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MULTIPHASE FLOW IN FRACTURED POROUS MEDIA

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CHAPTER 1

INTRODUCTION

The research work on fluid flow in porous media in the past several decades has been mainly focused on, 1) single phase flow in homogeneous, layered, and fractured media, and 2) two- and three-phase flow in homogeneous media. Major issues of two-phase immiscible and miscible flow in highly layered and especially fractured media are still unresolved. Since fractured porous media are comprised of matrix blocks and fracture network, the drainage and imbibition performance of a single matrix block have sometimes little relevancy to two-phase flow in fractured porous media. As an example, the drainage performance of a stack of matrix blocks is very different from the drainage performance of the individual matrix blocks. The main reason for the difference is the interaction between the matrix blocks as a result of two-phase flow in the fractures. Our work has shown conclusively that fracture capillary pressure, for example, is a key issue of gas-oil drainage in fractured porous media.¹⁻³

The major goal of this research project was to improve the understanding of the gas-oil two-phase flow in fractured porous media. Since our studies in the initial stage of this project revealed that gas-oil gravity drainage may not be an efficient process in some fractured porous media, miscible displacement process was also investigated. The main reason for the study of miscible displacement in fractured porous media was to evaluate its promise for improved recovery.

This report contains our wide-ranging investigation of miscible and immiscible flow in fractured porous media. In addition to miscible and immiscible displacement in fractured porous media, the issue of the solution gas drive process is also investigated. A major parameter of solution-gas-drive process is the critical gas saturation. This parameter strongly influences the oil recover of fractured reservoirs containing under saturated oil.

In Chapter 2, the evolution gas phase in a solution gas drive process is presented. The theory of gas phase formation and gas phase expansion are also discussed.

Chapter 3, presents our theoretical and experimental work on the topic of miscible displacement in fractured porous media. The results from both theory and experiments reveal that miscible displacement is an efficient displacement process. Concerns about production of large quantities of the injected fluid are ill-founded.

Chapter 4, discusses the major issues of immiscible gas-oil gravity drainage. All the results are based on experimental observations. The issues of fracture liquid transmissibility and reinfiltration are dealt with in detail.

In Chapter 5, our theoretical analysis of film flow in a two-phase gas-oil gravity drainage process describes the nature of liquid flow in a fracture. Based on theoretical analysis, numerical simulation follows. The analysis of reinfiltration process is also presented in Chapter 5.

Chapter 6, addresses the issue of gas-oil gravity drainage in layered porous media. The similarity between gas-oil gravity drainage in fractured porous media, and layered media is the most important feature of these two different permeable media.

References

1. Horie, T., Firoozabadi, A., and Ishimoto, K.: "Laboratory Studies of Capillary Interactions in Fracture/Matrix Systems," SPE Reservoir Engineering (Aug. 1990) 353-360.
2. Firoozabadi, A., and Hauge, J.: "Capillary Pressure in Fractured Porous Media," Journal of Petroleum Technology (June 1990) 784-791.
3. Firoozabadi, A., and Thomas, L.K.: "Sixth SPE Comparison Solution Project: Composition of Dual-Porosity Reservoir Simulators," Journal of Petroleum Technology (June 1990)

CHAPTER 2

SOLUTION GAS DRIVE MECHANISM

OVERVIEW

The formation of gas phase in porous media and its expansion prior to flow is important in solution gas drive process for unfractured and especially for fractured reservoirs. Prior to gas mobilization, capillary pressure has no effect on flow, and therefore the single phase flow equations prevail in the fractures.

In order to determine the gas saturation prior to mobilization (i.e., critical gas saturation), laboratory measurements may not apply to field conditions. Gas phase formation is a non-equilibrium process and, therefore, the amount of gas saturation may depend on rate of pressure decline. In this chapter, the process of gas evolution in porous media will be studied in detail.

2.1 KINETICS OF THE INITIAL STAGE OF ISOTHERMAL GAS PHASE FORMATION

Dimo Kashchiev and Abbas Firoozabadi

I. INTRODUCTION

Gas phase formation in supersaturated liquids is a first-order phase transition which occurs by bubble nucleation, growth and coalescence. Initially, the bubbles are small and far from each other and coalescence is of no importance. The kinetics of the initial stage of the process are therefore describable only in terms of the rates of bubble nucleation and growth. A description leading to better understanding of the overall process of gas phase formation is important not only scientifically, but also for the numerous applications of this process in technology. In petroleum engineering, for example, there is a need to know how gas builds up in oil reservoirs, since the factors controlling the process are directly related to the efficiency of oil recovery.¹⁻⁶

So far, considerable attention has been paid to the study of the rates of bubble nucleation⁷⁻¹² and growth.^{10,13-18} In contrast, relatively little is known about the role played by these quantities in the overall process of gas phase formation. This role is well understood and successfully analyzed in other cases of new phase formation, e.g., crystallization of melts and precipitation in solutions.¹⁹⁻²¹ It may be expected that the results known from the analysis of these cases can be a good basis for describing also the kinetics of gas phase formation.

