TRANSPORT PHENOMENA IN INDIUM ARSENIDE

AT LOW TEMPERATURES

APPROVED:

[Signatures]

Major Professor
David R. Eagle

Minor Professor
L. J. Connell

Director of the Department of Physics
Robert J. Flaherty

Dean of the Graduate School
TRANSPORT PHENOMENA IN INDIUM ARSENIDE
AT LOW TEMPERATURES

THESIS

Presented to the Graduate Council of the
North Texas State University in Partial
Fulfillment of the Requirements

For the Degree of

MASTER OF SCIENCE

By

Paul Jacob Luke, B. S.
Denton, Texas
August, 1968
TABLE OF CONTENTS

LIST OF TABLES ........................................ iv
LIST OF ILLUSTRATIONS ................................. v

Chapter

I. INTRODUCTION ..................................... 1
II. EXPERIMENTAL CONSIDERATIONS .................. 8
III. ANALYSIS AND CONCLUSION ...................... 24

APPENDIX ............................................... 40

BIBLIOGRAPHY ......................................... 42
<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Crystal Parameters</td>
<td>30</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The Experiment Chamber</td>
<td>11</td>
</tr>
<tr>
<td>2.</td>
<td>Crystal Holder</td>
<td>13</td>
</tr>
<tr>
<td>3.</td>
<td>Calibration Curves</td>
<td>16</td>
</tr>
<tr>
<td>4.</td>
<td>Bridge Arrangement</td>
<td>21</td>
</tr>
<tr>
<td>5.</td>
<td>Hall Resistivity</td>
<td>26</td>
</tr>
<tr>
<td>6.</td>
<td>Oscillatory Part Hall Resistivity</td>
<td>28</td>
</tr>
<tr>
<td>7.</td>
<td>Magnetoresistivity</td>
<td>32</td>
</tr>
<tr>
<td>8.</td>
<td>Thermal Conductivity</td>
<td>34</td>
</tr>
<tr>
<td>9.</td>
<td>Thermal Resistivity</td>
<td>38</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

Classically, when a magnetic field is applied to a conductor, the paths of the conduction electrons are curved so that the projections of their motion in a plane perpendicular to the field are circles of arbitrary radius, which are continuous functions of the applied field.

According to quantum theory (8), however, this motion is quantized so that only certain radii for the orbits are possible. These orbits, which correspond to energy levels, are magnetic-field dependent in energy, spacing, and degeneracy. As the field is increased, the energy of these levels increases causing them to expand (energy-wise) toward and through the Fermi level. This results in a variation of the density of electron states in the vicinity of the Fermi surface. Consequently a varying magnetic field gives rise to oscillations in all physical quantities that depend on the distribution of the electrons in the quantized levels (2).

Magnetic-field-dependent oscillations in the electrical conductivity or resistivity are known as Shubnikov-de Haas oscillations. The period of these oscillations is determined by the interval between the coincidence of the quantized levels with the Fermi surface.
The conditions for observation of the quantization effects in transport phenomena are (1)

\[ \omega \tau > 1 \quad 1 \]
\[ \hbar \omega > kT \quad 2 \]
\[ \zeta_0 > kT \quad 3 \]

where

\[ \omega = \frac{eH}{m^*c} \]

\( \tau \) is the collision time, \( \zeta_0 \) the Fermi energy in the absence of a magnetic field, and \( m^* \) is the effective mass.

Condition 1 above requires that an electron complete at least more than one radian before being scattered to another energy level. If this condition is not fulfilled, intra-orbit transitions will be so frequent that the density of states in the vicinity of the Fermi level will not depend entirely on the position of the quantized energy levels.

Condition 2 is due to the fact that if \( kT \) is large with respect to the separation of the Landau levels, these levels will be smeared to the extent that they overlap. Therefore, the density of states in the vicinity of the Fermi surface will not vary appreciably with a variation of the magnetic field. This condition also requires an effective mass which is approximately two orders of magnitude smaller than the electron rest mass, the reason being that only then are experimentally available magnetic fields large enough to satisfy this condition.
Condition 3 is also required to keep a sharp distribution function and hence well-defined Fermi level.

For materials with low carrier concentration, the Fermi surfaces are approximately spherical, with parabolic conduction bands. Then, according to theory (6), oscillatory electron-transport phenomena should exhibit a single period, as a function of reciprocal field strength, and the value of the period should be dependent only on the carrier concentration. Also, if the curvature of the conduction band is large, the effective mass will be small.

