ELECTROMAGNETIC HYSTERETIC RESPONSE CALCULATION FOR SUPERCONDUCTORS IN DEMAGNETIZING GEOMETRIES

L. S. Koo and K. L. Telschow
Idaho National Engineering Laboratory
P.O. Box 1625
Idaho Falls, ID 83415-2209

INTRODUCTION

The electromagnetic response of the new high T_c superconductors is similar to that of eddy currents in normal metals, except that in the superconductor induced currents are established nonlinearly at a single value known as the critical current density, J_c . These materials are extreme Type II superconductors where, in the presence of an external magnetic field and/or a transport current, magnetic flux exists in the material in the form of flux lines distributed on a lattice [1]. Individual flux lines become pinned at microstructural inhomogeneities such that only under a sufficient force caused by locally high current flows will they become depinned and flow throughout the material. The value of the local current density that causes depinning is the microscopic critical current density and is directly proportional to the pinning force strength. A phenomenological approach known as the critical state model [2,3] describes the pinned flux line distribution within the material quasi-statically, assuming the equilibrium distribution is achieved at each value of the externally applied field on a short time scale compared to experimental times. Operationally, whenever an external field is increased, flux lines enter the material from the surface and penetrate to a flux front boundary, whose position is determined by the value of the external field at the sample surface. An important nondestructive evaluation (NDE) task to aid the fabrication of high T_c superconductors is to develop methods for quantitatively determining the local current density. In the critical state the current density is either the critical value appropriate to the local value of the induction J_c , or it is zero. The electromagnetic response of the material is then determined by the extent of this critical state region and its measurement can be used to determine the local J_c . Therefore, a method that can predict the flux front profile with high spatial resolution, and also account for demagnetization effects, is essential. An integral equation technique dealing with a nonuniform applied magnetic field having azimuthal symmetry was presented at the last QNDE conference by the present authors [4]. The current paper shows results from the further development of this technique in two ways. Firstly, the superconducting sample is extended from a half-space to an infinite plate. This is an example of a nonuniform applied magnetic field having azimuthal symmetry. The second application is a sphere, that is a demagnetizing geometry, in a uniform applied magnetic field. In the following section, the general methodology of this technique is outlined. Then some results of both the plate and the sphere examples are given to illustrate this proposed approach. Since the study of the plate sample is still in progress, more results will be reported in future publications. For the sphere sample, detailed discussion and presentation of formulations are given in [5].

GENERAL METHODOLOGY

Whenever a superconducting material is placed under an applied magnetic field, a region of shielding current is induced. According to the Bean critical state model, the

MASTER

induced current is created starting at the material boundary next to the applied magnetic field and extending inward satisfying

$$\nabla \times \vec{B} = \mu_0 \vec{J}_c \quad (1)$$

where $\vec{B}$ is the magnetic induction vector and μ_0 is the free-space permeability. This leads to a decrease of the net magnetic flux density in the material from the surface to zero at the flux front boundary. The region inside this boundary is free of magnetic field, shielded completely by the induced screening currents. This flux front surface inside a superconducting material can be defined as the surface on which the total magnetic field is zero, which results in a vector equation defining the boundary [6]. However, as is proven in [5], it can also be defined as a surface of zero vector potential, $\vec{A}$, where $\nabla \times \vec{A} = \vec{B}$. The use of the vector potential simplifies the calculations since only one component is needed for problems of azimuthal symmetry. For problems with this symmetry, the induced currents inside the material can be modeled as forming loops, each carrying a current of constant value, J_c . The vector potential of a single current is well known and can be expressed in terms of either complete elliptical integrals [7] or orthogonal functions [8]. Hence, the vector potential due to all of the induced currents is a volumetric integral sum of this known vector potential function for the single loops. The unknown quantities in this integral are the flux front boundary, Ψ , which forms part of the integration limit, and the current density, J_c . In addition the flux front boundary, Ψ , is a function of both space and the applied magnetic field. To simplify the calculations, the external applied field is normalized by J_c and a characteristic length. With this normalizing scheme, the total vector potential, A_{tot} , becomes a dimensionless implicit function of $\Psi = \Psi(R, \beta)$, a general spatial variable, R , and the normalized applied field, β . Then the total vector potential is given by

$$A_{tot} = A_\beta - A_{J_c} \quad (2)$$

where A_β and A_{J_c} are the vector potentials due to the external field, β , and the induced critical currents, J_c . The minus sign in Eq. (2) results from the shielding effect. For a given β , the position of the flux front surface is determined by finding spatial points where the total vector potential in Eq. (2) is zero. This is a difficult problem in general, but can be simplified by reducing Eq. (2) to a single integral equation as follows. In general β is a function of time. This technique deals only with the quasi-stationary states of the critical state. The time scale for changes in the external field is typically very much longer than that exhibited by flux line motion, so the model always assumes a sequence of stationary states uniquely defined by the history and present value of the external field. Changes in the external field then produce a corresponding change in the flux line profile position, but at all times the total vector potential on this profile is zero. Therefore, a requirement determining the flux penetration profile is that

