MEASUREMENT OF THERMAL REGAIN
IN DUCT SYSTEMS LOCATED IN
PARTIALLY CONDITIONED BUFFER SPACES

John W. Andrews

July 1994

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Energy Efficiency & Conservation Division
Department of Applied Science
Brookhaven National Laboratory
Upton, New York 11973

This work was performed under the auspices of the U.S. Department of Energy
Washington, D.C. Under Contract No. DE-AC02-76CH00016

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

at

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency, contractor, or subcontractor thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

Thermal losses from duct systems have been shown to be a significant fraction of the heat or cooling energy delivered by the space-conditioning equipment. However, when the ducts are located in a partially conditioned buffer space such as a basement, a portion of these losses are effectively regained through system interactions with the building. This paper presents two methods of measuring this regain effect. One is based on the relative thermal resistances between the conditioned space and the buffer space, on the one hand, and between the buffer space and the outside, on the other. The second method is based on a measured drop in the buffer-space temperature when steps are taken to reduce the duct losses. The second method is compared with results of an extensive research project that are published in a major professional society handbook. The thermal regain fraction using the drop in basement temperature was found to be 0.68, while that obtained from an analysis of the system performance data, without using the basement temperature, was 0.59.

MEASUREMENT OF THERMAL REGAIN IN DUCT SYSTEMS LOCATED IN PARTIALLY CONDITIONED BUFFER SPACES

John Andrews
Brookhaven National Laboratory
July, 1994

INTRODUCTION

Central space heating and cooling systems require some means, such as piping or ductwork, to transport thermal energy to various parts of the building. During this transport process, a portion of this energy is usually lost. Thermal conduction through duct or pipe walls is one mechanism common to all types of thermal distribution systems. Air ducts in particular are prone to additional losses through air leakage and system interactions such as fan-induced infiltration during the on-cycle and thermosyphoning during the off-cycle.

The following discussion can be applied to both ducted and hydronic distribution systems. We will, however, refer to "ducts" both for reasons of brevity and because the methods proposed are likely to find their most common application in forced-air systems.

When duct systems are located within the conditioned space, losses to the conditioned space are fully recovered. Losses to a completely unconditioned space are completely lost. In many cases, however, the ducts are located within a partially conditioned "buffer space." Basement ducts are typical of this case. For ducts in such a location, some of the thermal energy lost may be effectively recovered because of system effects. For example, when heat is lost from ductwork to a basement, the basement becomes warmer than it otherwise would be. This retards heat losses from the house to the basement through the basement ceiling. The reduction in these losses can be credited to the ducts as a thermal regain effect.

Critical to any method of test for ductwork is a reliable, cost-effective procedure for determining the magnitude of this regain effect. It is also critical to any attempt at diagnosing duct problems. If 80% of the heat lost from ducts is recovered, it probably will not be cost-effective to reduce the losses. If only 20% is recovered, on the other hand, duct repair may be the best thing one could do.

Two approaches to determining the fraction of duct losses that are effectively regained are presented below. Because much of this

work is expected to be applied to basements, the word basement will often be used instead of the more general "buffer space."

FIRST APPROACH--THERMAL REGAIN COMPUTED FROM THERMAL RESISTANCES

Thermal Regain Defined--One Thermal Sink

In the simplest situation (Figure 1), heat flows from the conditioned space to the basement, where it is joined by heat lost from the ducts and both heat flows then pass out to the ambient. For this situation, the following heat-flow equations can be written:

$$Q_1 = \frac{T_c - T_b}{R_1} \quad (1)$$

$$Q + Q_1 = \frac{T_b - T_a}{R_2} \quad (2)$$

Eliminating T_b from these equations allows us to derive an expression for Q_1 :

$$Q_1 = \frac{T_c - T_a - Q R_2}{R_1 + R_2} \quad (3)$$

Since this expression is general, for any value of Q , we can determine what Q_1 would be if Q were zero. Call this baseline value Q_1' . Then

$$Q_1' = \frac{T_c - T_a}{R_1 + R_2} \quad (4)$$

The amount of thermal regain, or effective heat reclaimed from the duct losses, can be defined as the difference between the heat that flows from the house to the buffer space in the absence of duct losses (Q_1') and the heat that flows along this path when duct losses are present (Q_1). The **thermal regain fraction** f_{regain} is then the ratio of this difference to the heat lost from the ducts:

Figure 1. Temperatures, Resistances, and Heat Flows for Thermal Regain Calculation (One Thermal Sink).