Thermal Conductivity

When experiments are conducted concerning heat flow, the experimenter is usually concerned with the energy transport mechanisms and the properties of the material which retard the flow of heat (energy transport).

The chief modes of energy transport in an opaque solid at low temperature are free (conduction) electrons, and phonons, the quanta of lattice vibration. Scattering of the electrons and phonons is responsible for the thermal resistivity.

This scattering of electrons and phonons may be caused by their interaction with crystal boundaries, impurity atoms, atoms of different isotopes, dislocations, and Umklapp processes (5). These interactions limit the mean free path of
the electrons and phonons and thereby reduce their ability to transport heat, as can be seen from an approximation for the thermal conductivity which is derived from the kinetic theory of gases (2),

\[ K = \frac{1}{3} cv\lambda, \]

where \( C \) is the heat capacity per unit volume, \( v \) is the velocity of sound, and \( \lambda \) is the mean free path.

In semiconductors at low temperatures the electron concentration is small and the thermal conductivity is determined by heat transported by the phonons.

According to Mielczarek and Frederikse (6), the total thermal resistivity may be written as the sum of its components,

\[ \gamma_{\text{tot}} = \gamma_{\text{bnd}} + \gamma_{\text{iso}} + \gamma_{\text{imp}} + \gamma_{\text{umk}} + \gamma_{\text{dis}}. \]

where the above resistive components are due to boundary, isotope, point imperfection, Umklapp, and dislocation scattering respectively.

Theoretical expressions for the various components of equation 5 have been given by Mielczarek and Frederikse (3) and Slack (5): Thermal resistivity due to boundary scattering of phonons is

\[ \gamma_{\text{bnd}} = \frac{h^3 v^2}{4\pi k^4 T^3 L(25.658)} , \]

where \( L \) is the smallest linear dimension of the crystal, \( h \) is Planck's constant, \( k \) the Boltzmann constant, \( T \) the absolute temperature, \( v \) the velocity of sound, and the number
in parentheses is from evaluation of the integrals as given in references (3), and (5). The contribution from scattering by isotopes is

$$\gamma_{\text{iso}} = \frac{12\pi^2 TV_0 B}{hv^2}$$  \hspace{1cm} (7)

with

$$\Gamma = \sum f_i (1 - \frac{M_i}{M})^2,$$

where $M_i$ is the mass of a unit cell, $M$ is the average mass for all unit cells and $f_i$ is the fraction of unit cells with mass $M_i$. The volume of a unit cell is $V_0$, and $B$ is a numerical parameter evaluated by Slack (5) to be 0.33. Thermal resistivity due to scattering of phonons from impurities is

$$\gamma_{\text{imp}} = \frac{T(2\pi)^2 3v S^2}{hv^2 G(0.897)}$$  \hspace{1cm} (8)

where $S$ is a scattering parameter which can usually be taken as equal to 1 (3), and $G$ is the reciprocal of the concentration of impurities per unit cell. Umklapp scattering of phonons depends upon temperature as

$$\gamma_{\text{umk}} \propto \left(\frac{T^2}{\Theta}\right) \exp\left(-\frac{\Theta}{\alpha T}\right), \hspace{0.5cm} T < \Theta$$  \hspace{1cm} (9)

where $\alpha$ is a parameter approximately equal to 2 and $\Theta$ is the Debye temperature (3). The constant of proportionality has been calculated by Mielczarek and Frederikse (3) for the case of indium antimonide; it has been modified to the case of indium arsenide by Shalyt (4). Finally, an approximate
expression for the thermal resistivity due to scattering of phonons from dislocations is (7)

\[ \gamma_{\text{dis}} = \frac{BAvb^2 g^2 h^2}{60 kt(7.05)} \]

where \( A \) is the number of dislocation lines per unit area, \( b \) is the length of the Burger's vector (2) of the dislocation, \( B \) is a factor depending on the type of dislocation (4), and \( g \) is the Gruneisen constant.

An experimental curve showing \( \gamma \) as a function of \( T \) can be examined using the above equations to give values of the impurity concentration for the sample. Ideally, the number of impurities obtained in this manner should be directly related to the conduction-electron concentration obtained from measurements of the electrical resistivity and the Hall coefficient.
CHAPTER BIBLIOGRAPHY


CHAPTER II

EXPERIMENTAL CONSIDERATIONS

Sample Preparation

The indium arsenide crystal was cut from an ingot by a spark machine to dimensions of 1.53 X 0.312 X 0.118 millimeters. The crystal was then etched to remove the damaged surface material and x-rayed to determine its structure and orientation.

