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**SHEAR STRAIN LOCALIZATION
AND FRACTURE EVOLUTION IN ROCK**

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ABSTRACT

Work during this period has concentrated on refinements of a constitutive relation based on damage due to oriented microcracks. The goal of this portion of the study is to determine whether this relation satisfactorily describes axisymmetric and multi-axial (compression-torsion) test data on Tennessee marble. We have studied the predictions of the relation for the crack lengths and orientations in axisymmetric compression, for peak stress and for the damage surface for combined torsion and compression. A key result is that the damage surface for combined torsion and compression is initially elliptical but forms a vertex with axial loading beyond the initial damage stress. The vertex angle continues to narrow with further axial loading and depends additionally on the coefficient of friction and the range of angles of cracks that have been activated. A consequence of vertex formation is that the stiffness of response to an increment of shear imposed on axial compression, as occurs in localization, will be much less than the elastic modulus and will continue to diminish with further axial deformation.

INTRODUCTION

Shear fractures (faults) can occur at all scales from intragranular shear cracks to global fractures bounding tectonic plates. They can occur individually and in genetically related patterns. Understanding the localization of shear deformation into narrow zones is essential to understanding macroscopic rock fracture in compression, progressive reduction in load bearing ability (strain softening behavior) and the evolution of faults and certain other geologic structures, e.g., kink bands. Understanding highly localized deformation is also essential for predicting complex failures around underground excavations and boreholes where the mode and extent of rock decrepitation cannot be described by means of conventional yield or fracture criteria. Episodic slip on pre-existing faults or their dynamic extension may lead to seismic disturbances that can affect location of power plants, dams, and waste disposal sites. When faults act as barriers to fluid movement, they are one of the principal types of petroleum traps. If they act as fluid conduits, they may adversely impact waste disposal activities or allow hydrocarbon leakage from reservoirs. Their presence controls the mechanical response of rock masses causing strength and stiffness to be less than intact rock.

The onset of localization typically involves an abrupt change in the deformation pattern or, in other words, in the orientation of the principal axes of the rate of deformation. Consequently, attempts to predict the onset of localization in terms of the constitutive description of homogeneous deformation [*Rudnicki and Rice*, 1975] are strongly dependent on the moduli governing such abrupt changes in the pattern of deformation. These moduli are, however, difficult to measure in experiments. The goal of this work has been to develop a macroscopic

constitutive relation based on the inelastic deformation caused by the growth of tensile microcracks. Although the model is intended for applications where all the applied principal stresses are compressive, local tensile cracks are caused by a variety of microscale inhomogeneities, such as, mismatch of elastic constants and frictional slip. Our approach has its origin in a model of *Costin* [1983a, 1983b, 1985] based on an *ad hoc* expression for the local tensile stress due to microscale inhomogeneities. Numerical simulations using Costin's model [Costin and Stone, 1986] suggest, however, that the model inhibits realistic development of localization because it is too stiff in shear. Consequently, one of our goals is to improve on this feature of Costin's model.

MICROCRACK GROWTH MODEL

Costin [1983a, 1983b, 1985] proposed an *ad hoc* expression for the local tensile stress that arises due to microscale inhomogeneities. These inhomogeneities may include neighboring grains with different elastic constants, grain boundaries, or sliding frictional cracks with opening wing cracks. Regardless of the details of the microstructure, these inhomogeneities have the effect of creating local tensile stresses that drive crack growth even when all the applied principal stresses are compressive.

