The Gluon Propagator in non-Abelian Weizsäcker-Williams Fields

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Abstract

We carefully compute the gluon propagator in the background of a non-Abelian Weizsäcker-Williams field. This background field is generated by the valence quarks in very large nuclei. We find contact terms in the small fluctuation equations of motion which induce corrections to a previously incorrect result for the gluon propagator in such a background field. The well known problem of the Hermiticity of certain operators in Light Cone gauge is resolved for the Weizsäcker-Williams background field. This is achieved by working in a gauge where singular terms in the equations of motion are absent and then gauge transforming the small fluctuation fields to Light Cone gauge.

1 Introduction

In a recent work, two of the authors have argued that parton distribution functions may be computed for very large nuclei at small x [1]. They found that the valence quarks in these nuclei are the sources of non-Abelian Weizsäcker-Williams fields. This work is similar in spirit to that of A. Mueller who considers the distribution functions generated by heavy quark charmonium-like bound states [2]. In reference [1] the general form of these fields was found. It was argued that to compute the distribution functions, one should compute the fields associated with these valence quarks and then color average over these fields treating the valence quarks as classical charges. The weight for this averaging was shown to be

$$\exp\left(-\frac{1}{2\mu^2} \int d^2x_t \rho^2(x_t)\right) \quad (1)$$

where $\rho(x_t)$ is the distribution of transverse valence quark color charge and μ^2 is a parameter corresponding to the valence quark color charge per unit area.

The current for the valence quarks was found to be

$$J_a^\mu(x) = \delta^{\mu+} \delta(x^-) \rho_a(x_t, x^+). \quad (2)$$

Here the Light Cone variables are

$$y^\pm = (y^0 \pm y^3)/\sqrt{2}. \quad (3)$$

The approximation which leads to this form of the current is that $x \ll A^{-1/3}$ so that the small x partons see a nucleus which is essentially Lorentz contracted to a thin sheet. This is the origin of the factor $\delta(x^-)$ above. Further, since the small x partons are weakly coupled to the source, the source is "static" and only the $+$ component of the current is important.

The charge distribution in transverse coordinates is essentially random with a Gaussian weight as described above. This is true so long as we look at transverse

One can verify that the Light Cone Hamiltonian satisfies

$$P^- = 0 \tag{9}$$

so that these classical fields have vanishing Light Cone energy.

These fields are gauge transforms of vacuum fields with a discontinuity which is adjusted so as to reproduce the correct valence charge distribution. Because it is a pure gauge transform of the vacuum on either side of the infinitesimally thin sheet, we expect that the propagators are simple. In a previous paper, two of the authors argued that the fields which generate the propagator are just gauge transforms of the ordinary small fluctuation plane wave fields [3]. This argument is correct for matter fields such as scalars and fermions, but as we shall see is not quite correct for gluon fields.

The problem with the gluon fields is that there can be contact terms in the gluon small fluctuation equations of motion associated with the thin sheet of valence charge. These contact terms can generate contributions to the gluon field which are constant or grow linearly at large distances away from the sheet. The linearly growing terms will give rise to a constant electric field. In any case, neither of these terms were allowed for in the previous analysis which claimed to have determined the gluon propagator.

The terms which were ignored in the previous analysis are very singular. A direct consequence of their bad behavior at infinity is the non-self-adjoint character of the Light Cone gauge equations of motion.

The present work is organized as follows: in Section 2 we will see that the above mentioned terms are a consequence of the small fluctuation equations of motion in Light Cone gauge ($A^+ = 0$). To resolve the ambiguities inherently associated with the singular behavior of the small fluctuation fields, we choose to work in $A^- = 0$ gauge. In Section 3, we show that this gauge is non-singular and that the propagator can be computed directly. It turns out that the propagator in this

quark charge. If this gauge condition were not chosen, then the extended current conservation condition would require that the source evolve in time, and this would generate a correction to the above equation. This background field generates no magnetic field, but it does generate a non-zero electric field, F^{+i}

$$F^{+i} = \delta(x^-)\alpha^i(x_i). \quad (13)$$

On either side of $x^- = 0$, the background field is a gauge transform of the vacuum. We might therefore think that the solutions to the above equation, in the two different regions of x^- , are simply different gauge transforms of the plane wave small fluctuation field one would get in zero background field. Further, continuity at the origin could be assured by a matching condition. This is precisely what was done in the previous work of McLerran and Venugopalan [3].

