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**Reservoir and Injection Technology
and
Heat Extraction Project
Fifth Annual Report**

**Principal Investigators:
R.N. Horne, H.J. Ramey, Jr., F.G. Miller
W.E. Brigham, Paul Kruger**

December, 1989

**Department of Energy Contract Number
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For the Period
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INTRODUCTION

The Stanford Geothermal Program in the fiscal year 1989 was divided into several task areas, as defined by the Department of Energy contract No. AS07-84ID12529. One of tasks is carried out in the Department of Civil Engineering and rest at the Department of Petroleum Engineering.

The task areas include predictive modeling of reservoir behavior and tracer test interpretation and testing.

Predictive modeling of reservoir behavior consists of a multi-pronged approach to well test analysis under a variety of conditions. The efforts have been directed to designing and analyzing well tests in

- naturally fractured reservoirs
- fractured wells
- complex reservoir geometries
- gas reservoirs including inertial and other effects

The analytical solutions for naturally fractured reservoirs are determined using a fracture size distribution. In the study of fractured wells, an elliptical coordinate system is used to obtain semi-analytical solutions to finite conductivity fractures. Effort has also been directed to the modeling and creation of a user friendly computer program for steam/gas reservoirs including wellbore storage, skin and non-Darcy flow effects. This work has a complementary effort on modeling high flow rate wells including inertial effects in the wellbore and fractures. In addition, work on gravity drainage systems is being continued.

The task on the use of Boundary Element Method for a variety of reservoir characteristics and configurations has been successfully completed. This project shows the possibility of solving accurately for complex reservoir shapes including composite and piecewise homogeneous reservoirs.

The other task area consisting of tracer test interpretation and testing has been focussed towards monitoring both the tracer flow and thermal transients during reinjection. The work on mass and thermal transport in well-to-well, recirculating and injection-backflow tracer tests has been completed. Testing of the reinjection optimization algorithm is continuing and is

SECTION 1 : RESERVOIR TECHNOLOGY

Section 1.1 : Modeling Pressure Transient Behavior of Sectionally Homogeneous Reservoirs by the Boundary Element Method

Jitendra Kikani

Overview

Heterogeneities in petroleum and geothermal reservoirs complicate pressure transient testing. Conventional analytic techniques can only be used for fairly simple geometries. More complex situations usually require numerical simulation. This paper demonstrates a new approach using the Boundary Element Method (BEM) to solve for pressure transient behavior in complex reservoirs.

A Laplace space boundary element solution is proposed to determine the pressure solution in a piecewise homogeneous reservoir with arbitrary geometry of each region. Any number of such regions with different rock and fluid properties can be included in the solution procedure. This formulation can solve fluid injection problems which show composite behavior (as in water injection in geothermal reservoirs). In addition, impermeable barriers of any shape and orientation as well as large pressure support sources (aquifers) can be included. Both pressure and pressure derivative behavior for such systems is presented. Any number of line source wells can be included. Example solutions are verified against known analytical solutions.

At the same time as pressure transients are calculated, the method also determines fluid velocities. Injection fluid front velocity is therefore calculated as part of the solution and thus the fronts can be tracked accurately. The analytic aspects of this procedure are discussed.

BEM solutions are more accurate than other numerical methods such as finite element and finite difference techniques because the method retains the free space Green's function of the governing differential operator as a global weighting function in the integral equation.

been shown to provide accurate results. A survey of the literature on this subject is given in *Ambastha (1988)*.

In reality, the injection fronts are never truly radial due to a variety of reasons. Background drift caused by injection and production wells nearby can strongly affect the geometry of the propagating front and so can the outer boundary effects. Also, fractured injection wells could cause the fluid front to move in an elliptical or a rectangular fashion for a certain distance in the reservoir before the effect of inner boundary geometry is dissipated and the front becomes approximately radial.

The two categories of problems mentioned above are solved by the Boundary Element Method. This method holds promise for pressure transient type applications. The advantage of using this method to solve complex reservoir geometry problems in lieu of numerical methods stems from the fact that the analytical character of the solution is preserved in terms of the fundamental solution to the governing equations.

Boundary Element Method

Developed in the realm of potential theory, this method is now being used in a variety of engineering applications. The advantages of using this method for pressure transient type problems are :

- The accuracy of the method
- The reduction in the dimensionality of the problem, which leads to smaller size of matrix equations
- Flexibility of handling complex reservoir configurations and production schemes.

Applications of this method to reservoir engineering problems is recent. *Numbere and Tiab (1988)* and *Masukawa and Horne (1988)* looked at steady state problems for streamline generation. Transient problems have been investigated in the field of heat transfer (*Pina, 1984*) and groundwater flow (*Liggett and Liu, 1983*), in addition to reservoir engineering. *Kikani and Horne (1988a,b)* discuss the development of this method in other disciplines and also showed the accuracy of solutions for transient problems in homogeneous systems. Two techniques were presented. One was the solution in real space which involved dealing with the history dependent nature of the integrals and the other was a Laplace transform solution. The use of this method in streamline generation and well testing with some field type examples has been presented (*Kikani and Horne, 1989*).

Figure 1.1.1: Schematic of a reservoir with heterogeneities

interior sub-regions.

$\bar{G}$ in Eq. (1.1.1) corresponds to the free space Green's function in Laplace space. A two-dimensional free space Green's function is defined [Carslaw and Jaeger, 1959] as the pressure response at (x_D, y_D) at a time t_D due to an instantaneous line source of strength unity generated at coordinates (ξ, ζ) at the time τ . The medium is initially at zero pressure and infinite in extent. $\bar{G}$ is given by

$$\bar{G}(x_D, y_D, \xi, \zeta; s) = -K_0(|r_D| \sqrt{s}) \quad (1.1.3)$$

where K_0 is the modified Bessel function of the second kind and order zero and r_D is given by

$$r_D^2 = (x_D - \xi)^2 + (y_D - \zeta)^2 \quad (1.1.4)$$

The integral Eq. (1.1.1) is solved in a discrete form by assuming a fictitious or Green's function source to be placed at each nodal point on the boundary, in turn. The resulting contribution of each of the nodes is compiled to give $N_1 + N_2$ equations in as many unknowns. The solution in Laplace space is sampled a number of times and inverted to real space by the use of *Stehfest algorithm* (1970) which is usually robust for diffusion type operators. With a few modifications the algorithm for solving the diffusivity equation in homogeneous reservoirs can be used for sectionally homogeneous reservoirs.

Two Region Composite Reservoir

Figure 1.1.3 shows the schematic of a composite reservoir. The properties of the inner and outer regions are different. The interface between the two-regions is assumed stationary. This assumption can be relaxed in an approximate sense and will be discussed later. There are three parameters which govern the system behavior. These are mobility ratio (M), storativity ratio (F_s) and the distance to the discontinuity (R_D). The storativity ratio is defined as

$$F_s = \frac{M}{\eta} \quad (1.1.5)$$

where, η is the diffusivity ratio between the inner and outer regions.

The two regions can assume an arbitrary shape and consist of multiple injection and production wells. The non-dimensionalized governing equations and boundary conditions are as follows.

Inner Region

$$\frac{\partial^2 p_{D1}}{\partial x_D^2} + \frac{\partial^2 p_{D1}}{\partial y_D^2} = \frac{\partial p_{D1}}{\partial t_{DA}} + \sum_{i=1}^{n_w-1} Q_{D1_i} \delta(x_D - x_{D_i}) \delta(y_D - y_{D_i}) \quad [(x_D^2 + y_D^2 < R_D^2)] \quad (1.1.6)$$

Outer Region

$$\frac{\partial^2 p_{D2}}{\partial x_D^2} + \frac{\partial^2 p_{D2}}{\partial y_D^2} = \frac{M}{F_s} \frac{\partial p_{D2}}{\partial t_{DA}} + M \sum_{j=1}^{n_{w2}} Q_{D2j} \delta(x_D - x_{Dj}) \delta(y_D - y_{Dj}) \quad [(R_D^2 < x_D^2 + y_D^2 < R_{eD}^2)] \quad (1.1.7)$$

Initial Condition

$$p_{D1}(t_{DA} = 0) = p_{D2}(t_{DA} = 0) = 0 \quad (1.1.8)$$

Interface Conditions

$$p_{D1}(x_{D1}^2 + y_{D1}^2 = R_D^2) = p_{D2}(x_{D2}^2 + y_{D2}^2 = R_D^2) \quad (1.1.9)$$

$$\frac{\partial p_{D1}(R_D)}{\partial n} = -M \frac{\partial p_{D2}(R_D)}{\partial n} \quad (1.1.10)$$

The negative sign in Eq. (1.1.10) accounts for the opposite sense of the outward normal in the two regions.

