CYCLOTRON SUBHARMONICS RESONANT(CSR) HEATING

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ABSTRACT

The cyclotron subharmonics resonant (CSR) heating mechanism is studied using particle simulation codes with an emphasis on the relationship between CSR and the nonlinear Landau damping.

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I. INTRODUCTION

Efficient ion Bernstein wave heating (IBWH) was observed in the experiments¹ using the two-ion species plasma where H^+ majority and $^4He^{2+}$ minority ions were used in the Nagoya JIPPT-II-U tokamak. Ion heating should occur, near the center of the plasma column, where the rf frequency matches the ion cyclotron harmonics resonance, $f = 3f_{cHe}$. Interestingly, observed ion heating was found to be independent of the minority ion concentrations, even when there were no minority ions (0 % $^4He^{2+}$) which could be resonant with the rf wave. Therefore, the observed ion heating without minority ions was at first attributed to the resonance absorption by the fully stripped low-z impurity ions such as C^{6+} and O^{8+} which are deuterium-like. This experimental result and its physical interpretation were first presented in the original research report.¹

About the time of the report, in the plasma simulation study aiming at investigating the IBW physics in the JIPPT-II-U tokamak heating, the direct ion heating of single-ion species plasma due to the wave at the cyclotron subharmonic frequency ($f = 3/2 f_{cH}$) was observed. This result was explained using the cyclotron subharmonics resonant (CSR) heating,² because the presence of the minority ions was definitely eliminated, unlike the actual experiments. The results of the simulation were also applied to explain the experimental results.

Next, Porkolab³ proposed nonlinear Landau damping (NLD) as an explanation of the JIPPT-II-U tokamak results.

Following these two proposals, the experiments were examined again.⁴ Through the measurements of parallel as well as perpendicular ion energy spectra, the presence of a direct ion heating mechanism at $f = 3/2 f_{cH}$ was indeed confirmed.

In the IBWH particle simulations using two-ion species plasmas,^{5,6} CSR heating was shown to be comparable in efficiency with cyclotron harmonics resonance heating. Later, the NLD was investigated numerically by Sugaya⁷ and experimentally by Sugawa⁸ using electron Bernstein waves.

The Landau damping is understood from two different viewpoints in the linear theory: one is the fluid viewpoint by Landau⁹ and another is the particle viewpoint by Dawson.¹⁰ These two approaches are quite different but they describe the same phenomenon. Therefore, we were motivated to understand the relationship between these CSR and NLD viewpoints in the nonlinear heating mechanisms at half frequency of the cyclotron harmonics.

In the study for the CSR heating based on the particle viewpoint, we followed Dawson's assumption and started from the assumption that a monochromatic wave existed in the system and examined the temporal change of the particle energy gain or loss due to the nonlinear interactions between the particles and the wave with a frequency of the half harmonics.

Some may agree that the generation of the virtual wave for the NLD cannot be explained by the mechanism of CSR heating. Indeed, the role of the virtual wave is a complex issue here. However, if one may ignore the precise role of the virtual wave for a moment, NLD is indeed similar to the CSR heating from the heating characteristics such as the predicted power threshold, *etc.*

However, we should also emphasize that the understanding of the experimental and simulation results for the CSR and NLD mechanisms is still incomplete. Concerning the CSR mechanism to heat the plasma, we need additional mechanisms for energy relaxation, (as will be explained in detail in Sec. IIB), because the particle trapping mechanism itself which is the origin for the CSR heating is a reversible process and, thus, does

not lend itself to a true plasma heating. On the other hand, the NLD mechanism theory^{3,7,8} may not fully account for the experimental heating results,⁸ in which the observed virtual mode amplitude is extremely small.

In this paper, we will study these problems in detail, using the particle simulation method.

II. THEORETICAL REVIEWS AND SIMULATION MODELS

A. Propagation of Bernstein waves

The ion and electron Bernstein waves can be represented with the electrostatic approximation including the finite-Larmor-radius effects. The linear dispersion function can be written as¹¹

$$\varepsilon(k, \omega) = 1 + \sum_{\sigma} \frac{k_{\sigma}^2}{k^2} \left\{ 1 + \sum_n \frac{\omega}{\omega - n \Omega_{\sigma}} \times \left[W \left(\frac{\omega - n \Omega_{\sigma}}{k_{\parallel} (T_{\sigma}/m_{\sigma})^{1/2}} \right) - 1 \right] \Lambda_n(\lambda_{\sigma}) \right\}, \quad (1)$$

where $k_{\sigma}^2 = n_{\sigma} e^2 / \varepsilon_0 T_{\sigma \parallel}$, $\lambda_{\sigma} = k_{\sigma \perp}^2 / \Omega_{\sigma}^2 m_{\sigma}$, $\Lambda_n(x) = e^{-x} I_n(x)$, $W(x)$ is the plasma dispersion function, $I_n(x)$ is the modified Bessel function, and σ indicates the particles species. By numerically calculating $\varepsilon(k, \omega) = 0$, we can get the linear plasma dispersion relation between k and ω .

B. A review of the CSR heating

Assuming the particle mass and charge as m and q respectively, we consider the motion of the charged particles, *i.e.*, ions and electrons moving under the combined influence of the electrostatic wave $\phi = \phi_0 \cos(k_{\perp} x + k_{\parallel} z - \omega t + \theta_0)$ and a uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{z}}$ with cyclotron frequency Ω_0 in the magnetized plasma with an externally launched ion

or electron Bernstein wave (IBW or EBW). Then the equation of the motion of the charged particles is given by

$$\begin{aligned}\ddot{x} + \Omega_0^2 x &= \Omega_0 v_{y0} + \frac{q \phi_0}{m} k_{\perp} \sin \psi , \\ \ddot{z} &= -\frac{q \phi_0}{m k_{\parallel}} \sin \psi , \\ \psi &= k_{\perp} x + k_{\parallel} z + \theta_0 - \omega_0 t\end{aligned}\tag{2}$$

where v_{y0} is the initial y-directional velocity of the particle.

In order to approximate this equation of motion we assume nearly perpendicular wave propagation, *i.e.*, $k_{\perp} \gg k_{\parallel}$. In the case of the Bernstein wave, this assumption usually holds well. Then we get an approximation of $\ddot{z} \simeq 0$ and $\dot{z} \simeq v_{z0} = \text{const.}$ After the simple coordinate transformation, $x' = x - v_{y0}/\Omega_0$, we rewrite x' with x as

$$\begin{aligned}\ddot{x} + \Omega_0^2 x &= \frac{q \phi_0}{m} k_{\perp} \sin \psi , \\ \psi &= k_{\perp} x - \omega_d t , \\ \omega_d &= \omega - k_{\parallel} v_{z0} .\end{aligned}\tag{3}$$

Normalizing distance and time by the perpendicular wave vector $k_{\perp}$ and the wave frequency ω_0 , *i.e.*, $X = k_{\perp} x$ and $\hat{t} = \omega_0 t$, respectively, we get an approximated normalized equation of motion:

$$\begin{aligned}\ddot{X} + \hat{\Omega}^2 X &= \alpha \hat{\Omega}^2 \sin(X - \hat{\omega}_d \hat{t}) , \\ \hat{\Omega} &= \frac{\Omega}{\omega_0} , \quad \hat{\omega}_d = \frac{\omega_d}{\omega_0} , \\ \alpha &= \frac{q \phi_0 k_{\perp}^2}{m \Omega^2} .\end{aligned}\tag{4}$$

Changing the definition of some quantities for simplicity as

$$\hat{\Omega} \longrightarrow \Omega , \quad \hat{\omega}_d \longrightarrow \omega_d , \quad \hat{t} \longrightarrow t ,\tag{5}$$

we define a Hamiltonian for this motion as

$$\begin{aligned}
H(q_x, p_x, t) &= \frac{1}{2} p_x^2 + \frac{1}{2} \Omega^2 q_x^2 + \alpha \Omega^2 \cos(q_x - \omega_d t), \\
q_x &= X, \\
p_x &= \dot{X} = \frac{dX}{dt}.
\end{aligned} \tag{6}$$