The aim of this paper is (i) to develop a theoretical framework for description of the initial stage of the overall process of gas phase formation, (ii) to give a general expression for the time dependence of the total volume of the newly formed gas phase, and (iii) to apply the general results for determination of the critical supersaturation in the particular cases of isothermal gas phase formation at constant and steadily declining pressure.

II. PHYSICAL MODEL

We consider a liquid phase of volume V_0 at a constant absolute temperature T . At the initial moment $t=0$ the liquid is put under pressure $p \leq p_e$, $p_e(T)$ being the equilibrium pressure, i.e. the pressure at which the liquid and the corresponding gas phase can coexist. As time goes on, p may either remain constant or vary with time, but under the condition that it is always below p_e . In other words, the liquid is always supersaturated, the supersaturation s being defined as

$$s = p_e - p. \quad (2.1)$$

Thermodynamically, s is related to the driving force $\Delta\mu \equiv \mu_l(p) - \mu_g(p)$ for gas phase formation by the approximate formula $\Delta\mu = kT \ln(p_e/p)$ in which k is the Boltzmann constant, and μ_l and μ_g are the chemical potentials of the bulk liquid and gas phase, respectively. At $p = p_e$ there is no driving force for gas phase formation ($s=0$, $\Delta\mu=0$) and it is said that the liquid is saturated. Gas phase formation is motivated energetically only for $p < p_e$ when the liquid is supersaturated, i.e. when $s > 0$ so that $\Delta\mu > 0$. Under such conditions the build-up of the gas phase occurs by nucleation, growth and coalescence of bubbles into the bulk of the liquid and/or on the surface of the solid or liquid "walls" confining the liquid. At the beginning of the process coalescence is of no importance, since the bubbles are relatively small and sufficiently far from each other. The initial stage of the overall process of gas phase formation is, therefore, describable only in terms of the nucleation rate $J(\text{m}^{-3}\text{s}^{-1})$ and the growth rate $G \equiv dr/dt$ of the bubbles, r being the bubble radius.

III. RATE OF BUBBLE NUCLEATION

A. Constant supersaturation

When the liquid is kept at a constant supersaturation s , the rate J of bubble nucleation may be time-independent. Then it is reliably expressed by the stationary rate J_0 of bubble nucleation, which fairly generally may be written down as⁷⁻¹²

$$J_0 = A \exp(-W^*/kT) = A \exp(-B/s^2). \quad (2.2)$$

This formula applies to both single- and multicomponent liquids provided the pressure inside the nucleus bubble is practically equal to p_e . The quantities $A(\text{m}^{-3}\text{s}^{-1})$ and $B(\text{Pa}^2)$ are kinetic and thermodynamic parameters, respectively, related to the elementary act of nucleus growth and the nucleation work W^* . They are given by

$$A = \epsilon f^* Z \quad (2.3)$$

$$B = 16\pi\sigma_{ef}^3/3kT \quad (2.4)$$

where $\epsilon \approx 0.01$ to 0.1 is the so-called Zeldovich factor, $f^*(\text{s}^{-1})$ is the frequency of increasing the nucleus volume by the volume v_m of a single molecule, $Z(\text{m}^{-3})$ is the number density of points in space on which a bubble can be nucleated, and $\sigma_{ef}(\text{Jm}^{-2})$ is an effective specific surface energy.

Different physical conditions lead to different expressions and values for f^* , Z and σ_{ef} (and, thereby, for A and B),⁷⁻¹² but an important point is that A and B may be considered as practically independent of s . In the case of homogeneous nucleation of spherical bubbles σ_{ef} equals the specific surface energy σ of the gas/liquid interface and $Z = 1/v_m \approx 10^{28} \text{ m}^{-3}$. In this

case, for evaporation-mediated increase of the volume of the nucleus bubble f^* is given by the Knudsen-type formula⁹ $f^* = 4\pi r^{*2} p_e / (2\pi m k T)^{1/2}$ in which m is the molecular mass and $r^* \approx 10$ nm is the radius of the nucleus bubble. Thus, at $T=300$ K, with values of $\varepsilon=0.01$, $p_e=10^3$ to 10^7 Pa and $\sigma_{ef}=\sigma=3$ to 70 mJm⁻², from Eqs. (2.3) and (2.4) it follows that $A=10^{36}$ to 10^{40} m⁻³s⁻¹ and $B=10^{14}$ to 10^{18} Pa² for homogeneous bubble nucleation.