Exterior Boundary Condition

$$p_{D2}(R_{eD}, t_{DA}) = p_e \quad (1.1.11)$$

$$\frac{\partial p_{D2}}{\partial n}(R_{eD}, t_{DA}) = q_e \quad (1.1.12)$$

$$\alpha p_{D2} + \beta \frac{\partial p_{D2}}{\partial n} = \gamma_e \quad (1.1.13)$$

The non-dimensional variables in the above equations are defined as follows

$$p_{D1} = \frac{p_1 - p_0}{\bar{p}} \quad p_{D2} = \frac{p_2 - p_0}{\bar{p}} \quad (1.1.14)$$

$$x_D = \frac{x}{\sqrt{A_I}} \quad y_D = \frac{y}{\sqrt{A_I}} \quad (1.1.15)$$

$$t_{DA} = \frac{\eta_1 t}{A_I} \quad Q_{Di} = \frac{Q_i \mu_1}{k_1 \rho \bar{p}} \quad (1.1.16)$$

A_I in Eqs. (1.1.15) and (1.1.16) is the area of the inner region. The above equations can be normalized, alternatively, with respect to the exterior drainage area. $\bar{p}$ is a reference normalization pressure and p_0 is the initial reservoir pressure.

where ξ is the local coordinate varying along the element. Once the interpolation functions are defined, the integral equations are discretized in a local coordinate system in order to reduce the integrals in two-dimensions to a one-dimensional contour integral. Figure 1.1.4 shows the local coordinate system. The point 'i' is the fictitious source point and ξ_j and ξ_{j+1} are the nodal locations for the element, on which the contribution of the source is evaluated. The fictitious source point is moved, in turn, to all the nodes and the respective contributions at all nodal locations are calculated. The discrete form of the integral Eqs. (1.1.1), (1.1.17) or (1.1.18) is then, given symbolically as

$$0 = \sum_{j=1}^N \int_{\xi_j}^{\xi_{j+1}} d\xi \left[\bar{p}_{D_{j+1}} f_1(\xi, \zeta_i; s) + (1 - 2\theta_i \delta_{ij}) \bar{p}_{D_j} f_2(\xi, \zeta_i; s) + \bar{p}_{D_n} g_1(\xi, \zeta_i; s) + \bar{p}_{D_n} g_2(\xi, \zeta_i; s) \right] \quad i = 1, N \quad (1.1.22)$$

The integrals in Eq. (1.1.22) consisting of functions f_1 , f_2 , g_1 , and g_2 are evaluated either analytically or numerically. Rewriting Eq. (1.1.22) in a matrix form we obtain

$$\sum_{j=1}^N a_{ij} \bar{p}_{D_j} + b_{ij} \bar{p}_{D_n} = 0 \quad i = 1, N \quad (1.1.23)$$

where, a_{ij} and b_{ij} are the elements of the augmented coefficient matrix. Equation (1.1.23) is then sorted for the unknowns. The resultant matrix equation for the unknown boundary conditions is given by

$$\sum_{j=1}^N H_{ij} u_j = G_i \quad i = 1, N \quad (1.1.24)$$

where, u is the solution vector and G_i the right hand side vector. Solution of Eq. (1.1.24) gives the nodal values of the unknown at the boundaries. The solutions in the interior are obtained by using the same discretized integral equation again for an internal point.

The implementation of the integral equation for sectionally heterogeneous reservoirs is similar to that for homogeneous reservoirs and has been discussed in detail in *Kikani (1989)*. The only difference between the two is in the numbering of the nodes. For the multiply connected surface, the sub-domains have to be considered.

For two region composite reservoirs, Eqs. (1.1.17), and (1.1.18) must be solved separately and coupled through the interface conditions. The boundary of the inner zone is divided into N_1 nodes and the exterior boundary is divided in N_2 nodes. Figure 1.1.5 shows the partition of the boundaries in the two regions. Region I consists of the fluid interface and the domain

Figure 1.1.5: Partition of reservoir boundary into two regions

Figure 1.1.6 : Matrix structure for the composite reservoir

Figure 1.1.8 : Well producing external to an internal circular boundary

Figure 1.1.9: Pressure response at a well producing external to a circular sub-region

Figure 1.1.10: Pressure derivative behavior at a well producing external to a circular sub-region

Figure 1.1.12: Pressure response at a well producing external to an impermeable elliptic sub-region

radius

$$t_{D*} = t_D / R_D^2 \quad (1.1.25)$$

Early time radial infinite acting behavior corresponding to the inner region is seen as a constant value 0.5 on the pressure derivative plot. Depending on the value of storativity ratio, the pressure derivative deviates from the infinite acting behavior. The derivative passes through a maximum and goes to another radial infinite acting behavior, corresponding to the external region. The pressure derivative group is given by a value of $M/2$, where M is the mobility ratio of inner to the outer region. The solid lines in Fig. 1.1.13 are the analytical solution and the circles are the boundary element solution. Figure 1.1.14 shows the effects of a closed external boundary on the pressure transient response of a two-zone circular composite reservoir with a mobility ratio of 10 and storativity ratio of 1000. The composite reservoir effects are not seen if the ratio R_{eD}/R_D is ≤ 10 . For R_{eD}/R_D at 1000, the infinite acting behavior due to the outer region can be seen before pseudo-steady state begins, which appears as a unit slope straight line on the derivative plot.

Field data suggest that there are thermal injection wells which are intercepted by a vertical fracture. In such cases, the displacement front will move in a direction normal to the plane of the fracture at early times. The swept region at early times can be idealized as a low width to length ratio rectangle. Figure 1.1.15 shows the dimensionless pressure vs. time based on the length of the inner zone, for various width to length ratios of the inner swept region. The mobility and storativity ratios used were 200 and 16.67 respectively. *Teng (1984)* studied a similar problem for gas injection in a vertically fractured well with a finite difference simulator. The nonlinearities caused by the gas flow were included in the study. The solution matches qualitatively with Teng's results.

Computational Effort

The computations for composite reservoirs are quite intense and substantial computing effort is required. This is due to the fact that at every time step a fairly large matrix is inverted 6 to 8 times, depending on the number of sampling points in Laplace space. In addition, the geometry of the inner region must be represented quite accurately. For example, for a radial composite reservoir the phase discontinuity is a circle. If this circle is represented by a hexagon (6 nodes) then the pressure behavior is affected at late times significantly, because the isopotential lines are configured so that they are hexagons instead of circles.

No effort has been made to optimize the code for speedup. But, considerable improvement can be obtained by eliminating repetitive calculations, which are numerous, in favor of increased storage. Optimum number of nodes required varies from problem to problem. For a

Figure 1.1.15: Effect of W/L ratio on the pressure response of a rectangular composite reservoir

Section 1.2 : Comparison of Pressure Transient Response in Intensely and Sparsely Fractured Reservoirs

R.T. Johns and Y. Jalali-Yazdi

A comprehensive analytical model is presented to study the pressure transient behavior of a naturally fractured reservoir with a continuous matrix block size distribution. Geologically realistic probability density functions for matrix block size are used to represent reservoirs of varying fracture intensity and uniformity. Transient interporosity flow is assumed and interporosity skin is incorporated.

Drawdown and interference pressure transient tests are investigated. The results show differences in the pressure response from intensely and sparsely fractured reservoirs in the absence of interporosity skin. Also, uniformly and nonuniformly fractured reservoirs exhibit different responses, irrespective of the degree of fracture intensity. The pressure response of a nonuniformly fractured reservoir with large block size variability approaches a non-fractured (homogeneous) reservoir response. Type curves are developed to estimate matrix block size variability and the degree of fracture intensity from drawdown and interference well tests.

Introduction

The *Warren and Root* model (1963) and other pressure transient models for naturally fractured reservoirs assume fracture intensity is constant throughout the reservoir, i.e., they assume fracturing is uniform and matrix block size is constant. Geological and geomechanical studies of fractured reservoirs and outcrops, however, commonly report occurrences of nonuniform fracture patterns due to variability in lithology, bed thickness, degree of diagenesis, and stress environment (*Pollard and Aydin, 1988; Segall, 1981*). For reservoir engineering purposes, fracture patterns can be represented by simple geometric shapes or designs shown in Fig. 1.2.1. Skewed fracture patterns can also result due to variability in matrix block size and intersection angle. Some fractures are calcite cemented or mineral filled, which can restrict flow from the matrix blocks to the fractures. This phenomenon is termed interporosity skin (*Cinco-Ley et al., 1985; Moench, 1984*).