By making two successive canonical transformations of this Hamiltonian, we can handle this equation of motion using variables of the normalized Larmor radius $r = k_\perp v_\perp / \Omega_0$ and the phase ξ between the wave and the Larmor motion. By extending the multiple-time-scale method for the first order used in Ref. 12 to that for the second order, we average the rapid motion of the cyclotron frequency and get the equation of the averaged motion. Now the orbit of the center of the precession motion of the charged particle is given by a Hamiltonian H_0 :

$$H_0(\xi, r) = \alpha^2 \frac{\Omega^3 h}{\omega_d} \left[\frac{1}{2} F(r) \cos 2\xi + \frac{1}{2} G(r) \right] - \Delta\omega_d \left(\frac{\Omega r^2}{2 h} \right),$$

and

$$\begin{aligned}
F(r) &= \sum_{l=-\infty}^{\infty} \frac{(l-2h)}{(h-l)} \frac{1}{r} J_l'(r) J_{2h-l}(r), \\
G(r) &= \sum_{l=-\infty}^{\infty} \frac{l}{(h-l)} \frac{1}{r} J_l'(r) J_l(r),
\end{aligned} \tag{7}$$

where J_l is the Bessel function of the l -th order, $J_l'(r) = dJ_l/dr$, h is a half integer such as

$$h : \quad \frac{1}{2}, \quad \frac{3}{2}, \quad \frac{5}{2}, \quad \dots,$$

and the frequency mismatch $\Delta\omega_d$ is given by $\Delta\omega_d = h \Omega - \omega_d$.

This Hamiltonian H_0 is an invariant of time because it does not include time variable t .

The phase trajectories in phase space (r^2, ξ) derived from H_0 for $\Delta\omega_d = 0$ are illustrated in Fig. 1, where $h = \frac{3}{2}$, *i.e.*, $\omega_d = \frac{3}{2} \Omega_c$. Note that these trajectories are independent of the wave amplitude or α only in the case of no frequency mismatch or $\Delta\omega_d = 0$, judging from Eq. (7). The ordinate shows r^2 which is proportional to the perpendicular particle kinetic energy and the abscissa represents the phase ξ between the Larmor motion and the wave. The charged particle moves along this contour line, because this Hamiltonian is an invariant of time as described above. They can be classified into two kinds of trajectories: one is a closed trajectory and the other is an open trajectory in which ξ varies without limit. In the case of the closed contour, *i.e.*, when the contour forms an island shaped somewhat like a rounded triangle, the charged particle keeps going around this island. The particles with closed trajectories are trapped particles and those with open trajectories are untrapped particles;¹³ this is similar to the motion of particles in a large-amplitude wave with a frequency close to the cyclotron harmonics in a magnetized plasma. Correspondingly, phase space (r^2, ξ) can be generally classified into the trapped region and the untrapped region, which are composed of the trajectories of the trapped and the untrapped particles. These two regions are separated from each other by the separatrices which satisfy $\partial H_0 / \partial r = 0$ and $\partial H_0 / \partial \xi = 0$.

Let us consider here the possibility of plasma heating due to this trapping process. For the Maxwellian plasma, the majority of the particles are usually located at the bottom of the island or in the untrapped region due to the dispersion relation for the Bernstein wave.

After the wave is switched on, the particles in the trapped region move toward the top corner of the triangle island, *i.e.*, increase their kinetic energy extensively. This motion may lead to the possibility of the plasma heating, although the number of the trapped particles is considerably

smaller than that of the untrapped particles. After passing the triangular top corner of the island, the particles lose their kinetic energy. This means that the kinetic energy of the particle oscillates in time due to the presence of the Bernstein wave. Therefore, some energy relaxation mechanism must exist for this trapping to cause a true plasma heating.

We have examined the collisional effects and the fluctuations in plasma as causes of this energy relaxation process. These will be discussed in Sec. IIIF and Sec. IIIG.

C. A review of NLD

NLD is understood as a kind of parametric process in which a part of the pump wave energy with frequency ω_0 is up-converted to the wave energy with its double frequency $\omega' = 2\omega_0$, which is strongly coupled with thermal electrons and has a strong damping rate.

The resonance condition for NLD in a magnetic field is given by

$$\omega - \omega'' - (k_{\parallel} - k_{\parallel}'') v_{\parallel} = m \Omega_0 , \quad (8)$$

where $(\omega, \mathbf{k})$ and $(\omega'', \mathbf{k}'')$ are the frequencies and wave vectors for the two Bernstein waves, Ω_0 is the cyclotron frequency, and m is an integer.

The mechanism of the self-interaction for NLD proposed by Porkolab is a key point of this phenomenon. This mechanism requires the next condition:

$$\begin{aligned} \omega''(\mathbf{k}'') &= -\omega_0(\mathbf{k}_0), \\ \mathbf{k}'' &= -\mathbf{k}_0. \end{aligned} \quad (9)$$

Due to nonlinear interaction between these two normal waves which satisfy the linear wave dispersion relation, a virtual wave (beat wave or

quasimode) which does not satisfy any normal linear dispersion relation is produced and its frequency and wave vector are

$$\begin{aligned}\omega' &= \omega - \omega'' = 2\omega = 2\omega_0, \\ \mathbf{k}' &= \mathbf{k} - \mathbf{k}'' = 2\mathbf{k} = 2\mathbf{k}_0.\end{aligned}\tag{10}$$

Then Eq. (8) becomes

$$\omega' - k_{\parallel}' v_{\parallel} = m\Omega_0,\tag{11}$$

and the condition for occurrence of NLD is that there exist the particles with $v_{\parallel}$ which satisfies this equation.

The kinetic wave equation for the self-interaction of the Bernstein waves by NLD^{3,7,8} is

$$\frac{\partial |E_{\omega}|^2}{\partial x} + 2\gamma |E_{\omega}|^2 = \alpha_{\omega} |E_{\omega}|^4,\tag{12}$$

where γ is the linear spatial damping rate, $\gamma = \text{Im}(\varepsilon)/[\partial \text{Re}(\varepsilon)/\partial k_{\perp}]$, and α_{ω} is the nonlinear wave-particle coupling coefficient given by

$$\alpha_{\omega} = \frac{m\Omega_0}{k_{\parallel}' v_t} \frac{\omega_p^4}{\Omega_0^4} \frac{V}{4\pi^{1/2}} \frac{\exp[-(\omega' - m\Omega_0)^2/k_{\parallel}'^2 v_t^2]}{n T \partial \text{Re}(\varepsilon)/\partial k_{\perp}},\tag{13}$$

$$V = \sum_{p,r} \frac{A - BC/D}{[(\omega/\Omega_0 - p)^2 - 1][(\omega/\Omega_0 - r)^2 - 1]}.\tag{14}$$

Here

$$\begin{aligned}A &= \int_0^{\infty} d\xi (-1)^{p+r} (\partial f/\partial \xi) J_p(\rho) J_r(\rho) J_{p-m}(\rho') J_{r-m}(\rho'), \\ B &= \int_0^{\infty} d\xi (-1)^{p-m} (\partial f/\partial \xi) J_p(\rho) J_m(\rho') J_{p-m}(\rho), \\ C &= B(p \leftrightarrow r), \quad D = -\exp(-\lambda_e') \text{Im}(\lambda_e'), \\ f(\xi) &= \exp(-\xi), \quad \xi = v_{\perp}^2/v_t^2, \quad \lambda_e = k_{\perp}'^2 v_t^2/2\Omega_0^2, \\ \rho &= k_{\perp} v_{\perp}/\Omega_0, \quad \rho' = k_{\perp}' v_{\perp}/\Omega_0.\end{aligned}\tag{15}$$

Also, the nonlinear equation for the virtual waves is given by

$$|E_{\omega'}|^2 = \frac{|E_{\omega}|^4}{|\varepsilon(\omega', \mathbf{k}')|^2} \frac{\omega_p^6}{\Omega_0^6} \frac{\Omega_0^2}{k'^2 v_t^2} \frac{3U^2}{2\pi^3 n T} ,$$

$$U = \sum_{p,q,r} \frac{r[(p-q)(A' + B') + rB']}{(\omega/\Omega_0 - q)(\omega'/\Omega_0 - r)} , \quad (16)$$

where $\omega' = 2\omega_0$, $\mathbf{k}' = 2\mathbf{k}_0$,

$$A' = \int_0^\infty d\xi f(\xi) (\xi/\rho'^2)(1 - 2\xi^2) J_{p-r}(\rho) J_q(\rho) J_r(\rho') J_{p-q}(\rho') ,$$

$$B' = \int_0^\infty d\xi f(\xi)(\xi/\rho') J_{p-r}(\rho) J_q(\rho') J_r(\rho') J_{p-q}(\rho') . \quad (17)$$

Equations (12) - (17) complete the description of the spatial development of the system.