We have been studying the following modified form of the expression introduced by *Costin* [1985]:

$$\sigma_t = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} + f' g(a) \mathbf{n} \cdot \mathbf{S} \cdot \mathbf{n} \quad (1)$$

where σ is the applied stress tensor, S is the deviatoric stress tensor, n is the unit vector normal to the crack, f' is a constant and $g(a)$ is a function of the crack length a to be explained below. In the form used by *Costin* [1985], the first term, which is the normal traction on the crack, is replaced by the mean stress, $(1/3) \text{tr } \sigma$. *Holcomb* [unpublished research, 1991] has shown, however, that this form with the mean stress leads to a negative pressure sensitivity. The form used by *Costin* [1985] can be recovered by making the following substitutions:

$$f' = f \frac{d/a_0}{\sqrt{1 - (a_0/d_1)^2}} \quad (2)$$

$$g(a) = \frac{1}{a/a_0} \sqrt{\frac{1 - (a_0/d_1)^2}{1 - (a/d_1)^2}} \quad (3)$$

where a_0 is the initial cracklength, d is a measure of the size of the local tensile field, d_1 is a measure of the distance between cracks, and f is a constant that determines the proportion of the applied deviatoric stress that is converted to tensile stress. The term $(a/a_0)^{-1}$ describes the decrease of the local tensile stress as the crack grows away from the inhomogeneity. The term $[1 - (a/d_1)^2]^{-1/2}$ describes the increase in local tensile stress caused by crack interaction. *Costin* [1985] introduced this function as an approximation to numerical results of *Horii and Nemat-Nasser* [1983] for the dependence of the stress intensity factors for a row of colinear cracks.

More generally, the function $g(a)$ describes the decrease of local tensile stress with distance away from the inhomogeneity and the increase due to crack interaction. The first effect

causes initially stable growth and the second is the source of a peak in the stress strain curve. In anticipation that different forms of this function may be needed to describe data on different rock types and in order to investigate the effect of the form of this function on the model, we have considered possibilities other than the form suggested by *Costin* [1985]. In particular, *Chau*[1991] has suggested the following form:

$$g(x) = \frac{1}{x} e^{[(\alpha-1)/(d_1/a_0)]^\alpha} \quad (4)$$

where α is typically a large constant. We have also investigated the following generalization of *Costin's* [1985] form:

$$g(x) = \frac{1}{x^\beta} \left\{ \frac{1 - (a_0/d_1)^\gamma}{1 - [x/(d_1/a_0)]^\gamma} \right\}^{1/\gamma} \quad (5)$$

where β and γ are constants. This form reduces to that of *Costin* [1985] for $\beta = 1$ and $\gamma = 2$.

According to *Costin's* [1985] formulation, the stress intensity factor at the crack tip is determined by substituting the local tensile stress into the expression for the stress intensity factor of a circular crack of radius a in an infinite elastic body:

$$K = \frac{2}{\pi} \sqrt{\pi a} \sigma_t \quad (6)$$

Crack growth is predicted to occur when

$$K = K_{lc} \quad (7)$$

where K_{lc} is the critical stress intensity factor, which characterizes the material resistance to crack growth. For monotonically increasing loading, the set of stresses where (7) is first met defines an initial damage surface in stress space. If the initial crack length distribution is isotropic, the shape of the initial damage surface can be determined easily. To this end, it is convenient to write

$$\sigma = S + (1/3)\text{tr } \sigma \quad (8)$$

Then, for a fixed value of the hydrostatic stress, the damage condition is first met when (7) is met for the maximum of $n \cdot S \cdot n$. But the maximum of $n \cdot S \cdot n$ over all orientations is simply the maximum principal deviatoric stress. Consequently, in the three dimensional space of principal stresses, σ_1 , σ_2 , and σ_3 , the projection of the damage surface on planes perpendicular to the hydrostatic axis, $\sigma_1 = \sigma_2 = \sigma_3$, is an equilateral triangle, with each face corresponding to S_1 , S_2 , or S_3 being the maximum principal deviatoric stress. In a plane containing the hydrostatic axis, the projection of the initial damage surface is a cone with apex at

$$\frac{1}{3}\text{tr } \sigma = \frac{\pi}{2} \frac{K_{lc}}{\sqrt{\pi a_0}} = k_{lc} \quad (9)$$

and slope m , where

$$m = \frac{1}{1+f'} \quad (10)$$

Holcomb and Costin [1986] and *Holcomb* [1993] have discussed the projections of the initial damage surface in several stress spaces and its evolution for some specific loadings.