The distribution of fracture lengths commonly observed in outcrops are exponentially decaying (i.e. there are many short joint lengths and few large joint lengths). Figure 1.2.2 illustrates a probability density function constructed from an outcrop in the Mt. Abott quadrangle of the central Sierra Nevadas (*Segall, 1981*).

Figure 1.2.2: Construction of Probability Density Function from Outcrop, Central Sierra Nevadas. (Segall, 1981)

and for drawdown:

$$\bar{P}_{Dw} = \frac{K_0(x) + S_D x K_1(x)}{s[C_D s(K_0(x) + S_D x K_1(x)) + x K_1(x)]} \quad (1.2.4)$$

Parameter s is the Laplace variable related to dimensionless time (t_D) and the Bessel function argument is:

$$x = \sqrt{s f(s)}. \quad (1.2.5)$$

The function $f(s)$ embodies the reservoir parameters including the matrix block size distribution. For transient interporosity flow in the presence of interporosity skin:

$$f(s) = \omega_f + \omega_m \int_{h_{ratio}}^1 \frac{\sqrt{\frac{\lambda}{\omega_m s}} \tanh(\sqrt{\frac{\omega_m s}{\lambda}}) P(h_D)}{1 + S_{ID} \sqrt{\frac{\omega_m s}{\lambda}} \tanh(\sqrt{\frac{\omega_m s}{\lambda}})} dh_D, \quad (1.2.6)$$

where,

$$h_{ratio} = \frac{h_{min}}{h_{max}}, \quad (1.2.7)$$

$$S_{ID} = \frac{k_m h_s}{k_s h}. \quad (1.2.8)$$

The interporosity skin factor (S_{ID}) is a function of matrix block size distribution and, hence is constant if $\frac{h_s}{h}$ is constant. An alternate assumption is that the depth of skin damage (h_s) is constant for all matrix blocks, and hence, S_{ID} is a variable:

$$S_{ID} = S_{IDmin} \sqrt{\frac{\lambda}{\lambda_{min}}}, \quad (1.2.9)$$

where,

$$S_{IDmin} = \frac{k_m h_s}{k_s h_{max}}, \quad (1.2.10)$$

and now:

$$f(s) = \omega_f + \omega_m \int_{h_{ratio}}^1 \frac{\sqrt{\frac{\lambda}{\omega_m s}} \tanh(\sqrt{\frac{\omega_m s}{\lambda}}) P(h_D)}{1 + S_{IDmin} \sqrt{\frac{\omega_m s}{\lambda_{min}}} \tanh(\sqrt{\frac{\omega_m s}{\lambda}})} dh_D. \quad (1.2.11)$$

Probability Density Functions

Prediction of the pressure response requires the type of matrix block size distribution be known or assumed. When the PDF is selected, fracture intensity can be inferred from pressure transient data. Two types of probability density functions are used to represent

Figure 1.2.3: Probability Density Functions.

PDF	$f(s)$, where $\xi = \sqrt{\frac{\omega_m s}{\lambda_{\min}}}$
Exponential	$\omega_f + \frac{a\omega_m}{\xi(e^{-ah_{\text{ratio}}} - e^{-a})} \int_{\xi h_{\text{ratio}}}^{\xi} \frac{e^{-\frac{ay}{\xi}} \tanh(y)}{y[1 + S_{ID_{\min}} \xi \tanh(y)]} dy$
Linear	$\omega_f + \frac{\omega_m}{\xi[.5m(1 - h_{\text{ratio}}^2) + b(1 - h_{\text{ratio}})]} \int_{\xi h_{\text{ratio}}}^{\xi} \frac{[\frac{m}{\xi} + \frac{b}{y}] \tanh(y)}{1 + S_{ID_{\min}} \xi \tanh(y)} dy$
Rectangular	$\omega_f + \frac{\omega_m}{\xi(1 - h_{\text{ratio}})} \int_{\xi h_{\text{ratio}}}^{\xi} \frac{\tanh(y)}{y[1 + S_{ID_{\min}} \xi \tanh(y)]} dy$
Dirac delta	$\omega_f + \frac{\omega_m \tanh(\xi)}{\xi[1 + S_{ID} \xi \tanh(\xi)]}$, where $\lambda_{\min} = \lambda_{\max} = \lambda$

Table 1.2.1: Functions $f(s)$ for various PDF's.

Figure 1.2.5: Rectangular PDF: Varying h_{ratio} for Geometric Mean $\lambda = 10^{-6}$, $\omega_m = .9$.

Figure 1.2.6: Rectangular PDF: Solution by Stehfest Inversion versus Time Domain
 Approximation, $h_{ratio} = .1$, $\lambda_{min} = 10^{-7}$, $\omega_m = .9$.

decreases, the effect of matrix block size variability becomes less significant. Second, the effect of h_{ratio} on the pressure response is greatest for lower values of h_{ratio} (e.g. the pressure response changes more significantly for h_{ratio} values from 0.1 to 0.5 than from 0.5 to 1.0). This indicates that block size variability affects the pressure response significantly if h_{min} and h_{max} differ by at least one order of magnitude. Block size variability less than half an order of magnitude does not affect the pressure response significantly.

An example of the effect of interporosity skin ($S_{ID_{min}}$) on the pressure transient response is shown in Fig. 1.2.8. A significant change in the pressure derivative is seen for small changes in $S_{ID_{min}}$, and thus, the effect of the matrix block size distribution is masked. The derivative profile becomes symmetric and more pronounced which is typical of the PSS response of Warren and Root. A symmetric PSS type response develops even if the no skin profile is asymmetric. As interporosity skin increases, the derivative profile shifts in time, giving apparent λ values that are too small (more dormant matrix). Thus, if interporosity skin exists, interpretation of pressure transient tests by the Warren and Root model underestimates λ and fracture intensity.

Discussion-Interference Testing

Braester (1984) demonstrated that drawdown (or buildup) tests in naturally fractured reservoirs may not be influenced by matrix blocks significantly away from the wellbore. Interference testing, therefore, is preferred because the response is affected by matrix blocks between the active and observation wells. A simplified solution for interference testing in the absence of storage and wellbore skin is the line source solution:

$$\bar{P}_{D,f} = \frac{K_0(r_D \sqrt{s f(s)})}{s} \quad (1.2.25)$$

For any PDF distribution, it can be shown that $\theta = \lambda_{min} r_D^2$ is a correlating parameter *Deruyck et al* (1982). For instance, using the rectangular PDF:

$$f(sr_D^2) = \omega_f + \frac{\omega_m}{1 - h_{ratio}} \sqrt{\frac{\theta}{\omega_m s r_D^2}} \int_{h_{ratio}}^{\sqrt{\frac{\omega_m s r_D^2}{\theta}}} \frac{\tanh(y)}{y} dy. \quad (1.2.26)$$

Equation 1.2.25 can then be evaluated using the inverse Laplace transform relation:

$$\mathcal{L}^{-1}[\bar{P}_{D,f}(sr_D^2)] = \frac{1}{r_D^2} P_{D,f}\left(\frac{t_D}{r_D^2}\right). \quad (1.2.27)$$

A type curve (Fig. 1.2.9) is prepared using the rectangular PDF for $\omega_m = 0.9$. For each value of θ , h_{ratio} is varied from zero to one. If h_{ratio} determined from the type curve is equal to one, the PDF is a Dirac delta function and the type curve is similar to that presented by Deruyck et al (1982).

For large values of θ , the matrix block size variability becomes increasingly important and h_{ratio} can be better estimated. Thus, if the dimensionless distance (r_D) between the active and observation wells is large, or if λ_{min} becomes large (i.e. greater fracture intensity), then matrix block size variability becomes a key parameter in interference pressure transient analysis. Conversely, for smaller values of θ , matrix block size variability (or h_{ratio}) does not affect the pressure response significantly. Also, as θ becomes larger, the response approaches the line source solution for smaller values of $\frac{t_D}{r_D^2}$.