Attempts were made to solder a thermometer assembly to the crystal with many different types of solder and flux with unsatisfactory results; it was found that if the crystal was nickel plated, however, very good adhesion could be obtained. The crystal was nickel plated only at the point of probe attachment, and the area of the nickel spot was kept as small as practical. A small nickel spot was achieved by painting a doughnut shaped area on the crystal with Dow Corning silicon vacuum grease, and placing a drop of the plating solution inside the circle of grease. The plating solution consisted of a saturated solution of nickel ammonium sulfate.

A piece of platinum was used for the anode, and was hand held in the plating solution. It was found that 0.5 milliamperes driven by three volts for approximately one minute would give a good plate for a spot one millimeter in
diameter. The current was controlled by a resistance
decade box in the circuit and was limited to a value low
enough to avoid the dull burned look.

Before plating, the crystal was polished to give a
smooth surface so that the spot could be inspected under a
microscope. Care was taken to assure no surface contamination
before and after plating.

The adhesion was such that if the probes which were
soldered to the crystal were removed by force part of the
crystal was removed with the probe. Thirteen ounces of
stress were required to remove forceably a one millimeter
spot if no torque was applied.

Indium arsenide is a very fragile material and the
probes are easily broken off if torque is applied. If this
occurs a small hole is torn in the crystal. This hole can
sometimes be plated to facilitate reattachment. However,
if the first attempt fails, smoothing the hole with number
280 emory dust on a cotton swab and etching the hole with
a 5 per cent solution of bromine in alcohol usually facili-
tated nickel plating the hole.

Construction of the Sample Holder

The sample holder was machined from a one-inch-diameter
rod of half-hard yellow machining brass. The outer diameter
was relieved only enough to assure a comfortable fit in the
one-inch bore of the superconducting magnet.
The sample holder, Figure 1, was in the form of a cylindrical vacuum chamber consisting of an upper and lower part joined by a threaded section, with the top part acting as the crystal holder and the bottom part the experiment chamber.

A vacuum seal between the two halves was accomplished by means of indium tinned to the upper lip of the lower part of the chamber. The flange of the upper part of the crystal holder to which the indium was to seat to make a seal was lubricated lightly with Dow Corning silicon vacuum grease to reduce shearing stress on the indium. This indium seal was made very thin to avoid difficulties arising from expansion differentials.

The crystal holder was supported by a 0.25-inch-diameter stainless-steel tube which also served as the vacuum line to the crystal chamber. One and one-half inches above the crystal chamber the vacuum line was interrupted by a block of brass into which passageways were drilled so as to form a radiation trap. The radiation trap also served as a mounting bracket to which the junction boards were attached for the wires coming from the chamber and the larger wires leading to the external circuitry. The radiation trap, which was located at such a level that it was below the top of the magnet and therefore in the helium bath at all times, formed
Fig. 1—The experiment chamber
a constant-temperature bath for the junction boards to eliminate thermal emf's due to temperature differences at this junction.

The crystal-mounting stage, Figure 2, was formed from two pieces of brass separated by a 0.125-inch-diameter rod 0.250 inches long. The two outer surfaces of the stage were machined so as to be parallel. The crystal was held firmly against the lower surface of the stage by two brass machine screws which passed through a small brass plate and into the lower surface of the stage to form a clamp.

This stage was then bolted to a piece of brass which was silver soldered to the lower edge of the upper half of the vacuum chamber and machined so that its lower surface would be perpendicular to the walls of the chamber and therefore perpendicular to the magnetic field. This orientation provided crystal alignment perpendicular to the field.

A germanium resistance thermometer, calibrated by The National Bureau of Standards, was attached to the lower part of the crystal holder to provide a means of calibrating the carbon thermometers on the crystal and checking the calibration during each run.

Sixteen inches of number forty manganin resistance wire of 31.5 ohms per foot was wrapped around the 0.125 inch rod separating the two pieces of the stage. This resistance wire provided a means of elevating the temperature of the crystal
Fig. 2—Crystal holder
above that of the bath. One hundred milliamperes was sufficient to raise the temperature to 400 K.

Electrical leads entered the vacuum chamber through a brass tube in the top of the chamber. A vacuum seal for this tube was achieved by using Emerson and Cummings Sty-cast epoxy.

Thermometer Construction and Calibration

The thermometric material of the thermometers was carbon in the form of 0.1-watt, 47-ohm resistors manufactured by Allen Bradley Company. These resistors were designed to be temperature insensitive at room temperature. However, they are highly temperature sensitive below the temperature of liquid nitrogen.