Unfortunately, this procedure will not work here due to the presence of the background electric field term. Before seeing how this works, let us first express the small fluctuation equations in a slightly more transparent form (hereafter latin indices refer to transverse variables).

$$D^2(A)\delta A^\mu - D(A)^\mu D(A) \cdot \delta A = -2\delta^{\mu+}\delta(x^-)\alpha^i \times \delta A_i - 2\delta^{\mu j}\delta(x^-)\alpha^j \times \delta A^-.$$

The terms in the right-hand side of the above equation are called contact terms, they appear when the derivative $\partial/\partial x^-$ acts on the classical field which is proportional to $\theta(x^-)$. Notice that the last term vanishes in the gauge where $\delta A^-|_{x^-=0} = 0$.

The small fluctuation equation for the plus component of the above equations becomes

$$\partial^+(\partial^+\delta A^- - D(A)^i\delta A_i) = -2\delta(x^-)\alpha^i \times \delta A_i. \quad (14)$$

This equation requires that

$$\partial^+\delta A^- - D(A)^i\delta A_i = -2\theta(x^-)\alpha^i \times \delta A_i|_{x^-=0} + h(x^+, x_i) \quad (15)$$

shown that in this gauge J^+ is the only large component of the fermionic current. The transverse and minus components of the current are proportional to $\frac{1}{p^+}$ and are therefore small [1]. It is much easier to eliminate the dependent fermionic degrees of freedom in this gauge since we don't need to worry about inverting covariant derivatives. Finally, the gluon distribution function is most simply related to physical quantities measured in deep inelastic scattering experiments only in $A^+ = 0$ [8].

We will demonstrate in the following sections that the Green's functions in Light Cone gauge can be obtained by the following procedure: i) first, solve the small fluctuation equations in an alternative gauge ($A^- = 0$) and then ii) gauge transform the Green's functions thus obtained to Light Cone gauge. In the following section, we will show that the Green's function may be constructed in $A^- = 0$ gauge and that the equations of motion for the Green's function are self-adjoint. In subsequent sections we discuss the mechanics of how the Green's function may then be gauge transformed to Light Cone gauge. When this transformation is carried through, there is in general a singularity of the transformation function. This singularity is regulated using the well known Leibbrandt-Mandelstam prescription [4]. We find that the Green's function in Light Cone gauge does not satisfy the constraints which arise from self-adjointness of the gluon field. This lack of self-adjointness is not irretrievably damaging and has been dealt with in the past for Green's functions of gluon fields in Light Cone gauge [9]. Presumably the precise nature of the singularity is not important for gauge invariant quantities. When one chooses a prescription for regulating the singularity, if performed consistently in all quantities, it will give gauge invariant results for physical quantities. We are not aware that there is a proof of this, but believe it to be so.

3 The small fluctuations propagator in $A^- = 0$ gauge

What is the virtue of writing down the equations of motion in $A^- = 0$ gauge? In the small fluctuation equations for the transverse fields, the source term vanishes (the equation reduces to the sourceless Klein-Gordon field equation)

$$[-2\partial^+\partial^- + D_i^2(A)]\delta A_i = 0. \quad (16)$$

This equation is obtained by appropriately fixing the residual gauge freedom in $A^- = 0$ gauge. One of the other equations is also sourceless, and is the Gauss law constraint which must be imposed on initial field configurations. This constraint commutes with the equations of motion. It is

$$\partial^- (\partial^- \delta A^+ - D_i(A)\delta A_i) = 0. \quad (17)$$

We must now check that the remaining equation of motion is properly satisfied. If we use the Gauss law constraint that $D^\mu(A)\delta A_\mu = 0$, this equation is

$$[-2\partial^+\partial^- + \vec{D}^2(A)]\delta A^+ = -2\delta(x^-)\alpha^i \times \delta A^i. \quad (18)$$

Now from the Gauss law constraint,

$$\delta A^+ = \frac{1}{\partial^-} \vec{D}(A) \cdot \delta \vec{A}. \quad (19)$$

Although δA^i is continuous at $x^- = 0$, the spatial covariant derivative operator is not. The discontinuity gives precisely the source term on the right hand side of the equation, and the equations are consistent.

