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Tunneling Barrier

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Dependence of the Josephson coupling of unconventional superconductors on the properties of the tunneling barrier

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Abstract

The Josephson coupling between a conventional and an unconventional superconductor is investigated as a function of the properties of the tunneling barrier. A simple model is adopted for the tunneling probability and it is shown that its variation dramatically affects the $I_c R_n$ product of an s-d, as opposed to an s-s junction. Based on these conclusions, experiments are proposed to probe the symmetry of the order parameter in high temperature superconductors.

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One of the highly contested subjects in the condensed matter physics is the symmetry of the order parameter in high temperature superconductors, which is closely related to the question of the pairing mechanism. Recent experiments on photoemission in some of these superconductors suggest that the order parameter is very anisotropic [1], possibly with regions in k -space where it is exactly zero, although other interpretations have also been suggested [2]. The measurements of the magnetic penetration depth λ have given power law temperature (T) dependences that are in disagreement with the standard BCS result. In particular, the linear T -dependence of $\lambda(T)$ observed in very clean $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (YBCO) single crystals [3], has been interpreted in terms of an order parameter with line nodes on the Fermi surface. Both the photoemission and the penetration depth measurements have been widely cited as giving support to the contention that the order parameter in high temperature superconductors has d -wave symmetry. However, these two families of experiments can only suggest whether there are low lying excitations in the superconducting state, which is possible both in d -wave and anisotropic s -wave superconductors, and also in gapless, proximity-coupled SN multilayer models, [4], without addressing the question of the *sign* of the order parameter. This problem has recently been dealt with by several experimental groups [5–10], who considered Josephson coupling of high temperature superconductors in different geometries. All these experiments are based on a unique idea [11,12] that the critical current of a junction must decisively depend on the orientation of the crystallographic axes if at least one of the superconductors has d -wave symmetry. This feature brings about a qualitative difference from the case of Josephson coupling between two s -wave superconductors. Nevertheless, the experiments have so far provided contradicting conclusions, some of them suggesting s -wave [6,8] and some d -wave [5,9,10] symmetry (the experiments by the Urbana and ETH groups [5,9] have recently been given an alternative interpretation [13]).

In this Letter we address the problem from a different side. We show how the properties of the tunneling barrier can significantly alter the Josephson coupling if one of the superconductors involved has a gap function (such as that with d -wave symmetry) which averages to zero, thereby providing another possibility to test the symmetry of the order parameter.

From the quantum mechanical point of view a tunneling junction is usually described by the Hamiltonian which transfers electrons from one metal to the other:

$$\mathcal{H}_T = \sum_{\mathbf{p}, \mathbf{q}, \sigma} (T_{\mathbf{p}, \mathbf{q}} c_{\mathbf{p}\sigma}^+ d_{\mathbf{q}\sigma} + T_{\mathbf{p}, \mathbf{q}}^* d_{\mathbf{q}\sigma}^+ c_{\mathbf{p}\sigma}), \quad (1)$$

where $c_{\mathbf{p}\sigma}^+$ ($c_{\mathbf{p}\sigma}$) creates (destroys) an electron with momentum $\mathbf{p}$ and spin σ in metal 1, while $d_{\mathbf{q}\sigma}^+$ ($d_{\mathbf{q}\sigma}$) creates (destroys) an electron with momentum $\mathbf{q}$ and spin σ in metal 2. The quantity $T_{\mathbf{p}, \mathbf{q}}$ is a matrix element connected to the transition probability for an electron from a $\mathbf{p}$ -state on side 1 to a $\mathbf{q}$ -state on side 2. We assume units in which $\hbar = k_B = c = 1$.

The current through the junction is evaluated as $I = -e\dot{N}_i = -ie[\mathcal{H}_T, N_i]$, where N_i is the operator of the number of particles on the i^{th} side. To the lowest order in the perturbation theory, one gets for the current in the normal state:

$$I_n = 4\pi e \sum_{\mathbf{p}, \mathbf{q}} |T_{\mathbf{p}, \mathbf{q}}|^2 [f(\xi_{1\mathbf{p}} - eV) - f(\xi_{2\mathbf{q}})] \delta(\xi_{1\mathbf{p}} - \xi_{2\mathbf{q}}), \quad (2)$$

where $\xi_{1\mathbf{p}}$ and $\xi_{2\mathbf{q}}$ are the energy dispersions in the two metals relative to the Fermi level, and f is the Fermi function [14]. The Josephson critical current is obtained as