Figure 2. Temperatures, Resistances, and Heat Flows for Thermal Regain Calculation (Two Sinks at Different Temperatures).

$$f_{\text{regain}} = \frac{Q'_1 - Q_1}{Q} \quad (5)$$

Substituting the values from Equations 3 and 4 for the quantities in the numerator of Equation 5, one obtains

$$\begin{aligned} f_{\text{regain}} &= \left(\frac{Q R_2}{R_1 + R_2} \right) \frac{1}{Q} \\ &= \frac{R_2}{R_1 + R_2} \end{aligned} \quad (6)$$

This equation gives a particularly simple relationship between the effective thermal resistances to yield the thermal regain fraction. In the limiting case where there is no resistance between the buffer space and the conditioned space, the buffer space becomes part of the conditioned space and $f_{\text{regain}} = 1$. In the other limiting case where there is no resistance between the buffer space and the outside, the buffer space is totally unconditioned and $f_{\text{regain}} = 0$.

Two Thermal Sinks at Different Temperatures

In this situation (Figure 2) heat flows from the buffer space to the outside (surface) ambient and to the ground. These two sinks are in general at different temperatures. A set of four heat-flow equations can be written:

$$Q_1 = \frac{T_c - T_b}{R_1} \quad (7a)$$

$$Q_2 = \frac{T_b - T_a}{R_2} \quad (7b)$$

$$Q_3 = \frac{T_b - T_g}{R_3} \quad (7c)$$

$$Q_1 + Q = Q_2 + Q_3 \quad (7d)$$

Substituting the values for Q_2 and Q_3 from Equations 7b and 7c into Equation 7d, and then eliminating T_b with the help of Equation 7a, we obtain

$$Q_1 + Q = \frac{T_c - Q_1 R_1 - T_a}{R_2} + \frac{T_c - Q_1 R_1 - T_g}{R_3} \quad (8)$$

This equation can now be solved for Q_1 :

$$Q_1 = \frac{(T_c - T_a) R_3 + (T_c - T_g) R_2 - Q R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} \quad (9)$$

As in Case 1, we compare this with the situation in which there are no duct losses ($Q = 0$). As before, we put a prime on Q_1 :

$$Q_1' = \frac{(T_c - T_a) R_3 + (T_c - T_g) R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2} \quad (10)$$

The thermal regain fraction is then:

$$\begin{aligned} f_{\text{regain}} &= \frac{Q_1' - Q_1}{Q} \\ &= \left(\frac{Q R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} \right) \frac{1}{Q} \quad (11) \\ &= \frac{R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} \end{aligned}$$

These expressions can be used to derive theoretical values for the thermal regain fraction under various buffer-space configurations. It is to be noted that the thermal resistances should factor in all modes of heat transfer, including air infiltration between the spaces involved.

Conversion of Two Thermal Sinks to One

It is possible to select an effective sink temperature T_x and a basement-to-sink thermal resistance R_x that will allow the situation with two thermal sinks at different temperatures to be treated as if there were only one sink at one temperature. These values are obtained by setting the total heat flow from the buffer space ($Q_2 + Q_3$) equal to $(T_b - T_x)/R_x$ and setting this equal to the sum of the right-hand sides of Equations 7b and 7c:

$$\begin{aligned}
\frac{T_b - T_x}{R_x} &= \frac{T_b - T_a}{R_2} + \frac{T_b - T_g}{R_3} \\
&= \frac{(R_2 + R_3) T_b - R_3 T_a + R_2 T_g}{R_2 R_3} \quad (12) \\
&= \frac{T_b - (R_3 T_a + R_2 T_g) / (R_2 + R_3)}{R_2 R_3 / (R_2 + R_3)}
\end{aligned}$$

This permits us to identify the values $T_x = (R_3 T_a + R_2 T_g) / (R_2 + R_3)$ and $R_x = R_2 R_3 / (R_2 + R_3)$. Now it is possible to proceed on the basis of a single sink and a single sink temperature from this point on. (Note that, in particular, if one replaces R_2 with R_x in Equation 6 and evaluates it as defined here, one obtains Equation 11.)

Field Measurements on Basement Ductwork

Determining these resistances in the field will require estimation or measurement of conductive resistances and of infiltration rates between the conditioned space and the basement and between the basement and the outdoor ambient.

Basement ducts are usually installed near the ceiling. In the heating mode, this means that the basement air is likely to be stratified, unless the thermal drivers provided by pressure differences (infiltration) and temperature gradients near the basement walls (conduction) are sufficient to induce significant vertical mixing.

In an ideal case, there would be an upper region of basement air near the basement ceiling that is relatively well mixed, then a thermal gradient to a lower region of basement air near the basement floor. In that case, the temperature T_b in the above equations would be taken as that of the upper region of air, and the thermal resistance to the ground would be very high--perhaps high enough that one could ignore heat flow to the ground and treat all the duct-loss heat as going to ambient, either through infiltration or through the upper portion of the basement walls.