Conclusions

1. A formulation incorporating transient interporosity flow and interporosity skin is presented for fractured reservoirs with variable matrix block size. Exponential and linear probability density functions have been used to represent intensely and sparsely fractured reservoirs with varying degrees of fracture uniformity.
2. Type curves have been generated for drawdown and interference well tests based on the rectangular PDF and slab matrix block geometry. Type curves yield estimates of fracture intensity as well as fracture nonuniformity.
3. Fracture intensity determines the temporal position of the pressure response, while fracture uniformity affects the shape of the pressure response. For transient interporosity flow, uniformly fractured reservoirs exhibit asymmetric derivative profiles, whereas nonuniformly fractured reservoirs exhibit symmetric profiles.
4. The parameter h_{ratio} quantifies the degree of fracture uniformity. Uniform fracturing is indicated when h_{ratio} is near one, while nonuniform fracturing is indicated when h_{ratio} is near zero. For an extremely nonuniform fractured reservoir (h_{ratio} approaching zero), the pressure response is similar to a nonfractured homogeneous reservoir response.
5. Matrix block size variability (h_{ratio}) cannot be estimated in the presence of interporosity skin damage. The Warren and Root model overestimates matrix block size if interporosity skin is present.

Section 1.3 : Elliptical Finite Conductivity Fractures

Michael F. Riley

Movement of fluids in geothermal reservoirs takes place largely through a series of interconnected fractures. The reservoir pressure drops are due, in large part, to the fracture conductivity, i.e., how easily fluid flows in these fractures. The presence of material between the fracture walls will adversely effect their conductivity, increasing the pressure drop for a given flowrate.

The aim of the present research is to develop 'analytic' solutions for the pressure drop due to production from a single finite-conductivity fracture. It is assumed that a single fully penetrating fracture, of finite conductivity, produces from a homogeneous reservoir of infinite extent. This necessitates the solution of a two dimensional problem without the difficulty of including outer boundary effects. The novelty in the present approach is that the fracture cross-section will be modelled as a degenerate ellipse rather than a vanishingly thin rectangle. In both cases the resulting cross section is a line segment, but the equations governing the flow in the fracture are different.

The aim of this report is to show the problem as posed and solved for a rectangular fracture and to contrast it with the case of an elliptical fracture. The advantage of using elliptical coordinates will be shown to be the ease in coupling the flow in the reservoir with the flow in the fracture. This is independent of the fracture being more closely approximated by an ellipse than by a rectangle, especially in the case of hydraulically induced fractures.

Flow to a Rectangular Fracture

The classic work on finite conductivity fractures was done in a series of papers by Cinco, et al. The most important of these are *Cinco et al. (1978)* and *Cinco and Samaniego (1981)*. The first of these references shows the derivation and solution of the problem, while the second shows the application of the solution to pressure transient testing.

The following is a modification of the derivations given in *Cinco et al. (1978)*. Most of what follows in this section is shown in a paper by *Cinco and Meng (1988)*. These modifications are made in order to better contrast the resulting equations with those derived subsequently for the elliptical case.

$$\bar{P}_2(x,y;s) = \frac{1}{2s} \int_{-1}^{+1} q(x',s) K_0(\sqrt{s}\sqrt{(x-x')^2 + y^2}) dx'$$

$$\left. \frac{\partial^2 \bar{P}_1}{\partial x^2} + \frac{2}{F_D} \frac{\partial \bar{P}_2}{\partial y} \right]_{y=0} = \frac{1}{\eta_D} s \bar{P}_1$$

Figure 1.3.1 : Cinco et al's Model

Equation 1.3.2 can also be put in an integral form that also incorporates the two boundary conditions, Eqs. 1.3.3 and 1.3.4. In the integral form, Eq. 1.3.2 is

$$\bar{p}_f(x_D) = A(s) + \frac{(x_D - 1)^2}{2\pi s F_D} + \frac{1}{2s\pi F_D} \int_{-1}^{+1} \sum_{n=1}^{\infty} \bar{q}(x') \frac{\cos(n\pi x_D) \cos(n\pi x')}{n^2} dx' \quad (1.3.6)$$

This second integral formulation gives pressure anywhere in the fracture and includes the boundary conditions. Unfortunately, in this equation both $\bar{q}$ and $A(s)$ must be determined before the fracture pressure can be calculated. Equation 1.3.6 can be simplified somewhat but this is not necessary.

Equations 1.3.5 and 1.3.6, along with the condition that the integral of the flux over the fracture equals $1/s$, fully specify the problem. The first step in determining pressures is to set y_D equal to zero in Eq. 1.3.1 and to equate it to Eq. 1.3.2. This results in an integral equation for $\bar{q}$. For purposes of solution, $A(s)$ is treated as a constant, since it is independent of x_D . Once $\bar{q}$ is determined up to a multiplicative constant, $A(s)$ is determined by the constraint on the integral of the flux. The pressures are then determined from Eq. 1.3.5 or Eq. 1.3.6. So far we have made only relatively minor simplifications, and if we could find an equation form for $\bar{q}$ and $A(s)$, we would have the analytic solution we desire. However, these equations do not appear to be solvable by any known means.

In practice, the integral equation resulting from Eqs. 1.3.5 and 1.3.6 are solved numerically and the pressures are inverted to real space using the Stehfest algorithm. Although this is a numerical scheme, the resulting pressures seem to be very accurate. The main problem with the method is that it is time consuming and the lack of closed form representation hides the actual nature of the solution.

In order to see the behavior of the solutions, simpler approximate problems are solved and analyzed. From these, the regimes of fracture linear flow, bilinear flow and pseudolinear flow are determined. The fact that these are essentially linear flow regimes and that they were found by solving problems where linear flow dominates brings up the question of whether they result from the assumed shape of the fracture. In other words: are these linear flow regimes a result of assuming the fracture to be rectangular? This is something we hope to check by solving the problem with an elliptical fracture.

Flow to an Elliptical Fracture

In order to overcome most of the difficulties discussed in the last section, we will now consider the problem in elliptical coordinates. The first part will show the equations ex-

Figure 1.3.2: Elliptical Coordinate System

The pressure in the reservoir, $\bar{p}_r$, is found by determining the values of these constants. They are determined by coupling the reservoir solution to the fracture solution.

The solution to the fracture problem, Eq. 1.3.10, is approached in a similar way. Eigen-solutions are sought that satisfy the homogeneous fracture problem, i.e. the right hand side of Eq. 1.3.10 and the boundary conditions, Eqs. 1.3.12 and 1.3.13, all equal to zero. At this point the cosine eigenfunctions are multiplied by constants, δ_{2n} , and summed over positive n . Then, the series described in the last paragraph is inserted into the right hand side of Eq. 1.3.10 and treated as a known function, i.e., as an inhomogeneity. Some fairly delicate manipulation now has to be done to allow the cosine series to satisfy the inner boundary condition. The end result of all of these manipulations is a series for the reservoir pressure in terms of β_{2n} and Mathieu functions, a series for the fracture pressure in terms of δ_{2n} and cosines and the fracture equation which contains both sets of functions and constants and couples the two solutions.

The series representing the reservoir solution is

$$\bar{p}_r(\xi, \eta) = \sum_{n=0}^{\infty} \beta_{2n} Fek_{2n}(\xi, -\frac{s}{4}) ce_{2n}(\eta, -\frac{s}{4}), \quad (1.3.14)$$

where Fek_{2n} and ce_{2n} are Mathieu functions.

The series representing the fracture solution is

$$\bar{p}_f(\xi, \eta) = \sum_{n=0}^{\infty} \delta_{2n} \cos(2n\eta), \quad (1.3.15)$$

So far none of the constants in the above representations are specified. They are constrained by the fracture equation obtained by inserting the two series into Eq. 1.3.10. The result is

$$\sum_{n=0}^{\infty} \left(\frac{(-1)^n}{sF_E} - 4n^2 \delta_{2n} \right) \cos(2n\eta) + \frac{1}{\pi F_E} \sum_{n=0}^{\infty} \beta_{2n} Fek'_{2n}(0, -\frac{s}{4}) ce_{2n}(\eta, -\frac{s}{4}) = 0, \quad (1.3.16)$$

where $Fek'_{2n}(0, -\frac{s}{4})$ is the derivative of the radial Mathieu function evaluated at ξ equal to zero.

The fracture boundary conditions have been included in Eq. 1.3.16 and so it fully constrains the problem. The method of determining the constants is to remove one set of

Section 1.4 : Pressure Oscillations in a High Rate Well in Fractured Formation

Sanjay Bhatnagar

Overview

Geothermal reservoirs are highly fractured and the effect of high flow rates can be very important in the analysis of buildup tests. The focus of this study is limited to inertial effects on the pressure response of a fractured reservoir. A model is proposed for pressure transient response in high permeability, high rate, naturally fractured reservoir. Theoretical models presently available are inadequate for handling these systems because they do not include the inertial effects of the liquid/gas in the wellbore and the fractures.