Another case to note is the heterogeneous nucleation of bubbles, for instance, in a porous medium. If the medium is regarded as constituted of monodisperse grains (or droplets) of radius R and number density N_g (m⁻³) and if the nucleus bubbles are formed on the surface of the grains, σ_{ef} is known⁷⁻¹² to be given by $\sigma_{ef}=\phi^{1/3}\sigma$. The correction factor ϕ is defined by $\phi=V_{het}^*/V^*$ and is a number between 0 and 1, since the volume V_{het}^* of the heterogeneous nucleus bubble is always smaller than the volume $V^*=(4/3)\pi r^{*3}$ of the corresponding homogeneous nucleus bubble. This factor is a function of the grain radius R and of the liquid/grain wetting angle θ which is expressed by the Young formula

$$\cos\theta = (\sigma_l - \sigma_g)/\sigma$$

in terms of σ and the specific surface energies σ_l and σ_g of the liquid/grain and gas/grain interfaces, respectively. If the grains are large enough ($R \gg r^*$), ϕ is the following function of θ only⁷⁻¹²

$$\phi(\theta) = (1/4)(2 + 3\cos\theta - \cos^3\theta)$$

which shows that $\phi=1$, $1/2$ and 0 at complete ($\theta=0$), "half" incomplete ($\theta=\pi/2$) and no ($\theta=\pi$) wetting, respectively. This means that heterogeneous bubble nucleation is thermodynamically favored only when the liquid does not wet completely the grains, as only then is $\phi < 1$.

In the case of heterogeneous nucleation, a decrease in f^* is also possible because of the reduction of the area of the gas/liquid interface of the nucleus bubble. Also, Z will be given by $Z=4\pi R^2 Z_s N_g$, where Z_s (m⁻²) is the number density of these points on the grain surface on which heterogeneous bubbles can be nucleated. For densely packed grains $N_g \approx 1/(4/3)\pi R^3$, so that $Z \approx Z_s/R$. In the absence of nucleation-active sites on the surface of the grains $Z_s=1/s_a \approx 10^{19}$ m⁻² (s_a is the atomic area). This means that $Z \approx 10^{25}$ m⁻³, e.g. for grains of radius $R=1$ μ m. This is less than Z for homogeneous nucleation and can be further reduced to $Z=N_g \approx R^{-3}$ if every grain provides one nucleation-active site only (then $4\pi R^2 Z_s=1$). The general conclusion is, therefore, that for heterogeneous bubble nucleation the values of both A and B are smaller than those for homogeneous nucleation. For bubbles nucleated heterogeneously in a porous medium, A may increase with decreasing grain radius R to a power from 1 to 3.

B. Variable supersaturation

When nucleation takes place at variable supersaturation, in general, J is a complicated function of time t (Refs. 22-24). For slow enough changes of the supersaturation, however, J can be approximated by the quasi-stationary nucleation rate corresponding to the instantaneous values of the supersaturation. For bubble nucleation at time-dependent supersaturation $s(t)$ we shall, therefore, have $J(t)=J_0[s(t)]$ so that in view of Eq. (2.2)

$$J(t) = A \exp[-B/s^2(t)]. \quad (2.5)$$

According to Refs. 23 and 24, this formula is valid when the rate ds/dt of supersaturation change is low enough to satisfy the condition

$$|d\Delta\mu/dt| \ll kT/n^* [s(t)] \tau_n [s(t)]$$

which with $\Delta\mu=kT \ln(p_e/p)$ takes the form

$$|ds/dt| \ll p/n^* [s(t)] \tau_n [s(t)]. \quad (2.6)$$

Here n^* is the number of molecules in the nucleus bubble, and $\tau_n=4/\pi^3 \epsilon^2 f^*$ (Refs. 24 and 25) is the nucleation time-lag. For $p>0.1p_e$, with values of $p_e=10^3$ Pa, $n^*=100$ and $\tau_n=1 \mu s$ it thus follows from inequality (2.6) that Eq. (2.5) is a good approximation if

$$|ds/dt| \ll 10^6 \text{ Pa/s}, \quad (2.7)$$

a requirement which is usually met in practice.

IV. RATE OF BUBBLE GROWTH

After a bubble has surpassed the nucleus size (i.e. when $r>r^*$), it can grow irreversibly with a certain growth rate G . It is generally accepted^{10,13-15} that for an isolated supernucleus bubble G is controlled by mass, momentum and/or heat transfer across the bubble/liquid interface. Mass transfer occurs via evaporation and condensation at this interface and/or diffusion in the liquid when this is multicomponent, momentum transfer is governed by hydrodynamic forces connected with the bubble capillary pressure and the liquid inertia and viscosity, and heat transfer takes place by flow of heat from the liquid towards the bubble. It has been found^{10,13-18} that the different regimes of growth result in different $r(t)$ dependences which, in general, are rather complicated. However, when the effects of the bubble/liquid interface curvature on chemical potential are neglected, at constant supersaturation many of these dependences show exact or approximate proportionality of r to some power of t . This finding is supported also by available experimental $r(t)$ data. For example, the rate of bubble growth controlled only by the inertia of the liquid has a constant value G_0 given by the formula