The actual treatment of the basement still needs to be refined, but at a minimum the following would need to be done:

- o Estimate the conductive UA of the basement ceiling. This should be relatively straightforward, involving visual inspection of the insulation (if any) and measurement of the basement ceiling area.
- o Determine the infiltration rates between the basement and the conditioned space and between the basement and the outdoor ambient. One way to do this would be to employ the

perfluorocarbon tracer gas method using two different tracer gases. This provides a direct measurement under actual operating conditions, not only of the two infiltration rates needed for the thermal-resistance determination, but also of the infiltration rate between the conditioned space and the ambient. A recent cost estimate was approximately \$150 per house, plus an \$800 "project charge" pro-rated over all the houses of the project. Alternatively, fan-pressurization methods combined with the LBL infiltration model may be used.

- o Determine the conductive thermal resistance between the basement and the outdoor ambient. This is the most difficult one conceptually. Possibly one could take the basement wall down to the level where the vertical thermal gradient is a maximum, and then compute (through finite-difference or finite-element analysis) the overall thermal resistance of basement wall to that level, soil, and surface. The details of this process are beyond the scope of this article.

SECOND APPROACH--THERMAL REGAIN BASED ON DROP IN BASEMENT TEMPERATURE

Theory

As noted, it may often be difficult to determine the conductive thermal resistances needed to evaluate f_{regain} using the above theory. For this reason, an alternative method would be desirable. Such an alternative will be available if a replacement distribution system with lower losses is put in place as part of the test. This can be provided in at least two different ways:

- o In the system-comparison approach to measuring thermal distribution efficiency, a co-heating system is substituted for the as-found system for the second half of the test. [Andrews 1993] Such a system has no distribution losses at all.
- o The test protocol may involve a retrofit of the duct system, including sealing and insulation, which will greatly reduce the heat losses from the ducts.

In either case, a drop in the basement temperature is expected after the switchover from the as-found system to the alternative system. By measuring this temperature drop, one can obtain an independent value of the thermal regain fraction without having to measure thermal resistances.

In what follows, we will assume that acceptable results can be obtained by using a single heat sink. T_x and R_x will be used as the effective sink temperature and basement-to-sink thermal resistance,

as discussed. We write two sets of two equations analogous to Equations 1 and 2, one set for the as-found system (which we will call the base case) and the other for the retrofitted system or the co-heating system (which we will call the alternative case). Quantities referring to the alternative case will have primes. The four equations are:

$$Q_1 = \frac{T_c - T_b}{R_1} \quad (13a)$$

$$Q'_1 = \frac{T'_c - T'_b}{R_1} \quad (13b)$$

$$Q + Q_1 = \frac{T_b - T_x}{R_x} \quad (13c)$$

$$Q' + Q'_1 = \frac{T'_b - T'_x}{R_x} \quad (13d)$$

Normally, the conditioned-space temperature will be the same for both tests, so that $T'_c = T_c$. Whether or not that is the case, it is convenient to define

$$\alpha = \frac{T'_c - T'_b}{T_c - T_b} \quad (14a)$$

$$\beta = \frac{T'_b - T'_x}{T_b - T_x} \quad (14b)$$

We first eliminate R_1 and R_x from Equations 13:

$$\frac{Q'_1}{Q_1} = \alpha \quad (15a)$$

$$\frac{Q' + Q'_1}{Q + Q_1} = \beta \quad (15b)$$

These equations can then be solved for Q_1 and Q'_1 :

$$Q_1 = \frac{\beta Q - Q'}{\alpha - \beta} \quad (16a)$$

$$Q'_1 = \frac{\alpha \beta Q - \alpha Q'}{\alpha - \beta} \quad (16b)$$

A major difference between this treatment and the one performed earlier is that the duct losses under the alternative case are not necessarily zero. Since thermal regain was defined on the basis of a comparison between the base case and an alternative case where the duct losses are zero, we need to invoke the fact that the relation between Q and Q_1 is linear, which is equivalent to the statement that the thermal regain fraction is a constant as long as the thermal resistances do not change.

The linearity of the relationship between Q and Q' is shown by Equation 3, which is depicted schematically in Figure 3. As equation and figure show, when the heat lost from the ducts to the buffer space declines from Q to Q' , the heat lost from the house to the buffer space increases linearly from Q_1 to Q_1' . If the heat lost from the ducts to the buffer space is reduced further to zero, then the heat lost from the house to the buffer space increases to Q_1'' . As defined previously, the thermal regain fraction is the ratio of $(Q_1'' - Q_1)$ to Q , but because the graph is linear, it is also given by

$$f_{\text{regain}} = \frac{Q_1' - Q_1}{Q - Q'} \quad (17)$$

If we now substitute the expressions for Q_1 and Q_1' in Equations 16a and 16b into Equation 17, we obtain

$$f_{\text{regain}} = \frac{(\alpha - 1) (\beta Q - Q')}{(\alpha - \beta) (Q - Q')} \quad (18)$$

Field Measurements

For this approach, the main problems with field measurements are likely to be ones of accurately determining the values of Q , Q' , and the temperatures. The heat losses from the ducts to the buffer space are conductive and leakage. The conductive loss is found basically by dividing the temperature drop in the supply ducts by the temperature rise in the plenum, with care taken to consider that different branch ducts will have different temperature drops, that some conductive loss also takes place in the return ducts, and that under cycling conditions it will be necessary to average these quantities out.