$$A_{tot}(\Psi(R, \beta + \Delta\beta), R) = A_{tot}(\Psi(R, \beta), R) = 0 \quad (3)$$

which implies

$$\frac{\partial A_{tot}(\Psi(R, \beta), R)}{\partial \beta} = f\left(\Psi(R, \beta), \frac{\partial \Psi(R, \beta)}{\partial \beta}\right) = 0. \quad (4)$$

Eq. (4) is often a nonlinear first kind integral equation of Ψ . However, if Ψ is given but its derivative term remains as unknown, then Eq. (4) can be viewed as a linear first kind integral equation of the derivative term. This feature of Eq. (4) will be explored to form a solution procedure for the flux front boundary. As suggested by the critical state model, when the external field, β , is initially turned on, flux enters the superconducting material from the surface. Thus the initial profile of $\Psi(R, \beta) = 0$ is the material surface profile. With substitution of this known function into Eq. (4), it becomes a linear integral equation of the first kind for the unknown derivative of Ψ . This kind of linear integral equation is well understood and there are several algorithms available for resolving it. Most of these

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

algorithms use the iteration approach and convergence of the iteration is either guaranteed by rigorous mathematical arguments or shown to be so in applications where no mathematical proof is available. A simple convergent iteration scheme proposed and analyzed by Gold [9] is adopted to resolve the derivative term of Ψ from Eq. (4). As the external field is increased to $\beta = \Delta\beta$, the new flux front profile is approximated by

$$\Psi(R, \beta = \Delta\beta) = \Psi(R, \beta = 0) + \frac{\partial \Psi(R, \beta = 0)}{\partial \beta} \cdot \Delta\beta. \quad (5)$$

This approximation is acceptable as long as the increment $\Delta\beta$ is sufficiently small. With this approximation the flux front profile at $\beta = \Delta\beta$ is known but its derivative is yet to be determined. This is the same situation as at the beginning when $\beta = 0$. The above procedure is repeated and the new β value determined. Essentially, this methodology involves a progressively incremental numerical scheme in β and an iterative procedure for resolving Eq. (4) for each β . Once the flux front profile's dependence on the external field for the zero-field cooled (ZFC) case is known, then the response of the sample to a complete cycle of changes in external field can be readily calculated. An example of this calculation for a spherical sample is given in [5]. Two different configurations exhibiting azimuthal symmetry applicable to this calculational scheme are described below: an infinite plate in the field of an external single loop coil and a sphere in a uniform external field.

INFINITE PLATE

A single circular current coil above a superconducting infinite plate is depicted in Fig. 1. All geometrical lengths in this configuration are normalized by the coil radius, r_c . Normalized cylindrical coordinates (R, Z) are used with $Z = 0$ coinciding with the plate's top surface; its bottom edge is at $Z = D$. The driving coil is placed at (R_c, Z_c) , where $R_c = 1$ by normalization. The normalized external field is

$$\beta = \frac{I}{J_c r_c^2} \quad (6)$$

where I is current in the driving coil. Also in Fig. 1, flux front profiles for the ZFC case are shown. At $\beta = 0$, $\Psi = 0$ and coincides with the plate top surface. As β is increased to the next value, Ψ shows a profile that penetrates deepest directly under the driving coil. This minimum value of Ψ reaches the plate bottom edge at $\beta = 0.365$. Beyond this value, the flux front is broken up into two sections: an inner section, Ψ_1 , which approaches the center line $R = 0$, and an outer section, Ψ_2 , which approaches $R = \infty$ for increasing β . The broken flux front surfaces shown are for $\beta = 0.865$.

Two simulated measurement examples are given in Figs. 2a and 2b. In these examples, $r_c = 0.5$ mm, $Z_c = 0.4$, $D = -0.2$, and $\beta_{\max} = 9.065$. In Fig. 2a, the normalized potential vector amplitude, A_{J_c} , at the driving coil position (1, 0.4) is given, simulating the measurement results found above the sample with a balanced coil [10]. In Fig. 2b, the normalized total potential vector amplitude A_{tot} is given at (1, -0.6), simulating the measurement results found beneath the sample with a single unbalanced coil [11].

SPHERE

The configuration for a spherical sample in a uniform external field is shown in Fig. 3. In this example, all geometric lengths are normalized by the sphere radius, r_c , and the normalized external field β is

$$\beta = \frac{H_0}{J_c r_c} \quad (7)$$

Fig. 1 Measurement geometry for determining flux penetration in a superconducting infinitely wide plate. Flux front profiles for the ZFC case with $\beta = 0.1, 0.365, 0.865$ are shown (from top to bottom).