Solving these simultaneous equations numerically, Sugaya⁷ got the spatial variations of the plasma temperature, the pump wave E_{ω} , and the virtual wave $E_{\omega'}$ for a typical set of parameters in the JIPPT-II-U tokamak plasma.^{2,4}

One of his major observations is that the efficiency of the ion heating may be decreased remarkably with $k_{||}R = k_{||}v_t/\Omega_0$. He saw that $k_{||}R \leq 0.1$ must be satisfied for efficient ion heating.

Sugawa also calculated the spatial variations of the plasma temperature, the pump wave E_{ω} , and the virtual wave $E_{\omega'}$ for a typical set of parameters in his EBW experiment and measured them to compare the experimental results with the numerical ones.⁸ He pointed out that the virtual wave was always observed for $k_{||}R \leq 0.1$ but was not observed for $k_{||}R \geq 0.15$ at a frequency near $\omega_0/\Omega_0 \simeq 1.5$.

D. The threshold values for CSR and NLD

We discuss more about the relation between the CSR and the NLD mechanisms, here.

The threshold for the CSR mechanism is given by²

$$\begin{aligned}\alpha &= \frac{q \phi_0 k_{\perp}^2}{m \Omega^2} \\ &= \frac{E_{\perp}/B}{\Omega/k_{\perp}},\end{aligned}\tag{18}$$

where α is the normalized wave amplitude for motion of the charged particles in the resultant magnetic field and the pumping electric field. This threshold may be common throughout the experiments.

This definition of the threshold may be subtly different with the NLD threshold^{3,7,8} which will be reviewed later. To be precise, this threshold value is defined as the minimum value of the normalized wave amplitude α , to the fourth order of which the heating rate is scaled [see Fig. 14(b) in Sec. IIIG].

The threshold for the NLD mechanism⁷ is given by

$$\epsilon = \frac{\frac{\epsilon_0}{2} |E|^2}{m n_{\sigma} T_{\sigma}}.\tag{19}$$

Because this threshold is related closely to the experimental condition or ϵ is a ratio of the pressure due to the rf field to the plasma pressure, its values have been much different from each other throughout the experiments, as shown in Table I.

As a trial, we rewrite ϵ using the normalized quantities:

$$\epsilon = \left(\frac{\Omega_{\sigma}}{\omega_{\sigma}}\right)^2 \frac{\alpha^2}{(k_{\perp} \rho)^2}.\tag{20}$$

In Eq. (19), $(\Omega_{\sigma}/\omega_{\sigma})^2$ may be associated with the wave dispersion, therefore, we introduce another definition of the threshold for the NLD mech-

anism,

$$\begin{aligned}\epsilon' &= \left(\frac{\omega_\sigma}{\Omega_\sigma}\right)^2 \epsilon \\ &= \frac{\alpha^2}{(k_\perp \rho)^2}.\end{aligned}\tag{21}$$

Then, we get a universal threshold value which is comparably common throughout the experiments, in other words, $\epsilon' \sim 10^{-3}$.

E. Simulation model

The Bernstein wave is essentially an electrostatic wave. Therefore, we have adopted an electrostatic option of the $2\frac{1}{2} - \text{D}(x, y, v_x, v_y, v_z)$ code PS2M,^{14,15} which is an electromagnetic code for the plasma with the boundaries.

The left half in the simulation system aims to represent experimental devices in the most simplified way, while its right half is set to prevent the wave reflections. The details will be explained in Sec. III.

The density and the temperature are also slowly increased with x proportionally to $\exp(x/l_n)$ and $\exp(x/l_T)$, where l_n and l_T are characteristic lengths for density and temperature.

In the x direction, this system is bounded by a pair of conducting walls, where the electrostatic potentials must be zero. In the z direction, the system is assumed to be periodic, so that the wave propagates through it. On the inner side of the conducting walls, a pair of limiters near the conducting walls are assumed to reflect the particle. The particle reaching the left limiter at $x = L_1$ is reflected into the system with random velocities whose standard deviation is the initial surface temperature, and those reaching the right limiter at $x = L_2$ are reflected such that no macroscopic current is generated.

The antenna for excitation of the electrostatic wave is located between the left wall and the left limiter, as shown in Fig. 4(a). This antenna

is expressed with the external charge which is distributed along the y -direction:

$$\rho_{ext}(x, y, t) = \rho_0 \delta(x - x_s) \sin(k_{\parallel} y - \omega_0 t). \quad (22)$$

This antenna changes the potential temporally and spatially as described in Ref. 16 and is a single-mode exciter.

Because the parallel wave length is much longer than the perpendicular wave length in the Bernstein wave and the perpendicular grid spacing Δ_x is chosen as approximately one Debye length λ_D , the grid size Δ_y is usually much larger than λ_D . When we use the linear (first-order spline) spatial interpolation in this case, we may encounter trouble with the growth of numerical instability, which is called the grid instability. As in the previous work,^{2,5,6} we suppressed this numerical instability by using the fourth-order spline spatial interpolation¹⁷ along the y -direction and used the stretching rate Δ_y/Δ_x of 16. On the other hand, we have used the second-order spline spatial interpolation¹⁷ along the x -direction where $\Delta_x \sim \lambda_D$ in order to save computer time without sacrificing accuracy.

F. Single particle simulation model

This model was previously used in Ref. 13 and is referred to as the single particle simulation model here. Let us explain this model simply. The single particle simulation model used here is not a self-consistent model but traces the motion of the equation of the charged particles with a Maxwellian initial velocity distribution in the prescribed uniform magnetic fields $\mathbf{B} = B_0 \hat{y}$ and the monochromatic, electrostatic potential $\phi = \phi_0 \cos(k_{\perp} x + k_{\parallel} y - \omega_0 t)$. The equation of motion of a test particle is given by Eq. (2).

Therefore, this model can probably only play an auxiliary role for understanding the heating results.

G. A binary collision model

In the conventional self-consistent particle simulation, the collisional effects should exist and change the physics predicted in the collisionless case. In the gridded simulation adopted in this work, a particle charge is not localized spatially but has finite-size in space due to the weighting or interpolation of the particles onto the grid and vice versa for the forces. These finite-sized particles are much different in their properties from the natural point particle treated in classical electromagnetic theory, but are able to pass through each other. Therefore, the interparticle force does not become singular as the distance of the separation of the particle midpoints goes to zero, while it does for point particles as their separation becomes infinitesimal. This means that the average scattering angle for the interaction of the finite-sized particle is much smaller than that for the point particle. Hence the collisionality for finite-sized particles is less than that for point particles.

As a result of these modifications of actual interparticle forces, the so-called long-wave phenomena and the macroscopic collisional properties such as resistivity in the MHD theory are fortunately unchanged, while the microscopic properties of the collision such as the velocity dependence of the test particles are changed. Therefore, in order to investigate subtle properties such as nonlinear wave heating, it is appropriate to introduce a method in which the effects of binary collisions are added to the finite-sized particle model or the single particle simulation model described in Sec. IIF. The binary collision model presented in this work is nonrelativistic and employs Monte-Carlo techniques to simulate the scattering of plasma particles. This model was first proposed by Takizuka and Abe.¹⁸ Since the code for the single particle model simulation presented in Sec. IIF has no spatial dependence (with three velocity components), we cannot simulate the details of Coulomb scattering based upon impact

parameters. On the other hand, we can scatter particles through an angle which is chosen randomly from a distribution of angles. The mean scattering angle is zero and the width or variance of the distribution is a function of the particle charges, density, reduced mass, and relative velocity, as well as the simulation time step. The scattered relative velocity of a pair of particles is not changed during the process of this binary collision. Therefore, this collision model conserves both the total energy and momentum of the system.

