which are not possible by any other method, and contribute significantly to the study of suspension flows.

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APPENDIX A

DATA REDUCTION SCHEME FOR DETERMINING TRACER LOCATION

The mathematical description of the scheme used for determination of tracer location follows. A monotonic relationship between count rate and distance of the tracer to each detector is established by a prior calibration. The density dependence of gamma ray attenuation through the bed makes it necessary to calibrate in-site because of the inhomogeneity of the bubbling fluidized bed. The availability of distance measurements from 16 scintillation detectors results in data redundancy. To take advantage of this planned redundancy, a weighted least squares method based on a linearization scheme is employed. The distance between the tracer and the i th detector is given by

$$(\xi - x_i)^2 + (\eta - y_i)^2 + (\zeta - z_i)^2 = r_i^2 \quad (\text{A.1a})$$

where ξ , η , and ζ denote the coordinates of the tracer to be determined, x_i , y_i , and z_i denote coordinates of the center of the i th crystal, r_i is its distance to the tracer, and $i = 1, 2, \dots, n$, n being the total number of detectors used. Physically the tracer position (ξ, η, ζ) can always be determined uniquely by four equations, employing one of them to resolve the ambiguity of multiple solutions if only three equations are used. However, the solutions obtained by solving any set of four equations out of n equations vary because of the inherent fluctuation of gamma ray count rates and the changing attenuation, however small, due to random motion of bubbles in the bed. Expanding Eq. (A.1a) gives

$$\mu - 2\xi x_i - 2\eta y_i - 2\zeta z_i = r_i^2 - x_i^2 - y_i^2 - z_i^2 \quad (\text{A.1b})$$

where $\mu = \xi^2 + \eta^2 + \zeta^2$. Rewriting Eq. (A.1b) in matrix form, we have

$$|X| \cdot |B| = |Z| \quad (\text{A.2})$$

in which

$$|X| = \begin{bmatrix} 1 & -2x_1 & -2y_1 & -2z_1 \\ 1 & -2x_2 & -2y_2 & -2z_2 \\ \cdot & \dots & \dots & \dots \\ \cdot & \dots & \dots & \dots \\ 1 & -2x_n & -2y_n & -2z_n \end{bmatrix} \quad \text{nx4} \quad (\text{A.2a})$$

$$|B| = \begin{bmatrix} \mu \\ \xi \\ \eta \\ \zeta \end{bmatrix} \quad 4 \times 1 \quad (\text{A.2b})$$

and

$$|Z| = \begin{bmatrix} r_1^2 - x_1^2 - y_1^2 - z_1^2 \\ r_2^2 - x_2^2 - y_2^2 - z_2^2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ r_n^2 - x_n^2 - y_n^2 - z_n^2 \end{bmatrix} \quad \text{nx1} \quad (\text{A.2c})$$

It should be noted that $|X|$, a hardware constant, depends only on the coordinates of the centers of the n detectors; $|B|$ consists of the

instantaneous coordinates of the tracers and $|Z|$ is determined by the observed distances from each detector. Since errors in the measurements of $r_1, r_2, \dots, r_n$ invariably exist, Eq. (A.2) is modified to become

$$|Z| = |X| \cdot |b| + |E| \quad (A.3)$$

where

$$|E| = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}_{n \times 1} \quad (A.3a)$$

which is the error vector for the estimation of $|b|$.

As pointed out previously, due to the statistical nature of gamma rays, the detector measurements are not equally accurate. The closer the tracer is to the detector, the better the signal-to-noise ratio. Therefore in order to obtain an optimal solution for $|b|$, appropriate weighting functions should be introduced. To this end, use is made of linear regression theory (Draper and Smith, 1966) and Eq. (A.2) is rewritten as

$$|X|^T \cdot |W|^{-1} \cdot |X| \cdot |b| = |X|^T \cdot |W|^{-1} \cdot |Z| \quad (A.4)$$

where $|b|$ is the estimated vector for $|b|$ which would minimize the sum of the matrix squared, $|E|^T \cdot |E|$ and $|W|$ is the weighting function matrix defined by

$$|W|^{-1} = \begin{bmatrix} 1/\sigma_1^2 & & & \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ & & & 1/\sigma_n^2 \end{bmatrix}. \quad (\text{A.5a})$$

A simple empirical relation has been assumed for σ :

$$\sigma_i = \text{constant} \times r_i^m \quad (\text{A.5b})$$

and, by trial and error, it was found that $m = 3$ would minimize the mean of $|\mu - \xi^2 - \eta^2 - \zeta^2|/\mu$. The tracer location is given by the solution vector $|b|$ which can be formally represented by

$$|b| = (|X|^T \cdot |W|^{-1} \cdot |X|)^{-1} \cdot |X|^T \cdot |W|^{-1} \cdot |Z|. \quad (\text{A.6})$$

Note the increased computational complexity which arises due to the addition of the weighting function. Rather than computing $(X^T X)^{-1}$ once, $(X^T W^{-1} X)^{-1}$ must be computed each time the tracer position is to be determined due to the fact that the weighting factor depends on the tracer distances r_i , $i = 1 \dots, n$. However prudent use of the purposely introduced redundancy enhances the accuracy of measurements through mitigation of the effect of secondary emission due to interaction of the tracer's primary radiation with system materials.

APPENDIX B GOVERNING EQUATIONS FOR THE RADIATION DETECTION

A.1 Bed and Detector Geometries

Referring to Fig. A.1, the origin of the bed coordinate system is located at the bottom center of the bed, the z axis being the center line of the cylindrical bed. Other parameters are defined as follows:

(x_i, y_i, z_i) = coordinates of the center of the i th detector crystal

$(\bar{x}, \bar{y}, \bar{z})$ = coordinates of the tracer particle

d_1 = internal radius of the bed

d_2 = thickness of the wall of the bed

D_1 = half length of the crystal

D_r = radius of the crystal

R = $[(x_i - \bar{x})^2 + (y_i - \bar{y})^2 + (z_i - \bar{z})^2]^{1/2}$

R_1 = length of the line segment inside the wall measured along the line connecting tracer and crystal center

R_2 = length of the line segment inside the bed measured along the line connecting tracer and crystal center

A.2 Determination of R_1 and R_2

The surface equations for the inner and outer bed walls are, respectively

$$x^2 + y^2 = d_1^2 \quad (A.1)$$

$$x^2 + y^2 = (d_1 + d_2)^2. \quad (A.2)$$

Figure A.1 Coordinates of the fluidized bed column and relative positions of the tracer and the i th detector

The equation of the line connecting the tracer and the center of the crystal is

$$\begin{aligned}x &= \bar{x} + (x_i - \bar{x})t \\y &= \bar{y} + (y_i - \bar{y})t \\z &= \bar{z} + (z_i - \bar{z})t\end{aligned}\tag{A.3}$$

where t is a variable. When $t = 0$, Eq. (A.3) gives the coordinates of the tracer. When $t = 1$, Eq. (A.3) gives the coordinates of the crystal center.

Substituting Eq. (A.3) into (A.1), we have

$$t_1 = \frac{\bar{x}^2 + \bar{y}^2 - (\bar{x}x_i + \bar{y}y_i) + \{[(x_i - \bar{x})^2 + (y_i - \bar{y})^2] d_1^2 - (\bar{x}y_i - x_i\bar{y})^2\}^{1/2}}{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}\tag{A.4}$$

By inserting t_1 into Eq. (A.3), the coordinates of point 1, illustrated in Fig. A.1, are obtained. Substituting (A.3) into (A.2), the equation for t_2 can be written as

$$t_2 = \frac{\bar{x}^2 + \bar{y}^2 - (\bar{x}x_i + \bar{y}y_i) + \{[(x_i - \bar{x})^2 + (y_i - \bar{y})^2] (d_1 + d_2)^2 - (\bar{x}y_i - x_i\bar{y})^2\}^{1/2}}{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}\tag{A.5}$$

Then inserting t_2 into Eq. (A.3), it gives the coordinates of point 2.