PREDICTIONS FOR AXISYMMETRIC COMPRESSION

Now consider the predictions of (6) with (7) for axisymmetric compression. Align a rectangular cartesian coordinate system with the specimen so that the x_3 direction is axial. Relative to this coordinate system, the nonzero stress components are $\sigma_{33} = -\sigma_a$ and $\sigma_{11} = \sigma_{22} = -\sigma_c$, where σ_a is the axial stress and σ_c is the confining stress (both positive in compression). The components of the normal to the crack plane n are given as follows:

$$n_1 = \sin\phi \cos\psi, n_2 = \sin\phi \sin\psi, n_3 = \cos\phi \quad (11)$$

Because the initial distribution of cracks is isotropic, the distribution remains axisymmetric and $\psi = 0$ without loss of generality. When these results are substituted into (1), the resulting expression for the local tensile stress is

$$\sigma_t = \frac{1}{3}(\sigma_a - \sigma_c)[(1 - 3\cos^2\phi)(1 + f'g(a)) - 1] - \sigma_c \quad (12)$$

Substituting (12) into the expression for K and setting equal to K_{lc} yields

$$\frac{1}{3}(\sigma_a - \sigma_c)[(1 - 3\cos^2 \phi)(1 + f'g(a)) - 1] - \sigma_c = k_{lc} \quad (13)$$

where k_{lc} is given in (9). Since all cracks are assumed to initially have length a_0 and $g(a_0) = 1$, that crack growth first occurs in the axial direction ($\phi = 90_0$) when

$$\frac{1}{3}(\sigma_a - \sigma_c)_0 = \frac{\sigma_c}{f'} + c_0 \quad (14)$$

where

$$c_0 = \frac{\pi K_{lc}}{2f'\sqrt{\pi a_0}} \quad (15)$$

Thus, $1/f'$ is the slope of the axial stress difference versus confining stress at the initiation of crack growth, which can be taken to coincide with the deviation of the stress strain curve from linearity. The constant c_0 is the intercept of this curve. By converting values given by *Costin* [1985], we find that values of f' and c_0 given in the table below

Rock Type	f'	c_0 (MPa)
Westerly granite	1.14	36.0
Tennessee marble	2.09	20.1
Nevada Tuff	1.28	13.1

For continued increase of the axial stress difference, the relation between axial stress difference and the length of an axial crack is given by

$$\Sigma_a = \frac{\Sigma_c + (a/a_0)^{-1/2}}{1 + \Sigma_c} \frac{1}{g(a)} \quad (16)$$

where

$$\Sigma_a = \frac{\sigma_a - \sigma_c}{(\sigma_a - \sigma_c)_0}$$

is the normalized axial stress difference and

$$\Sigma_c = \frac{2}{\pi} \sigma_c \frac{\sqrt{\pi a_0}}{K_{Ic}} \quad (18)$$

is the normalized confining stress. As the axial stress increases beyond the value needed to cause initial crack growth, cracks with planes making larger angles with the axial direction

(smaller values of ϕ) begin to grow. The minimum angle at which crack growth occurs is given in terms of the normalized axial stress difference by

$$\cos \phi_{\min} = \sqrt{\frac{f'(\Sigma_a - 1)}{3\Sigma_a(1 + f')}} \quad (19)$$

Thus, the values for ϕ for cracks that have undergone some growth lie in the range $\phi_{\min} \leq \phi \leq 90^\circ$. The minimum value of ϕ is typically about 75° at peak stress.

The stress intensity factor initially decreases with crack growth as the crack grows away for the source of the local tensile stress. Thus, initial crack growth is stable; the axial stress difference must be increased to cause further growth. With continued growth, the effect of crack interaction causes K to increase with crack growth; at this point, growth can occur with no increase in axial stress and the process is unstable. Thus, the crack length at which this transition occurs satisfies

$$\frac{\partial K}{\partial a} = 0 \quad (20)$$

For axisymmetric compression, this condition can be written as follows:

This equation is satisfied when $a = a_{\text{peak}}$ where a_{peak} is the length of an axial crack at peak stress; substituting a_{peak} into (16) yields the peak axial stress difference for a given (normalized)

confining stress. In general, (21) requires numerical solution, but for the *Costin* [1985] form of $g(a)$ (3) and $\Sigma_c = 0$, $a_{peak} = d_1/3^{1/2}$.