These thermometers respond well to temperature cycling and an approximate formula for the resistance as a function of temperature is available (1),

$$\left(\frac{\log R}{T}\right)^a = a + b \log R.$$  \hfill (11)

This formula, however, is not accurate enough to be used with values of a and b taken from two calibration points for precision experimental purposes, but can be used to get an estimate of the resistance at a specific temperature before calibration.

For this experiment two resistors were selected from a group of eight which were rough calibrated. Selection was
made on the basis of matching temperature-versus-resistance curves. The resistors selected were fitted with brass sleeves of their own length having a wall thickness of .003 inches. The sleeves were drilled under-size and then lapped to a perfect fit with the resistors. The lapping procedure was accomplished by applying a lapping compound to the shank of a wire drill and passing it through the sleeve until the proper fit was achieved.

To achieve point contact of the thermometers with the crystal, a number twenty-eight-gauge copper wire was flattened on one end and bent around the sleeve and soldered in place. The other end was soldered to the crystal. This wire was only long enough to hold the thermometers above the crystal, since excess length retards the thermometers reaction time and makes them more susceptible to being broken from the crystal.

The thermometer assembly was soldered to the nickel spots on the crystal with Cerroseal solder.

For the temperature range between two and five degrees Kelvin the thermometers were calibrated by the helium-vapor-pressure method. (See Figure 3.) For temperatures between five and thirty degrees calibration was achieved by use of the germanium resistance thermometer on the heat sink. (See Figure 2.)

Care was taken to keep the current in the resistors small (less than ten microamperes between two and ten degrees,
Fig. 3—Calibration curves
and less than fifty microamperes between ten and fifty degrees Kelvin) so that Joule heating would be negligible. The current through the thermometers was known to 0.1 percent by measuring the voltage drop across a precision resistor in series with the thermometers.

The current and sensing leads of the thermometers were number forty copper and manganin wires respectively. For the first calibration attempt these wires were taken directly from the epoxy seal of the chamber to the thermometers, a distance of about two inches. It was determined that the heat loss through these wires was such that it could not be tolerated. As a remedy, the leads were lengthened to about six inches and first brought in contact with the heater on the heat sink and then the crystal at the base of the thermometer it served.

A Typical Run With the Superconducting Magnet

The stainless-steel Dewar was pumped to a pressure of less than $10^{-4}$ torr before precooling procedure was started. Since, by the procedure outlined here, precooling of the magnet is done directly with liquid nitrogen and not by conduction through the inner Dewar walls, the lower the Dewar pressure the better. However, since the volume between the Dewar walls is large and the pumping port is small, the pumping rate is quite slow beyond $10^{-4}$ torr.
If a fast warm up procedure had been performed previously by introducing helium gas into the vacuum jacket, it was flushed with air several times by pumping down with a fore pump for several minutes and then allowing air to enter the jacket. This procedure insured that a minimum amount of helium remained in the vacuum jacket.

After the magnet was placed in the Dewar and secured, liquid nitrogen was introduced into the Dewar magnet space through the magnet support tube, which has an extension reaching to the bottom center of the Dewar. The liquid nitrogen level was maintained at a depth of about six inches until the evolved gas indicated the magnet had reached equilibrium temperature with the liquid nitrogen. The depth of six inches was not exceeded because the top portion of the magnet has a hollow top hat which will fill with the liquid and cannot be removed while the magnet is in the Dewar. Liquid nitrogen was then added to the outer Dewar space provided for it. At this point the outer Dewar wall temperature was checked by touch to be sure that its temperature was only slightly below room temperature indicating a good Dewar pump-down procedure.

The system was precooled for a length of time indicated by the amount of gas being evolved from the liquid nitrogen, and with the liquid level maintained at about six inches, pressure on the nitrogen was reduced to about 105 torr to
reduce its temperature to approximately 64 degrees Kelvin. The pressure on the nitrogen was not allowed to go below 95 torr, corresponding to the temperature at which nitrogen becomes solid and therefore cannot be removed from the Dewar by the means available.

After the pressure was maintained at approximately 105 torr for fifteen to twenty minutes, an over pressure of helium gas was applied to force the liquid nitrogen out through the same port through which it was introduced. After being checked with a small diameter stainless steel tube to be certain that all the nitrogen had been expelled, the magnet space pressure was reduced to about 50 torr in order to freeze any nitrogen that might have remained in the inner Dewar. The inner Dewar was then brought up to atmospheric pressure with helium gas and transferring of liquid helium was begun.

Care was taken to prevent air from entering the magnet support tube through which the transfer tube was inserted, since air can freeze the transfer tube in the magnet support tube and prevent its extraction once transferring has been completed.