$$G_0 = (2v_m s / 3m)^{1/2}. \quad (2.8)$$

following from the Rayleigh equation.¹⁶ Hence, in this regime the bubble radius obeys a linear growth law as long as s remains time-independent: neglecting the bubble initial radius $r(0)$ which is just above the nucleus radius r^* , we have

$$r(t) = (2v_m s / 3m)^{1/2} t. \quad (2.9)$$

For instance, with $m/v_m = 1 \text{ g/cm}^3$, Eq. (2.8) yields $G_0 = 8 \text{ m/s}$ at $s = 10^5 \text{ Pa}$. In the same regime, in the important case of supersaturating the liquid at constant rate $c(\text{Pa/s})$, i.e. when

$$s(t) = p_e - p = ct, \quad (2.10)$$

Eq. (2.8) predicts the following time-dependent growth rate if c is small enough:

$$G(t) = (2v_m c / 3m)^{1/2} t^{1/2}. \quad (2.11)$$

Accordingly, for the bubble growth law we find

$$r(t) = (8v_m c / 27m)^{1/2} t^{3/2} \quad (2.12)$$

provided $r(0) \equiv 0$.

Another case characterized by power dependences of G and r on t is the case of bubble growth in the regime of mass transfer. For instance, for bubble growth controlled by evaporation and condensation, at constant s the growth rate is also constant and when $s \ll p_e$, it is given by⁹

$$G_0 = (kTs^2 / 2\pi m p_e^2)^{1/2}. \quad (2.13)$$

Accordingly, the bubble radius obeys a linear growth law:

$$r(t) = (kTs^2 / 2\pi m p_e^2)^{1/2} t \quad (2.14)$$

provided $r(0) \equiv 0$. For example, with $m = 10^{-22} \text{ g}$, at $T = 300 \text{ K}$ and $s/p_e = 0.1$, Eq. (2.13) yields $G_0 = 8 \text{ m/s}$. Equations (2.13) and (2.14) change in the case of increasing supersaturation at constant, sufficiently low rate c : using Eq. (2.10) in Eq. (2.13) results in a time-dependent growth rate of the form ($ct \ll p_e$)

$$G(t) = (kTc^2 / 2\pi m p_e^2)^{1/2} t \quad (2.15)$$

which for $r(0) \equiv 0$ corresponds to the following growth law:

$$r(t) = (kTc^2 / 8\pi m p_e^2)^{1/2} t^2. \quad (2.16)$$

Again in the regime of mass transfer, when growth is diffusion-controlled and $dr/dt = DK_s kTs / p_e r$, at constant supersaturation $s \ll p_e$ known dependences are^{14,18}

$$G(t) = (DK_s kTs / 2p_e)^{1/2} t^{-1/2} \quad (2.17)$$

$$r(t) = (2DK_s kTs / p_e)^{1/2} t^{1/2} \quad (2.18)$$

provided $r(0)$ is negligible. Here D is the diffusion coefficient of dissolved gas molecules, and K_s is the solubility constant in the supposed Henry-type proportionality between pressure and gas concentration in the liquid. If, e.g., $T = 300 \text{ K}$, $D = 10^{-9} \text{ m}^2/\text{s}$ and $K_s = 10^{19} \text{ Pa}^{-1} \text{ m}^{-3}$, at $s/p_e = 0.1$ and $t = 1 \text{ s}$, Eq. (2.17) predicts $G = 1 \text{ } \mu\text{m/s}$. When the supersaturation increases steadily but slowly enough, in view of Eq. (2.10), as long as $ct \ll p_e$, the growth rate from Eq. (2.17)

will change to a time-independent one given by

$$G_0 = (DK_s k T c / p_e)^{1/2}, \quad (2.19)$$

and the growth law Eq. (2.18) will become linear ($r(0)$ is again neglected):

$$r(t) = (DK_s k T c / p_e)^{1/2} t. \quad (2.20)$$

Having in mind the above results, it seems reasonable to assume that at both constant and linearly increasing supersaturation the rate of bubble growth may be represented (accurately or approximately) in the following fairly general form

$$G(t) = v K_s \omega_t^{v-1} \quad (2.21)$$

for constant supersaturation and

$$G(t) = (\omega + v) K_c \omega_t^{\omega+v-1} \quad (2.22)$$

for steadily increasing supersaturation. These growth rates correspond to power-type growth laws given by

$$r(t) = K_s \omega_t^v \quad (2.23)$$

$$r(t) = K_c \omega_t^{\omega+v} \quad (2.24)$$

for constant and steadily increasing supersaturation, respectively. Here the powers $v > 0$ and $\omega > 0$ and the growth constant K have to be specified for each particular regime of growth. As before, Eqs. (2.21)-(2.24) imply that the supersaturating rate c is low enough and that the bubble initial radius $r(0)$ is vanishingly small. Also, the $v=1$ and $\omega+v=1$ cases correspond to bubbles growing linearly with time at a constant growth rate $G_0 = K_s \omega$ or $K_c \omega$.