We can now solve the above equations of motion in the following way. We assume that the transverse field on the side where the background field vanishes is a simple plane wave. On the other side of the boundary where the background field is a gauge transform of the vacuum, we take the small fluctuation field to be a linear superposition of plane waves. The linear superposition is done in a manner

For the pieces of the propagator which involve the component δA^+ we have

$$\langle \delta A^i \delta A^+ \rangle = \langle \delta A^i \frac{1}{\partial^-} \bar{D}(A) \cdot \delta \bar{A} \rangle \quad (22)$$

and

$$\langle \delta A^+ \delta A^+ \rangle = \langle \frac{1}{\partial^-} \bar{D}(A) \cdot \delta \bar{A} \frac{1}{\partial^-} \bar{D}(A) \cdot \delta \bar{A} \rangle - i \delta G^{++} \quad (23)$$

where in our convention $\langle A_\mu A_\nu \rangle = -i G_{\mu\nu}$ and $\langle A_\mu A_\nu \rangle$ represents the correlation of fields in the path integral sense.

4 On deriving the $A^+ = 0$ propagator from the $A^- = 0$ propagator

In the previous section we derived an explicit expression for the small fluctuation Green's function in $A^- = 0$ gauge. In this section, we discuss general arguments which allow us to compute expectation values of operators in one gauge in terms of the expectation values of operators in another gauge. The formal arguments will be illustrated by the specific example of the free gluon propagator. We are of course interested in these arguments because we want to find the gauge transform which will relate the Green's function in Light Cone gauge to the Green's function in $A^- = 0$ gauge.

Let $O(A)$ be some function of the field operators evaluated in Light Cone gauge, $A^+ = 0$. We will here consider QED and later generalize to the case at hand. We have to evaluate the expectation value

$$\langle O(A) \rangle = \frac{\int [dA] \delta(A^+) e^{iS} O(A)}{\int [dA] \delta(A^+) e^{iS}}. \quad (24)$$

We now introduce the identity in the form

$$1 = \frac{\int [d\Lambda] \delta(A^- + \partial^- \Lambda)}{\int [d\Lambda] \delta(A^- + \partial^- \Lambda)}. \quad (25)$$

to obtain a purely quadratic action in $\delta\bar{A}^+$ and $\delta\bar{A}_i$. The minimum thus corresponds to the fields that make the action vanish. We label the fields as follows: type I fields satisfy

$$\begin{aligned}\partial^- \delta\bar{A}^+ - \nabla \cdot \delta\bar{A} &= 0 \\ \square \delta\bar{A}^i &= 0\end{aligned}\quad (32)$$

whereas type II fields satisfy

$$\begin{aligned}\delta\bar{A}^i &= 0 \\ \partial^- \delta\bar{A}^+ &= 0.\end{aligned}\quad (33)$$

The contributions of different components to the free propagator are then

$$\begin{aligned}\langle \delta\bar{A}^i \delta\bar{A}^j \rangle &= i \frac{\delta^{ij}}{k^2} \\ \langle \delta\bar{A}^i \delta\bar{A}^+ \rangle &= i \frac{k^i}{k^- k^2} \\ \langle \delta\bar{A}^+ \delta\bar{A}^+ \rangle &= i \left(\frac{k_i^2}{(k^-)^2 k^2} - \frac{1}{(k^-)^2} \right).\end{aligned}\quad (34)$$

The second term in the parenthesis above corresponds to the contribution of the expectation value $\langle \delta\bar{A}_{II}^+ \delta\bar{A}_{II}^+ \rangle$ to $\langle \delta\bar{A}^+ \delta\bar{A}^+ \rangle$. Notice that the contribution from terms like $\langle \delta\bar{A}_I^+ \delta\bar{A}_{II}^+ \rangle$ vanish. If we now combine all the above terms together we obtain the familiar expression for the gluon propagator

$$G^{\mu\nu} = -\frac{1}{k^2} \left(g^{\mu\nu} - \frac{n^{*\mu} k^\nu + k^\mu n^{*\nu}}{k \cdot n^*} \right), \quad (35)$$

where $n^{*\mu} = \delta^{\mu+}$.