The Measuring Circuit and Technique for the Thermal Conductivity

Measurement of the thermal conductivity involves measuring two variables, the temperature gradient and heat flow.
The temperature gradient was determined by the use of what was basically a Wheatstone bridge and null detector. The temperature sensing elements were carbon resistors which formed two legs of the bridge. (See Figure 4.)

The technique used to find the temperature gradient was to select the temperature at which the thermal conductivity was to be determined, and bring both thermometers to this temperature by use of the heater on the heat sink. The bridge was then balanced and a small temperature gradient was established (ΔT less than 0.01 degrees Kelvin) by employing the heater on the crystal. (See Figure 2.) The difference in temperature of the two thermometers caused the bridge to become unbalanced by an amount ΔV. Care was taken to maintain the warmer thermometer at the preselected temperature. The imbalance of the bridge was then just due to the change in resistance of the colder thermometer.

For small temperature differences the difference in temperature is directly proportional to the change in resistance, the proportionality constant being \( \left( \frac{\Delta T}{\Delta R} \right)_T \) which is the slope of the calibration curve at the temperature T. Therefore

\[
\Delta T = \left( \frac{\Delta T}{\Delta R} \right)_T \Delta R.
\]
\[ \Delta T = \left( \frac{\Delta T}{\Delta V} \right)_T \Delta V \]

\[ \gamma = \kappa' = \frac{\Delta T}{s \omega^\star} = \frac{\left( \frac{\Delta T}{\Delta V} \right)_T}{s \frac{P}{A}} = \left[ \frac{A}{i \left( \frac{\Delta R}{\Delta T} \right)_T} \right] \Delta V \]

**A** = CROSS-SECTIONAL AREA of In As CRYSTAL

**i** = CURRENT THROUGH THERMOMETER

\( \left( \frac{\Delta R}{\Delta T} \right)_T \) = SLOPE OF CALIBRATION CURVE AT T

**P** = POWER INTO HEATER

**S** = SEPARATION OF THERMOMETERS

---

**Fig. 4**—Bridge arrangement
However, $\Delta V$ is the measured quantity, but since $\Delta V$ is $I_R \Delta R$,

$$
\Delta T = \left( \frac{\Delta T}{\Delta R} \right) \frac{\Delta R I_R}{T I_R} \quad 13
$$

$$
= \left( \frac{\Delta T}{\Delta R} \right) \frac{\Delta V_R}{T I_R}, \quad 14
$$

where $I_R$ is the current in the warmer thermometer for that particular measurement. The current in the thermometers was held constant to 0.01 per cent for any particular measuring procedure.

The absolute temperature of each thermometer could also be determined by measuring the voltage drop across the thermometer and knowing the current through it. With this information the resistance of the thermometer could be found. The temperature was then determined by consulting the temperature-versus-resistance calibration curve.

The heat delivered to the crystal was determined by measuring the voltage drop across the heater and the current through it very precisely.
CHAPTER BIBLIOGRAPHY

CHAPTER III

ANALYSIS AND CONCLUSION

Shubnikov-de Haas Effect

Lifshitz and Kosevich (6) have related the oscillatory part of the galvanomagnetic effect to Landau's theory of the oscillations of the magnetic moment (de-Haas-Van Alphen effect). One can use the Lifshitz-Kosevich results to write

\[ \Delta \rho_{21} \propto H^2 \frac{\lambda}{\sinh (\lambda)} \exp(i2\pi \frac{S_m c}{eH}), \]

where \( \lambda = \frac{2\pi^2 kT}{\hbar \omega^*} = \lambda_0 T, \quad \omega^* = \frac{eH}{m^* c} \)

and \( \rho_{21} \) is the oscillatory part of the Hall resistivity.

From this relationship one can see that the period of the oscillatory part of \( \rho_{21} \) is controlled by the factor \( 1/H \) in the exponential. The same phase will be attained when \( H \) changes such that

\[ 2\pi \left( \frac{S_m c}{eH\Delta H} \right) = 2\pi \]

\[ \Delta \left( \frac{1}{H} \right) = \frac{eH}{S_m c} \]

where \( S_m \) is the extremal orbital area of an electron at the Fermi surface.
If $S_m$ is defined as the area in momentum space of a spherical Fermi surface, one can write

$$S_m = \frac{4}{3} p^2 = \frac{4}{3} m^* \zeta_0$$

and

$$\zeta_0 = \frac{h^2 k_F^2}{2 m^*}$$

$$\Delta \left( \frac{1}{H} \right) = \frac{2e}{\hbar k_F}$$

$$k_F = (3\pi^2 n)^{1/3} = (3\pi^2 n)^{1/3}$$

where $n$ is the carrier concentration. Then

$$\Delta \left( \frac{1}{H} \right) = \frac{2e}{\hbar c (3\pi^2)^{2/3}} \frac{1}{n^{2/3}} = 3.18 \times 10^6 \text{ G}^{-1} \frac{1}{n^{2/3}}$$

Thus the period of the oscillations is dependent only on the carrier concentration.