The model is designed to handle any combination of test and field particles: electron-electron and electron-ion. [In the case of the imaginary one component plasma such as the electron plasma in which the ion is treated as an immobile charged particle, the ion is assumed to have sufficiently heavy mass ratio, as will be seen in a later section (see Table VI used in Sec. IIIG)].

Now we will explain the method to implement this Monte-Carlo type collision model. First we determine pairs of particles at random in a cell. Next we calculate the changes in the velocities due to binary collisions in the time interval Δt . As an example of a pair, one particle of species α with mass m_α and charge e_α has a velocity $\mathbf{v}_\alpha^t$ at the time t , and the other of species β with mass m_β and charge e_β has a velocity $\mathbf{v}_\beta^t$. Then the relative velocity in the laboratory frame is defined as

$$\mathbf{u}^t = \mathbf{v}_\alpha^t - \mathbf{v}_\beta^t = (u_x, \quad u_y, \quad u_z)^t. \quad (23)$$

This is transformed into that in the frame of the relative velocity at the time t ;

$$\begin{pmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \phi \\ -\sin \phi & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}^t = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \quad (24)$$

where the angles θ and ϕ are shown in Fig. 2(a). As the result of a binary collision, the magnitude of the relative velocity is unchanged but its direction is altered by the scattering angle Θ as illustrated in Fig. 2(b):

$$(0, \quad 0, \quad u)^t \rightarrow (u \sin \Theta \cos \Phi, \quad u \sin \Theta \sin \Phi, \quad u \cos \Theta)^{t+\Delta t}, \quad (25)$$

where the angle Φ takes values from 0 to 2π . The postcollision relative velocity in the frame of the precollision relative velocity is transformed into that in the laboratory frame. It can be expressed in the form

$$(u_x, \quad u_y, \quad u_z)^{t+\Delta t} = (u_x, \quad u_y, \quad u_z) + (\Delta u_x, \quad \Delta u_y, \quad \Delta u_z), \quad (26)$$

i.e.,

$$\begin{aligned} \Delta u_x &= \frac{u_x^t}{u_\perp^t} u_z^t \sin \Theta \cos \Phi - \frac{u_y^t}{u_\perp^t} u_z^t \sin \Theta \sin \Phi - u_x^t (1 - \cos \Theta), \\ \Delta u_y &= \frac{u_y^t}{u_\perp^t} u_z^t \sin \Theta \cos \Phi - \frac{u_x^t}{u_\perp^t} u_z^t \sin \Theta \sin \Phi - u_y^t (1 - \cos \Theta), \\ \Delta u_z &= -u_\perp^t \sin \Theta \cos \Phi - u_z^t (1 - \cos \Theta), \end{aligned} \quad (27)$$

where $u_\perp = (u_x^2 + u_y^2)^{1/2}$. In the cases of $u_\perp = 0$, Eq. (27) is changed to those given by Ref. 18, although these cases rarely occur.

By the use of these expressions we obtain the postcollision velocity of the particle α and that of the particle β ;

$$\mathbf{v}_a^{t+\Delta t} = \mathbf{v}_a^t + \frac{m_{\alpha\beta}}{m_\alpha} \Delta \mathbf{u}$$

and

$$\mathbf{v}_a^{t+\Delta t} = \mathbf{v}_a^t - \frac{m_{\alpha\beta}}{m_\alpha} \Delta \mathbf{u}, \quad (28)$$

where the reduced mass $m_{\alpha\beta}$ is defined by

$$m_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}. \quad (29)$$

It is easily seen that both the total momentum and total energy are conserved for a collision. In order to give the scattering angle in Eqs. (25), we introduce the relations:

$$\sin \Theta = \frac{2\delta}{1 + \delta^2}$$

and

$$1 - \cos \Theta = \frac{2\delta^2}{1 + \delta^2}. \quad (30)$$

The variable $\delta \equiv \tan(\Theta/2)$ is chosen at random with the Gaussian distribution. The mean is zero and the variance $\langle \delta^2 \rangle$ is given by

$$\langle \delta^2 \rangle = \frac{e_\alpha^2 e_\beta^2 n_L \lambda}{8\pi \epsilon_0^2 m_{\alpha\beta}^2 u^3} \Delta t. \quad (31)$$

H. Measurement of collision frequencies

In order to verify the accuracy of the Monte-Carlo type binary collision model, several simulations¹⁸ were performed by using the code which is made by combining the single particle model with the binary collision model. In these simulations, the magnetic field and the external wave were assumed to vanish. In other words, these simulations dealt only with velocity space transport of particles in a spatially uniform plasma for the confirmation of the collision model.

Let us introduce here two types of plasma relaxation processes: the transverse diffusion and the slowing-down, which is governed by the following differential equations describing the transverse diffusion and the slowing down in velocity space of a beam of charged particles as

$$\frac{d}{dt} \langle \mathbf{v}(t) \rangle_\perp^2 = \nu_d \mathbf{v}^2(t)$$

and

$$\frac{d}{dt} \langle \mathbf{v}(t) \rangle = \nu_s \mathbf{v}(t), \quad (32)$$

where $\mathbf{v}(t)$ is the velocity of a particle in the beam at time, $\langle \mathbf{v}(t) \rangle$ is the mean velocity of a particle, and ν_d and ν_s are the deflection and the slowing-down frequencies.

The deflection and the slowing-down frequencies are defined in terms of the mass ratio of the incident and target particles (m_α and m_β), the energy of the incident particle (E_α), the temperature of the target particle (T_β) and the basic collision frequency ν_0 as¹⁸

$$\nu_d \equiv 2 \left[\mu(x) \left(1 - \frac{1}{2x}\right) + \mu'(x) \right] \nu_0$$

and

$$\nu_s \equiv \left(1 + \frac{m_\alpha}{m_\beta}\right) \mu(x) \nu_0, \quad (33)$$

where the Maxwell integral is defined as

$$\mu(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^\pi \sqrt{t} \exp^{-t} dt \quad (34)$$

with

$$x \equiv \left(\frac{m_\beta}{m_\alpha} \frac{E_\alpha}{T_\beta} \right) \quad (35)$$

and the basic collision frequency is defined as

$$\nu_0 \equiv \frac{e^4 n \lambda}{8 \sqrt{2} \pi \varepsilon_0^2 m_e^{1/2} T_e^{3/2}}. \quad (36)$$

As in the test for the binary collision model,¹⁸ we measured the collision frequency in the simulated finite-sized particle plasma, fitting the velocity dependence with Eqs. (32) and (33) and obtained their basic collision frequency ν_0 which will be presented in Sec. IIIF (see Table V).

The simulation results agreed well with the theoretical prediction. Equations (32) and (33) could be compared with the results of the test

of the binary collision as observed before.¹⁸ In other words, the velocity dependence fitted well with these equations, although the simulation used the $2\frac{1}{2}$ dimensional model while the theory assumed the 3 dimensional model.

III. SIMULATION RESULTS

In Refs. 2, 5, and 6, we investigated CSR plasma heating in the IBW plasma heating configurations, motivated by the results of JIPPT-II-U experiment.^{1,4} However, the computer time is too large to explore the various parameters in detail. Therefore, we switched from the IBW heating configuration to the EBW heating configuration since the underlying heating mechanisms are essentially the same.

We shall now give the simulation parameters. The two-dimensional space of $512\Delta \times 512\Delta$ including the magnetized plasma is simulated by using the 512×32 spatial grids which means that the z-directional grid number is 1/16 of the x-directional number or the stretching rate Δ_y/Δ_x is 16, as explained in Sec. IIE.

This system is immersed in a z-directional static magnetic field $\mathbf{B} = B(x) \hat{y}$, which is increased with x . In the left half region, the function $B(x)$ is given by

$$B(x) \propto \frac{1}{(x - x_0)}. \quad (37)$$

Because of this, the magnetic field changes spatially: $\omega_0/\Omega_e = 1.90$ at the left limiter, $x = L_1$, $\omega_0/\Omega_e = 1.50$ at $x \simeq 110$ corresponding to the electron subharmonic resonance position, and $\omega_0/\Omega_e = 0.99$ at $x \simeq 252$. The magnetic field strength corresponding to $\omega_0/\Omega_e = 1.90$ at the left edge of the plasma is optimized as a weak-field-side excitation and that to

$\omega_0/\Omega_e = 0.99$ at $x = 252$ is the strength near the center of the system. In the right half, $B(x)$ remains steady for ω_0/Ω_e to be close to 1.0. Because the pump frequency ω_0 is close to the cyclotron frequency Ω_e there, the cyclotron damping causes the wave passing from the left half to damp sufficiently from the middle part of the system to the right half and prevents it from arriving at the right conducting wall. Therefore, the wave reflection, which makes the phenomena so hard to understand, does not occur at the right wall, as will be seen in the wave spectra [Fig. 7(c)] in Sec. IIIC.