Problem Definition

Figure 1.4.1 shows a pressure buildup test performed in an oil well in Mexico. The well was shut in at the wellhead and pressure as well as the flowrate were recorded downhole. An expanded pressure and time scale in Fig. 1.4.2 shows underdamped oscillations with a time period of about 70 seconds. For high production rate in fractured reservoirs, the kinetic energy of the liquid moving in the fractures and the wellbore may be important and contribute to the early time pressure response. The relative importance of inertial effects in the wellbore and in the formation fractures is also investigated.

Saldana and Ramey (1986) considered inertial effects in the wellbore and coupled a momentum balance equation for the wellbore to the diffusivity equation in the reservoir. They presented criteria useful to evaluate the magnitude of inertial and frictional effects. *Hotzhausen and Gooch (1985)* observed a definite change in the period of oscillations before and after hydraulic fracturing. This suggests that the natural fractures and/or the wellbore could explain the Mexican well test data.

Solution Methodology

Two models are considered to identify the source of oscillations. The first model considers inertia in the wellbore coupled with diffusivity equation in the reservoir while the second model considers inertial effects of the liquid in a single fracture in the reservoir. The fracture is coupled to the wellbore but does not account for inertial effects in the wellbore.

Figure 1.4.2: Pressure Buildup for Mexican Oil Well with Expanded Pressure and Time Scales

Figure 1.4.3: Schematic diagram of well and reservoir

Figure 1.4.4: Schematic of a single fracture in the Linear Flow with Inertia Model

Figure 1.4.5: Dimensionless Pressure vs. Dimensionless Time for $\beta=1.0$ and for a Transducer Time Constant of 0.5 Seconds

- Evaluation of wellbore storage effects under both build-up and draw-down tests.
- Further verifications of correlations developed by *Jenkins and Aronofsky (1954)* for ideal gas flow in a one dimensional radial reservoir.

The project is still undergoing development in the following areas :

- Testing the usability of the program for the following five configurations of well tests: (1) draw-down, (2) build-up, (3) interference, (4) injection and (5) multiple rate tests.
- Providing any combination of three different outer boundary conditions (infinite acting, no-flow, constant pressure) and two different inner boundary condition (constant rate, constant pressure) for each type of well test.
- Create access to instant graphic display of usual log-log, semi-log or cartesian plot analysis of simulated data.

The main problems in implementing these changes are related to obtaining accurate data close to the wellbore. In addition, numerical difficulties and improving user friendliness are important considerations.

field approach (*Pickens and Grisak, 1981*). This is appropriate if the scale of heterogeneities is smaller than the scale of flow.

In geothermal reservoirs, extremely fast fluid movements (up to 100 m/hr in some instances (*Horne, 1982a; Horne, 1982b*)) and asymmetric tracer return profiles indicate that flow takes place mainly in fractures between injection well and producing wells. Since the scale of fractures is in the order of the scale of flow, the very near field approach is appropriate to model the tracer transport.

There are three commonly used mathematical models to represent tracer transport through fractures. The first of these models, the convection-dispersion(CD) model, assumes a purely dispersive flow in the fracture. The form of the dispersion parameter depends on how the fracture is modelled.

The governing differential equation of the convection-dispersion model is given by

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} = 0 \quad (2.1.1)$$

The variables used in Eq. 2.1.1 and in all other equations in this report are defined in the nomenclature.

The second model is called the matrix diffusion(MD) model. It represents a system consisting of a fracture in which the tracer fluid is mobile located in a porous matrix in which the reservoir fluid is virtually immobile. The model considers the diffusion of the tracer from the fracture into the adjacent porous matrix as the main mechanism spreading the tracer along the transport region. It neglects longitudinal dispersion and assumes that tracer concentrations across the fracture are equalized before any significant effect of the convection appears.

Two coupled one-dimensional equations are used to represent tracer transport. The equations are coupled by using the continuity of the flux and concentration across the fracture-matrix interface. The equation of tracer transport in the fracture (*Neretnicks, 1980*) is

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + q = 0 \quad (2.1.2)$$

A fairly complete discussion of the analysis of interwell tracer tests without recirculation can be found in the previous annual report of Stanford Geothermal Program. Also, for a detailed discussion of the results of the field experiments, refer to the dissertation by Kocabas, (1989).

Interwell Tracer Tests with Recirculation

The purpose of the interwell tests with recirculation is to estimate the total reservoir volume accessible to the tracer as well as the parameters affecting the tracer transport. The total reservoir volume accessible to the tracer consists of the recirculating fluid volume, which corresponds to the fracture volume, and the stationary fluid volume, which corresponds to the matrix pore volume. In general, while the recirculating fluid volume affects the breakthrough time, the stationary fluid volume determines the final level of concentration reached after adequate recirculations.

In the study of return profiles, it is convenient to normalize the concentration by a reference concentration, C_r , the final concentration that would be reached if all the tracer were to mixed only in the recirculating volume. The normalized concentration functions are given in Table 2.1.1. The generated return profiles indicate (see Figs. 2.1.1 to 2.1.5) as many local minima and maxima as the number of recirculations. If we try to determine the times of the local minima and maxima by equating the time derivative to zero, only the first peak time occurs in a linear equation. However, the equation of the time of the first peak allows us to make close initial estimates of the parameters.

The return profiles generated by the CD model in Fig. 2.1.1 display almost periodic patterns, and the amplitude of the peaks decreases with increasing time. The second peak occurs almost exactly at a time equal to twice the time of the first peak. Also the first local minimum occurs virtually at the midpoint of the first and second peaks. These features can be used to identify the number of recirculations in an experiment and to determine if a recirculation is completed or not. Therefore in cases where there is not enough data, we can look for distinguishing features such as the slope after the first peak. It should be negative until the time is close to $1.5t_p$, and then positive until the time is close to $2t_p$.

The MD model return profiles shown in Figs. 2.1.2 and 2.1.3 also have almost periodic patterns. On the other hand, their features are different from the features of the CD model profiles. First the peaks occur immediately after t_w , and the section between t_w and t_p are characterized by a steep slope. Second, the influence of time is significant in determining the shape of the return profile. For example, both a system with a small λ and large t_w and a system with a large λ and a small t_w yield similar tracer return profiles. The parameter t_w affects the injection period because the tracer reaches the observation point when $t > t_w$.

Figure 2.1.1: Normalized Solutions to CD Model For Recirculating Flow

Figure 2.1.3: Normalized Solutions to the MD Model for Large Amount of Diffusion

Figure 2.1.4: Normalized Solutions to the AD Model for Short Period Tests

Convection-Dispersion Model

In this study we consider four cases of injection-detection modes, namely C_{CRR} , C_{CRF} , C_{CFR} and C_{CFF} .

The solutions may be expressed in dimensionless variables as shown in Table 2.1.2. The variable t_D is an inverse Peclet number scaled to the distance (ut_j) travelled by the convective front during the injection period. Other variables are $x_D = x/(ut_j)$ and $t_{Dp} = t/t_j$.

Numerical differences between injection-backflow solutions are similar to numerical differences between the solutions for unidirectional flow. While for small t_D differences between profiles are small (see Fig. 2.1.6), for large values of t_D differences increase (see Fig. 2.1.7). In Fig. 2.1.6, the C_{CRR} solution yields the highest concentration values as opposed to C_{CFF} in the case of unidirectional flow. This is because, convective and dispersive fluxes are in opposite directions.

Matrix Diffusion Model

The solutions of transport equations were obtained by using a double Laplace transformation method. The equations were transformed first with respect to the injection period time variable t_j , and then with respect to the backflow period time variable, t . Using the Laplace transformation, the difficulties caused by the effect of the step function, which is in the solutions for the injection period, have been avoided.