Then,

$$R_1 = R(t_2 - t_1) \quad (A.6)$$

$$R_2 = R t_1 \quad (A.7)$$

A.3 Attenuation of Gamma Rays through Particles

As mentioned in Section 2.1.2.1, the density of the glass particles is a function of position. The corresponding attenuation of gamma rays in Eq. (2.2) should therefore be evaluated using numerical integration. Referring to Fig. A.1, the line equation is given by (A.3) where $0 \leq t \leq R_2/R$. Then, the average density is found by

$$\bar{\rho}_2 = \frac{1}{R_2} \int_0^{R_2} \rho_2 dl. \quad (A.8)$$

To compute this integration, we divide R_2 into L segments, i.e. setting $\Delta t = R_2/R \cdot L$, then the above equation can be rewritten as

$$\bar{\rho}_2 R_2 = \sum_{j=1}^L \rho_2 (\bar{R}_j, \bar{z}_j) \Delta l \quad (A.9)$$

where

$$\Delta l = R \Delta t$$

$$\bar{R}_j = \{ [\bar{x} + (x_i - \bar{x}) (j - \frac{1}{2}) \Delta t]^2 + [\bar{y} + (y_i - \bar{y}) (j - \frac{1}{2}) \Delta t]^2 \}^{1/2}$$

$$\bar{z}_j = \bar{z} + (z_i - \bar{z}) (j - \frac{1}{2}) \Delta t.$$

$\bar{R}_j$ is measured from the axis of the bed.

Density ρ_2 is obtained from the corresponding experimental results. These data, however, were taken only in a system of positions. To simplify the computation for Eq. (A.9), an analytical expression of ρ_2 as a function of r and z is desired. This is accomplished by fitting the ρ_2 data with a two-dimensional polynomial surface, i.e.

$$\rho_2(r, z) = a_1 + a_2 r + a_3 z + a_4 r^2 + a_5 r z + a_6 z^2 + \dots \quad (A.10)$$

The program DENSE used to evaluate these coefficients is listed in A.6 of this appendix.

A.4 Computation of the Detector Efficiency

For convenience, define the bed coordinate as (x', y', z') .

(i) Transformation of (x', y', z') into particle coordinate (x, y, z)

Referring to Fig. A.2, the particle coordinate is selected such that, origin 0 is at the position of the tracer; y axis is parallel to the crystal axis; z axis intersects with the crystal axis; x axis makes the (x, y, z) coordinate system as a right-handed coordinate system.

Figure A.2 Position of NaI crystal with respect to the tracer coordinates

This coordinate transformation is a linear, rotational transform, which does not change the scale of the coordinates for the bed. Furthermore, since only the detector efficiency is to be computed, R and θ are the only parameters needed for this calculation. Therefore, the transformation equations are not given here.

θ is found from

$$\cos \theta = \frac{x_i^2 + y_i^2 - (x_i \bar{x} + y_i \bar{y})}{R \cdot (x_i^2 + y_i^2)^{1/2}} . \quad (\text{A.11})$$

(ii) Equations of the crystal surfaces

Denote the crystal front flat surface as I, back flat surface as III and the cylindrical surface as II.

Equation of surface I is

$$y = R_y - D_1 . \quad (\text{A.12})$$

Equation of surface II is

$$(z - R_z)^2 + x^2 = D_r^2 . \quad (\text{A.13})$$

Equation of surface III is

$$y = R_y + D_1 \quad (\text{A.14})$$

where

$$R_z = R \sin \theta \quad (\text{A.15})$$

$$R_y = R \cos \theta. \quad (\text{A.16})$$

(iii) Determination of gamma ray penetrating distance r in the crystal

There are four cases where r should be evaluated differently as shown in Figs. A.3a-d.

Case (a) gamma ray enters the crystal from I and exits from II.

The line equation is:

$$\begin{aligned} x &= x_1 t \\ y &= y_1 t \\ z &= z_1 t \end{aligned} \quad (\text{A.17})$$

where (x_1, y_1, z_1) is the coordinates of point 1 as shown in Fig. A.3a.

It is found to be

$$\begin{aligned} y_1 &= R_y - D_1 \\ x_1 &= y_1 \cot \phi \\ z_1 &= y_1 \cot \theta (1 + \cot^2 \phi)^{1/2}. \end{aligned} \quad (\text{A.18})$$

Figure A.3 Penetration length of gamma rays in the crystal

In Eq. (A.17), $t = 1$ gives the coordinates of point 1.

Substituting (A.17) into (A.13), the solution is

$$t_2 = \frac{z_1 R_z + [z_1^2 R_z^2 - (R_z^2 - D_r^2)(x_1^2 + z_1^2)]^{1/2}}{(x_1^2 + z_1^2)} \quad (A.19)$$

Inserting t_2 into Eq. (A.17) gives the coordinates of point 2 as

$$\begin{aligned} x_2 &= x_1 t_2 \\ y_2 &= y_1 t_2 \\ z_2 &= z_1 t_2 \end{aligned} \quad (A.20)$$

Then

$$r = [x_1^2 + y_1^2 + z_1^2]^{1/2} (t_2 - 1). \quad (A.21)$$

Case (b) gamma ray enters the crystal from I and exits from III.

The coordinates of point 1 are given in Eq. (A.18), the coordinates of point 2 are

$$\begin{aligned} y_2 &= R_y + D_1 \\ x_2 &= y_2 \cot \phi \\ z_2 &= y_2 \cot \theta (1 + \cot^2 \phi)^{1/2}. \end{aligned} \quad (A.22)$$

Then

$$r = 2 D_1 [(1 + \cot^2 \theta) (1 + \cot^2 \phi)]^{1/2}. \quad (\text{A.23})$$

Case (c) gamma ray enters the crystal from II and exits from II.

The line equation is:

$$x = t \sin \theta \cos \phi,$$

$$y = t \sin \theta \sin \phi, \quad (\text{A.24})$$

$$z = t \cos \theta.$$

Substituting (A.24) into (A.13), the solution is:

$$t_{1,2} = \frac{\cos \theta R_z \pm [\cos^2 \theta R_z^2 - (\cos^2 \theta + \sin^2 \theta \cos^2 \phi) (R_z^2 - D_r^2)]^{1/2}}{\cos^2 \theta + \sin^2 \theta \cos^2 \phi} \quad (\text{A.25})$$

Then

$$r = 2 \frac{[\cos^2 \theta R_z^2 - (\cos^2 \theta + \sin^2 \theta \cos^2 \phi) (R_z^2 - D_r^2)]^{1/2}}{\cos^2 \theta + \sin^2 \theta \cos^2 \phi} \quad (\text{A.26})$$

Case (d) gamma ray enters the crystal from II and exits from III.

Similar as in case (a), the solution is

$$t_1 = \frac{z_2 R_z - [z_2^2 R_z^2 - (R_z^2 - D_r^2)(x_2^2 + z_2^2)]^{1/2}}{x_2^2 + z_2^2} \quad (\text{A.27})$$

and

$$r = (x_2^2 + y_2^2 + z_2^2)^{1/2} (1 - t_1) \quad (\text{A.28})$$

where (x_2, y_2, z_2) is given from (A.22).