The density and the temperature are also slowly increased with x proportionally to $\exp(x/l_n)$ and $\exp(x/l_T)$, l_n and l_T are characteristic lengths for density and temperature.

In Fig. 3, the schematic illustration of the system simulated is presented.

A. Propagation of the electron Bernstein wave

In Figs. 4(a) and (b), we show the contour of the interferogram in (x, z) space between the measured wave potential and the antenna signal, or E_x vs x which is an interferogram between the observed electric field and the antenna signal. In accordance with our expectation, the Bernstein wave is excited at the left edge, propagates through the left half and then, in the middle of the system, is subjected to powerful cyclotron damping. In Fig. 4(c), we show a comparison between $\text{Re } k_\perp$ vs x measured in the steady state and the values calculated using the linear theory, or Eq. (1); the circles represent measured values, and the solid curve, the theoretical ones. The measured values agree well with the theoretical values except for both sides of plasma edge where the dispersion equation Eq. (1) is strongly affected by the nonuniformity of space due to antenna or by the finiteness of $\text{Im } k_\perp$. Therefore, the excited wave

may be identified as the Bernstein wave. All these values are measured at $t = 80T$, where T is $2\pi/\omega_0$.

B. Energy deposition in the plasma

In Fig. 5(a), the perpendicular energy deposition vs x is shown by the bar graph in each of 12 cells which are divided from the right edge of plasma, $x = 4\Delta$, to the middle of the system, $x = 256\Delta$. Each cell is specified from 1 to 12 beginning at the left or the lower density side [see these numbers in Fig. 5(a)]. This perpendicular energy deposit is measured as a difference between the energy at $t = 80 T$ and the energy at $t = 32 T$, when the wave arrives at the right edge of the plasma. Figure 5(b) is a phase space plot showing the relation on the energy distribution vs x at $t = 80 T$.

The point indicated by the arrow corresponds to the cyclotron sub-harmonic resonance point. As seen in both Figs. 5(a) and (b), there is clearly present a heating which cannot be predicted by the linear dispersion relation Eq. (1) and its peak is located around the point where ω_0/Ω_e is a half-integer or $\frac{3}{2}$. This may be heating due to a nonlinear process. The heating in the right edge in Figs. 5(a) and (b) is due to cyclotron resonance heating, which is a conventional linear phenomenon. We can see from this figure that the nonlinear heating is considerable compared with linear cyclotron heating.

C. Observation of the virtual wave

As the mechanism of this nonlinear heating, we should examine two candidates: One is nonlinear CSR heating and the other is NLD which predicts that the mode with $(2\omega_0, 2\mathbf{k}_0)$ appears around the region associated with this nonlinear heating. This is called the virtual mode, because it does not satisfy the linear dispersion relation, Eq. (1). Due

to this property, it does not propagate and therefore should be localized around the position where this mode is produced.

In Refs. 2, 5, and 6, we did not observe this mode during CSR heating. As mentioned in Sec. IIC, the theory predicts quenching of the virtual mode for $k_{\parallel}R = k_{\parallel}v_t/\Omega_0 \geq 0.15$ even for a frequency near $\omega_0/\Omega_0 = 1.5$, while this parameter $k_{\parallel}R$ is 0.05 for a typical run.² Although it may be difficult to attribute the lack of observation of the virtual mode to the value of $k_{\parallel}R$, we note that the growth rate of the virtual mode depends sensitively on the parameters. The optimum condition for growth is easily detuned due to other condition of the dispersion relation.

Therefore, in our simulation study, we installed a special diagnostic for which we can extract the mode with $(2\omega_0, 2k_{\parallel 0})$ in the interferogram outputs and catch any signs of even a small growth. Finally, by optimizing the simulation parameter, we found clear evidence of the growth of this mode from a large number of preliminary runs.

In Fig. 6(a), we show the spatial pattern (x, y) of the interferogram of E_x with frequency $2\omega_0$. From this figure we can see the presence of the mode with $2\omega_0$ at $x = 62 \sim 110$; in other words, we can see the growth of the virtual mode with a shorter wavelength than that of the pump wave. Then we extracted the mode with $(2\omega_0, 2k_{\parallel 0})$ from these data which are shown in Fig. 6(b). This mode has twice the wave number of the pump wave or $2k_{\perp 0}$ as the perpendicular wave number and the maximum amplitude around $x = 100$.

Now, we can say that we succeeded in observing and identifying the virtual mode which is excited through NLD. When optimized, it is about 0.0625 times in the power or in the square magnitude of the electric field, E_x^2 , and is 25 % in the amplitude, compared with the pump wave with $(\omega_0, k_{\perp 0}, k_{\parallel 0})$. It is interesting to compare this result with that of the EBW experimental results,⁸ where the wave amplitude for the virtual

wave was reported to be as much as 30dB below that for the pump wave.

We show the x -dependent wave spectra of the perpendicular electric field $E_x^2(x, \omega)$ which is Fourier-analyzed with respect to both y and t in Figs. 7(a) and (b) for the fundamental and second Fourier modes in the y direction, respectively. From these figures, we can see that the virtual wave with $(2\omega_0, 2k_{||0})$ exists and is strongly localized around the position of $\omega_0/\Omega_e = 1.5$.

Figure 7(c) shows the frequency spectrum, which is calculated by summing up in k -space. Waves with a plus ω are those with a plus phase velocity. Excited waves are localized closely to the position of $\omega_0/\Omega_e = 1$ and the reflected waves with $\omega_0/\Omega_e = -1$ are almost nonexistent. We can see that wave reflection is almost completely prevented by the method explained in the introductory part of Sec. III.

Here we notice from Fig. 7(a) that the pump wave changes its amplitude spatially along the x -direction. Therefore, we plot it more accurately in Fig. 8 which shows $|E_x|$ vs x for the waves with frequencies ω_0 and $2\omega_0$. This spatial amplitude oscillation is not due to the reflected waves, because they are scarcely seen in Fig. 7(c) and this oscillation is seen even before the wave reflected in the right-half can reach the left-half.

D. A hypothetical virtual wave experiment

Now we will proceed to discuss the effects of this virtual wave on the heating. As explained in the previous section, we have sometimes observed CSR heating without the virtual wave in the preliminary runs and in the previous works.^{2,5,6} The virtual wave has a frequency of $\omega' = 2\omega_0 = 3\Omega_e$ and a wave number of $k'_\perp = 2k_{\perp 0}$ which may interact with the thermal particle, because of $k'_\perp/\omega' = v_{th}$. From our experience, however, we have set a hypothesis that almost all of the initial heating in the cyclotron subharmonic regime originates from the CSR heating

effects not from the virtual wave (or NLD).

In order to prove this hypothesis, we have made comparison runs in which the virtual mode is prevented from growing by the computational technique essentially turning off the NLD heating mechanism. This is achieved by cutting down the higher- k_y modes than k_y of the fundamental mode with $k_{||0}$ by the use of the spatial filter, because the virtual wave is a mode with $2k_{||0}$, as derived from Eq. (10). This technique is easy and popular in particle simulation, because the Poisson solver in the particle code uses FFT (Fast Fourier transform).¹⁴

Even though we adopt these artificial techniques, no physical principles should be violated. In order to show this, we have carried out an energy checkup using the following equation¹⁶:

$$H(t) = \sum_j \frac{1}{2} m_j v_j^2 + \sum_j \frac{1}{2} q_j \phi_i(x_j, y_j) + \sum_j q_j \int_0^t d\tau \mathbf{v}_j \cdot \nabla \phi_e(x_j, y_j, \tau), \quad (38)$$

where the first term is the particle kinetic energy, the second term is the internal potential energy in the system and the third term is the energy absorbed in the plasma by the external electric field (ϕ_i is the electric potentials produced by the plasma charge and ϕ_e is the external source current at the antenna).