The leakage loss is obtained by determining the leakage to the buffer space in CFM under normal operating conditions, and multiplying this by 1.08 times the average difference between the

Figure 3. Relation of Duct Losses to Heat Flows Between Conditioned Space and Buffer Space.

Figure 4. Heat Flows Characterized by the ASHRAE System Performance Factors.

air temperature in the supply ducts T_{sup} and the basement temperature T_b . The basement temperature is used as the base temperature in calculating the temperature difference for the following reasons.

1. If return-duct leakage is equal to supply-duct leakage, then the amount of heat added to the basement is proportional to $(T_{sup}-T_b)$, and there are no infiltration effects in the conditioned space unless the supply registers are isolated from the returns (a common situation).

2. If return-duct leakage is greater or less than supply-duct leakage, then infiltration between the conditioned space and the outside will be promoted by the operation of the air handler. This fan-induced infiltration loss is intertwined with duct leakage loss, and to some extent how one separates them is arbitrary. It may be advantageous to use the conditioned space temperature T_c as the temperature base rather than T_b , but in view of (1) above we have chosen to use T_b . The following addendum suggests that the error involved in making a "wrong" choice is likely to be small.

Addendum on Base Temperature for Leakage Losses

Leakage losses are computed as the product of a mass-flow rate, a specific heat, and a difference between the temperature of the leaked air and a base temperature used as the "zero point" for enthalpy calculations. In most work the temperature in the conditioned space should probably be used as this base temperature for figuring leakage losses, but for the purposes of defining Q and Q' here, it is argued that T_b be used instead. This issue needs further thought. Fortunately, for furnaces the difference between these possibilities is not too great. As a benchmark, consider the following case in which pre- and post-retrofit testing is done:

	Pre	Post
Flow rate at plenum:	1200 scfm	1200 scfm
Duct leakage--supply:	200 scfm	50 scfm
--return:	200 scfm	100 scfm
Temperatures:		
Conditioned space	70 F	70 F
Buffer space	60 F	55 F
Sink	30 F	30 F
Return plenum	65 F	68 F
Supply plenum	140 F	143 F
Supply registers (avg)	125 F	135 F

We first calculate alpha and beta as follows:

$$\alpha = (70-55)/(70-60) = 1.50$$

$$\beta = (55-30)/(60-30) = 0.833$$

Next we calculate the conductive and leakage components of Q and Q' . For the purposes of this exercise, we will ignore the return ducts and consider the two choices for the base temperature to use in computing leakage losses, $T_c = 70$ F and $T_b = 60$ F.

Q_{cond}	$= 1100 \times 1.08 \times (140-125)$	$= 17820$ Btuh
$Q_{leak-70}$	$= 200 \times 1.08 \times (132.5-70)$	$= 13500$
$Q_{leak-60}$	$= 200 \times 1.08 \times (132.5-60)$	$= 15660$
Q_{tot-70}		$= 31320$
Q_{tot-60}		$= 33480$

This represents a 6% difference in calculated duct loss depending on which base temperature is used.

Q'_{cond}	$= 1175 \times 1.08 \times (143-135)$	$= 10152$ Btuh
$Q'_{leak-70}$	$= 100 \times 1.08 \times (139-70)$	$= 7452$
$Q'_{leak-60}$	$= 100 \times 1.08 \times (139-60)$	$= 8532$
Q'_{tot-70}		$= 17604$
Q'_{tot-60}		$= 18684$

Again, the difference in calculated losses is about 6%.

The fractional regain can now be calculated. Using the values obtained with a 70 F base temperature on leakage,

$$f_{regain-70} = (0.50/0.833)(0.833 \times 31320 - 17604)/(31320-17604) = 0.371$$

For the 60 F choice of base temperature,

$$f_{regain-60} = (0.50/0.833)(0.833 \times 33480 - 18684)/(33480-18684) = 0.373$$

This represents less than a 0.5% difference, so for this case at least the choice of base temperature does not appear to be a major issue. Further study and generalization is needed on this point.

RELATION TO THE ASHRAE SYSTEM PERFORMANCE FACTORS

Heat Flows as Described in the ASHRAE Handbook

We now turn to the system published by ASHRAE for taking account of the energy that passes through furnaces and ducts. The terminology is explained in Chapter 29 of the 1992 HVAC Systems and Equipment Handbook, and will not be repeated here. Figure 4 diagrams the heat flows between the various parts of the system. The major components are the equipment (furnace or heat pump), the duct, the conditioned space, the buffer space, and the ambient.