V. GENERAL FORMULATIONS

Knowing the bubble nucleation and growth rates J and G makes it possible to determine theoretically the following important characteristics of the overall process of gas phase formation:

- The number N of bubbles nucleated in the liquid until time t :

$$N(t) = V_0 \int_0^t J(t') dt'. \quad (2.25)$$

This formula is valid until the initial volume V_0 of the liquid is not significantly exhausted and neglects the compressibility of the liquid at time-dependent s . These two restrictions apply also to Eqs. (2.26)-(2.28) and (2.30) below and to all results following from them.

- The probability P to form at least one nucleus bubble until time t :

$$P(t) = 1 - \exp[-N(t)] = 1 - \exp[-V_0 \int_0^t J(t') dt']. \quad (2.26)$$

This formula was derived by Toshev et al.²⁶ by using the fact that the nucleation events are random in time and obey the Poisson distribution law. With the help of Eq. (2.26), the mean time τ for the appearance of at least one nucleus bubble can be found:

$$\tau \equiv \int_0^{\infty} t dP(t) = \int_0^{\infty} \exp[-V_0 \int_0^t J(t') dt'] dt. \quad (2.27)$$

- The total volume V_g of the gas phase at time t :

$$V_g(t) = V_0 \int_0^t J(t') v_b(t, t') dt'. \quad (2.28)$$

Here $v_b(t, t')$ is the volume at time t of a bubble nucleated at time $t' \leq t$ and is given by

$$v_b(t, t') = a r^3(t-t') = a \left[\int_{t'}^t G(t''-t') dt'' \right]^3 \quad (2.29)$$

provided the initial volume of the bubble is vanishingly small (a is a time-independent shape factor, e.g. $a=4\pi\phi(\theta)/3$ for bubbles of equilibrium shape, and $r(t-t')$ is the effective radius at t of a bubble formed at $t' \leq t$). Eq. (2.28) corresponds to the case of the so-called progressive nucleation (PN) when in the liquid new bubbles are continuously nucleated among the growing ones. A limiting case of Eq. (2.28) is the case called instantaneous nucleation (IN) when bubble nuclei of number density $N_0(\text{m}^{-3})$ are formed at once at $t'=0$, so that later they only grow. In this case nucleation and growth are decoupled in time and, since $J(t')$ is formally expressed by the Dirac δ -function as $J(t')=N_0\delta(t')$, Eq. (2.28) becomes

$$V_g(t) = V_0 N_0 v_b(t, 0). \quad (2.30)$$

Here, according to Eq. (2.29),

$$v_b(t, 0) = a r^3(t) = a \left[\int_0^t G(t'') dt'' \right]^3 \quad (2.31)$$

when the initial volume of the nucleus bubble is negligible.

The above formulations are general enough to describe the initial stage of the overall process of gas phase formation in many cases of physical significance. In every concrete case the dependences of J and G on time t , supersaturation s and the material parameters of the studied system have to be specified in order to obtain the corresponding dependences of N , P , τ and V_g . An important case to note is the case of gas phase formation at time-independent rates J_0 and G_0 of nucleation and growth. Such a case may be realized when the liquid is kept under constant pressure $p \leq p_e$, so that the supersaturation s does not change with time. In this case Eqs. (2.25)-(2.31) lead to the following simple kinetic dependences:

$$N(t) = V_0 J_0 t \quad (2.32)$$

$$P(t) = 1 - \exp(-V_0 J_0 t) \quad (2.33)$$

$$\tau = 1/V_0 J_0 \quad (2.34)$$

$$V_g(t) = (a/4) V_0 J_0 G_0^3 t^4 \quad (2.35)$$

$$V_g(t) = a V_0 N_0 G_0^3 t^3. \quad (2.36)$$

These equations are well known in other cases of new phase formation.^{19-21,26,27} Equations (2.35) and (2.36) show that separate information about the kinetics of bubble nucleation and growth cannot be obtained by analyzing the $V_g(t)$ dependence only, since J_0 and G_0 or N_0 and G_0 appear combined into a single kinetic parameter governing the gas phase build-up in the case of PN or IN, respectively. We emphasize, however, that in the latter case a complete determination of G_0 is possible if N_0 is independently known, e.g., if it equals the number density of sufficiently small particles (seeds) introduced deliberately into the liquid as nucleation-active sites generating one nucleus each. A similar process of crystal phase formation in solutions is known as seeded precipitation and has been recently used for determination of crystal growth rates.²⁸⁻³⁰

VI. CRITICAL SUPERSATURATION FOR GAS PHASE FORMATION

Since the bubbles do not nucleate and grow infinitely fast, some time always elapses from the moment of supersaturating the liquid until the appearance of the gas phase. This so-called induction time (or period) depends on the supersaturation and on the experimental technique used for detecting the first portion of the newly formed gas phase. That is why, given the experimental technique, a critical supersaturation $s_c \equiv p_e - p_c$ exists at which the induction time equals a previously specified arbitrarily chosen value (p_c is the corresponding critical supersaturation pressure). Physically, this means that for supersaturations between $s=0$ and $s=s_c$ the liquid remains effectively in a metastable state. It must be stressed, however, that the critical supersaturation is not a universal quantity, since it depends on how it is theoretically defined and experimentally measured. We shall now consider three possible definitions of the critical supersaturation, which are relevant to different experimental conditions and techniques.