In Fig. 9, we show the total energies vs time for run 1 and run 1', which is a run where the virtual mode is artificially eliminate for the same simulation parameter as in run 1. The total energies are well conserved for both runs and to the same extent. Therefore, the results of these two runs may be useful to understand the physics.

In Fig. 10, the perpendicular energy deposition vs x is shown for run 1', while that for run 1 is shown in Fig. 5(a) previously. For ease of comparison, the methods for measurement and representation are the same for

both runs; for example, both measurements were made at time $t = 80T$ earlier than the final time $t = 240T$. The arrows in both the figures indicate the CSR point of $\omega_0/\Omega_e = 1.5$. The difference between these two figures may be attributed to whether the effect of the virtual wave (or NLD) to this heating exists or not. The peak of the deposition is low compared with that in Fig. 5(a) for run 1. However, there still remains the deposition peak located at region 6 near $\omega_0/\Omega_e = 1.5$. In addition, this difference becomes smaller as the time elapses and may be neglected later, around $t = 240 T$ in Fig. 11(a), while Fig. 5(a) and Fig. 10 are plotted for the data at $t = 80 T$, as is noted already. This is the nonlinear heating referred as to CSR heating which has been first proposed by Abe and Okada *et al.*² This demonstrates that there exists a direct nonlinear heating mechanism in addition to the indirect heating through the virtual wave.

Next we present the deposit energy in the cyclotron subharmonic resonance region vs time in Fig. 11(a), which shows the deposit energy in regions 5 and 6 where the nonlinear heating is the greatest and where there exists the point of $\omega_0/\Omega_e = 1.5$. The abscissa and the ordinate represent the elapsed time as periods of the pump wave and the kinetic energy increases from the initial time ($t = 0 T$), respectively. The circles and the triangles stand for run 1 and run 1', respectively. We will explain Figs. 11(b) and (c) in Sec. IIIE.

No increase is observed until $t = 20 T$ because the wave does not arrive at region 5. Starting at $t = 50 T$, there is a difference between the circles and the triangles and tendency toward saturation of increase for the triangles. After that, the triangles take over the circles and so we see little difference between the circles and triangles at $t = 200 T$. In simple CSR heating, the heating should be saturated after the time near the average bounce time, because of the particle trapping in the velocity

phase-space. In these two runs (runs 1, and 1'), this average trapping time is estimated about $40 T$ from the single particle simulation results.

A series of these results suggest that there exist some energy relaxation mechanisms which destroy the pure phase relation between the particle Larmor motion and the wave motion in the single particle due to some fluctuations in the plasma. Furthermore, initially untrapped particles enter the trapped region due to this destruction of the phase relation.

We have considered two possible mechanisms to explain the energy relaxation: One is due to the collisional effects and the other is due to the fluctuations in the plasma and their enhancement due to the rf electric field, which are shown in Fig. 12. The effects on the plasma heating due to both the mechanisms will be presented in Sec. IIIF and Sec. IIIG.

E. Dependence of the wave amplitude

In order to study the dependence of the wave amplitude, we have added 4 runs (runs 2 $\sim$ 5) in which the magnitude of the external source current is changed. In runs 1 to 5, all the simulation parameters are the same except for the external source current. Each amplitude of the electric field measured at the CSR point is presented in Table III. The values in the braces are those normalized by the value for run 1.

In Fig. 13(a), we show the deposit energy in regions 5 and 6 vs. time for each run, where the CSR point is located close to the center of both regions 5 and 6. The circles, squares, diamonds, triangles, and plus signs stand for runs 1, 2, 3, 4, and 5, respectively. This figure suggests that the profile of the heating may be separated into two stages: one is the first stage or the rapid heating stage after the wave reaches the CSR resonance region $\omega_0/\Omega_e = 1.5$ and the other is the second stage or the moderate heating stage following the rapid heating stage.

Then we have measured the temporal heating rate for each run in the rapid heating stage by adopting the least square method. We show this result in Fig. 13(b), which shows that the measured point on the heating rate lies on the line proportional to the fourth power of the electric field in the first stage of the heating (the rapid heating stage).

We may assume that CSR heating occurs when the measured heating rate resides on this line. The threshold value might be smaller than that for the measured point which lies on the left edge in Fig. 13(b) or run 2 with the wave amplitude $\alpha = 0.13$. In other words, the threshold for CSR heating might be $\alpha \leq 0.13$.

F. Effect of collision on CSR heating

In order to examine the collisional effects on CSR heating, we have carried out supplementary runs 6 and 7, in which only the particle numbers are changed compared to the simulation parameters in run 1. In other words, the simulations in these three runs are the same except for the particle numbers used, as seen in Table II. Therefore, it is expected that the results in runs 6 and 7 reproduce those in run 1 including their wave amplitude excitation. Actually, the measured amplitude in run 7 is the same as that in run 1, but that in run 6 is slightly less than that in run 1. This result is good enough for our purpose.

Corresponding to these two runs, we have carried out two additional runs, 6' and 7', in which the modes of the virtual wave are cut, in order to make the same comparison as in Sec. IIID, the virtual experiments.

The collisionality or the collision frequency is expected to be inversely proportional to the particle number used in the particle simulation. The particle numbers for runs 6 and 7 are double and quadruple with that in run 1 as shown in Table II. Therefore, their collision frequencies should be $\frac{1}{2}$ and $\frac{1}{4}$ compared with that for run 1.

In order to confirm this theoretical prediction, we have carried out six supplementary runs, 1", 1'", 6", 6'", 7" and 7'".

We have adopted the following special system for measurement of collision: The magnetic field and the applied external electric field for pumping the rf are set to vanish, and the plasma density and the temperature are set to be spatially uniform at the average values in regions 5 and 6. The parameters are presented in Tab. IV. Then, we measured the basic collision frequency ν_0 , adopting the method which was originally developed for the purpose of examination of the Monte-Carlo type collision model¹⁸ which is explained in Sec. IIH.

The measured results are presented in Tab. V. Dependence of the collision on the particle numbers roughly agrees with the theoretical prediction. Therefore, it may be better to consider that the collision normalized by the plasma frequency or ν_0/ω_{pe} is roughly of order or less than 10^{-4} .

In order to see the effects of the collision on CSR heating and to compare the results of run 1, the deposition energy vs. time at the cyclotron subharmonic resonance regions for runs 6 and 7 are shown in Figs. 11(b) and (c), respectively. In the figures, the circles stand for runs with the virtual wave retained (runs 6 and 7) and the triangles for runs with the virtual wave cut out in the simulations (runs 6' and 7'). Compared to the results of run 1 in Fig. 11(a), we recognize the following fact: the results represented with the circles are not very different, while the energy deposition represented with the triangles begins to be saturated at a certain time and then to increase again. The longer the saturation time is, the less the collisions are, and the saturation level is almost the same for each run.

When we artificially eliminate the virtual wave in k-space, the NLD mechanism disappears in the simulation leaving only the CSR heating mechanism. Then, the heating may be saturated after about the bounce

time of the particle trapping in phase space.

Although this behavior in the simulation runs was imagined a priori as a scenario of this virtual experiment, the results showed that the heating seemed to be saturated at once but then began to be strongly heated again seeming to catch up to the runs in which the full modes were retained. The time which it was kept longer, the less the collision was. Therefore, the simulation result showed that our CSR scenario correctly describe the initial phase of heating which was comparably short. However, the unexpected slower heating evolution beyond the saturation for the later heating phase remained not fully understood.

G. Results of the single particle simulations

When we study the effects of collisions on this nonlinear heating including the anomalous effect if it exists, we should be able to change the collisionality at an arbitrary magnitude. However, we could not exclude the effect of the collisionality entirely in the self-consistent particle simulation, although we are able to reduce the collisional effects to some extent by increasing the particle numbers under the condition of the constant plasma density, as explained in Sec. IIIF.

In addition, the fluctuation amplified by the external electric field disturbs the classical collisional effects on this nonlinear heating. In order to extract the phenomena based on collision, we have carried out a series of the following single particle simulations.