Fig. 2 Calculated vector potential, A_{J_c} , for a balanced coil above the superconducting plate ($R = 1, Z = 0.4$) (a) and calculated vector potential, A_{tot} , for an unbalanced coil beneath the plate ($R = 1, Z = -0.6$) (b). The vertical dotted line marks $\beta = 0.3648$ at which the flux front reaches the bottom boundary and $D = -0.2, r_c = 0.5$ mm.

Figure 3. Two penetrating flux surface profiles corresponding to $\Psi_1 = \Psi_1(Z, \beta_{\max})$ and $\Psi_2 = \Psi_2(Z, \beta)$.

where H_0 is the applied magnetic field amplitude. Inside the sphere, two flux front surfaces are also given. Their presence illustrates the response of the sample due to a complete cycle of changes in external field. The first flux front surface, Ψ_1 , penetrates to the current position as the external field is increased in the ZFC state to some value $\beta_{\max} < \beta^*$, where β^* is the value of β that fills the sphere with current loops. When the external field is subsequently reduced to β from $\beta_{\max}$, a second flux front boundary, Ψ_2 , enters the sphere from the outer surface with currents flowing at $+J_c \hat{e}_\phi$, where $\hat{e}_\phi$ is the unit vector for the azimuthal direction. During this time, the functional dependence of these two flux fronts are $\Psi_1(Z, \beta_{\max})$ and $\Psi_2(Z, \beta)$. The first front is filled with negative currents (defined as flowing in the direction that produces a magnetic moment that opposes the external field) and remains fixed as the second front fills with positive currents as depicted in Fig. 3. The profiles of these flux fronts agree with results given in [6]. They all possess a sharp intersection at both ends with the Z-axis. For the calculation of the second flux front, Ψ_2 , it can be shown [5] that Ψ_2 retraces all the locations of Ψ_1 but at a slower pace with respect to β . This accounts for the hysteresis observed in the critical state model. The hysteresis curves of four independent cycles corresponding to different $\beta_{\max}$ values are shown in Fig. 4.

CONCLUSIONS

A method has been outlined for calculating the flux front profile for a superconducting sample in either a uniform or nonuniform applied magnetic field possessing azimuthal symmetry. This technique relies upon finding a surface with zero vector potential. This surface is determined by simple integration of its derivative with respect to the external field, found by resolving a linear integral equation of the first kind. Measurement induced voltages and the entire hysteresis loop response can be found by extension of the ZFC magnetization response with increasing external field. Other experimentally measured quantities relating to the critical state can be calculated directly from the hysteresis loop if the time dependence of the external field is known.

The technique shown in this report for solving the critical state model in the Bean approximation can be extended to field-dependent critical currents and other azimuthally symmetrical external fields [10]. This work is presently in progress.

ACKNOWLEDGMENTS

This work was sponsored by the U.S. Department of Energy, Office of Energy Research, Office of Basic Energy Sciences, under DOE Idaho Operations Office Contract DE-AC07-76ID01570.

Figure 4. Normalized magnetization (M/M^*) versus $\beta = H_0/J_c r_c$ of the sphere where $M^* = M(\beta^*)$. These hysteretic curves (starting from the smallest) correspond to $\beta_{\max} = 0.15, 0.36, 0.57$ and 0.7893 (full penetration).

REFERENCES

1. A. M. Campbell and J. E. Evetts, *Critical Currents in Superconductors* (Taylor & Francis Ltd., London, 1972).
2. C. P. Bean, Phys. Rev. Lett. 8, 250 (1962).
3. C. P. Bean, Rev. Mod. Phys. 36, 31 (1964).
4. L. S. Koo and K. L. Telschow, in *Review of Progress in QNDE*, Vol. 13, eds. D. O. Thompson and D. E. Chimenti (Plenum, New York, 1994), p. 1677.
5. K. L. Telschow and L. S. Koo, accepted for publication in Phys. Rev. B.
6. R. Navarro and L. J. Campbell, Phys. Rev. B 44, 10146 (1991).
7. W. R. Smythe, *Static and Dynamic Electricity*, 2/ed (McGraw-Hill, New York, 1950), pp. 270-1.
8. C. V. Dodd and W. E. Deeds, J. Appl. Phys. 39, 2829 (1968).
9. R. Gold, ANL-6984, Argonne National Laboratory, December 1964.
10. K. L. Telschow and T. K. O'Brien, Appl. Phys. Lett. 59, 730 (1991).
11. K. L. Telschow and K. Haulenbeek, private communication.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.