The heating load of the house is equal to a "no-dist" base load L (which has no thermal distribution system losses or impact), divided by the Load Modification Factor F_{LM} for the situation being considered. This load is mostly met by heat from the ducts, but a small amount may come directly from the equipment (for example via jacket losses if the equipment is in the conditioned space). The possibility of direct heat transfer to the load from the equipment, bypassing the ducts, is accounted for by the Miscellaneous Gain Factor, F_{MG} . (Unless the furnace is in the conditioned space, F_{MG} will usually be within a fraction of a percentage point of 1.00.) The heat from the ducts is $L/(F_{MG}F_{LM})$, while that from the equipment directly is the difference between this quantity and the modified load, or $(F_{MG}-1)L/(F_{MG}F_{LM})$.

The heat delivered to the ducts is the duct delivered heat divided by the duct efficiency, or $L/(E_D F_{MG} F_{LM})$. This is split three ways, in quantities proportional to the duct efficiency E_D and two duct loss factors, E_A , the loss fraction to the ambient, and E_B , the loss fraction to the buffer space. (Thus, $E_D + E_A + E_B = 1$.)

The duct is fed by the equipment, whose efficiency is referred to as furnace efficiency E_F . This efficiency is defined as the ratio of duct input to furnace input. Thus, direct gains from the furnace to the load (included in F_{MG}) are not taken into account in figuring furnace efficiency in this scheme.

The fuel requirement is then the bare heating load divided by the product of the four factors $E_F E_D F_{MG} F_{LM}$. The system index is defined as the product of these four factors. It is an overall figure of merit.

The heat flows Q , Q_1 , and Q_2 used in this discussion are also shown in Figure 4. This puts these heat flows in the broader context of the ASHRAE system performance factors.

Thermal Regain in the SP-43 Project

The ASHRAE SP-43 Project (which was co-funded by DOE, GRI, and ASHRAE in the 1980's) used a validated computer model to project the values of the above-described performance factors under a variety of conditions. Among these were insulation and sealing of ductwork located in a basement.

We will use the basement-temperature drop (BTD) method discussed above to calculate the thermal regain fraction. In addition, we will use an alternative calculation procedure that uses only the reported values of the performance factors (such as furnace efficiency, duct efficiency, etc.) but not the basement temperature drop. This will serve as a check on the BTD method.

Our purpose in looking at the SP-43 results is to determine to what extent the BTD results, which reflect what a field experiment might obtain, agree or disagree with the results based on performance factors only, which presumably reflect everything the SP-43 model knew about the system.

Performance-Factor Comparison

We will do the calculation based strictly on performance factors first. Our determination of f_{regain} by this approach will depend on one key assumption, namely that the only effect on the load of insulating the ducts will be to reduce heat loss through the basement floor. This allows us to conclude that the reduction in this heat loss is equal to the reduction in heat input to the conditioned space from the ducts:

$$Q'_1 - Q_1 = \frac{L}{F'_{LM}F'_{MG}} - \frac{L}{F_{LM}F_{MG}} \quad (19)$$

The heat lost by the ducts to the buffer space, in the as-found system, is given (in terms of the performance factors discussed above) by

$$Q = \frac{L E_B}{E_D F_{MG} F_{LM}} \quad (20)$$

while Q' is given by a similar expression with all quantities except L being primed as well. We now are in a position to write an expression for the thermal regain fraction in terms of system performance factors:

$$\begin{aligned} f_{\text{regain}} &= \frac{Q'_1 - Q_1}{Q - Q'} \\ &= \frac{\frac{L}{F'_{LM}F'_{MG}} - \frac{L}{F_{LM}F_{MG}}}{\frac{L E_B}{E_D F'_{LM}F'_{MG}} - \frac{L E'_B}{E'_D F'_{LM}F'_{MG}}} \quad (21) \end{aligned}$$

We note that E_B is not defined in the SP-43 performance parameters; we introduced it to make the distinction between heat

lost from the ducts to the buffer space and heat lost directly to ambient. In the SP-43 Project, however, all duct losses were modeled as going to the basement; none went directly to the outside. Hence, in what follows we can set $E_B = (1 - E_D)$. [Jakob 1994]

Using $I_s = E_F E_D F_{MG} F_{LM}$, we then can write f_{regain} in terms of ASHRAE-recognized system performance factors:

$$\begin{aligned}
 f_{regain} &= \frac{\frac{L}{F'_{LM} F'_{MG}} - \frac{L}{F_{LM} F_{MG}}}{\frac{(1-E_D) L}{E_D F'_{LM} F'_{MG}} - \frac{(1-E'_D) L}{E'_D F'_{LM} F'_{MG}}} \\
 &= \frac{\frac{E'_F E'_D}{I'_s} - \frac{E_F E_D}{I_s}}{\frac{(1-E_D) E_F}{I_s} - \frac{(1-E'_D) E'_F}{I'_s}} \quad (22) \\
 &= \frac{E'_F E'_D - E_F E_D (I'_s / I_s)}{E_F (1-E_D) (I'_s / I_s) - E'_F (1-E'_D)}
 \end{aligned}$$

We now turn to the results of SP-43 as shown in Table 12 of Chapter 29 of the 1992 ASHRAE HVAC Systems and Equipment Handbook. (ASHRAE 1992) This table shows seven duct treatments, all in a basement having R8 wall insulation and no ceiling insulation.