The simulation parameters are presented in Tables VI and VII. These are all determined from the parameters in run 1 and their results.

First, we present the results for the collisionless case or runs A and A'. By the solid curve in Fig. 14(a), we show the deposit energy vs time for run A with the monochromatic wave as the external electric field. Its frequency ω_0 is the same as in run 1 and its wave number k is the value

measured at the cyclotron subharmonics resonant point in run 1. By the broken curve in Fig. 14(a), we present the results for run A' with the pump wave and the virtual wave. For the amplitude, the frequency, and the wave number of the wave assumed to be virtual waves, the values measured in run 1 are substituted [$E_x = 0.68$, $E_x' = 0.17$ and $k_{\perp 0}$ can be seen in Fig. 3(c)]. As in the results on the full simulation (Sec. IIID), we can see that in the rapid heating stage the virtual mode influences the heating results for the run with the monochromatic wave as the external electric field, but fails to break the saturation of heating in the following stage, as seen in Fig. 14(a).

Then, we present simulation results for the collisional case in Fig. 14(b). We have carried out five runs, runs A'', B'', C'', D'', and E''. Only the magnitudes of the collisions are changed in these 5 runs, which are presented in Table VII. In the figure, the curves for these five runs are located in the order of the magnitude of the collision. In other words, the curves for run A'' and E'' are located in the lowest and the uppermost positions at the end, respectively.

Run A'' with the weakest collision of $\nu_o/\omega_{pe} = 2.16 \times 10^{-4}$ corresponds to the collision in the full simulation run 1 and 1'' (see Table V). This run increases their perpendicular energy at the same ratio as that in the run without collision in the first stage of heating (see Sec. IIIE) and decreases the heating rate considerably in the second stage of heating unlike the results of run 1 of the full simulation [compare the solid curve in Fig. 14(a) for run A and the circle points in Fig. 11(a)].

As collisions increase, the heating is not saturated in the second stage. Finally, we can see that introduction of the weak collision of $\nu/\omega_0 \sim 10^{-3}$ to the system may approximate the results of the full simulation of run 1.

IV. SUMMARY, DISCUSSION, AND CONCLUSION

The cyclotron subharmonics resonant heating is studied using particle simulation. Here, we will summarize (a) the full simulation results, (b) the results of the virtual experiments using the artificial technique, and (c) the single simulation results:

[a] the full simulation results:

- (a.1) The Bernstein wave was identified as a wave which causes the subharmonics heating (Sec. IIIA).
- (a.2) Subharmonics heating was observed and could not be neglected compared with harmonics heating (Sec. IIIB).
- (a.3) The virtual modes generated by the NLD were observed and identified (Sec. IIIC).
- (a.4) The direct nonlinear heating was observed to be larger than the indirect heating caused by the virtual wave (Sec. IIID).
- (a.5) The evolution of the heating may be separated into two stages: one is the rapid heating stage after the wave reaches the CSR resonance region $\omega_0/\Omega_e = 1.5$ and the other is the moderate heating stage following the rapid heating stage (Sec. IIIE).
- (a.6) The heating rate is proportional to the fourth power of the electric field in the first stage of the heating (the rapid heating stage). (Sec. IIIE).
- (a.7) The threshold α might be equal to or smaller than 0.13 (Sec. IIIE).

[b] the results of the hypothetical virtual wave experiment:

- (b.1) When we artificially retained only the CSR mechanism by eliminating the NLD mechanism, we observed that the heating was saturated immediately after the bounce time of the

trapped particles, but began to be strongly heated again (Sec. IIIF).

- (b.2) The longer the saturation time is, the less the collisions are but its saturation level is nearly identical for each run (Sec. IIIF).
- (b.3) As a possible scenario, the relaxation could occur due to the fluctuating electric fields perhaps enhanced by the presence of the rf electric field (Sec. IIIF).

[c] the single simulation results:

- (c.1) The virtual mode only slightly influences the heating results for the run with the monochromatic wave as the external electric field. (Sec. IIIG).
- (c.2) The weak collision of $\nu/\omega_0 \sim 10^{-3}$ destroys the regular motion of the trapped and the untrapped particles. Its introduction may reproduce the results of the full simulation (Sec. IIIG). This result is not so unusual. In the one-dimensional beam-plasma system, the small collision of order $\nu/\omega_{pe} \sim 10^{-3}$ also destroys the regular amplitude oscillation.¹⁹
- (c.3) It is possible for the plasma to be heated resonantly by a wave with the CSR frequency without the growth of the virtual wave in the slightly-collisional plasma.

Next we comment on the relation among these simulation results and the theoretical and experimental results.

The collective effects seem not to be included in the particle viewpoint, as Porkolab criticized our first report:² “the simulation results are interpreted in terms of a single-particle, single-wave force equation which does not correctly include collective (plasma shielding) effects.” In accordance with this criticism, he introduced the fluid viewpoint to predict the growth of the virtual modes and to calculate the heating rate.

However, as we have demonstrated, the heating mechanism itself is mostly dominated by the particle viewpoint, as stressed in the Sec. I. In other words, the heating rate may be well approximated within the particle viewpoint. This was why we could not recognize the growth of the virtual mode in the previous simulation.^{2,5,6}

As the threshold wave amplitude for this nonlinear phenomena in the present study using the simulation, we may adopt $\alpha \leq 0.13$, as discussed in Sec. IIIE. Substituting this value into Eq. (21), we can calculate ϵ' as $\epsilon' \leq 17 \times 10^{-3}$ for $k_{\perp}\rho \sim 1$ which is the value at the cyclotron subharmonic frequency. This value may be larger by one order of magnitude than those in the experimental results, which are presented in Table I. Conversely, the threshold α calculated from $\epsilon' = 10^{-3}$ in Tab. 1 is about 0.03. We feel that this value is too small to explain our simulation results. However, we may not attach exaggerated importance for this discrepancy, because the significant changes in ΔT_i vs input power for the IBW heating experiments took place at the critical power levels, whose normalized wave amplitudes α were slightly larger than 0.3, which was the range of α used in the simulation as pointed out in Ref. 2 and an error of a factor of 6 was reported to be included in the EBW experiment, as noted in Ref. 8. In addition, the definition of the threshold for both the mechanisms may be subtly different from each other, as discussed in Sec. IID.

The NLD theory predicts presence of the virtual modes unlike CSR heating. In the EBW experiments,⁸ we believe that such low level virtual wave [see Sec. IIIC.] may scarcely affect the heating results, comparing them with our simulation results in Sec. IIID. To the best of our knowledge, the power level of the virtual wave has been never measured in the subharmonics wave heating experiments using the IBW in the tokamaks.

The NLD theory predicts that the condition of $k_{\parallel}R \leq 0.1$ is necessary for the efficient heating,⁷ as introduced in Sec. IIB. On the other hand,

the CSR theory predicts the efficient heating due to the Bernstein wave, only if its wave frequency is half harmonics of the cyclotron frequency even in the case of $k_{\parallel}R > 0.1$.

Finally, we may summarize these results and conclude that CSR and NLD are divided into two types of physical process: The CSR heating is a kind of stochastic process, in which its resonance feature is a result of the resonant change of the step length in phase space or energy change due to trapping (the correlation time does not show any resonance in this heating) can be responsible for heating at cyclotron subharmonics at any time only if the half-harmonics resonance region exists in the plasma, while NLD is a parametric process in which its resonance feature results from the resonant condition involved in this process, but needs the complex physical condition in addition to the frequency resonance condition for the efficient heating.