The carrier concentration can be obtained from the Hall coefficient $R_H (3)$;

$$R_H = \frac{E_y}{J_x H} = \frac{\rho_{21}}{H} = \alpha \frac{V_{21}}{H}$$

Thus $R_H$ is the slope of the gross Hall effect. When $J_y$ is zero, as it is in this case,

$$E_y = -\omega_c v E_x \quad J_x = ne^2 v E_x$$

$$R_H = -\frac{1}{ne c}$$
HALL RESISTIVITY
n-TYPE IN As

Fig. 5—Hall resistivity
If the above equations for $\Delta\left(\frac{1}{H}\right)$ and $R_H$ are combined, the period of the oscillations can be determined. The value of $\Delta\left(\frac{1}{H}\right)$ calculated by this method using $n = 6.35 \times 10^{16} \text{ cm}^{-3}$ is $18.5 \times 10^{-6}$ per gauss.

Since the carrier concentration can be determined from the slope of the gross Hall curve, one can see from Figure 5 that, since both curves have the same slope, the carrier concentration does not vary appreciably over this range of temperature. Also, since the gross curve is more or less a straight line, the carrier concentration is not a function of the magnetic field strength. Therefore the period of oscillations in $1/H$ should not change with temperature or magnetic field.

Figure 6 shows the oscillatory part of the Hall resistivity. These data were obtained by computer processing of the raw data. The difference between the polynomial curve and the Hall data was plotted as a function of reciprocal field strength.

From the Lifshitz-Kosevich equation for the oscillatory part of $\rho_{21}$ given by Miller (8)

$$\Delta\rho_{21} = H^k \frac{\lambda_0 T}{\text{SINH}(\lambda_0 T)} \exp\left(\frac{i2\pi}{\Delta\left(\frac{1}{H}\right)}\right)$$

the effective mass of the electrons $m^*$ can be determined by taking the ratio of $\Delta\rho_{21}$ at two different temperatures.
Fig. 6—Oscillatory part Hall resistivity
where \( \lambda_0 = \frac{2\pi k}{\beta^*\hbar} \) \hspace{1cm} \beta^* = \frac{\hbar e}{m^*c}

The equation above containing the hyperbolic sine function was solved for \( \lambda_0 \) by a substitution process using a computer (see Appendix). From the value of \( \lambda_0 \), \( \beta^* \) could be found, and from this \( m^* \) was determined. The evaluation of \( m^* \) was made a 27 kilogauss (at a point where the oscillations are pronounced and most accurate), the two temperatures being 4.2 and 2.2 degrees Kelvin as indicated in Figure 5. The value obtained for \( m^* \) was 0.028 \( m_0 \).

With the value for \( m^* \), \( \zeta_0 \), the Fermi energy can be obtained from

\[
\zeta_0 = \frac{\epsilon h}{m^*c \Delta(\frac{T}{H})}.
\]

The mobility, \( \mu \), was obtained from the resistivity at zero field (see Figure 7), and the Hall coefficient.

\[
\mu = c |R_H| \frac{1}{\rho_{110}}
\]

The relaxation time can be obtained from the effective mass, zero-field resistivity, carrier concentration and electronic charge