For the base case (BC) we use the same case called base case in ASHRAE 1992, namely Case 2, which has no duct insulation and 10% leakage. This percentage reflects the percentage of total air flow leaking from the basement into the return ducts and also from the supply ducts into the basement. [Jakob 1994] Thus, SP-43 assumed balanced duct leakage.

For the first two alternative cases (AC) we use the two cases with better insulation but with the same amount of leakage. This was because of our concern that a different leakage level could lead to different heat loss from the portion of the envelope not adjacent to the basement ceiling, arising from fan-induced infiltration. Case 5 has R5 insulation on the duct exterior, while Case 7 has R5 insulation on the duct interior, which gives better thermal performance because of reduced storage losses. Two other alternative cases were also considered, Case 1 with no insulation but with no leakage, either, and Case 4 with R5 (exterior) insulation and no leakage. A matter of interest was whether these

cases with less leakage would behave differently from the ones that varied conductive losses only.

Table 1 shows the performance factors from ASHRAE 1992. E_F , E_D , and the system index values were taken directly from Table 12.

Table 1. Computation of f_{regain} from ASHRAE System Performance Factors.

Case Duct Insul. Leakage % Factor	BC-2 None 10	AC-5 R5 10	AC-7 R5 Int 10	AC-1 None 0	AC-4 R5 0
E_F	0.755	0.752	0.750	0.754	0.750
E_D	0.609	0.704	0.796	0.668	0.774
I_s' / I_s	1.000	1.044	1.085	1.026	1.070
f_{regain} (Eq. 22)	---	0.577	0.587	0.607	0.605

It can be seen that the four values of f_{regain} are quite consistent, with an average of 0.594. The closeness of the AC-1 and AC-4 values to those for AC-5 and AC-7 is consistent with the absence of fan-induced infiltration in the SP-43 model.

Basement-Temperature Drop Method

This section computes f_{regain} using the BTM method. The conditioned-space setpoint in the SP-43 model runs was 68 F. (Jakob et al. 1987, p. V-14) The average ambient temperature for September through May in the simulation was 43.6 F. (Jakob et al. 1987, p. V-17) There is some question whether this is the best ambient temperature to use. It could be argued that a heating-load-weighted average should be used instead (which would result in a lower value). For now, we have elected to use this value as the best available.

In the SP-43 simulations, the bulk of the heat transfer from the basement to the outside went to the ambient rather than to the ground. In the base case (ASHRAE 1992, Table 6), 5298 kBtu passed conductively through the portion of the basement walls above grade (i.e. to ambient), 6922 kBtu went via infiltration (again to ambient), and 5905 kBtu passed conductively through the portion of the basement walls below grade (some going to ambient via upper layers of earth and the remainder to ground). The heat flow through the basement floor was negligible (8 kBtu). Assuming all the below-grade heat flow was to ground would mean that the ground received 33% of the heat flow from the basement. Taking $T_g = 55$ F, this leads to $T_x = 49$ F. The actual value of T_x will be somewhat less than this, to the extent that some of the heat flow through

the wall below grade ends up at the surface. We use $T_x = 48$ F in what follows.

In a later section, the uncertainty in f_{regain} caused by error in T_x (and in T_c) will be calculated.

Since the duct loss to the buffer space was not reported as such (except for the base case), it was necessary to construct this quantity from the system performance factors in the same manner as in the previous section. What is different here is that instead of using the system performance factors to compute $Q_1' - Q_1$, as in Equation 17, the drop in basement temperature is used to determine this difference, as in Equation 18.

Table 2 gives the performance factors F_{LM} and F_{MG} , which, along with E_F and E_D from Table 1, are used in this reconstruction. The F_{LM} values are taken directly from ASHRAE 1992, Table 12, while F_{MG} is inferred from the given values of E_F , E_D , F_{LM} , and I_s .

Table 2. Values of F_{LM} and F_{MG} from ASHRAE 1992.

Case Duct Insul. Leakage % Factor	BC-2 None 10	AC-5 R5 10	AC-7 R5 Int 10	AC-1 None 0	AC-4 R5 0
F_{LM}	1.00	0.91	0.84	0.94	0.85
F_{MG}	1.000	1.004	1.005	1.004	1.003

The value of Q (base case) or Q' (alternative cases) is then given by the expression $(1-E_D)L/(E_DF_{LM}F_{MG})$, with primed or unprimed values as appropriate. This is as illustrated in Figure 4.

The cases considered for the BTD approach are the same as those in Table 1. Steps of the process are shown in Table 3. The values of α and β are calculated from the temperatures, with the unprimed values taken from the base case and the primed values taken from the particular alternative case under consideration.