$$I_c = 2e \sum_{\mathbf{p}, \mathbf{q}} |T_{\mathbf{p}, \mathbf{q}}|^2 \frac{\Delta_1(\mathbf{p})\Delta_2(\mathbf{q})}{\varepsilon_{1\mathbf{p}}\varepsilon_{2\mathbf{q}}} \frac{\varepsilon_{1\mathbf{p}} \tanh[\varepsilon_{2\mathbf{q}}/(2T)] - \varepsilon_{2\mathbf{q}} \tanh[\varepsilon_{1\mathbf{p}}/(2T)]}{\varepsilon_{1\mathbf{p}}^2 - \varepsilon_{2\mathbf{q}}^2}, \quad (3)$$

where $\Delta_1(\mathbf{p})$ and $\Delta_2(\mathbf{q})$ are the gap functions in the two metals, with $\varepsilon_{1\mathbf{p}} = \sqrt{\xi_{1\mathbf{p}}^2 + |\Delta_1(\mathbf{p})|^2}$ and similarly for the other metal. We allow the two metals to have different Fermi surfaces as well as gap functions.

Hereafter we will restrict ourselves to $T = 0$. Under the assumption that both superconductors are of the BCS type with spherical Fermi surfaces and constant gaps [$\Delta_1(\mathbf{p}) = \Delta_1$, $\Delta_2(\mathbf{q}) = \Delta_2$], Ambegaokar and Baratoff derived from (2) and (3) the following expression for the product of the critical current and the normal state resistance of the junction [15,14]:

$$eI_c R_n = \frac{2\Delta_1\Delta_2}{\Delta_1 + \Delta_2} K \left(\frac{|\Delta_1 - \Delta_2|}{\Delta_1 + \Delta_2} \right), \quad (4)$$

with K being the complete elliptic integral of the first kind. The most important feature of this formula is that the measurable quantity $I_c R_n$ depends on the gap values of the two

metals and is independent of the properties of the barrier. We shall show that this result also holds for Josephson tunneling between s -wave superconductors with arbitrary Fermi surfaces and tunneling directions. However, the situation changes dramatically when the order parameter in at least one of the two superconductors has d -wave symmetry.

In what follows we assume that an isolated metal 1 has a free-electron-like dispersion $\xi_{1\mathbf{p}} = p^2/(2m_1) - E_{F1}$ with the s -wave (isotropic) gap $\Delta_1(\mathbf{p}) \equiv \Delta_1$. An insulated metal 2 has either an uniaxially ellipsoidal Fermi surface with a dispersion $\xi_{2\mathbf{q}} = (q_x^2 + q_y^2)/(2m_2) + q_z^2/(2M_2) - E_{F2}$, or an open Fermi surface appropriate for a layered conductor, $\xi_{2\mathbf{q}} = (q_x^2 + q_y^2)/(2m_2) + J(1 - \cos q_z s) - E_{F2}$, where $J \ll E_{F2}$, and a gap function which depends only on the x and y components of the wave-vector, $\Delta_2(\mathbf{q}) \equiv \Delta_2[q \equiv (q_x^2 + q_y^2)^{1/2}, \varphi]$. Bringing these metals into contact changes their Fermi energies so that $E_{F1} = E_{F2}$.

The properties of the tunneling barrier are buried in the matrix elements $|T_{\mathbf{p},\mathbf{q}}|^2$ giving the transition probability for an electron tunneling from the state $\mathbf{p}$ on one side to the state $\mathbf{q}$ on the other side of the barrier. To get an insight into the importance of the properties of the barrier for the Josephson tunneling, we assume that the transition probability has the following simple form [16]:

$$|T_{\mathbf{p},\mathbf{q}}|^2 = \exp \left[-\frac{p^2 - (\mathbf{p} \cdot \hat{n})^2}{(\mathbf{p} \cdot \hat{n})^2 \theta_0^2} \right] \exp \left[-\frac{q^2 - (\mathbf{q} \cdot \hat{n})^2}{(\mathbf{q} \cdot \hat{n})^2 \theta_0^2} \right]. \quad (5)$$

The unit vector $\hat{n}$ is perpendicular to the plane of the junction. The two exponents restrict the wave-vectors of the tunneling electrons to those that are nearly perpendicular to the plane of the junction. In that respect, the parameter θ_0 roughly corresponds to the tangent of the maximum angle between the quasimomentum of the electrons which are able to tunnel with a nonnegligible probability, and the normal to the junction. For $\theta_0 \ll 1$, only those electrons that are perpendicular to the junction will be able to tunnel, whereas for $\theta_0 \gg 1$ there will be practically no restrictions in that the probability of the electrons to tunnel will be almost the same regardless of the angle of their quasimomenta relative to the plane of the junction.