(iv) Limiting angles of θ and ϕ for solid angle Ω .

Referring to Fig. A.4. There are four special angles of θ at points 1, 2, 3 and 4. They are found as

$$\theta_1 = \tan^{-1} \left(\frac{R_y - D_1}{R_z + D_r} \right) \quad (\text{A.29})$$

$$\theta_2 = \tan^{-1} \left(\frac{R_y - D_1}{R_z - D_r} \right) \quad (\text{A.30})$$

$$\theta_3 = \tan^{-1} \left(\frac{R_y + D_1}{R_z + D_r} \right) \quad (\text{A.31})$$

$$\theta_4 = \tan^{-1} \left(\frac{R_y + D_1}{R_z - D_r} \right) \quad (\text{A.32})$$

And there is a minimum angle of ϕ at point 5, which is

$$\phi_5 = \tan^{-1} \left(\frac{R_y - D_1}{D_r} \right) \quad (\text{A.33})$$

The edge of the surface I is found along the semi-circle 1-5-2, where ϕ is a function of θ . It defines the minimum ϕ for the integration of solid angle. This edge belongs to surfaces I and II, therefore, by solving Eqs. (A.18) and (A.13) simultaneously, the minimum angle ϕ_1 is found from

$$\cot^2 \phi_1 = \left(\frac{R_z \cot \theta + [(y_1^2 + D_1^2)(1 + \cot^2 \theta) - R_z^2]^{1/2}}{y_1 (1 + \cot^2 \theta)} \right)^2 - 1 \quad (\text{A.34})$$

where y_1 is given in Eq. (A.18).

For the case where the lines enter from surface II, the minimum angle ϕ_2 is obtained by setting $t_1 = t_2$ in Eq. (A.25). It is found from

$$\cos \phi_2 = \frac{D_r \cot \theta}{(R_z^2 - D_r^2)^{1/2}} \quad (\text{A.35})$$

(v.) Schemes for Computing the Detector Efficiency

Equation (2.6) can be written as

$$\Omega \cdot \epsilon = \iint_{\Omega} (1 - e^{-\mu r}) \sin \theta \, d\theta \, d\phi. \quad (\text{A.36})$$

Since the crystal geometry is symmetric to the y-z plane, the above integration can be performed in the first and fourth quadrants only.

Dividing the integration for θ into N segments and ϕ into M segments, i.e.

$$\Delta\theta = \frac{\theta_{\max} - \theta_1}{N}$$

$$\Delta\phi = \frac{\frac{\pi}{2} - \phi_5}{M}$$

where $\theta_{\max} = \text{Max}(\theta_2, \theta_4)$. Then the numerical integration formula for (A.36) is

$$\Omega\epsilon = 2 \sum_{i=1}^N \sum_j [1 - \exp(-\mu r(\theta_i, \phi_j))] \sin \theta_i \Delta\theta \Delta\phi \quad (\text{A.37})$$

where

$$\theta_i = \theta_1 + (i - \frac{1}{2}) \Delta\theta$$

$$\phi_j = \frac{\pi}{2} - (j - \frac{1}{2}) \Delta\phi$$

where the j summation is restricted by ϕ_1 or ϕ_2 from Eqs. (A.34) or (A.35), i.e., $\phi_j \geq \text{Min}(\phi_1, \phi_2)$, r is calculated as follows.

In the range of (θ_1, θ_2) and $\phi_j > \phi_1$, r is found from the smaller value of Eqs. (A.21) and (A.23). In the range of (θ_2, θ_4) , or in the range of (θ_1, θ_2) and $\phi_2 < \phi_j < \phi_1$ when $\theta_2 < \theta_4$, r is found from the smaller value of Eqs. (A.26) and (A.28).

A.5 Correction to the Crststal Container

As mentioned in Section 2.1.4.1, the aluminum wall of the detector also causes gamma ray attenuation and scattering. Since the wall is very thin, only the attenuation effect is considered here. This introduces a factor $e^{-\mu r_{12}}$ into the integral (A.36) where μ is the linear attenuation coefficient of aluminum (which is 0.0166 per millimeter [10]) and r_{12} is the gamma ray penetrating distance in the wall.

Referring to Fig. A.5, R_d is the outside radius of the detector, L_d is the length from crystal center to the front edge of the detector, δ_1 and δ_2 are wall thicknesses. r_{12} can be computed in two cases distinguished by R_r

$$R_r = \{ [R_z - (R_y - L_d) \frac{\cot \theta}{\sin \phi}]^2 + [(R_y - L_d) \cot \phi]^2 \}^{1/2} \quad (\text{A.38})$$

Performing procedures similar to those in the previous section, it was found that when $R_r \leq R_d$,

$$r_{12} = \frac{\delta_1}{\sin \theta \sin \phi} \quad (\text{A.39})$$

and when $R_r > R_d$,

$$r_{12} = \frac{R_d \delta_2}{[\cos^2 \theta R_z^2 - (\cos^2 \theta + \sin^2 \theta \cos^2 \phi)(R_z^2 - R_d^2)]^{1/2}} \quad (\text{A.40})$$

The numerical values of the parameters are $R_d = 28.4$ mm, $L_d = 30.9$ mm, $\delta_1 = 5.5$ mm, and $\delta_2 = 3.0$ mm.

(a)

(b)

Figure A.5 Penetration length of gamma rays in the detector container

A.6 Programs

Program DENSE is used to fit the discrete density distribution of particles by a two-dimensional polynomial function expressed by Eq. (A.10).

```

PROGRAM DENSE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
  DIMENSION U(20,50),V(20,50),RO(20,50),IRO(20,50),X(20),
1          Y(50),Z(28),XY(28,28),A(28)
C***** H --- Length of the Compartment *****
C***** JMEAN --- Compartment No. below which Density Is Fitted ****
  DATA H,JMEAN/0.92,25/
C***** INPUT DATA *****
C* IN --- No. of Columns of the Input Data *
C* JM --- No. of Rows of the Input Data *
C* L --- Order of the Polynomial Fit for Density *
C* PM --- Total Mass of Particles inside the Bed (Kg) *
C* U,V --- Velocity Distributions in X and Y direction *
C* IRO --- Occurrence Distribution of Tracer inside the Bed *
C*****
  READ(5,*) IN,JM,L,PM
  READ(5,*) ((U(I,J),I=1,IN),J=1,JM),((V(I,J),I=1,IN),J=1,JM)
  READ(5,*) ((IRO(I,J),I=1,IN),J=1,JM)
C***** Find the Positions where Data Are Defined *****
  DO 40 I=1,IN
    X(I)=0.5*FLOAT(2*I-1)*H
40  CONTINUE
  DO 50 J=1,JM
    Y(J)=0.5*FLOAT(2*J-1)*H
50  CONTINUE
  WRITE(6,60) ((IRO(I,J),I=1,IN),J=1,JM)
C***** Determine the Relation between Occurrence and Mass *****
  JMM=0
  DD=0.
  DMEAN=0.
  DO 9 I=1,IN
    DO 5 J=1,JM
      RRR=FLOAT(IRO(I,J))
      DD=DD+RRR
      IF(J.LE.JMEAN) DMEAN=DMEAN+RRR
      RO(I,J)=RRR/(X(I)*H)
      IF(IRO(I,J).GT.10) II=J
5    CONTINUE
      IF(II.GT.JMM) JMM=II
9    CONTINUE
  VV=3.1415926*H**3*FLOAT(IN*IN*JMEAN)
  DMEAN=DMEAN*PM/(DD*VV)
C***** Determine Matrix for Polynomial Fitting *****