Now perform the gauge transformation

$$\delta A^\mu = \delta\bar{A}^\mu + \partial^\mu \Lambda. \quad (36)$$

Since we are gauge transforming to Light Cone gauge, we require

$$\partial^+ \Lambda = -\delta\bar{A}^+. \quad (37)$$

distortion of the small fluctuation fields by the background field. The contribution of the type II modes to the gluon propagator in momentum space is

$$\begin{aligned}\delta G_{(II)}^{++} &= \frac{1}{(k^-)^2} \\ &= \frac{1}{k^2} \left(\frac{k_i^2}{(k^-)^2} - 2 \frac{k^+}{k^-} \right)\end{aligned}\quad (41)$$

The second of the two expressions in the above equations is algebraically equivalent to the first but corresponds to a different way of regulating the singular behavior of the type II modes at $k^- = 0$. In the next section, we will see that the latter choice is the one aptly suited to regularization.

5 Regularization prescriptions and results

We shall now use the method described in the previous section to derive the gluon propagator in Light Cone gauge from the propagator in $\tilde{A}^- = 0$ gauge (equation (21)). Note that the gluon propagator here is the gluon propagator in the background field. It will shortly become clear that we need to choose an appropriate regularization prescription to obtain the final expression for the propagator.

If we have a fluctuation field in $\tilde{A}^- = 0$ gauge, the fields in $A^+ = 0$ gauge are given in terms of the tilde fields as

$$\delta A^+ = 0 \quad (42)$$

$$\delta A^- = -\frac{\partial^-}{\partial^+} \delta \tilde{A}^+ \quad (43)$$

$$\delta A^i = \delta \tilde{A}^i - D^i(A) \frac{1}{\partial^+} \delta \tilde{A}^+. \quad (44)$$

To derive these equations, we note that a small gauge transformation with a fixed background field A , where A has only transverse components, is

$$\delta A^+ \rightarrow \delta A^+ + \partial^+ \Lambda \quad (45)$$

$$\delta A^- \rightarrow \delta A^- + \partial^- \Lambda \quad (46)$$

$$\delta A^i \rightarrow \delta A^i + D(A)^i \Lambda. \quad (47)$$

It is now straightforward to construct similar expressions for non-diagonal components of the Green's function. Even though only transverse components of the Light Cone gauge Green's function are needed to compute the gluon distribution function, we will give an implicit expression for all components for the sake of completeness. It is

$$\begin{aligned}
G^{\mu\nu}(x, y) = & \left[\delta^{\mu\rho} \delta^{\nu\lambda} + \delta^{\rho\lambda} \left(D^\mu \frac{1}{\partial_- \partial_+} D_\rho \right)_{(x)} \left(D^\nu \frac{1}{\partial_- \partial_+} D_\lambda \right)_{(y)} \right. \\
& + \delta^{\mu\rho} \left(\delta^{\lambda+} - 1 \right) \left(D^\nu \frac{1}{\partial_- \partial_+} D_\lambda \right)_{(y)} \\
& \left. + \delta^{\nu\lambda} \left(\delta^{\mu+} - 1 \right) \left(D^\mu \frac{1}{\partial_- \partial_+} D_\rho \right)_{(x)} \right] \bar{G}^{\rho\lambda}(x, y). \quad (53)
\end{aligned}$$

In this expression $\mu, \nu = 1, 2, -$ while $\rho, \lambda = 1, 2, -, +$ and again $\bar{G}^{\rho\lambda}(x, y)$ is the Green's function in $A^- = 0$ gauge. From here on, we will concentrate on the transverse components only.

It is evident from the above equations that in relating the fields in the two gauges, one has to define $1/\partial^+$ and $1/\partial^-$. These operators are not well defined unless some prescription is adopted for treating the singularity at $k^+ = 0$ and at $k^- = 0$. Presumably, there is no physics in these singularities –when one computes gauge invariant quantities, different prescriptions should give the same results. It has however proven most convenient to choose a Feynman like $i\epsilon$ prescription for regularizing this singularity, the so called Leibbrandt–Mandelstam prescription [4]. This prescription however spoils the Hermiticity of the field δA^i . This is not a new problem since even in the absence of our background field, such a prescription will destroy the reality properties of A^- in $A^+ = 0$ gauge. It is indeed unpleasant, but it does make for a Green's function which is manifestly causal. The price to be paid for a small lack of Hermiticity of the fields is small compared to the price of dealing with non-causal Green's functions, albeit all such ambiguities must come to naught when computing gauge invariant quantities.

We must now establish our prescriptions for defining the inversion of $\partial^\pm$. We

from infinity [10]. In the Leibbrandt-Mandelstam prescription, positive frequencies propagate forward in time and negative frequencies propagate backward in time. For our problem where we have a nucleus acting as a source of charge, the fields do not vanish at $x^- = \pm\infty$. There is therefore no advantage to choosing an integration prescription with one of its integration limits at either $\pm\infty$.

