BROOKHAVEN NATIONAL LABORATORY

December 1994

BNL-61139

Update on B_K with Wilson Fermions*

Claude Bernard

Department of Physics
Washington University
St. Louis, MO 63130, USA

and

Amarjit Soni

Physics Department
Brookhaven National Laboratory
Upton, NY 11973, USA

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* Talk presented by A. Soni at the XII International Symposium on Lattice Field Theory, Bielefeld, Germany, Sept. 27-Oct. 1, 1994.

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Update on B_K with Wilson Fermions

Claude Bernard ^a and Amarjit Soni ^{a, b}

^aDepartment of Physics, Washington University,
St. Louis, MO 63130, U.S.A.

^bDepartment of Physics, Brookhaven National Laboratory,
Upton, NY 11973, U.S.A.

The method proposed earlier for Wilson fermions is used with data accumulated at $\beta = 5.7, 6.0, 6.3$. We find $B_K^{a=0}(2 \text{ GeV}) = .70 \pm .10$ in the quenched approximation, where the error includes statistical and systematic errors.

The $K-\bar{K}$ mixing matrix element:

$$M_{LL} \equiv \langle \bar{K}^0 | (\bar{s}d)_L (\bar{s}d)_L | K^0 \rangle \quad (1)$$

where $(\bar{s}d)_L \equiv \bar{s}\gamma_u(1 - \gamma_5)d$ is an hadronic matrix element that is of crucial importance to electroweak phenomenology [1,2]. For convenience, we introduce the so called "bag parameter", B_K :

$$B_K = M_{LL} / \left(\frac{8}{3} f_K^2 m_K^2 \right) \quad (2)$$

The denominator in eqn. (2) represents the value of M_{LL} in the so called "vacuum saturation approximation (VSA)". It has been known for a long time that B_K is amenable to lattice Monte-Carlo methods. Indeed, recently Sharpe and collaborators, using staggered formulation for fermions have made a remarkable progress in calculating B_K and have reported a result with an impressive accuracy [3]:

$$B_K(NDR, 2 \text{ GeV}) = .616 \pm .020 \pm .017 \quad (3)$$

In this work we will present our results for B_K using the Wilson formulation for fermions.

Calculation of B_K with Wilson fermions has been very difficult due to the fact that Wilson fermions break chiral symmetry explicitly on the lattice through the so called "r-term". To overcome this difficulty we have proposed a method that is described in Lat'89 [4]. We will use that

method with the data set accumulated over the past many years.

We have analyzed the data at three values of the gauge couplings, namely $\beta = 5.7, 6.0$, and 6.3 . The lattice spacings have been previously determined by calculating f_π , the pion decay constant, and normalizing it to the experimental value of 130 MeV. Thus for $\beta = 5.7, 6.0$ and 6.1 we use $a^{-1} = 1.17 \text{ GeV}, 2.2 \text{ GeV}$ and 3.2 GeV respectively [5]. At $\beta = 5.7$ we have two lattices of sizes $16^3 \times 25$ and $16^3 \times 33$, each with sixteen gauge configurations. At $\beta = 6.0$, we used two lattices (namely $16^3 \times 39$ and $24^3 \times 39$). Comparison of the results emerging from these two lattice gives a good indication of the finite size effects. The 16^3 lattice consists of two sequences of gauge configurations (8+11) totaling 19. The 24^3 ($\beta = 6.0$) lattice has four sequences (8+8+9+15) with a total of 40. At $\beta = 6.3$ the lattice is of size $24^3 \times 61$ with four sequences (5+5+5+5) totaling 20 in all.

Results obtained for the lattices are summarized in Table 1. We have used degenerate as well as nondegenerate $q\bar{q}$ pairs. At $\beta = 5.7, 6.0$, and 6.3 we have used 7, 6, and 4 such mesons with meson mass $\lesssim 1 \text{ GeV}$. Errors indicated are obtained by adding in quadrature statistical and systematics of the chiral restoration procedure.

Fig. 1 shows the results from the three gauge couplings renormalized to the common scale $\mu = 2 \text{ GeV}$ which is the case to a good approximation for $\beta = 6.0$. The points shown for $\beta = 5.7$ and 6.3

*Presenting author

Table 1

β	a_f^{-1} (GeV)	Size	Configurations	# of m_M	$B_K \pm \sqrt{st^2 + sy^2}$
5.7	1.2	$16^3 \times 25$	16	7	$.93 \pm .09$
		$16^3 \times 33$	16		
		$16^3 \times 39$	19		
6.0	2.2	$24^3 \times 39$	40	6	$.78 \pm .06$
6.3	3.2	$24^3 \times 61$	20	4	$.73 \pm .05$

on this figure thus include a correction factor of $g^{-4/9}$ since $g = 1$ for $\beta = 6.0$. Since we anticipate $O(a)$ corrections to the standard Wilson action, we have fitted the three results linearly. We thus find

$$B_K^{a=0}(2 \text{ GeV}) = .70 \pm .09 \quad (4)$$

Figure 1. $B_K(2 \text{ GeV})$ vs a with Wilson Fermions. Data at $\beta = 5.7, 6.0$ and 6.3 is fitted to a straight line to get $B_K^{a=0}(2 \text{ GeV})$.

where the error includes statistical errors and systematic errors due to our procedure. The remaining systematic errors are all very small. 1) The scale uncertainty is expected to lead to an error of $< .01$. 2) In our data we find very little difference whether we use degenerate or nondegenerate $q\bar{q}$; we estimate the related systematic error $< .01$. Note however that the difference in the quenched approximation between the results for degenerate and nondegenerate $q\bar{q}$ is not necessarily a good

indication of the corresponding difference in the full theory. This is because the chiral logs for this difference in the quenched approximation do not reproduce those of the full theory. The full theory chiral logs indicate a correction of a few percent, perhaps as much as 5% [3,6]. 3) Comparison of data ($\beta = 6.0$) of $24^3 \times 39$ and of $16^3 \times 39$ lattices indicates finite volume effects $< .03$. 4) The perturbative uncertainty can be estimated by replacing $1/a$ by π/a for the scale of the coupling constant used in computing the one-loop tadpole-improved [7] perturbative corrections. This gives an error of $< .04$. Adding all the errors in quadrature we then get:

$$B_K^{6.0}(2 \text{ GeV}) = .70 \pm .10$$

This result is consistent with the staggered result given above. For the sake of completeness we note also that the value of the corresponding renormalization group invariant parameter is:

$$\hat{B}_K = .91 \pm .13$$

We thank Steve Sharpe for discussions. This work was supported in part by the DOE under grant numbers DE2FG02-91ER40628 and DE-AC0276CH00016. The computing for this project was done at NERSC and at SDSC.

REFERENCES

1. For a recent review of the status of the weak matrix element effort on the lattice see G. Martinelli, these proceedings.
2. C. Bernard and A. Soni in "Quantum Fields on the Computer", Ed. M. Creutz, p.362
3. S. Sharpe, Nucl. Phys. B (Proc. Suppl.) **34**, 403 (1994).

4. C. Bernard and A. Soni, Nucl. Phys. B (Proc. Suppl.) **17**, 495 (1990) .
5. C. Bernard, J. Labrenz, and A. Soni, Phys. Rev. **D49**, 2536 (1994).
6. C. Bernard, Nucl. Phys. B (Proc. Suppl.) **34**, 47 (1994).
7. G.P. Lepage and P.B. Mackenzie, Phys. Rev. **D48** (1993) 2250.