We will assume for the moment that the gap in the second metal has d -wave symmetry:

$\Delta_2(\mathbf{q}) \equiv \Delta_2 \cos 2\varphi$, where φ is the azimuthal angle in the $q_x - q_y$ plane, and Δ_2 is the maximum value of the gap. Let us consider two different geometries. We assume there is a single domain of this gap function, which is locked onto the crystal lattice.

First, let the plane of the junction be parallel to the a - b (x - y) plane of the second metal: $\hat{n} = \hat{z}$ (this is the geometry probed recently in the experiment on YBCO-Pb junctions [8]). The form of $|T_{\mathbf{p},\mathbf{q}}|^2$, Eq. (5), then implies that the tunneling electrons do not have a preferential quasimomentum orientation in the x - y plane [the exponents in (5) are φ -independent]. It is easy to see from Eq. (3) that this produces zero critical current since there is exact cancellation of the positive and negative regions in $\mathbf{q}$ -space. This conclusion does not depend on the value of θ_0 .

Next, let the plane of the junction contain the z -axis, with β being the angle between the normal to the plane of the junction and the x axis: $n_z = 0$, $\cos \beta = n_x$. Then, using Eqs. (2), (3) and (5), after some algebra, one obtains for the critical current the following expression:

$$eI_c R_n = \frac{2 \int_0^{2\pi} d\varphi \frac{\Delta_1 \Delta_2 \cos 2\varphi}{\Delta_1 + \Delta_2 |\cos 2\varphi|} K \left(\frac{|\Delta_1 - \Delta_2| |\cos 2\varphi|}{\Delta_1 + \Delta_2 |\cos 2\varphi|} \right) g(\varphi - \beta)}{\int_0^{2\pi} d\varphi g(\varphi)}. \quad (6)$$

Here, both Δ_1 and Δ_2 are taken to be positive, with

$$g(\varphi) = \exp \left(-\frac{\tan^2 \varphi}{\theta_0^2} \right) \int_0^1 \frac{d\rho}{\sqrt{\rho}} \exp \left(-\frac{M_2/m_2}{\theta_0^2 \cos^2 \varphi} \frac{\rho}{1-\rho} \right) \quad (7)$$

for the uniaxially ellipsoidal dispersion, and

$$g(\varphi) = |\cos \varphi| \operatorname{erf} \left(\frac{\pi / s q_F}{|\cos \varphi| \theta_0} \right) \exp \left(-\frac{\tan^2 \varphi}{\theta_0^2} \right) \quad (8)$$

for the open Fermi surface (q_F is the Fermi wave-vector). First thing to notice is that the Ambegaokar-Baratoff result, Eq. (4), is recovered if $\cos 2\varphi$ in (6) is formally replaced with unity, which would correspond to both metals having (isotropic) s -wave gaps. That holds true even if all three effective masses are different. The most remarkable feature of (6) is the way in which the angular dependence of the gap in the d -wave metal is coupled to

the angular dependence of the tunneling probability contained in the function $g(\varphi)$. This coupling comes from the fact that the geometry of the junction now breaks the x - y symmetry in the d -wave superconductor, producing nonvanishing critical current. To discern the role of the barrier parameter θ_0 , the product $I_c R_n$ is shown in Fig. 1 as a function of the angle β for several values of θ_0 for the case of the open Fermi surface (the conclusions for the uniaxially ellipsoidal dispersion are the same). All the curves are qualitatively similar – they are antisymmetric with respect to $\beta = 45^\circ$ – the angle when the node direction of the d -wave gap is perpendicular to the junction. However, there are substantial quantitative differences for different θ_0 . The $I_c R_n$ product is the largest in the limit $\theta_0 \rightarrow 0$. In that case the function $g(\varphi)$, Eqs. (7) and (8), behaves like a delta function so that the product $I_c R_n$ becomes:

$$eI_c R_n = \frac{2\Delta_1 \Delta_2 \cos 2\beta}{\Delta_1 + \Delta_2 |\cos 2\beta|} K \left(\frac{|\Delta_1 - \Delta_2| |\cos 2\beta|}{\Delta_1 + \Delta_2 |\cos 2\beta|} \right). \quad (9)$$