```

```

K1=L+1
NUM=0
DO 1 KKK=1,K1
  DO 1 III=1,KKK
    II=III-1
    KI=KKK-III
    NUM=NUM+1
    NUM1=0
    DO 2 MMM=1,K1
      DO 2 NNN=1,MMM
        KP=II+NNN-1
        LQ=KI+MMM-NNN
        NUM1=NUM1+1
        PP=0.
        DO 3 I=1,IN
          DO 3 J=1,JMM
            PP=PP+X(I)**KP*Y(J)**LQ
3          CONTINUE
          XY(NUM,NUM1)=PP
2        CONTINUE
        PP=0.
        DO 4 I=1,IN
          DO 4 J=1,JMM
            PP=PP+RO(I,J)*X(I)**II*Y(J)**KI
4          CONTINUE
          Z(NUM)=PP
1        CONTINUE
C***** Solve Matrix Equation to Get Coefficients in the Polynomial
        NN=K1*(K1+1)/2
        NN1=NN-1
        DO 6 K=1,NN1
          KK=K+1
          DO 6 I=KK,NN
            Z(I)=Z(I)-Z(K)*XY(I,K)/XY(K,K)
            DO 6 J=KK,NN
              XY(I,J)=XY(I,J)-XY(K,J)*XY(I,K)/XY(K,K)
6          CONTINUE
          DO 7 I=1,NN
            A(I)=Z(I)/XY(I,I)
7          CONTINUE
          DO 8 I=1,NN1
            NNI=NN-I
            DO 8 J=1,I
              NNIJ=NNI+J
              A(NNI)=A(NNI)-XY(NNI,NNIJ)*A(NNIJ)/XY(NNI,NNI)
8          CONTINUE
C***** Calculate the Total Occurrence by the Polynomial *****
        DD1=0.
        DO 70 I=1,IN
          DO 70 J=1,JMM
            NUM=0
            RO(I,J)=0.

```

```

      DO 80 II=1,K1
        DO 80 JJ=1,II
          NUM=NUM+1
          JP=JJ-1
          IQ=II-JJ
          RO(I,J)=RO(I,J)+A(NUM)*X(I)**JP*Y(J)**IQ
80      CONTINUE
        IF(RO(I,J).GT.O.) DD1=DD1+RO(I,J)*X(I)*H
70      CONTINUE
C***** Normalize Those Coefficients *****
      CC1=DD/DD1
      CC=6.283185*DD1/PM
      DO 30 I=1,NN
        A(I)=A(I)/CC
30      CONTINUE
C***** Calculate Occurrence and Density Distributions *****
      DO 90 I=1,IN
        DO 90 J=1,JMM
          IRO(I,J)=RO(I,J)*X(I)*H*CC1
          RO(I,J)=RO(I,J)/CC
90      CONTINUE
C***** OUTPUT DATA *****
C* IRO --- Modified Occurrence Distribution *
C* RO --- Density Distribution *
C* K1-1 --- Order of the Polynomial Defining Density *
C* NN --- No. of Coefficients for the Polynomial *
C* DMEAN --- Mean Density within the Compartment Column JMEAN *
C* JMM --- Compartment No. below which Output Data Are Given *
C* A --- Coefficients in the Polynomial (Density Unit g/cm**3) *
C*****
      WRITE(6,60) ((IRO(I,J),I=1,IN),J=1,JMM)
      WRITE(6,61) ((RO(I,J),I=1,IN),J=1,JMM)
      WRITE(6,*) K1,NN,DMEAN,JMM
      WRITE(6,*) (A(I),I=1,NN)
60      FORMAT(10I8)
61      FORMAT(10F8.4)
      STOP
      END

```

Program G22 performs calculations of count rate as a function of tracer position and various parameters.

```

PROGRAM G22(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
DIMENSION R(156),THETA(156),PHI(156),K(156),CRN(156),CRM(156)
COMMON /BL1/X(156),Y(156),Z(156)/BL2/XI,YI,ZI,LL
COMMON /BL3/NORDER,DEN(28)
C***** DATA *****
C* D1 --- Inner Radius of the Bed (mm) *
C* D2 --- Thickness of the Bed Wall (mm) *
C* DR --- Radius of the NaI Crystal (mm) *
C* DL --- Half Length of the NaI Crystal (mm) *
C* DELTA1 --- Length from Front Edge of Detector to Crystal (mm) *
C* RD --- Radius of the Detector (mm) *
C* APGLAS,ANAI,AALUMI --- Linear Attenuation Coefficients of *
C*                               Plexiglass (bed wall), NaI (crystal) *
C*                               and Aluminum (detector container) (1/mm) *
C* AGLASS --- Mass Attenuation Coefficient of Glass (cm*dm/g) *
C* AE,BE --- Constants for the Buildup Function *
C* LL,NN,MM --- Segments for Integrations of Density *
C*                               and Detector Efficiency *
C*****
      DATA D1,D2,DR,DL,DELTA1,RD,PI/92.,8.,25.4,25.4,5.5,
1      28.4,3.1415926/
      DATA APGLAS,ANAI,AALUMI,AGLASS,AE,BE,LL,NN,MM/0.0081,
1      0.0213,0.0166,0.0064,0.9,0.03,20,20,10/
C***** INPUT DATA *****
C* ICH --- Detector Number *
C* AA --- Two Times of the Proportional Constant for True Rate n *
C* ACT --- Activity of the Tracer (micro-Curie) *
C* CO --- Background Counts *
C* TAU1,TAU2,ALPHA --- Parameters for Dead Time *
C* (XI,YI,ZI) --- Coordinates of ith Crystal Center (mm) *
C* NORDER-1 --- Order of the Polynomial Defining Glass Density *
C* NCOEF --- No. of Coefficients for the Polynomial *
C* DEN --- Coefficients in the Polynomial (density unit g/cm**3) *
C* (X,Y,Z) --- Coordinates of the Tracer (mm) *
C*****
      READ(5,*) ICH,AA,ACT,CO,TAU1,TAU2,ALPHA,XI,YI,ZI
      READ(5,*) NORDER,NCOEF
      READ(5,*) (DEN(I),I=1,NCOEF)
      READ(5,*) (X(I),Y(I),Z(I),I=1,156)
      C1=SQRT(XI**2+YI**2)
      DO 30 I=1,156
C***** Determine Relative Position of Tracer w.r.t. the Detector **
      XP=C1-(XI*X(I)+YI*Y(I))/C1
      YP=(YI*X(I)-XI*Y(I))/C1
      ZP=ZI-Z(I)
      C2=XP**2+YP**2