(a) The critical supersaturation to obtain one nucleus bubble.

This definition originates from Volmer⁷ and reads $N(t_1)=1$, where t_1 is the moment of appearance of the first nucleus bubble, i.e. the induction time. In view of Eq. (2.25), it may be represented in the form

$$1 = V_0 \int_0^{t_1} J(t') dt'. \quad (2.37)$$

As seen, knowing the time dependences of J and s allows determination of the so-defined

critical supersaturation s_c for an arbitrarily chosen value of t_1 .

(b) The critical supersaturation to form on the average at least one nucleus bubble.

This definition is in fact given by Eq. (2.27):

$$\tau = \int_0^\infty \exp[-V_0 \int_0^t J(t') dt'] dt. \quad (2.38)$$

Again, choosing arbitrarily a value for the mean time τ for appearance of at least one nucleus bubble (which is now the induction time) makes it possible to determine the so-defined critical supersaturation s_c .

(c) The critical supersaturation to form a detectable gas phase volume V_d which is a given fraction $\alpha = V_d/V_0$ of the initial volume of the liquid.

This definition reads $\dot{V}_g(t_v) = V_d$, t_v being the moment at which the total volume of the gas phase is V_d and having the physical significance of induction time. Using Eqs. (2.28) and (2.30) leads to the following formulae in the cases of PN and IN, respectively:

$$\alpha = \int_0^{t_v} J(t') v_b(t_v, t') dt' \quad (2.39)$$

$$\alpha = N_0 v_b(t_v, 0). \quad (2.40)$$

As before, determination of the so-defined critical supersaturation s_c is possible upon assigning an arbitrary value to t_v . However, in the PN case it is necessary to know not only the time dependences of J and s , but also of v_b (i.e. of G). In the case of IN, only knowledge of v_b and s as functions of t is required.

Each of the above three definitions is physically acceptable, but has a limited experimental application. The use of definitions (a) and (b) is restricted to experimentation with techniques which are so sensitive that the detection of the very first nucleus bubble is possible. Using these definitions is also correct in the case of gas phase formation by the so-called mononuclear mechanism (e.g. Ref. 27) which is operative for such small V_0 and J and/or large G that after the appearance of the first nucleus bubble virtually no time is lost for the formation of the detectable volume V_d of the gas phase. For time-independent nucleation and growth rates the condition for the operativeness of the mononuclear mechanism is²⁷

$$(J_0/G_0) \alpha^{1/3} V_0^{4/3} \ll 1. \quad (2.41)$$

Physically, definitions (a) and (b) are closely related and become equivalent in the case of stationary nucleation at constant rate J_0 , as then Eqs. (2.37) and (2.38) lead to identical expressions for t_1 and τ and, accordingly, to the same critical supersaturation.

Definition (c) has a much less restricted experimental usage, since it is relevant to techniques detecting also a macroscopic volume V_d of the newborn gas phase. The condition

to apply it is the gas phase to be formed by the so-called polynuclear mechanism (e.g. Ref. 27). This mechanism is almost always effective in practice, since it corresponds to such large V_0 and J and/or small G that many bubbles nucleate and grow in the liquid until the detection of V_d . In the case of time-independent nucleation and growth rates the condition for the operativeness of the polynuclear mechanism is the opposite to Eq. (2.41) (Ref.27):

$$(J_0/G_0)\alpha^{1/3}V_0^{4/3} \gg 1. \quad (2.42)$$

An important point concerning the above definitions is that at a given choice of J , G and V_0 they may predict substantially different values of s_c . This means that uncritical employment of any of these definitions for analysis of experimental data may result in misleading conclusions about the mechanism of bubble nucleation and growth and/or very erroneous values for the parameters characterizing these processes.

VII. PARTICULAR CASES

A. Constant supersaturation

1. Kinetic dependences

We shall now apply the general results from Section V to the case of isothermal gas phase formation at constant supersaturation s . This case is realized when at $t=0$ the liquid is put under pressure $p < p_e$ which is afterwards kept constant so that $s > 0$ for $t \geq 0$. This condition allows stationary bubble nucleation to take place at time-independent rate J_0 given by Eq. (2.2). Assuming that the bubble growth rate and radius can be expressed by Eqs. (2.21) and (2.23), from Eqs. (2.25)-(2.31) we find explicitly the dependences of N , P and V_g on supersaturation and time and of τ on supersaturation:

$$N(t) = V_0 A \exp(-B/s^2)t \quad (2.43)$$

$$P(t) = 1 - \exp[-V_0 A \exp(-B/s^2)t] \quad (2.44)$$

$$\tau = (1/V_0 A) \exp(B/s^2) \quad (2.45)$$

$$V_g(t) = [a/(3v+1)] V_0 A K^3 s^3 \omega \exp(-B/s^2) t^{3v+1} \quad (2.46)$$

$$V_g(t) = a V_0 N_0 K^3 s^3 \omega t^{3v}. \quad (2.47)$$

Equations (2.46) and (2.47) pass into Eqs. (2.35) and (2.36), respectively, when the bubble growth rate is constant (i.e. at $v=1$). Also, they show that the total gas volume V_g depends more strongly on t and especially on s in the PN than in the IN case.

2. Critical supersaturation

We can now apply definitions (a)-(c) from Section VI to the above equations in order to determine the critical supersaturation for gas phase formation. Since J_0 is time-independent,

$t_1 = \tau$ so that definitions (a) and (b) are equivalent and in conjunction with Eqs. (2.43) and (2.45) yield the same result:

$$\begin{aligned} t_1 &= (1/V_0 A) \exp(B/s^2) \\ s_c &= [16\pi/3kT \ln(V_0 A t_1)]^{1/2} \sigma_{ef}^{3/2}. \end{aligned} \quad (2.48)$$

Here s_c is the critical supersaturation for the appearance of the first nucleus bubble and, as seen, it decreases with decreasing σ_{ef} and/or increasing T , but depends relatively weakly on the nucleation kinetic factor A , the initial volume V_0 of the liquid and the choice of value for t_1 . Eq. (2.48) is known from the classical nucleation theory^{7,9} and has been found to be in conformity with experiment.^{7,9,11,12}

According to definition (c), from Eqs. (2.46) and (2.47) we find that the induction time t_v and the critical supersaturation s_c to form a detectable volume $V_d = \alpha V_0$ of the gas phase are given by

$$\begin{aligned} t_v &= [\alpha(3v+1)/aAK^3 s_c^{3\omega}]^{1/(3v+1)} \exp[B/(3v+1)s_c^2] \\ s_c &= \{16\pi/3kT \ln[aAK^3 s_c^{3\omega} t_v^{3v+1}/(3v+1)\alpha]\}^{1/2} \sigma_{ef}^{3/2} \end{aligned} \quad (2.49)$$

in the case of PN and by

$$\begin{aligned} t_v &= (\alpha/aN_0 K^3)^{1/3} v_s^{-\omega/v} \\ s_c &= (\alpha/aN_0 K^3 t_v^{3v})^{1/3\omega} \end{aligned} \quad (2.50)$$

in the case of IN. Although Eq. (2.49) is implicit in s_c , it shows that the dependence of the critical supersaturation on σ_{ef} and T is essentially the same as that from Eq. (2.48), since the logarithmic factor changes relatively little with s_c . Numerically, however, s_c from Eq. (2.49) may be considerably higher than s_c from Eq. (2.48) especially for smaller K -values when the bubble growth is relatively slow. This is seen in Fig. 1 where s_c from Eqs. (2.48) and (2.49) is plotted as a function of $\sigma_{ef} = \phi^{1/3} \sigma$ for heterogeneous nucleation and parabolic growth ($v=1/2$) of bubbles with $\omega=1/2$ (e.g., in the regime of mass transfer by diffusion according to Eq. (2.18)). The other parameters used are $\phi=1/2$ (hemispherical bubbles, $a=2\pi/3$), $\alpha=0.001$, $V_0=10^{-3} \text{ m}^3$, $A=10^{25} \text{ m}^{-3} \text{ s}^{-1}$, $t_1=t_v=100 \text{ s}$, $T=300 \text{ K}$, and K is given two values, 10^{-7} and $10^{-9} \text{ m/s}^{1/2} \text{ Pa}^{1/2}$ (as indicated in Fig. 1), to illustrate the effect of the growth rate on s_c . Fig. 1 shows the decrease of s_c from both Eqs. (2.48) and (2.49) with decreasing σ_{ef} and the shift of s_c from Eq. (2.49) towards s_c from Eq. (2.48) with increasing K (i.e. at faster bubble growth). The latter is understandable, as in accordance with Eqs. (2.41) and (2.42), at higher growth rates the polynuclear mechanism (resulting in s_c from Eq. (2.49)) is bound to be replaced by the mononuclear one (resulting in s_c from Eq. (2.48)). A general consideration of the interrelation between the critical supersaturations for the mononuclear and the polynuclear mechanisms is given elsewhere.²⁷

Finally, we note that in contrast to s_c from Eq. (2.49), Eq. (2.50) shows that when gas

phase formation occurs by IN, s_c is determined only by the parameters of bubble growth provided N_0 is s -independent, as it is for instance in the case of seeded new phase formation.²⁸⁻³⁰ Concerning the induction time t_v , we note that in the case of IN it is a much weaker (only power) function of s than in the case of PN. In its turn, the $t_v(s)$ dependence for PN is weaker than the $t_1(s)$ dependence because of the $3v+1$ factor in the exponential function.