We can now combine terms together to get an expression for the propagator. The derivation of $G_{ij(II)}$ from equation (51) is rather subtle and we shall elaborate a little on it. We can easily do the k^+ and k_t integrations in equation (52) to get

$$\delta\tilde{G}_{(II)}^{++}(x, y) = \tau_a \tau_a \delta^{(2)}(x_t - y_t) \delta(x^- - y^-) \int \frac{dk^-}{2\pi} \frac{e^{-ik^-(x^+ - y^+)}}{(k^-)^2 - i\epsilon}.$$

It is clear that we have locality in both transverse and minus coordinates. Hence there will not be any contributions from the regions $\theta(x^-)\theta(-y^-)$ and $\theta(-x^-)\theta(y^-)$. Then

$$\begin{aligned} (D^i)_{(x)}(D^j)_{(y)}\delta\tilde{G}_{(II)}^{++} &= \left[\theta(-x^-)\theta(-y^-) \tau_a \tau_a \partial_i \partial_j \delta^{(2)}(x_t - y_t) \right. \\ &\quad \left. + \theta(x^-)\theta(y^-) F_a(x_t) F_a(y_t) \partial_i \partial_j \delta^{(2)}(x_t - y_t) \right] \\ &\quad \times \delta(x^- - y^-) \int \frac{dk^-}{2\pi} \frac{e^{-ik^-(x^+ - y^+)}}{(k^-)^2 - i\epsilon}. \end{aligned} \quad (59)$$

We now write the δ -functions in their Fourier representations and replace

$$\frac{1}{(k^-)^2} \rightarrow \frac{1}{k^2} \left(\frac{k_t^2}{(k^-)^2} - 2 \frac{k^+}{k^-} \right). \quad (60)$$

Finally, making use of the identity

$$\begin{aligned} \int \frac{dk^+}{2\pi} (2k^+) e^{-ik^+(x^- - y^-)} &= i \left[\frac{\partial}{\partial x^-} \int \frac{dk^+}{2\pi} e^{-ik^+(x^- - y^-)} \right. \\ &\quad \left. - \frac{\partial}{\partial y^-} \int \frac{dk^+}{2\pi} e^{-ik^+(x^- - y^-)} \right], \end{aligned} \quad (61)$$

we can write equation (51) as

$$G_{ij(II)}^{\alpha\beta;\alpha'\beta'}(x, y) = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - i\epsilon} \left[\theta(-x^-)\theta(-y^-) \tau_a^{\alpha\beta} \tau_a^{\alpha'\beta'} + \theta(x^-)\theta(y^-) \right]$$

an amusing coincidence specific to $O(\alpha_s^2)$.

6 Summary

In this paper we have computed the propagator for transverse gluons in the Light Cone gauge $A^+ = 0$ and in the presence of a background Weizsäcker-Williams field produced by the valence quarks of a heavy nucleus. We have argued that such a propagator can be obtained from the propagator in the $A^- = 0$ gauge by a gauge transformation. This method has the advantage of avoiding the difficulties introduced by the singular behavior of the fields at $x^- = \pm\infty$ caused by the presence of the sources. The gauge transformation requires a choice for the inverse of the operators ∂^- , ∂^+ . We chose these to be such that the gluon fields satisfy the boundary condition $\delta A^-|_{x^-=0} = 0$ and the usual Feynman causal behavior. The expression for the gluon propagator in $A^+ = 0$ gauge is necessary if we want to extract from it the information about the gluon density which can be compared to experimental measurements.

In the above analysis, there are still many subtle questions remaining about the independence of physical quantities from the precise choice of the regularization prescription for the small k^- and k^+ singularities. We have shown that there is a gauge where there are no singularities and that the subsequent singularities in the Light Cone gauge Green's function arise from the gauge transformation relations. Then, it is only sensible that the physical quantities computed here will be independent of the nature of these singularities.

At the very least it seems we have a consistent prescription. It should be noted that because of the gauge choice, there is no ambiguity about the limit $k^+ \rightarrow 0$. The propagator is smooth in this limit as a consequence of the way we have specified the residual gauge degree of freedom. If it were only the free transverse propagator which was of interest, we would have a non-singular propagator at small k^+ . The problem

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