The physical meaning of this formula is the following: when θ_0 is small, only the Cooper pairs with quasimomenta almost perpendicular to the junction take part in tunneling, and the gap value for that particular direction is the only one relevant; that is why the formula looks very much like the Ambegaokar–Baratoff result, Eq. (4). Note that it differs somewhat from the widely adopted formula $I_c \propto \cos 2\beta$ [12]. The $I_c R_n$ product drops significantly when θ_0 increases (see also Fig. 2). The reason for this is that Cooper pairs with a range of the quasimomentum angles contribute now to the Josephson current, so that there is a substantial cancellation of the positive and negative regions of $\Delta(\mathbf{q})$ stemming from its d -wave nature. In the limit $\theta_0 \rightarrow \infty$, the function $g(\varphi)$ tends to a constant, causing the Josephson current to vanish, since in that case the junction does not act anymore as an x - y symmetry-breaker for the d -wave superconductor.

As an interesting comparison, we give in Fig. 3 the $I_c R_n$ product for a Josephson junction characterized by the same tunneling probability (5), but with the d -wave superconductor being replaced with an extremely anisotropic s -wave superconductor having $\Delta_2(\mathbf{q}) \equiv \Delta_2 |\cos 2\varphi|$. For small θ_0 the critical current has some angular dependence due to

the selectivity of the barrier with respect to the electron quasimomenta. With increasing θ_0 the critical current gets “isotropized” without losing its magnitude, since the gap function now has the same sign everywhere in $\mathbf{q}$ -space and there is no cancellation.

How can we justify the use of the tunneling probability as given in Eq. (5)? The simplest thing to assume that the longer the “path” of the electrons through the barrier, the smaller the tunneling probability. Therefore, the electrons tunneling in the perpendicular direction should have the highest probability, which is what (5) implies. Another interpretation would involve possible scattering inside the barrier which tends to “isotropize” the tunneling probability, whereby effectively increasing the parameter θ_0 . In that respect, the effect of junction scattering on the s - d Josephson current would be very much like that of *bulk* scattering upon the transition temperature of a d -wave superconductor.

How can we use the main result of this paper (Fig. 1 as opposed to Fig. 3) to probe the symmetry of the order parameter in high temperature superconductors? Although the theoretical approach used here assumes the contact to be of SIS’ type, the physical idea applies equally well to SNS’ type junctions. What we propose to be done is to progressively introduce (nonmagnetic) impurities into the normal layer of an SNS’ contact where one of the superconductors is conventional (believed to be s -wave), while the other one is an high T_c superconductor. The depression of the critical current due to impurities should be much stronger if the high T_c metal has d -wave symmetry, as opposed to both metals having s -wave order parameters. What exactly “much stronger” means, is best determined experimentally: together with the proposed series of SNS’ Josephson junctions, one ought to make a series of SNS Josephson junctions with both superconductors being conventional, and with varying concentrations of impurities in the normal junction layers; the same metal and the same impurities should be used for the normal junction layers in both series.

A less reliable possibility is to use point contacts. If a tip made of a conventional superconductor is put in contact with a flat surface of a high temperature superconductor, the parameter θ_0 in the tunneling probability (5) could be made larger or smaller depending on the distance as well as the opening angle of the tip: the larger the opening angle, the

larger θ_0 , assuming that the tip touches the surface. However, this method is difficult, at best, to control.

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$I_c R_n.$

FIGURES

FIG. 1. The reduced $I_c R_n$ product of the s-d junction as a function of the angle β between the x axis of the d-wave superconductor and the normal to the junction, for $\Delta_1/\Delta_2 = 0.1$, $\pi/sq_F = 1$, and for three different values of the barrier parameter θ_0 .

FIG. 2. The $I_c R_n$ of the s-d junction for $\beta = 0$, scaled with $I_c R_n$ for an s-s junction (Ambegaokar-Baratoff, both s-wave superconductors having isotropic BCS gaps) for the same choice of the transition temperatures of the two metals. The limiting value for $\theta_0 \rightarrow 0$ is greater than 1 because $\Delta_2 \approx 2.14T_{c2}$ for a d-wave metal, while $\Delta_2 \approx 1.76T_{c2}$ for an s-wave metal.

FIG. 3. The reduced $I_c R_n$ product of an s-s junction as a function of the angle β between the x axis of the anisotropic s-wave superconductor (see text) and the normal to the junction, for $\Delta_1/\Delta_2 = 0.1$, $\pi/sq_F = 1$, and for three different values of the barrier parameter θ_0 .