The Laplace space solution for equal injection and backflow rates were presented in a previous work (Kocabas, 1986; Kocabas and Horne, 1987). Defining a new variable, λ_D

$$\lambda_D = \lambda\sqrt{t_j} \quad (2.1.8)$$

and normalizing the time variables by t_j , Laplace space solution becomes

$$\bar{C} = \frac{1}{s + p + 2\lambda_D(\sqrt{s} + \sqrt{p})} \left[\frac{1}{s} + \frac{2\lambda_D}{s\sqrt{p}} + \frac{2\lambda_D}{s - p} \left(\frac{1}{\sqrt{s}} - \frac{1}{\sqrt{p}} \right) \right] \quad (2.1.9)$$

In Eq. 2.1.9, s corresponds to the variable $t_j/t_j = 1$ and p correspond to the variable t_{Dp} . Using the numerical inversion method, functions of functions, discussed by *Ditkin and*

Figure 2.1.6: Solutions for Different Injection and Detection Modes-Medium t_D

Prudnikov (1962), and the inversion formula given by Voelker and Doetsch (1950), the real space function of Eq. 2.1.9 is

$$\begin{aligned}
C_f = & \int_0^{\min(1, t_{Dp})} \left\{ \operatorname{erfc} \left(\frac{\lambda_D \theta}{\sqrt{1-\theta}} \right) \frac{\lambda_D \theta}{\sqrt{\pi(1-\theta)^3}} \exp \left(-\frac{\lambda_D^2 \theta^2}{t_{Dp} - \theta} \right) + \right. \\
& \left. 2\lambda_D \operatorname{erfc} \left(\frac{\lambda_D \theta}{\sqrt{1-\theta}} \right) + \frac{1}{\sqrt{\pi(1-\theta)}} \exp \left(-\frac{\lambda_D^2 \theta^2}{t_{Dp} - \theta} \right) \right\} d\theta \\
& + 2\lambda_D \int_0^1 d\eta \int_0^{\min(1-\eta, t_{Dp}+\eta)} \left\{ \frac{1}{\sqrt{\pi(1-\eta-\theta)}} \exp \left(-\frac{\lambda_D^2 \theta^2}{1-\eta-\theta} \right) \right. \\
& \quad \left. \frac{\lambda_D \theta}{\sqrt{\pi(t_{Dp}+\eta-\theta)^3}} \exp \left(-\frac{\lambda_D^2 \theta^2}{t_{Dp}+\eta-\theta} \right) \right\} d\theta \\
& - 2\lambda_D \int_0^1 d\eta \int_0^{\min(1-\eta, t_{Dp}+\eta)} \left\{ \frac{\lambda_D \theta}{\sqrt{\pi(1-\eta-\theta)^3}} \exp \left(-\frac{\lambda_D^2 \theta^2}{1-\eta-\theta} \right) \right. \\
& \quad \left. \frac{1}{\sqrt{\pi(t_{Dp}+\eta-\theta)}} \exp \left(-\frac{\lambda_D^2 \theta^2}{t_{Dp}+\eta-\theta} \right) \right\} d\theta \quad (2.1.10)
\end{aligned}$$

Eq. 2.1.10 was evaluated numerically for several values of λ_D , and Eq. 2.1.9 was inverted numerically by using a double numerical inversion technique based on the *Stehfest* (1970) algorithm.

Figs. 2.1.8 to 2.1.10 show the results of numerical integration of the solution as well as the numerically inverted Laplace space solution. From these figures, the profiles computed by both methods preserve a material balance. The material balance can be checked by drawing a vertical line at $t_{Dp} = 1$, and determining whether the areas above the curve before the line and under the curve after the line are equal or not. For small values of λ_D , the curves for the two methods differ considerably. For large values of λ_D , the profiles become smooth, and both methods produce the same result.

Thermal Injection-Backflow Tests

A thermal injection-backflow test may be the best way to estimate the thermal characteristics of a system to avoid high cost or extremely long test period of thermal interference tests. The heat and the tracer transport mechanisms in porous media are nearly identical in form. The transport of both heat and tracer may be modelled by the same differential

Figure 2.1.9: Solutions to MD Model for Medium λ_D

equations. Therefore, the solution to the MD model, which was developed for the analysis of injection-backflow tracer tests is used to interpret temperature return profiles from thermal injection-backflow tests. In this case, the dependent variable of the system is temperature, and the parameter of the model, λ_D , for flow in a porous stream tube, is

$$\lambda_D = \frac{\sqrt{k_m \rho_m c_m}}{\phi_f b \rho_w c_w} \sqrt{t_j} \quad (2.1.11)$$

In heat transport problems, a dimensionless temperature variable is

$$T_D = \frac{T - T_o}{T_{in} - T_o} \quad (2.1.12)$$

The thermal breakthrough time, if equal the breakthrough time of $T_D = 0.75$, is (*Pruess and Bodvarsson, 1984*)

$$t_i = \frac{\rho_1 c_1}{\rho_w c_w} \frac{t_w}{\phi_f} + \frac{\lambda_D^2}{t_j} \frac{t_w^2}{0.8134^2} \quad (2.1.13)$$

The contribution of the first group of terms in Eq. 2.1.13 is small compared to the contribution of the second group of terms, because the lateral heat conduction is the main mechanism retarding propagation of the thermal front. In other words, it is the parameter $\lambda = \lambda_D / \sqrt{t_j}$ which determines the thermal breakthrough time.

Using the values given in Table 2.1.3 for the rock and fluid properties (*Reynolds and Perkins, 1977*) and for $t_j = 1 \text{ hr}$ and $t_w = 50 \text{ hrs}$, corresponding breakthrough times were calculated and are given in Table 2.1.4.

While the effect of temperature on some of the rock and the fluid properties such as $\rho_w c_w$ and $\rho_r c_r$ are not important, others such as k_w and k_r may be affected by temperature. In Table 2.1.3 the properties of water were evaluated at 176°C , and the reservoir rock was assumed to be granitic. It is reported (*Reynolds and Perkins, 1977*) that k_r of granite may have values ranging from 1.73 to 3.98. If k_r is taken to be 1.73, then for $\phi_f b = 1 \text{ mm}$, the corresponding breakthrough time in Table 2.1.4 would have decreased from 633 to 398 years. This demands an accurate determination of the thermal conductivity of the rock. An in-situ determination of thermal conductivity, and other parameters as well, may be achieved by using thermal injection-backflow tests.

Eq. 2.1.13 requires two important parameters, namely, water transit time t_w , and λ which is a measure of thermal interaction between fracture and matrix. The parameter t_w is estimated from interwell tracer tests and λ is estimated from thermal injection-backflow tests.

Conclusions

Any method of estimating the thermal breakthrough time must be based on both tracer and thermal data. This work involved quantitative interpretation techniques of tracer tests and thermal injection-backflow tests, and development of a new method to estimate thermal breakthrough times.

Depending on the scale of heterogeneities relative to the scale of flow, one of the four approaches namely, the very near field, the near field, the far field and the very far field may be used to model tracer and heat transport through fractures. In geothermal reservoirs, extremely fast fluid movements and asymmetric tracer return profiles of interwell tracer tests indicate that the scale of fractures is in the order of the scale of flow. Therefore, tracer return profiles should be interpreted by using the very near field approach. Based on this approach, quantitative interpretation techniques of interwell tracer tests with and without recirculation, and injection-backflow tests are presented.

Tracer return profiles are matched with the solutions of three mathematical models (CD, MD and AD) developed to study tracer transport through a single vertical fracture. Matching tracer return profiles with solutions of mathematical models provides estimates of three parameters:

- t_w , a measure of flow speed,
- D , a measure of dispersive characteristics,
- λ , a measure of fracture-matrix interaction.

Estimating the thermal breakthrough time may be based solely on tracer data. The two parameters D , and λ can be used to estimate the fracture aperture b , the most important parameter controlling the propagation of thermal front. However, estimating b from D requires that $P_e \simeq 500 - 1000$. For most of the tracer return profiles $P_e \ll 500$. Estimating b from λ is also questionable, since D_a and ϕ are usually unknown.

Alternatively, estimation of the thermal breakthrough time may be based solely on thermal interference tests. Thermal interference tests are infeasible, because durations of

Section 2.2 : Optimal Selection of Flow Rates at Palinpinon Geothermal Field Based on Tracer Return Data

Ma. Elena G. Urbino

Overview

The Palinpinon Geothermal Field is one of the two producing steamfields currently operated by the Philippine National Oil Company (PNOC). The requirements of the commercial plant, known as Palinpinon 1, are met by 21 production wells, 10 reinjection wells and 1 condensate reinjection well. The need for reinjection of liquid waste effluent has been primarily dictated by environmental constraints which prohibit full disposal into the river systems that are used for rice field irrigation. In addition to this, the other benefits of injection, such as maintaining reservoir pressures and increasing thermal recovery from rocks have been recognized.