It is to be noted that in this computation, the factor L occurs in all terms of the equation, both in the numerator and the denominator. Hence it cancels out. The values of f_{regain} determined in the four cases are quite close, with an average of 0.720.

Table 3. Computation of f_{regain} from Basement Temperature Drops. Temperatures $T_c = 68.0$ F and $T_x = 48$ F for all cases.

Case Duct Insul. Leakage % Quantity	BC-2 None 10	AC-5 R5 10	AC-7 R5 Int 10	AC-1 None 0	AC-4 R5 0
T_b (F)	67.8	66.3	65.1	66.8	65.2
$T_c - T_b$ (F)	0.2	1.7	2.9	1.2	2.8
$T_b - T_x$ (F)	19.8	18.3	17.1	18.8	17.2
α	---	8.5	14.5	6.0	14.0
β	---	0.924	0.864	0.949	0.869
Q/L	0.642	0.460	0.304	0.527	0.342
$(\beta Q - Q') / (Q - Q')$	---	0.732	0.742	0.715	0.720
$(\alpha - 1) / (\alpha - \beta)$	---	0.990	0.990	0.990	0.990
f_{regain} (Eq. 18)	---	0.725	0.735	0.708	0.713

Note: Q is given by the expression $(1 - E_D)L / (E_D F_{MG} F_{LM})$. The factor L drops out in the ratio computed on the next line.

Comparison of the Methods

The values of f_{regain} as calculated above differ by nearly 13 percentage points. We have looked at three possible causes of the difference in f_{regain} values:

- o The presence of heat inputs to the buffer space other than duct losses--specifically direct losses from the furnace to the basement;
- o Departure of the actual value of T_c from the assumed 68 F;
- o Departure of the actual value of T_x from the assumed 48 F.

Furnace Losses to the Buffer Space. Because of the structure of Equation 18, it is possible for a source of heat to the buffer space other than duct losses to influence the value of f_{regain} calculated from the basement temperature drop, even if this other source is the same in the alternative case as it is in the base case. Direct losses from the furnace (jacket losses, plenum losses, and direct losses from that part of the stack that passes through the basement) are the major possibility for such a source. We now calculate a reasonable estimate for this effect.

Table 6 of ASHRAE 1992 enables us to conclude that in the base case, these direct furnace losses to the basement (labeled QJAC, QPLEN, and QSTACB) totaled 5.5% of total energy input to the furnace. Although these numbers are not given in such detail for the other cases, we will assume that the percentage of input energy lost directly to the basement remains nearly constant across the cases considered. That is, an amount of heat q given by the expression

$$q = \frac{0.055 L}{E_F E_D F_{MG} F_{LM}} \quad (23)$$

should be added to Q when doing the computations shown in Table 3. Steps of the procedure are shown in Table 4.

Table 4. Impact on the Thermal Regain Fraction of Direct Furnace Losses to the Basement .

Case Duct Insul. Leakage % Quantity	BC-2 None 10	AC-5 R5 10	AC-7 R5 Int 10	AC-1 None 0	AC-4 R5 0
β (from Table 3)	---	0.924	0.864	0.949	0.869
q/L (see Note)	0.120	0.114	0.109	0.116	0.111
Q/L (Table 3)	0.642	0.460	0.304	0.527	0.342
$Q_m = (Q+q)/L$	0.762	0.574	0.413	0.643	0.453
$(\beta Q_m - Q_m') / (Q_m - Q_m')$	---	0.692	0.703	0.673	0.677
$(\alpha-1)/(\alpha-\beta)$ (from Table 3)	0.990	0.990	0.990	0.990	0.990
f_{regain}	---	0.685	0.696	0.666	0.670

Note: $q = 0.055 L / ((E_F E_D F_{MG} F_{LM}))$

Including the direct furnace losses causes the average calculated value of f_{regain} to drop from 0.72 to 0.68, even though these losses are nearly constant from case to case.

Error Caused by Uncertainty in Conditioned-Space Temperature.
To investigate the impact of errors in T_c on the value of the thermal regain fraction calculated using basement temperature drops, we proceed to derive an expression for the fractional change in f_{regain} for each degree of error in T_c .