As an example of the predictions for axisymmetric compression, Figure 1 shows some calculations using the form of $g(a)$ suggested by *Costin* [1985]. Figure 1a plots the normalized axial stress difference, $\Sigma_a - 1$, against the normalized axial crack growth, $(a - a_0)/a_0$ (where a refers to a crack in the axial direction), for $a_0/d_1 = 0.3$ and three values of the normalized confining pressure Σ_c : 0, 0.5, and 1.0. Figure 1b plots the $\Sigma_a - 1$ against $(a - a_0)/a_0$ for $\Sigma_c = 0.5$ and three values of the crack interaction parameter, a_0/d_1 : 0.25, 0.30 and 0.35. Figure 1c illustrates the dependence of the damage surface on confining pressure at the onset of damage and at peak stress. Figure 1c plots $(\sigma_a - \sigma_c)/k_c$ against Σ_c for $a = a_0$ (initial damage surface) and for $a = a_{peak}$ (damage surface at peak stress) for a_0/d_1 equal to 0.25 and 0.3 (Plots are for $f' = 2.0$).

COMBINED AXIAL COMPRESSION AND TORSION

We now consider the predictions of this theory for combinations of axial compression and torsion or, more specifically, small increments of torsion superimposed on axial compression. Ultimately, the goal here is to compare predictions of the theory with the results of experiments by *Olsson* [1992, and unpublished research, 1933], but, thus far, we have investigated the evolving shape of the damage surface.

Consider an element of a thin-walled tube subjected to axial compression σ_a , confining stress σ_c , and torsional shear stress τ . For monotonic loading (increase of $\sigma_a - \sigma_c$ and τ in a specified ratio with σ_c held constant), the condition for crack growth (7) can be written as follows:

$$\frac{1}{3} S [(1 - 3 \cos^2 \phi)(1 + f' g(a)) - 1] + T \sin \phi \cos \phi (1 + f' g(a)) = 1 \quad (22)$$

where

$$S = \frac{(\sigma_a - \sigma_c)}{k_{lc}(\Sigma_c + 1)} \quad (23)$$

and T is given by the same expression with τ replacing $\sigma_a - \sigma_c$. The initial damage surface is the locus of values of S and T at which crack growth first occurs if S and T are increased in fixed proportion. This surface can be determined by setting $S = s \cos \theta$ and $T = s \sin \theta$, setting $a = a_0$ so that $g = 1$, substituting into (22) and determining the value of ϕ yielding the smallest value of s for each θ . That value of ϕ is $(\pi - \theta)/2$. Alternatively, it can be recognized, as mentioned earlier, that the initial damage surface corresponds to a critical value of the maximum tensile principal deviatoric stress. Thus, in the S - T plane the initial damage surface is a circle, with center at $T = 0$, $S = S_0$, where

$$S_0 = \frac{1}{2} \frac{(1 + f'/3)}{(1 + f')} \quad (24)$$

and radius

$$R = \frac{1}{2(1 + f')} \sqrt{8(1 + f') + (1 + f'/3)^2} \quad (25)$$

Now, we wish to examine the local structure of the damage surface when axial loading is continued beyond the initial damage point and small increments of torsion are added. We will see, however, that the exploration of the local structure reveals the global structure as well, that is, the current shape of the damage surface. To this end, let the current axial stress difference be $(\sigma_a - \sigma_c)_c > (\sigma_a - \sigma_c)_0$. At the axial stress level $(\sigma_a - \sigma_c)_c$, all cracks with normals within the range

$$\phi_{\min} \leq \phi \leq 180 - \phi_{\min} \quad (26)$$

where $\phi_{\min}$ is given by (19), have extended beyond their initial length. Thus, for all these cracks the stress intensity factor is equal to the critical value K_{Ic} . Denote the distribution of crack lengths within the range (26) be $a_c(\phi)$.