Although injection wells had been drilled on the periphery of the field, preferably at the identified outflows, initial chemical monitoring of the produced fluids have shown increases in well fluid chloride measurements. These have been interpreted as evidence of the return of reinjected fluids to the production sector. The premature thermal breakthrough of cooler injected fluids at producing wells could cut short the economic life of the field. Hence, guidelines for the safe and efficient management of the Palinpinon reservoir have been established. These include the requirements of 1) minimizing fluid residence times in the surface and downhole piping while operating reinjection wells at or near, maximum capacity, 2) minimizing steam wastages brought about by varying steam demand and supply, and, 3) adopting a production and reinjection well utilization scheme, under any given load demand, such that the rate and magnitude of reinjection fluid returns could be minimized, if not avoided.

The first of these requirements is the answer to the problem of silica deposition which occurs on injection of a fluid that is supersaturated with respect to amorphous silica. The second requirement which is economical in nature, has been satisfied by prioritizing high enthalpy production wells for peak steam requirements and choosing injection wells with additional capacity. The decisions on well utilization (the third requirement) to achieve optimum well configurations have been arrived at, on a relative-basis, by the confluence of production and reinjection fluid chemistry, downhole measurements of pressure and temperature, interference testing, tracer tests, and the interpreted field model. It is to this requirement that the study is directed.

Problem Formulation and Approach

This study aims to provide answers to two questions. First, given an existing configuration of production and reinjection wells, as well as operating requirements in the Palinpinon Geothermal Field, is it possible to choose in a systematic fashion, an optimum number and

SECTION 3.1: HEAT EXTRACTION PROJECT

Progress Report on Heat Extraction Research

Paul Kruger

Introduction

The primary objective of the SGP Heat Extraction Project is to provide a means for estimating thermal behavior of geothermal fluids reinjected into fractured hydrothermal reservoirs under sustained production. The methods are based on estimated thermal properties of the reservoir components, reservoir management planning of production and reinjection, and the mixing of reservoir fluids: geothermal resource fluid cooled by drawdown and infiltrating groundwater, and reinjected recharge heated by sweep flow through the reservoir formation. Several reports and publications, listed in Appendix A, describe the development of the analytical methods which were part of five Engineer and PhD dissertations, and the results from many applications of the methods to achieve the project objectives.

During FY89, the Heat Extraction Project carried out several thermal cooldown applications in geothermal fields in a number of countries. Heat sweep joint studies are underway for reinjection evaluation at the Los Azufres, Los Humeros, and La Primavera fields in Mexico under the DOE-CFE Geothermal Agreement. The location of these three geothermal fields in relation to Mexico City was shown in the *FY89 Annual Report* (1989). The timing of these studies is especially important because the development at each of these fields is at the stage when decisions must be made on the selection of brine disposal wells at each of the production zones.

Also during the year, initial estimates of thermal effects by heat sweep for the Rosemanoes Hot Dry Rock experiment were made as a joint study with the Camborne School of Mines to match the observed cooldown over the life of the long-term test. Excellent match agreement of the cooldown data is expected to provide values for key reservoir parameters.

A broad cooperative program was initiated with the Leningrad Mining Institute in Leningrad, USSR on the development of hot water supply recirculation systems to be developed in the USSR, the first of which is scheduled for the Zakarpate Oblast in western Ukraine in 1990. During the invited two-month visit to the Leningrad Mining Institute in the spring of 1989, a paper was prepared on the joint efforts on the Zakarpate field and presented (*Dyadkin and Kruger, 1989*) at the Camborne School of Mines International Conference on Hot, Dry Rock Resources, in Cornwall, England, June, 1989. Cooperative efforts were also carried out for the initial tracer test at the Mutnovsky Geothermal Field in Kamchatka, far eastern USSR. During FY89, a joint study on the comparison of thermal decline during the tracer flow test was prepared (*Kiryukhin and Kruger, 1990*) for the Fifteenth SGP Geothermal Workshop in January, 1990.

A main effort of the Heat Extraction Project is the early evaluation of thermal drawdown by small wellhead electrical generating units on potentially large geothermal resources. Such evaluations are of use in the planning of large central power plant units. The long-term

evaluation of several Laplace transform inversion methods to optimize the accuracy of the output cooldown curves and compilation of the heat sweep source code as a microcomputer version for use in field applications.

In FY88, the 1-D Heat Sweep Model was improved to include doublet flow from a pair of injection-production wells. The doublet flow geometry is useful in new geothermal zones where only exploratory production wells are available and flow boundaries, such as faults, are not identified. The difficulty encountered in using doublet flow geometry in a one-dimensional model is its intrinsic two-dimensional behavior. The approach taken was to accomplish reduction in dimensionality by division of the total flow field into a number of linear channels, and integration of the one-dimensional linear heat sweeps over the domain of the two-dimensional flow. For a sufficiently large number of flow channels (e.g., $N > 80$), the crescent flow channels between streamlines can be expressed as equivalent rectangles having the same heat transfer surface and mean fluid velocity. With the mean channel length and mean fluid velocity in each channel, the heat transfer between fluid and reservoir rock is modeled as a 1-D linear heat sweep in each channel. Summing of the N channel data in an array of residence time intervals provides the time dependent cooldown curve for the total flow at a production well. If the reinjection rate is only part of the production rate, mixing of the sweep fluid with reservoir resource fluid at the appropriate cooldown rate is also modeled.

The first application of the 1-D Doublet Heat Sweep Model was made for the El Chino zone of the Los Azufres geothermal field. This zone is between the steam reservoir in the Tejamaniles (south) zone and the two-phase reservoir in the Maritaro (north) zone. One production well (Az-9) and one potential reinjection well (Az-3) have been drilled in this zone. CFE has decided to locate one of the four new 5-MW wellhead units at well Az-9 and reinject at Az-3. The application of the 1-D Doublet Heat Sweep Model allows a comparison of the linear and radial models for flow bounded by the El Chino and Laguna faults. The results of this study were presented by *Lam and Kruger* (1987).

During FY89, work continued on development of the doublet flow model for non-uniform temperature distribution in the production zone of the reservoir. For a uniform initial reservoir temperature, the heat content available above a useful abandonment temperature increases rapidly with increasing crescent number. The result is a very long cooldown time to the abandonment temperature relative to results obtained with a uniform temperature in the bounded flow geometry inherent in the linear and radial flow models. An improvement in the 1-D Doublet Heat Sweep Model was achieved by limiting the formation volume of the reservoir at mean initial temperature to a defined geometry. Unfortunately, little temperature distribution data exist in the new geothermal fields covered under the current joint projects. During FY89, efforts were continued to acquire temperature data, and an initial temperature cross section was prepared by the CFE staff at the La Primavera field. To obtain early insight on the effect of bounded temperature distribution on doublet recharge return flow geometry, an exercise was carried out for several potential initial temperature distributions at the La Primavera field and the El Chino zone of the Los Azufres field. The output temperature was evaluated as a function of crescent number, where crescent number

in philosophy of CFE to change to plastic (polyethylene) surface tubing, which costs less but has a lower maximum allowable fluid temperature for the recharge lines. CFE is evaluating the option of using the nearby Laguna Verde lake for cooling and silica deposition before reinjection. Therefore, the studies, revised to reflect the specific objectives outlined in the original DOE-CFE Task 5.1 joint study for heat sweep analysis in the Los Azufres field, should be useful in determining the optimum reinjection time and temperature. During the site visits in FY89, review of the new objectives for heat sweep analyses were discussed and compilation of the input data sets was initiated under the Renewed 5-year DOE-CFE Geothermal Agreement. Continuation of these joint studies is expected by the pertinent CFE co-investigators when the reorganization of CFE Gerencia is completed.

C. La Primavera

Heat sweep studies for the La Primavera field in FY88 were centered on the plans for the first 5-MW portable wellhead unit planned to be installed at well PR9 with reinjection into existing well PR2. The joint studies for this unit were presented at the SGP 12th annual Workshop (*Kruger et al.*, 1987). During FY89, the joint project was adjusted to examine the effects of a second 5-MW unit to be installed concurrently with Unit 1. Steam for the two wells are expected to be supplied from three existing production wells, PR9, PR1, and PR8. During FY89, CFE suspended plans to develop the La Primavera field pending clarification of environmental and other local problems. The joint study is scheduled to proceed only in project preparation until CFE plans are changed.