```



```

C3=(C1*YP)**2
C4=SQRT(YP**2+ZP**2)
IF(C4.EQ.0.) C4=1.
C***** The Relative Position in Spherical Coordinate *****
R(I)=SQRT(C2+ZP**2)
THETA(I)=57.29578*ACOS(XP/R(I))
PHI(I)=180.
IF(ZP.NE.-C4) PHI(I)=57.29578*ACOS(ZP/C4)
C***** R1 --- Length of Gamma-ray Penetrating in the Bed Wall *****
R1=R(I)*(SQRT(C2*(D2+D1)**2-C3)-SQRT(C2*D1**2-C3))/C2
T3=(XP*C1-SQRT(C2*D1**2-C3))/C2
C***** R2 --- Length of Gamma-ray Penetrating in the Particle *****
R2=R(I)*(1.-T3)
C***** Get the Average Density of the Glass Particle *****
C***** along the Path of Gamma-ray Penetration *****
CALL GMASP(R(I),R2,I,RHO2R2)
C***** 2*CR --- Product of the Solid Angle and Detector Efficiency
CR=0.
C***** K --- No. of Compartments for Solid Angle integration *****
K(I)=0
Z11=R(I)*SQRT(1.-(XP/R(I))**2)
XS=XP-DL
XA=XP+DL
C***** Determine the Limit Angles of Theta for Solid Angle *****
Q1=ATAN(XS/(Z11+DR))
Q2=ATAN(XS/(Z11-DR))
Q4=ATAN(XA/(Z11-DR))
IF(Q2.LT.0.) Q2=PI+Q2
IF(Q4.LT.0.) Q4=PI+Q4
C***** Determine the Limit Angle of Phi for Solid Angle *****
P5=ATAN(XS/DR)
C***** Determine Size of Compartment for integration of Solid Angle
DP=(0.5*PI-P5)/FLOAT(MM)
QMAX=Q2
IF(Q4.GT.Q2) QMAX=Q4
DQ=(QMAX-Q1)/FLOAT(NN)
DO 140 N=1,NN
C***** Q --- Theta(i) (within the solid angle) *****
Q=Q1+DQ*(FLOAT(N)-0.5)
BBB=SIN(Q)
BB=COS(Q)/BBB
B=Z11*BB
IF(Q.GT.Q2) GOTO 150
D=1.+BB**2
E=SQRT(D*(DR**2+XS**2)-Z11**2)
C***** P1,P2 --- Limit Angles of Phi for integration of Solid Angle
P1=ATAN(D*XS/SQRT((E+B)**2-(D*XS)**2))
IF(Q4.LE.Q2) GOTO 156
150 DRBBZ1=DR*BB/SQRT(Z11**2-DR**2)
IF(ABS(DRBBZ1).GE.1.) GOTO 156
P2=ACOS(DRBBZ1)
IF(P2.GT.P5) GOTO 155

```

```

156   P2=0.5*PI
155   DO 160 M=1,MM
C***** P --- Phi(j) (within the solid angle) *****
      P=0.5*PI-DP*(FLOAT(M)-0.5)
      F=SQRT(1.+1./TAN(P)**2)
      A=(BB**2+1.)*F**2-1.
C***** R4 --- Length of Gamma-ray Penetrating in Detector Container
      R4=DELTA1/(SIN(P)*BBB)
      IF(Q.GT.Q2) GOTO 100
      IF(P.LE.P1) GOTO 100
      T2=(B*F+SQRT((B*F)**2-A*(Z11**2-DR**2)))/(XS*A)
C***** RR1,RR2 --- Length of Gamma-ray Penetrating in the Crystal *
      RR1=2.*DL*SQRT(1.+A)
      RR2=XS*SQRT(1.+A)*(T2-1.)
      GOTO 110
100   IF(P.LE.P2) GOTO 140
      C=(BB*BBB)**2+(BBB*COS(P))**2
      T1=(B*F-SQRT((B*F)**2-A*(Z11**2-DR**2)))/(XA*A)
      RR1=2.*SQRT((B*BBB)**2-C*(Z11**2-DR**2))/C
      RR2=XA*SQRT(1.+A)*(1.-T1)
C***** RR --- Rr (for determining R4) *****
      RR=SQRT((Z11-(XP-DL-DELTA1)*BB/SIN(P))**2+
1      (F**2-1.)*(XP-DL-DELTA1)**2)
      IF(RR.GT.RD) R4=SQRT((B*BBB)**2-C*(Z11**2-RD**2))/C-0.5*RR1
110   IF(RR2.LT.RR1) RR1=RR2
      CR=CR+(1.-EXP(-ANAI*RR1))*BBB*DQ*DP*EXP(-AALUMI*R4)
      K(I)=K(I)+1
160   CONTINUE
140   CONTINUE
C***** Evaluation of Buildup Function *****
      FL=(AGLASS*RHO2R2+APGLAS*R1)*SQRT(1.-(ZP/R(I))**2)
      BF=1.+AE*FL*EXP(BE*FL)
C***** CRN --- True Rate n *****
      CRN(I)=AA*ACT*BF*CR*EXP(-APGLAS*R1-AGLASS*RHO2R2)
C***** CRM --- Recorded Rate m *****
      CRM(I)=CRN(I)*((1.-ALPHA)*EXP(-TAU1*CRN(I))+
1      ALPHA*EXP(-TAU2*CRN(I)))+CO
30   CONTINUE
      WRITE(6,40) (K(I),CRM(I),R(I),CRN(I),I=1,156)
40   FORMAT(I4,F16.5,F13.2,F16.5)
      STOP
      END
C*****
C***** Subroutine GMASP determines the mean density of glass *****
C***** particles along the path of gamma-ray penetration *****
C*****
      SUBROUTINE GMASP(R,R2,I,RHO2R2)
      COMMON /BL1/X(156),Y(156),Z(156)/BL2/XI,YI,ZI,LL
      COMMON /BL3/NORDER,DEN(28)
      X1=XI-X(I)
      Y1=YI-Y(I)
      Z1=ZI-Z(I)

```

```

C***** Determine the size of integration for density *****
      DL=R2/FLOAT(LL)
      DT=DL/R
C***** RHO2R2 --- Product of Mean Density and Penetrating Length **
      RHO2R2=0.
      DO 1 L=1,LL
C***** Determine the Position for Integration of Density *****
      ZB=Z(I)+Z1*DT*(FLOAT(L)-0.5)
      RB=SQRT((X(I)+X1*DT*(FLOAT(L)-0.5))**2+(Y(I)+
1      Y1*DT*(FLOAT(L)-0.5))**2)
C***** Calculate the Density at Position (RB,ZB) *****
      NUM=0
      RHO2=0.
      DO 2 K=1,NORDER
        DO 2 J=1,K
          NUM=NUM+1
          JJ=J-1
          KK=K-J
          RHO2=RHO2+DEN(NUM)*((RB/10.)**JJ)*((ZB/10.)**KK)
2      CONTINUE
      IF(RHO2.LT.0.) RHO2=0.
      RHO2R2=RHO2R2+RHO2*DL
1    CONTINUE
      RETURN
      END

```

APPENDIX C: STREAM FUNCTION CALCULATION

The following equations are derived for a two-dimensional, cylindrical coordinate case. These equations can also be used for the Cartesian coordinate by some simple changes as stated later.

As illustrated in Fig. C.1, the experimental fluidized bed data, $\rho_{i,j}$, $u_{i,j}$, $v_{i,j}$, were given on the centers of the set of compartments where $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$.

C.1 Smoothing Density Distribution

The density averaging scheme is

$$\bar{\rho}_{i,j} = \frac{1}{8} (4\rho_{i,j} + \rho_{i,j-1} + \rho_{i,j+1} + \rho_{i-1,j} + \rho_{i+1,j}). \quad (C.1)$$

Its boundary conditions are assumed to be symmetric, i.e.,

$$\rho_{0,j} = \rho_{1,j}$$

$$\rho_{n+1,j} = \rho_{n,j}$$

(C.2)

$$\rho_{i,0} = \rho_{i,1}$$

$$\rho_{i,m+1} = \rho_{i,m}$$

C.2 Computation of the Two Stream Functions

Using a midpoint integration formula, Eqs. (3.16) and (3.17) can be expressed as

Figure C.1 Locations of velocity and density data in the fluidized bed

$$\psi_{1,i,j+1} = \psi_{1,i,j} + r_i \rho_{i,j} u_{i,j} \Delta z \quad (C.3)$$

$$\psi_{2,i+1,j} = \psi_{2,i,j} - r_i \rho_{i,j} v_{i,j} \Delta r. \quad (C.4)$$

For the Cartesian coordinate, r_i is replaced by unity. The corresponding boundary conditions are

$$\psi_{1,i,1} = 0 \quad i=1, 2, \dots, n \quad (C.5)$$

$$\psi_{2,1,j} = 0 \quad j=1, 2, \dots, m. \quad (C.6)$$

$\psi_{1,i,j}$ and $\psi_{2,i,j}$ are defined on different points not coinciding with the data point (i,j) , as illustrated in Fig. C.2.