\[
\tau = \frac{m^*}{\rho_{110}ne^2}
\]
# TABLE I

**CRYSTAL PARAMETERS**

<table>
<thead>
<tr>
<th>GROSS PHENOMENA</th>
<th>OSCILLATORY PHENOMENA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EXPRESSION</strong></td>
<td><strong>VALUE</strong></td>
</tr>
<tr>
<td>( n = \frac{1}{R \nu c} )</td>
<td>( 5.58 \times 10^{16} \text{ cm}^{-3} )</td>
</tr>
<tr>
<td>( \Delta(T_H) )</td>
<td>( \frac{2e}{\hbar c (3\pi^2)^{2/3}} \frac{1}{n^{2/3}} )</td>
</tr>
<tr>
<td>( \frac{m^*}{m} )</td>
<td>( \frac{\Delta E_{g1}}{\Delta E_{g2}} \frac{T_{g1} \sin h (\lambda_3 T_1)}{T_{g2} \sin h (\lambda_2 T_2)} )</td>
</tr>
<tr>
<td>( \mathcal{L} )</td>
<td>( \mathcal{L} = \frac{e \hbar}{m^* c \Delta(T_H)} )</td>
</tr>
<tr>
<td>( \mu = c \left</td>
<td>R_H \right</td>
</tr>
<tr>
<td>( \tau )</td>
<td>( \tau = \frac{m^*}{p_{n_e} n e^2} )</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>( \Lambda = \nu \tau )</td>
</tr>
</tbody>
</table>
From this equation and the Fermi energy one may determine the mean free path

\[ \Lambda = v_f T, \]

where

\[ v_f = \left( \frac{2\xi_0}{m^*} \right)^{\frac{1}{2}} \]

The values for the above parameters are summarized in Table I.

In comparing the carrier concentration as given by the gross Hall slope and that given by the oscillatory part of the Hall data, a discrepancy of about 8 to 9 per cent is found, probably due to poor resolution of the period as measured because of the small number of oscillations obtained. This discrepancy is also evident in the values obtained for the period of oscillations since the same data are used to obtain both.

The value of \( m^* \), evaluated as given below, agrees with that of Shalyt (10) of 0.025 \( m_0 \) within 9 per cent. Since the value of \( \xi_0 \), the Fermi energy, is evaluated from the oscillatory part of the Hall data it may also have an error of about 9 per cent.

The mobility also seems to be somewhat low as compared to other experimenters (1) which may be due to impurity concentration being larger than that given for reasons that will be given in the section on thermal conductivity. The relaxation time and mean free path seem to be too small for the same reason.
Fig. 7—Magnetoresistivity
Thermal Conductivity

The thermal conductivity (see Figure 8) has been analyzed in terms of mechanisms which retard the flow of heat through the crystal, that is, in terms of the components of the thermal resistivity. These resistive components, being due to phonon scattering, fall into three main categories: (1) Umklapp processes; (2) crystal boundaries, and dislocations; (3) point imperfections such as isotopes and impurity atoms. The thermal conductivity was observed to be independent of the magnetic field and therefore the electronic contribution to thermal conductivity is insignificant.

The contribution of the various scattering mechanisms to the thermal resistivity has been discussed in many papers, as summarized by Holland (2). According to Elements (4), the main phonon-scattering mechanisms may be found, to within an accuracy of coefficients which may be assumed to be the same for crystals having the same structure, from the following expressions:

\[ \gamma_{\text{imp}} \propto \frac{TV}{v^2} N, \]
\[ \gamma_{\text{iso}} \propto \frac{TV}{v^2} \sum_i (\frac{\Delta M_i}{M})^2, \]
\[ \gamma_{\text{umk}} \propto (\frac{T}{\Theta})^2 \exp\left(-\frac{\Theta}{2T}\right), \]
\[ \gamma_{\text{bnd}} \propto \frac{V}{T^3 L}, \]
\[ \gamma_{\text{dis}} \propto \frac{v^2 A}{T^2}, \]
THERMAL CONDUCTIVITY of n-InAs

Fig. 8—Thermal conductivity
where the symbols indicate the same parameters as in Chapter I except that \( N \) is now \( G^{-1} \).

According to Klements (4) and Shalyt (9), one may assume that the proportionality constants for materials of like structure are the same. Therefore, it will be assumed that between ten and thirty degrees Kelvin the resistivity components may be analyzed by comparing the thermal resistivity of indium arsenide with that of indium antimonide which was measured and analyzed between these temperatures by Mielczarek and Frederikse (7).

Designating the parameters of indium arsenide as "A" and those of indium antimonide as "S", one obtains for the ratio of the resistive components

\[
\gamma_{\text{a imp}} = \gamma_{\text{s imp}} \frac{v_a}{v_s} \left( \frac{v_s}{v_a} \right)^2 \frac{N_a}{N_s},
\]

\[
\gamma_{\text{a iso}} = \gamma_{\text{s iso}} \frac{v_a}{v_s} \left( \frac{v_s}{v_a} \right)^2 \frac{\Gamma_a}{\Gamma_s},
\]

\[
\gamma_{\text{a umk}} = \gamma_{\text{s umk}} \left( \frac{\Theta_s}{\Theta_a} \right)^2 \frac{\Theta_s - \Theta_a}{2T},
\]

\[
\gamma_{\text{a bnd}} = \gamma_{\text{s bnd}} \left( \frac{v_a}{v_s} \right)^2 \frac{S_s}{S_a},
\]

\[
\gamma_{\text{a dis}} = \gamma_{\text{s dis}} \frac{v_a}{v_s} \frac{A_a}{A_s}.
\]
The first three of these expressions may be evaluated by using the resistivities for InSb as found by Mielczarek and Frederikse (7) and the ratios of $v_a/v_s$, $\nu/v_a$, and $\Gamma_a/\Gamma_s$ as given by Shalyt (9) leaving only $N_a/N_s$ to be evaluated.