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FIGURE CAPTIONS

- Fig. 1 The phase trajectories obtained from Eq. (7). The parameters are $h = 3/2$, *i.e.*, $\omega_d = 3/2 \Omega_c$ and $\Delta\omega_d = 0$. The ordinate shows r^2 which is proportional to the perpendicular particle kinetic energy and the abscissa represents the phase ξ between the Larmor motion and the wave.
- Fig. 2 A schematic illustration of the velocity coordinates; (a) the relative velocity u at time t and (b) the change in the relative velocity.
- Fig. 3 A schematic illustration of the simulation model: (a) the slab model, in which the broken line indicates the position of the antenna expressed with by Eq. (22), (b) the spatial(x-directional) variation of the magnetic field intensity as ω/Ω , where ω is the rf frequency and Ω is the local cyclotron frequency, (c) the spatial variation of the initial temperature, and (d) the spatial variation of the initial density as ω_{pe}^2 , where ω_{pe} is the local plasma frequency.
- Fig. 4 The wave propagation profiles in run 1 can be seen in the interferograms of the electric field with the rf frequency ω_0 : (a) Wave potential contours at $t = 80T$. Distance in the z-direction is scaled down compared with that of the x-direction; (b) The $E_\perp$ vs x observed; (c) $k_\perp$ vs x. The solid line and the circles are the real part of $k_\perp$ calculated from Eq. (1) and the values measured from the simulation results, respectively.
- Fig. 5 The observed heating results, where the arrows indicate the cyclotron subharmonic resonance point or $X = 110$: (a) The x-

dependent energy deposition from $t = 32T$ to $80T$ where the number from 1 to 12 on the bar graph indicates each region and (b) The instantaneous phase space plot in $(x, v_{\perp})$ space at $t = 80T$.

Fig. 6 The generation of the virtual wave in run 1 can be seen in the interferograms of the electric field with the rf frequency $2\omega_0$ where the arrows indicate the cyclotron subharmonics resonant point or $X = 110$: (a) Wave potential contours at $t = 80T$. Distance in the z -direction is scaled down compared with that of the x -direction; (b) The $E_{\perp}$ vs x observed.

Fig. 7 The wave spectra of the squared magnitude of the electric field: (a) The x -dependent wave spectra for the first Fourier mode with $k_{\perp 0}$. The dashed line indicates the position of the frequency of the wave source, (b) The x -dependent wave spectra for the second Fourier mode with $2k_{\perp 0}$. Two dashed lines indicate the positions of the frequencies of the wave source and its double, and (c) The wave spectra spatially averaged.

Fig. 8 The wave amplitudes of the electric field. The solid and broken curves indicate the amplitudes for the frequency of the wave source and its doubles, respectively. : (a) for $t = 20T$ and (b) for $t = 80T$, the arrow indicates the cyclotron subharmonics resonant point.

Fig. 9 A test of the total energy conservation of the system. In the figure, the abscissas show the time and the ordinates show the ratio of the total energy error, which is defined as $\Delta H/H_0 \times 100 = (H - H_0)/H_0 \times 100[\%]$, where H_0 is the initial H or $H_0 = H(0)$ in Eq. (38): (a) for run 1, where the full Fourier modes are retained and (b) for run 1', where the higher modes are cut.

Fig. 10 The observed heating in run 1', where the higher modes are cut [compare this with Fig. 5(a)]. The x-dependent energy deposition from $t = 32T$ to $80T$. The arrows indicate the cyclotron subharmonics resonant point or $X = 110$ and the number from 1 to 12 on the bar graph indicates each region.

Fig. 11 The perpendicular energy increase for the cyclotron subharmonic resonance region vs time: (a) for runs 1 and 1'; (b) for runs 6 and 6'; (c) for runs 7 and 7'. The circles stand for runs 1, 6, and 7 and the triangles stand for 1', 6', and 7'.

Fig. 12 The wave frequency spectra of the squared magnetic fields: (a) for run 1 with rf on; (b) for the run which has the same parameters as in run 1 other than those with rf off.

Fig. 13 The results of the single particle simulations: (a) the energy deposition vs time; (b) The energy increase rate vs the wave amplitude.

Fig. 14 The results of the single particle simulations: (a) the energy deposition vs time with the virtual wave (the dotted curve) and with the pump wave only (the solid curve); (b) The energy increase rate vs the wave amplitude for various magnitudes of the collision frequency.

TABLE I. Experimental parameters in the experiments

	$\frac{\omega_0^2}{\Omega_e^2}$	ϵ	ϵ'
Tokamak(JIPP TII-U)	1000	10^{-6}	10^{-3}
EBW experiments	10	10^{-4}	10^{-3}

TABLE II. Simulation parameters

Run name	run 1 to run 5	run 6	run 7
	run 1'	run 6'	run 7'
λ_i	512	512	512
$\left(\frac{\omega_0}{\Omega_e}\right)_{\max}$	1.90	1.90	1.90
$\left(\frac{\omega_0}{\Omega_e}\right)_{\min}$	1.00	1.00	1.00
$\frac{\omega_{pe \max}^2}{\Omega_{e \max}^2}$	3.65	3.65	3.65
$\frac{\omega_{pe \min}^2}{\Omega_{e \min}^2}$	0.9	0.9	0.9
$T_{e \max}$	14.9	14.9	14.9
$T_{e \min}$	2.56	2.56	2.56
System length (X-direction)	512	512	512
System length (Y-direction)	512	512	512
No. of electrons	225,216	451,794	900,864

TABLE III. Wave amplitude for each run

$k_{||} = 0.012$, $k_{\perp} = 0.16$, $\Omega_e = 0.69$, and $v_{te} = 2.1$ at the measurement point

run name	run symbol	E_x	(E_x / E_{xrun1})	α
run 1	○	0.680	(1.00)	0.23
run 2	□	0.375	(0.55)	0.13
run 3	◇	0.874	(1.29)	0.29
run 4	△	1.17	(1.71)	0.39
run 5	+	1.21	(1.78)	0.41
run 6		0.593	(0.87)	0.20
run 7		0.680	(1.00)	0.23

TABLE IV. Simulation parameters for measurement of the basic collision frequency ν_0

Run name	run 1"	run 6"	run 7"
	run 1'''	run 6'''	run 7'''
λ_i	512	512	512
$\frac{\omega_{pe}^2}{\omega_0^2}$	1.4	1.4	1.4
T_e	4.0	4.0	4.0
System length (X-direction)	256	256	256
System length (Y-direction)	512	512	512
No. of electrons	176,000	352,000	704,000

TABLE V. Measurement of collision frequency

Run name	run 1"	run 6"	run 7"
$\frac{\nu_0}{\omega_{pe}}$	1.6×10^{-4}	1.0×10^{-4}	0.6×10^{-4}
Run name	run 1'''	run 6'''	run 7'''
$\frac{\nu_0}{\omega_{pe}}$	1.0×10^{-4}	0.8×10^{-4}	0.5×10^{-4}

TABLE VI. Simulation parameters of the single particle simulations

λ_i	512
$\left(\frac{\omega_0}{\Omega_e}\right)_{\max}$	1.59
$\left(\frac{\omega_0}{\Omega_e}\right)_{\min}$	1.51
$\frac{\omega_{pe,\max}^2}{\Omega_e^2}$	1.55
$\frac{\omega_{pe,\min}^2}{\Omega_e^2}$	1.29
$\frac{m_i}{m_e}$	10000.0
$T_{e\max}$	4.71
$T_{e\min}$	4.45
System length (X-direction)	256
System length (Y-direction)	512
No. of electrons	4,700
No. of ions	4,700

TABLE VII. Basic collision frequencies ν_0

run name	E_x	E'_x	$\frac{\nu_0}{\omega_{pe}}$
run A	0.68	0.0	0.0
run A'	0.68	0.17	0.0
run A''	0.68	0.0	2.16×10^{-4}
run B''	0.68	0.0	4.30×10^{-4}
run C''	0.68	0.0	6.45×10^{-4}
run D''	0.68	0.0	8.59×10^{-4}
run E''	0.68	0.0	1.07×10^{-3}

Fig. 1

Fig. 2 (a) and (b)

Fig. 3 (a)

Fig. 3 (b) - (d)

Fig. 4 (a)

Fig. 4 (b)

Fig. 4 (c)

Fig. 5 (a)

Fig. 5 (b)

Fig. 6 (a)

Fig. 6 (b)

Fig. 7 (a)

Fig. 7 (b)

Fig. 7 (c)

Fig. 8 (a)

Fig. 8 (b)

Fig. 9 (a)

Fig. 9 (b)

Fig. 10

Fig. 11 (a)

Fig. 11 (b)

Fig. 11 (c)

Fig. 12 (a)

Fig. 12 (b)

Fig. 13 (a)

Fig. 13 (b)

Fig. 14 (a)

Fig. 14 (b)

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