C.3 Correction of ψ_1 and ψ_2

The correction of the stream function ψ_2 , Eq. (3.21), can be expressed as

$$\bar{\psi}_{2,i,j} = \psi_{2,i,j} - \psi_{2,n+1,j} \left(\frac{i-1}{n}\right)^2. \quad (C.7)$$

For the Cartesian coordinate, the power 2 in the last term is replaced by unity.

For the stream function ψ_1 , there are two cases for determining its upper boundary condition.

Figure C.2 Positions of the stream functions Ψ_1 and Ψ_2

(i.) The upper boundary can be found within the data domain. That is, for a fixed r_i , if at $j = k$, ($k \leq m$), $u_{i,k} = v_{i,k} = 0$ (further examination is made to make sure that $u_{i,k+1} = v_{i,k+1} = 0$), the upper boundary is found at

$$z_0(r_i) = (k-1) \Delta z \quad (C.8)$$

where ψ_1 should be zero. Therefore, ψ_1 is corrected by

$$\bar{\psi}_{1,i,j} = \psi_{1,i,j} - \psi_{1,i,k} \left(\frac{j-1}{k-1} \right) \quad (C.9)$$

for $j = 1, 2, \dots, k-1$, and $\bar{\psi}_{1,i,j} = 0$ for $j = k, k+1, \dots, m$. where k is a function of i .

(ii.) The upper boundary cannot be found within the data domain. That is, for a fixed r_i , if at $j = m$, $u_{i,m} \neq 0$ or $v_{i,m} \neq 0$, then, $\bar{\psi}_{1,i,m+1}$ needs to be prescribed. It is determined as follows:

The "total" change of ψ_1 from $j = m$ to $j = m+1$ is

$$\Delta \psi_{1\Sigma} = \sum_{i=1}^n (\psi_{1,i,m+1} - \psi_{1,i,m}). \quad (C.10)$$

The "total" value of $\bar{\psi}_2$ at $j = m$ is

$$\bar{\psi}_{2\Sigma} = \sum_{i=1}^n \bar{\psi}_{2,i,m}. \quad (C.11)$$

Smoothing the distribution of ψ_1 on line $j = m + 1$ by

$$\bar{\psi}_{1,i,m+1} = \frac{1}{4} (\psi_{1,i-1,m+1} + 2\psi_{1,i,m+1} + \psi_{1,i+1,m+1}) \quad (C.12)$$

where at $i=1$

$$\bar{\psi}_{1,1,m+1} = \frac{1}{4} (2.5 \psi_{1,1,m+1} + 0.5 \psi_{1,2,m+1}) \quad (C.13)$$

and at $i=n$

$$\bar{\psi}_{1,n,m+1} = \frac{1}{4} (0.5 \psi_{1,n-1,m+1} + 2.5 \psi_{1,n,m+1}). \quad (C.14)$$

The "total" value of $\bar{\psi}_1$ at $j = m+1$ is

$$\bar{\psi}_{1\Sigma} = \sum_{i=1}^n \bar{\psi}_{1,i,m+1}. \quad (C.15)$$

Then the upper boundary condition for $\bar{\psi}_1$ at $j = m+1$ is

$$\bar{\psi}_{1,i,m+1} = \bar{\psi}_{1,i,m+1} \frac{\bar{\psi}_{2\Sigma} + \Delta\psi_{1\Sigma}}{\bar{\psi}_{1\Sigma}}. \quad (C.16)$$

Therefore ψ_1 is corrected by

$$\bar{\psi}_{1,i,j} = \psi_{1,i,j} - (\psi_{1,i,m+1} - \bar{\psi}_{1,i,m+1}) \left(\frac{j-1}{m} \right) \quad (C.17)$$

for $j = 1, 2, \dots, m$.

The stream functions $\bar{\psi}_1$ and $\bar{\psi}_2$ satisfy all the boundary conditions (3.12) through (3.15).

C.4 Combining $\bar{\psi}_1$ and $\bar{\psi}_2$ by Smoothing Scheme

The smoothing scheme was discussed in Section 3.2.2. The mathematical formulas are

$$\begin{aligned} \bar{\psi}_{1,i,j} = \frac{1}{8} [4\bar{\psi}_{1,i,j} + \bar{\psi}_{2,i,j} + \bar{\psi}_{2,i+1,j} \\ + \bar{\psi}_{2,i,j-1} + \bar{\psi}_{2,i+1,j-1}] \end{aligned} \quad (C.18)$$

for $i = 1, 2, \dots, n$; $j = 2, 3, \dots, m$.

$$\begin{aligned} \bar{\psi}_{2,i,j} = \frac{1}{8} [4\bar{\psi}_{2,i,j} + \bar{\psi}_{1,i,j} + \bar{\psi}_{1,i,j+1} \\ + \bar{\psi}_{1,i-1,j} + \bar{\psi}_{1,i-1,j+1}] \end{aligned} \quad (C.19)$$

for $i = 2, 3, \dots, n$; $j = 1, 2, \dots, m$.

C.5 Programs

Program SMOOTH calculates and smoothes the two stream functions defined in the fluidized bed.

```

PROGRAM SMOOTH(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
  DIMENSION U(21,51),V(21,51),RHO(21,51),JN(51),PSI2(21,51),
1          PSI1(21,51),IRHO(21,51),R(20),P(51),P1(51),
2          RHO1(2,52)
C***** INPUT PARAMETERS *****
C* IN --- NUMBER OF COLUMNS OF THE INPUT DATA *
```

```

C* JM --- NUMBER OF ROWS OF THE INPUT DATA *
C* MNI --- CONTROL VARIABLE (1 FOR CARTESIAN, 2 FOR CYLINDRICAL) *
C* IH --- ROW ABOVE IT THE UPPER BOUNDARY IS SEARCHED *
C* IRP --- TIMES OF SMOOTHING PROCESS FOR VELOCITY DISTRIBUTION *
C* IRO --- TIMES OF SMOOTHING PROCESS FOR DENSITY DISTRIBUTION *
C* ZO --- HIGHT OF THE BALL CENTER (IN.) *
C* RO --- RADIUS OF THE BALL (IN.) (ZERO IF NO BALL INSIDE THE BED) *
C* TMASS --- TOTAL MASS INSIDE THE BED (KILOGRAMS) *
C***** INPUT DATA *****
C* U(IN,JM) --- VELOCITY DISTRIBUTION OF X OR R DIRECTION *
C* V(IN,JM) --- VELOCITY DISTRIBUTION OF Y OR Z DIRECTION *
C* IRHO(IN,JM) --- OCCURRENCE DISTRIBUTION (RELATED TO DENSITY) *
C*****
      READ(5,*) IN,JM,MNI,IH,IRP,IRO,ZO,RO,TMASS
      READ(5,*) ((U(I,J),I=1,IN),J=1,JM),((V(I,J),I=1,IN),J=1,JM)
      READ(5,*) ((IRHO(I,J),I=1,IN),J=1,JM)
C***** SET GRID SIZE FOR XY OR RZ COORDINATES *****
      DX=9.69
      DY=15.0
      DXO=9.5
      IF(MNI.EQ.1) GOTO 39
      DX=9.21
      DY=DX
      DXO=0.
39  CONTINUE
      DO 40 I=1,IN
        R(I)=1.
        IF(MNI.EQ.2) R(I)=0.5*FLOAT(2*I-1)*DX
40  CONTINUE
C***** CALCULATE THE TOTAL OCCURRENCE AND THE "DENSITY" DISTRIBUTION *
      TOCCUR=0.
      DO 5 I=1,IN
        DO 5 J=1,JM
          TOCCUR=TOCCUR+FLOAT(IRHO(I,J))
          IF(IRHO(I,J).EQ.0) IRHO(I,J)=1
          RHO(I,J)=FLOAT(IRHO(I,J))/(DX*R(I))
5    CONTINUE
      CCC=1.E9*TMASS/(6.283185*DY*TOCCUR)
      IF(IRO.EQ.0) GOTO 11
C***** SMOOTHING THE "DENSITY" DISTRIBUTION *****
      DO 9 IK=1,IRO
        DO 6 I=1,IN
          RHO1(2,1)=RHO(I,1)
          DO 7 K=1,2
            KI=I+K-2
            IF(I.EQ.1) KI=1
            DO 7 J=1,JM
              RHO1(K,J+1)=RHO(KI,J)
7    CONTINUE
          RHO1(2,JM+2)=RHO(I,JM)
          I1=I+1
          IF(I.EQ.IN) I1=I