In the range of temperature considered above one assumes that

$$\gamma_{tot} = \gamma_{iso} + \gamma_{umk} + \gamma_{imp}$$
$$\gamma_{imp} = \gamma_{tot} - (\gamma_{iso} + \gamma_{umk}).$$

Evaluating $\gamma_{tot}$, $\gamma_{iso}$, and $\gamma_{umk}$ from Figure 9 of the thermal resistivity, one finds

$$\gamma_{imp} = 16.5 \times 10^{-2} \, \text{cm} \, \text{deg} \, \text{watt}$$

Then from equation 27, \( \frac{N_a}{N_s} = 0.906 \).

Taking $N_s$ as the average impurity concentration as given in (7)

$$N_a = 2.7 \times 10^{17} \, \text{cm}^{-3}$$

This compares within a factor of 4 with the current-carrier concentration of $6.35 \times 10^{16} \, \text{cm}^{-3}$ as given by the Hall data. Discrepancies of this order and larger have been reported in the literature (8). The exact cause of the discrepancy is not known. However, in InSb at low temperatures electrical conductivity is extrinsic, the conduction electrons being supplied by impurity atoms. Since the thermal conductivity
gives a carrier concentration larger than that given by the electric data, and since the electric data only give the lower limit to the impurities, it seems reasonable to assume that the impurities may be highly compensated (8) and/or electrically inactive. This view seems to be substantiated by the low mobility, short relaxation time and mean free path as given by the electric data.

That part of the thermal resistivity below ten degrees Kelvin has been analyzed by assuming that at very low temperatures boundary scattering will predominate since this component is dependent on $T^3$, and dislocation scattering, the other important scattering factor below $10^9$K, is dependent on $T^3$. The difference between the total thermal resistivity and the boundary resistivity is assumed to be dislocation scattering since other scattering contributions below ten degrees Kelvin are negligible (see Figure 9).

This method of analysis gives a boundary scattering value of about 25 per cent of that as calculated by the theoretical expression as given in Chapter I.

These data were reproducible within about 10 per cent.
THERMAL RESISTIVITY
OF INAs

\[ \gamma = \frac{1}{\alpha} \left(10^{-2} \text{ K-cm}^2/\text{Watt}\right) \]

Fig. 9—Thermal resistivity
CHAPTER BIBLIOGRAPHY


APPENDIX

PROGRAM TO EVALUATE m*

I=0
Read X T1 T2 A R
A=A+X
X1=EXP(A*T1)-1./(EXP(A*T1))
X2=EXP(A*T2)-1./(EXP(A*T2))
Q=X1/X2
IF(Q-R)20,21,10
20 I=I+1
IF(I-4)90,90,21
90 A=A-X
X=X/10.0
IF(SENSSE SWITCH 1)26,10
26 PRINT 101, A,Q
GO TO 10
21 PRINT 101,A,Q
101 FORMAT (/2E14.8)
STOP
END

This program was used to evaluate $\lambda_0$ of Equation 22, Chapter III. $\lambda_0$ was then used to find $m^*$. In this program $R$ is the ratio $\frac{\Delta \rho_{21} T_2}{T_2} / \frac{\Delta \rho_{21} T_1}{T_1}$, $T_1$ is the low temperature, and $T_2$ is the higher temperature. $A$ is the value for $\lambda_0$, which must be guessed low for the initial value. $X$ is the amount by which $A$ is increased for each new substitution. Successively larger values of $A$ will be substituted until $Q$, which is the ratio of the hyperbolic sine functions, is larger than $R$. This value of $A$ is the first approximation for $\lambda_0$. The program then directs the computer to return to the last value of $A$ which was too small, decrease $X$ (the
increment of $A$) by a factor of ten, and continue substituting. This process continues until $I$, which controls the accuracy of the approximation for $A$ ($\lambda_0$), has been satisfied.

For this program the exponential form of the hyperbolic sine function has been used.
BIBLIOGRAPHY

Books


Articles


Unpublished Material