DOE-CFE Los Azufres Well Startup Project

A. Startup Analysis of the First Seven Years of Production in the Maritaro Zone Reservoir

The response of the Los Azufres geothermal field to the onset of continuous production from the original five wells to supply the five 5-MW wellhead units has been examined in previous annual reports. Publications were prepared after 2, 2.5, 4, and 5 years of operation. The final report under the original DOE-CFE Agreement is published (*Kruger and Aragon*, 1989). The accumulation of the many production, chemical, and reservoir data has allowed a detailed examination of the effects of the cumulative production on the thermal drawdown of the reservoir. An important aspect of this analysis is the thermal history of the produced water, especially in the two-phase Maritaro zone, where geothermometer temperature data allows the estimation of reservoir specific volumes for the chemical components.

During FY89, the major effort on this joint study was the compilation and analysis of the additional cumulative data from the now seven years of continuous operation and initiation of new startup analysis at the new wells in the El Chino and Tejamaniles zones. In continuation of the joint study on thermal drawdown effects, the general objective remains to observe the thermodynamic changes in the reservoir and to evaluate the extent of changes observable by operation of small wellhead units in potentially large geothermal fields. Data for all years of production are accumulated in the form of chemical data for Na, K, Ca, Cl, and SiO₂. Production data include wellhead pressure P(wh), separator pres-

Other Joint Studies

A. Heat Sweep Joint Study with CSM, England

In June, 1989, during a visit to Cornwall to present a paper from the Leningrad Mining Institute, a joint effort was initiated with D. Nicol on examination of the cooldown observed during the current long-term experiment at the Rosemanoes Hot, Dry Rock Experiment reservoir. All of the temperature, production, and other data were provided in disk form and preliminary analysis of the cooldown has been completed. It was possible to obtain an excellent match of the observed cooldown by estimation of two key parameters, (1) the reservoir dimensions, which determine the mean residence time of the circulating fluid, and (2) the mean fracture spacing, which determines the rock thermal time constant for providing the heat content to the circulating fluid. In the SGP 1-D Heat Sweep Model, the primary parameter, the Number of Heat Transfer Units, is the ratio of these two times. The present match is expected to be further refined with evaluation of the reservoir volume estimated by seismic monitoring techniques.

B. Heat Sweep Joint Study with LMI, Leningrad, USSR

During the Fulbright Lectureship visit to the Stanford Geothermal Program in the Spring Quarter of 1987, Prof. Yuri D. Dyadkin of the Leningrad Mining Institute offered the PE269 course in Geothermal Technology in which he reviewed the plans for geothermal energy development in the USSR. He published an article in the May, 1987 issue of the Geothermal Resources Council Bulletin (*Dyadkin, 1987*) describing the initiation of two closed geothermal circulation systems in petrogeothermal areas as part of the USSR national program to provide hot water supplies to remote towns and villages. As a result of these exercises, a joint analysis was undertaken to examine the lifetime of hydrofractured reservoirs in providing large flowrates of 100+ C artificially circulated water to an abandonment temperature of 90-100 C with recharge water of 30 C. The case study for the hot water supply was a comparison of the calculations made with the LMI heat extraction model developed from basic principles of heat transfer from oblate slabs of parallel hydrofractured rock using heat transfer coefficients derived from simulations and regression analysis of Soviet oil and gas well experience and the SGP 1-D heat sweep model using the same input data. The results were presented by *Dyadkin and Kruger (1987)*. The agreement between the two models was satisfactory.

During FY89, communications continued with the Leningrad Mining Institute for the execution of a cooperative agreement with the Stanford Geothermal Program for joint efforts in low- temperature petrothermal resources noted by USSR authors to be abundant throughout the Soviet Union. A visit by the principal investigator of the Heat Extraction Project for two months was carried out from April to June, 1989. A formal Geothermal Agreement is being executed between Stanford University and the Leningrad Mining Institute. Four technical joint projects are contemplated under this Agreement. One of the projects is the experimental hydrofractured reservoir at the Russkie Komarovtsy field in the Zakarpate Oblast of the Ukraine SSR. A joint paper on these studies was presented at the

system temperature levels; (2) steady-state rock temperatures with the axial heaters on and the cores at thermal equilibrium with the total system at each temperature level; (3) transient natural cooldown temperatures initiated intermittently during the system heatup process; and (4) transient cooldown temperatures during cold-water heat sweep cooling. System temperatures were also recorded for the complete system cooldown test. These temperature data are needed to derive rock thermal conductivity values for the pre- and post-sweep states and as a function of the number of sweeps.

The temperature data have been compiled and organized in FY88 for analysis with both steady and transient thermal models of the cylindrical rock blocks. Several thermal models are being used to interpret the experimental data from the three thermal stressing cycles and the final long-term natural cooldown test (LTCT).

Section 3.2 : Appendix A

Heat Extraction Project Reports and Publications

Aragon, A. and Kruger P., *Uso del Modelo de Barrido Termico en el Campo Geotermico de Los Humeros, Puebla, Geotermia 3, No.3, 263-273 (1987).*

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Kiryukhin, A. and Kruger P., *Preliminary Analysis of Tracer Response and Thermal Cooldown Estimates for the Mutnovsky Geothermal Field in Kamchatka, USSR, Abstract submitted to 15th SGP Workshop in Geothermal Engineering, Jan, 1990.*

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Molinar, R., Aragon, A., Kruger, P., and Lam, S., One-Dimensional Heat Sweep and Fluid Mixing at the Los Azufres Geothermal Field, Proceedings, 8th New Zealand Geothermal Workshop, November, 1986.

Robinson, B.A. and Kruger, P., A Comparison of Two Heat Transfer Models for Estimating Thermal Drawdown in Hot Dry Rock Reservoirs, Proceedings, 13th SGP Geothermal Workshop, TR-113, 113-120, 1988.

NOMENCLATURE

A	= Area of the problem domain
A_i	= Area of the region i
C	= Wellbore storage, resident concentration variable of fracture transport equations
C_r	= A reference concentration (concentration that would be obtained if all the injected tracer were to mix in the recirculating fluid volume)
D	= Longitudinal dispersion coefficient
D_a	= Apparent diffusion coefficient (diffusion coefficient in a porous matrix in this study)
F_s	= Storativity ratio, $\frac{\phi c_{t1}}{\phi c_{t2}}$
$G(x, y, \xi, \zeta; s)$	= Free Space Green's Function, Laplace Space
G_j	= Right hand side vector
H	= Length of the wellbore
H_{ij}	= Coefficient matrix
$K_0(x)$	= Modified Bessel function, second kind, order zero
$K_1(x)$	= Modified Bessel function, second kind, order one
L	= Length of the fracture
M	= Mobility ratio, $\frac{(k/\mu)_1}{(k/\mu)_2}$
$P(h)$	= Block size distribution function
P_e	= Peclet number
Q	= Flow rate of a well
$Q(h)$	= Flow contribution from matrix size h
R	= Distance to the discontinuity (phase front) or external boundary
S	= Wellbore skin factor
T	= Temperature of the fracture
T_{in}	= Temperature of the injected fluid
T_0	= Initial temperature of the system
U	= Velocity in the fracture at the wellbore
a	= Exponential PDF constant
a_{ij}	= Coefficient matrix for pressure
b	= Intercept of linear PDF, Fracture aperture
b_{ij}	= Coefficient matrix for fluxes
c	= Single phase fluid compressibility
c_s	= Wave speed in fluid
c_t	= Total compressibility
c_m	= Specific heat of the immobile(matrix) phase
c_r	= Specific heat of the reservoir rock
c_w	= Specific heat of water
c_f	= Specific heat of the mobile(fracture) phase

ξ, ζ	= Local coordinate system based on the fictitious source point
η	= Diffusivity = $k_{rx}/\phi\mu c$
ϵ	= Belongs to, or is an element of
μ	= Viscosity
ρ	= Density
ρ_m	= Density of immobile(matrix) phase
ρ_r	= Reservoir rock density
ρ_f	= Density of mobile(fracture) phase
λ	= Dimensionless interporosity flow coefficient
τ	= Dimensionless matrix response time coefficient, = Transducer time constant
ϕ	= Porosity
ω	= Dimensionless storativity ratio
η	= Integration variable of the solution to the MD model for injection-backflow tests

Subscripts

D	= Dimensionless
I	= Interporosity
eD	= Dimensionless, based on external boundary area
DA	= Non-Dimensionalized with respect to area
f	= Fracture
i	= Initial
m	= Matrix
min	= Minimum
max	= Maximum
t	= Partial derivative with respect to t (also Total)
w	= Wellbore
wf	= Wellbore flowing
x, y	= Partial derivative with respect to x or y
1, 2, 3	= Sections of the boundary
i, j, j + 1	= Nodal locations on the boundary

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