```

```

      DO 8 J=1,JM
      DO=SQRT(((FLOAT(J)-0.5)*DY-25.4*ZO)**2+
1      ((FLOAT(I)-0.5-DX0)*DX)**2)/25.4
      IF(DO.LT.RO) GOTO 8
      RHO(I,J)=0.125*(RHO1(1,J+1)+RHO1(2,J)+4.*RHO1(2,J+1)+
1      RHO1(2,J+2)+RHO(I1,J))
8      CONTINUE
6      CONTINUE
9      CONTINUE
11     CONTINUE
C***** RESTORE THE MODIFIED OCCURRENCE AND DENSITY DISTRIBUTION *****
      DO 12 I=1,IN
      DO 12 J=1,JM
      IRHO(I,J)=RHO(I,J)*DX*R(I)
      RHO(I,J)=CCC*RHO(I,J)
12     CONTINUE
C***** FIND THE UPPER BOUNDARY FOR THE STREAM FUNCTIONS *****
      JNN=0
      JM1=JM-1
      DO 10 I=1,IN
      DO 20 J=IH,JM1
      IF(U(I,J).NE.0.0.OR.V(I,J).NE.0.0) GOTO 20
      IF(U(I,J+1).EQ.0.0.AND.V(I,J+1).EQ.0.0) GOTO 30
20     CONTINUE
      IF(U(I,JM).NE.0.0.OR.V(I,JM).NE.0.0) J=JM+1
30     JN(I)=J
      IF(JN(I).GT.JNN) JNN=JN(I)
10     CONTINUE
C***** CALCULATE STREAM FUNCTIONS PSI1 AND PSI2 *****
      IN1=IN+1
      JNN1=JNN-1
      DO 50 J=1,JNN
      IF(J.EQ.(JM+1)) GOTO 50
      PSI2(1,J)=0.
      DO 52 I=2,IN1
      I1=I-1
      PSI2(I,J)=PSI2(I-1,J)-DX*R(I-1)*RHO(I-1,J)*V(I-1,J)
      IF(J.EQ.1) GOTO 51
      PSI1(I1,J)=PSI1(I1,J-1)+DY*R(I1)*RHO(I1,J-1)*U(I1,J-1)
      GOTO 52
51     PSI1(I1,J)=0.
52     CONTINUE
C***** SET PSI2 TO SATISFY ITS UPPER BOUNDARY CONDITION *****
      DO 53 I=2,IN1
      RR=FLOAT(I-1)/FLOAT(IN)
      IF(MNI.EQ.2) RR=RR**2
      PSI2(I,J)=PSI2(I,J)-PSI2(IN1,J)*RR
      IF(J.GE.JN(I-1)) PSI2(I-1,J)=0.
53     CONTINUE
50     CONTINUE
      IF(JNN.LE.JM) GOTO 60
C***** CALCULATE VALUES OF THE STREAM FUNCTION PSI1 ON THE UPPER *****

```

```

C***** BOUNDARY WHEN IT REACHES THE TOP ROW OF THE DATA *****
  PSI2M=0.
  PSI1M=0.
  DO 70 I=1,IN
    PSI1(I,JM+1)=PSI1(I,JM)+DY*R(I)*RHO(I,JM)*U(I,JM)
    P(I)=PSI1(I,JM+1)
    IF(JN(I).NE.JM+1) P(I)=0.
    PSI2M=PSI2M+PSI2(I,JM)
    PSI1M=PSI1M+PSI1(I,JM+1)-PSI1(I,JM)
70  CONTINUE
  DP=PSI2M+0.5*PSI1M
  IF(IRP.EQ.0) GOTO 80
C***** SMOOTHING THE STREAM FUNCTION ON THE UPPER BOUNDARY *****
  DO 81 II=1,IRP
    DO 82 I=1,IN
      P1=P(I)
      IF(JN(I).NE.JM+1) GOTO 85
      IF(I.EQ.1) GOTO 83
      IF(I.EQ.IN) GOTO 84
      P(I)=0.25*(P2+2.*P(I)+P(I+1))
      GOTO 85
83    P(I)=0.25*(2.5*P(I)+0.5*P(I+1))
      GOTO 85
84    P(I)=0.25*(0.5*P2+2.5*P(I))
85    P2=P1
82  CONTINUE
81  CONTINUE
80  DP1=0.
    DO 90 I=1,IN
      DP1=DP1+P(I)
90  CONTINUE
C***** SET PSI1 TO SATISFY ITS UPPER BOUNDARY CONDITION *****
60  DO 100 I=1,IN
      JN1=JN(I)
      IF(JNN.NE.JM+1) P(I)=0.
      IF(JNN.EQ.JM+1) P(I)=P(I)*DP/DP1
      DO 101 J=2,JN1
        PSI1(I,J)=PSI1(I,J)-FLOAT(J-1)*(PSI1(I,JN1)-P(I))/FLOAT(JN1-1)
101  CONTINUE
      IF(JN1.EQ.JM+1) GOTO 100
      DO 102 J=JN1,JNN
        PSI1(I,J)=0.
102  CONTINUE
100 CONTINUE
    IF(IRP.EQ.0) GOTO 123
C***** SMOOTHING THE STREAM FUNCTIONS PSI1 AND PSI2 *****
C***** TO FORM A SINGLE STREAM FUNCTION *****
  DO 122 II=1,IRP
    DO 111 K=1,JNN
      P(K)=0.
111  CONTINUE
    DO 120 I=1,IN