As a preliminary, we define $\gamma = (\alpha-1)/(\alpha-\beta)$ and proceed to evaluate this quantity in terms of the temperatures:

$$\gamma = \frac{\alpha-1}{\alpha-\beta} = \frac{\frac{T_c-T_b'}{T_c-T_b} - \frac{T_c-T_b}{T_c-T_b}}{\frac{T_c-T_b'}{T_c-T_b} - \frac{T_b'-T_x}{T_b-T_x}} \quad (24)$$

$$= \frac{T_b-T_x}{T_c-T_x}$$

Then the derivative of γ with respect to T_c is

$$\frac{d\gamma}{dT_c} = - \frac{T_b-T_x}{(T_c-T_x)^2} \quad (25)$$

Because f_{regain} is equal to γ multiplied by an expression that does not contain T_c , it is the case that

$$\frac{1}{f_{\text{regain}}} \frac{df_{\text{regain}}}{dT_c} = \frac{1}{\gamma} \frac{d\gamma}{dT_c}$$

$$= - \frac{1}{T_c-T_x} \quad (26)$$

This is easily evaluated; in fact it is the same for all of the above cases because T_c and T_x are the same for all. Substituting $T_c=68$ F and $T_x=48$ F yields $f_{\text{regain}}^{-1} df_{\text{regain}}/dT_c = -0.05$. This means that for every degree F error in T_c there is a 5% error in f_{regain} . The minus sign means that if the actual value of T_c were greater than 68 F, this would move the BTD value (Table 4) of f_{regain} closer to the Table 1 value obtained with performance factors only.

Error Caused by Uncertainty in the Sink Temperature. We follow a similar procedure to evaluate $f_{\text{regain}}^{-1} df_{\text{regain}}/dT_x$. It is a little more complicated, since both parts of the expression for f_{regain} are functions of T_x . We can make use of the following:

Using these facts and Equation 18 for f_{regain} , one obtains (after some algebra)

$$\begin{aligned}\frac{dy}{dT_x} &= -\frac{T_c - T_b}{(T_c - T_x)^2} \\ \frac{d\beta}{dT_x} &= -\frac{T_b - T_b'}{(T_b - T_x)^2} \quad (27) \\ \frac{d}{dT_x} \frac{\beta Q - Q'}{Q - Q'} &= \left(\frac{Q}{Q - Q'} \right) \frac{d\beta}{dT_x}\end{aligned}$$

$$\begin{aligned}\frac{1}{f_{\text{regain}}} \frac{df_{\text{regain}}}{dT_x} &= \\ &= -\frac{1}{T_b - T_x} \left[\frac{T_c - T_b}{T_c - T_x} + \frac{T_b - T_b'}{T_c - T_x} \frac{Q}{Q - Q'} \frac{1}{f_{\text{regain}}} \right] \quad (28)\end{aligned}$$

This quantity is calculated for each of the alternative cases, relative to the base case, as shown in Table 5.

Table 5. Calculation of Uncertainty in f_{regain} Due to Error in T_x .

Case Duct Insul. Leakage % Quantity	BC-2 None 10	AC-5 R5 10	AC-7 R5 Int 10	AC-1 None 0	AC-4 R5 0
$1/(T_b - T_x)$	0.050				
$T_c - T_x$	20.0				
$T_c - T_b$	0.2				
$T_b - T_b'$		1.5	2.7	1.0	2.6
$Q_m / (Q_m - Q_m')$		4.05	2.18	6.40	2.47
f_{regain}		0.685	0.696	0.666	0.670
$(T_c - T_b) / (T_c - T_x)$	0.01				
$\frac{(T_b - T_b') Q}{(T_c - T_x)(Q - Q') f_{\text{regain}}}$		0.44	0.42	0.48	0.48
$f_{\text{regain}}^{-1} df_{\text{regain}} / dT_x$		0.022	0.022	0.024	0.024

The result of these calculations is that, consistently over the four alternative cases, relative to the base case, there is about a 2.3% error in f_{regain} for each degree F uncertainty in T_x . It is fortunate that the error percentage per degree is less with T_x than with T_c , because the uncertainty in T_x in this case is (intuitively) larger than that in T_c . Even a 5 F uncertainty in T_x still yields

only an 11% uncertainty in f_{regain} , which brackets it within a band of about seven percentage points.

SUMMARY

This article presented two approaches to determining the fraction of duct losses to a basement or other partially conditioned buffer space. One approach is based on relative thermal resistances and the other is based on the drop in buffer-space temperature following a duct retrofit or a temporary switch to a co-heating system as part of a test.

This was followed by application of these methods to published data from the ASHRAE SP-43 Project. Results from the basement-temperature drop approach and from a direct calculation based on system performance factors yielded thermal regain fractions of 0.68 and 0.59, respectively.

REFERENCES

1. Andrews, J. Test Procedures and Protocols: Their Relevance to the Figure of Merit for Thermal Distribution Systems. BNL 60317, September, 1993.
2. ASHRAE 1992. American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Atlanta, GA. 1992 HVAC Systems and Equipment Handbook, Chapter 29.
3. Batey, J. and McDonald, R., 1994. Oil Heat Technology Efficiency Training Manual. In press.
4. Jakob, F.E., R.D. Fischer, L.J. Flanigan, R.A. Cudnik, and D.W. Locklin. Dynamic and Seasonal Performance of Central Forced-Air Systems Installed in Residences. Summary Report of ASHRAE Special Project SP-43: Phases I and II. Heating. Battelle Columbus Division, Columbus, OH, June 1987.