Add to this axisymmetric compressive stress state a small increment of combined axial compression and torsion such that

$$(\sigma_a - \sigma_c) = (\sigma_a - \sigma_c)_c + \delta\sigma \cos\alpha \quad (27)$$

and

$$\tau = \delta\sigma \sin\alpha \quad (28)$$

where the magnitude of the increment $\delta\sigma$ is assumed to be small. This increment could cause an increment of crack growth δa , such that

$$a = a_c + \delta a \quad (29)$$

We wish to determine the range of α for which

$$\frac{\delta \sigma}{\delta a} > 0 \quad (30)$$

for all ϕ in the range (26). This range defines the forward loading cone: for any loading such that α is in this range, every crack that has experienced growth continues to grow. Substituting (27) and (28) into (22), expanding for small δa , neglecting terms beyond the first order, and enforcing the condition (30) yields

$$|\alpha| \leq \arctan \left[\frac{2 [(1 - 3 \cos^2 \phi_{\min})(1 + f') - 1]}{3 \sin 2\phi_{\min} (1 + f')} \right] \quad (31)$$

Alternatively, it is possible to determine the range of α for which there is no additional crack growth of any orientation. This defines the unloading cone and the interior of the damage surface. If the forward loading cone is extended back toward the initial damage surface, it defines the region of unloading; that is, the range of α for which no crack growth occurs, neither on those cracks that have been extended nor those that are still at the initial length. Figure 2 shows a sketch of the initial damage surface in the T versus S plane and the formation of a vertex after loading in the axial direction.

DISCUSSION

We have investigated the predictions for microcrack growth and orientation and for the macroscopic damage surface based on a modification of an expression proposed by *Costin* [1983a, 1983b, 1985] for the local tensile stress induced in an inhomogeneous material. A primary result is that for combined axial compression and torsion, the damage surface, which is initially a smooth ellipse, develops a vertex with axial loading. Although the vertex angle depends on the details of the functions entering the expression for the local tensile stress, the prediction of the vertex does not. Indeed, general considerations [*Hill*, 1968] suggest that a vertex is to be expected for rate-independent models of inhomogeneous materials.

The vertex divides the area of the axial stress versus shear stress plane near the current stress point into three regions: a forward loading cone, a cone of unloading and a region separating them. Stress increments directed into the forward loading cone cause continued loading on all microcracks within the range of orientations that have been loaded previously. Stress increments directed into the unloading cone cause no loading on microcracks of any orientation. Stress increments directed into the region between the loading and unloading cones cause both new microcrack orientations to be activated and unloading of some orientations. Consequently, the response is expected to be fully nonlinear in the sense that the direction of the inelastic increment of strain depends on the direction of the stress increment. This contrasts with incrementally linear response for which the magnitude of the inelastic increment of strain depends on the magnitude of the stress increment but the direction is fixed by the current stress state. Such fully nonlinear response is known to enhance localization by comparison with predictions based on the incrementally linear behavior predicted by models with smooth yield or damage

surfaces [Rudnicki and Rice, 1975].

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Figure 1

$$T = \frac{\tau}{\sqrt{\frac{\pi}{2} \frac{K_{Ic}}{\sqrt{\pi a_0}} + \sigma_c}}$$

$$R = \left\{ \frac{2}{(1+f')^2} + \frac{(1+f'/3)^2}{4(1+f')^2} \right\}^{1/2}$$

initial damage surface
current damage surface

$$S = \frac{(\sigma_a - \sigma_c)}{\frac{\pi}{2} \frac{K_{Ic}}{\sqrt{\pi a_0}} + \sigma_c}$$

$$\leftarrow \frac{1}{2} \frac{(1+f'/3)}{(1+f')} \rightarrow$$

Figure 2

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