```

```

      JN1=JN(I)
      JN11=JN1-1
      IF(I.EQ.1) GOTO 140
      DO 130 J=1,JN11
      D1=SQRT(((FLOAT(J)-0.5)*DY-25.4*ZO)**2+
1      ((FLOAT(I-1)-DX0)*DX)**2)/25.4
      IF(D1.GT.R0) GOTO 133
      PSI2(I,J)=0.
      P(J)=0.
      GOTO 130
133      P(J)=PSI2(I,J)
      PSI2(I,J)=0.125*(4.*PSI2(I,J)+P1(J)+PSI1(I,J)+
1      P1(J+1)+PSI1(I,J+1))
130      CONTINUE
      DO 131 J=JN1,JNN
      IF(J.EQ.JM+1) GOTO 131
      P(J)=PSI2(I,J)
131      CONTINUE
140      P1(1)=0.
      DO 150 J=2,JN11
      D2=SQRT((FLOAT(J-1)*DY-25.4*ZO)**2+
1      ((FLOAT(I)-0.5-DX0)*DX)**2)/25.4
      IF(D2.GT.R0) GOTO 153
      PSI1(I,J)=0.
      P1(J)=0.
      GOTO 150
153      P1(J)=PSI1(I,J)
      PSI1(I,J)=0.125*(4.*PSI1(I,J)+P(J-1)+PSI2(I+1,J-1)+
1      P(J)+PSI2(I+1,J))
150      CONTINUE
      DO 151 J=JN1,JNN
      P1(J)=PSI1(I,J)
151      CONTINUE
120      CONTINUE
122      CONTINUE
123      CONTINUE
C***** FIND THE MAXIMUM AND MINIMUM VALUES OF THE STREAM FUNCTION *****
      PMAX=0.
      PMIN=0.
      DO 171 J=1,JNN
      DO 171 I=1,IN
      IF(PSI1(I,J).GT.PMAX) PMAX=PSI1(I,J)
      IF(PSI1(I,J).LT.PMIN) PMIN=PSI1(I,J)
      IF(J.EQ.JM+1) GOTO 171
      IF(PSI2(I,J).GT.PMAX) PMAX=PSI2(I,J)
      IF(PSI2(I,J).LT.PMIN) PMIN=PSI2(I,J)
171      CONTINUE
      DP=(PMAX-PMIN)/6.
C***** CALCULATE THE MODIFIED VELOCITY DISTRIBUTIONS *****
      DO 160 I=1,IN
      DO 160 J=1,JNN1
      C=DX*R(I)*RHO(I,J)

```

```

      C1=DY*C/DX
      V(I,J)=(PSI2(I,J)-PSI2(I+1,J))/C
      U(I,J)=(PSI1(I,J+1)-PSI1(I,J))/C1
160  CONTINUE
C***** FOLLOWING OUTPUT PARAMETERS ARE USED FOR STREAMLINE PLOTTINGS *
      NPO=0
      JJ=JNN
      IF(JNN.EQ.JM+1) JJ=JM
      WRITE(6,*) IN,JJ,MNI,NPO,PMAX,PMIN,DP
C***** OUTPUT DATA *****
C*  PSI1+PSI2 --- STREAM FUNCTION DISTRIBUTION *
C*  U(IN,JM) --- MODIFIED VELOCITY DISTRIBUTION IN X OR R DIRECTION *
C*  V(IN,JM) --- MODIFIED VELOCITY DISTRIBUTION IN Y OR Z DIRECTION *
C*  IRHO(IN,JM) --- MODIFIED OCCURRENCE DISTRIBUTION *
C*  RHO(IN,JM) --- DENSITY DISTRIBUTION (GIVEN IN CYLINDRICAL CASE) *
C*****
      DO 124 J=1,JNN
        WRITE(6,201) (PSI1(I,J),I=1,IN)
        IF(J.EQ.JM+1) GOTO 124
        WRITE(6,202) (PSI2(I,J),I=1,IN)
124  CONTINUE
      IF(MNI.EQ.1) GOTO 164
      WRITE(6,200) ((U(I,J),I=1,IN),J=1,JM),((V(I,J),I=1,IN),J=1,JM)
      WRITE(6,203) ((IRHO(I,J),I=1,IN),J=1,JM)
      WRITE(6,200) ((RHO(I,J),I=1,IN),J=1,JM)
      GOTO 165
164  WRITE(6,204) ((U(I,J),I=1,IN),J=1,JM),((V(I,J),I=1,IN),J=1,JM)
      WRITE(6,205) ((IRHO(I,J),I=1,IN),J=1,JM)
165  CONTINUE
200  FORMAT(10F8.2)
201  FORMAT(5X,10F11.1/15X,9F11.1)
202  FORMAT(11F11.1/10X,9F11.1)
203  FORMAT(10I8)
204  FORMAT(19F7.1)
205  FORMAT(19I6)
      STOP
      END

```

```
DO 60 J=1,65
  DO 60 I=1,NM2
    I1=NW-I+1
    F(J,I)=F(J,I1)
60  CONTINUE
61  DO 50 I=1,65
    X(I)=DX*FLOAT(I-1)
50  CONTINUE
    DO 70 I=1,41
      Y(I)=5.*FLOAT(I-1)/FLOAT(NW-1)
70  CONTINUE
    CALL TEK
    CALL PLOTS(0.,0.,10)
    CALL BOX(0.,8.,2.,-10,0.,5.,1.,-10)
    CALL LEGEND(0.,0.,0.,0.)
    IK=II
    IF(II.EQ.0) IK=5
    DO 80 I=1,IK
      VALUE=PMIN+DP*FLOAT(I)
      IF(II.NE.0) VALUE=VAL(I)
      CALL ISOPLT(X,65,Y,41,F,VALUE,WORK)
80  CONTINUE
    CALL PLOT(0.,0.,999)
    STOP
    END
```


Program STRMLN is cooperated with the program SMOOTH to obtain the streamline plottings.

```

PROGRAM STRMLN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
DIMENSION F(65,41),P(20,50),P1(20,50),X(65),Y(41),WORK(91),
1      VAL(20)
READ(5,*) IN,JNN,MNI,II,PMAX,PMIN,DP
IF(II.EQ.0) GOTO 3
READ(5,*) (VAL(I),I=1,II)
3  IN1=IN+1
   DO 20 J=1,50
     DO 20 I=1,20
       P(I,J)=0.
       P1(I,J)=0.
20  CONTINUE
   DO 10 J=1,JNN
     READ(5,1) (P1(I,J),I=1,IN)
     READ(5,2) (P(I,J),I=1,IN1)
10  CONTINUE
1  FORMAT(5X,10F11.1/15X,9F11.1)
2  FORMAT(11F11.1/10X,9F11.1)
   DO 90 I=1,65
     DO 90 J=1,41
       F(I,J)=0.
90  CONTINUE
   NM2=21
   NW=41
   NH=32
   DX=0.125
   IF(MNI.EQ.2) GOTO 15
   NM2=1
   NW=39
   NH=20
   DX=5./24.56
15  DO 30 J=1,NH
     J2=2*J
     J1=J2+1
     DO 30 I=1,IN
       I2=NM2+2*I
       I1=I2-1
       F(J2,I1)=0.25*(P(I,J)+P(I+1,J)+P1(I,J)+P1(I,J+1))
       IF(P1(I,J).EQ.P1(I,J+1)) F(J2,I1)=P1(I,J)
       F(J1,I1)=P1(I,J+1)
       IF(I.EQ.IN) GOTO 30
       F(J2,I2)=P(I+1,J)
       F(J1,I2)=0.25*(P1(I,J+1)+P1(I+1,J+1)+P(I+1,J)+P(I+1,J+1))
       IF(P(I+1,J).EQ.P(I+1,J+1)) F(J1,I2)=P(I+1,J)
30  CONTINUE
   IF(MNI.EQ.1) GOTO 61

```

END

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