B. Steadily increasing supersaturation

1. Kinetic dependences

The general results from section V are relatively easily applicable to the important case of gas phase formation when the supersaturation increases with time at a constant rate c . According to Eq. (2.10), this is the case, for instance, when the pressure in the liquid decreases steadily while p_e remains practically unchanged. For this case, with the help of Eqs. (2.5), (2.22) and (2.24), from Eqs. (2.25)-(2.31) we find the dependences of N , P and V_g on t and c and of τ on c in the following form:

$$N(t) = V_0 A t \{ \exp(-B/c^2 t^2) - [(\pi B)^{1/2}/ct] \operatorname{erfc}(B^{1/2}/ct) \} \quad (2.51)$$

$$P(t) = 1 - \exp[-N(t)] \quad (2.52)$$

$$\tau = \int_0^\infty \exp[-N(t)] dt \quad (2.53)$$

$$V_g(t) = a V_0 A K^3 c^{-(3v+1)} \int_0^{ct} (ct-s')^{3(\omega+v)} \exp(-B/s'^2) ds' \quad (2.54)$$

$$V_g(t) = a V_0 N_0 K^3 c^3 \omega^3 (\omega+v). \quad (2.55)$$

In Eqs. (2.52) and (2.53) $N(t)$ is given by Eq. (2.51) in which $\operatorname{erfc}(x)$ is the complementary error function defined by³¹

$$\operatorname{erfc}(x) = (2/\pi^{1/2}) \int_x^\infty \exp(-y^2) dy.$$

The above $N(t)$ dependence has also been found in Ref. 3.

Since the integration in Eq. (2.53) cannot be carried out analytically, in combination with Eq. (2.51) this equation gives only implicitly τ as a function of the supersaturating rate c . Analytical integration is also impossible in Eq. (2.54) at arbitrary $(\omega+v)$ -value and for this reason, in the case of PN, the general dependence of V_g on t and c is not obtainable explicitly. In contrast, in the case of IN, V_g is an explicit function of t and c : according to Eq. (2.55), V_g increases with increasing t and c following a power law.

The complicated Eq. (2.51) simplifies considerably for times close enough to the beginning of the process. When

$$t \ll (2B/3c^2)^{1/2}, \quad (2.56)$$

$N(t)$ from Eq. (2.51) becomes

$$N(t) = (V_0 A c^2 / 2B) t^3 \exp(-B/c^2 t^2) \quad (2.57)$$

which follows from the series expansion of the erfc-function for large $B^{1/2}/ct$ values.³¹

Accordingly, if Eq. (2.56) is satisfied, the $P(t)$ function from Eq. (2.52) has the form

$$P(t) = 1 - \exp[-(V_0 A c^2 / 2B) t^3 \exp(-B/c^2 t^2)]. \quad (2.58)$$

With a relatively low $B=10^{14}$ Pa², from Eq. (2.56) it follows that the $N(t)$ and $P(t)$ dependences are described reliably by Eqs. (2.57) and (2.58) as long as $ct \ll 8 \times 10^6$ Pa.

In contrast to the linear $N(t)$ dependence, Eq. (2.43) at constant supersaturation, Eq. (2.57) shows that N increases sharply with t after a certain time delay which, physically, plays the role of induction time. This threshold rise of the number of bubble nuclei is due to both the intuitively expected exponential term and the extra t^2 -factor in Eq. (2.57).

Figure 2 depicts the time dependences of N and P from Eqs. (2.57) and (2.58) at three supersaturating rates $c=1, 5$ and 25 Pa/s (as indicated) and $V_0=10^{-3}$ m³, $A=10^{25}$ m⁻³s⁻¹ and $B=10^{14}$ Pa² (this B -value corresponds to $\sigma_{ef}=3$ mJ/m²). It is seen that N and P vary very strongly with time and that, also, they are very sensitive to the supersaturating rate c .

We shall now analyze the t, c -dependence, Eq. (2.54), of the total volume V_g of the gas phase in the case of PN. The integration in Eq. (2.54) can be carried out analytically for $(\omega+v)$ -values satisfying the condition $3(\omega+v)=1, 2, 3, \dots$. The $\omega+v=1$ case corresponds to bubble growth with time-independent rate G_0 exemplified by Eq. (2.19) when growth is diffusion-controlled. In this case the integral in Eq. (2.54) can be expressed in a closed form and the final result is

$$V_g(t) = (a/4)V_0 A K^3 c^{3\omega-4} [(c^4 t^4 - 7Bc^2 t^2) \exp(-B/c^2 t^2) - 4(\pi B)^{1/2} (c^3 t^3 - 2Bct) \operatorname{erfc}(B^{1/2}/ct) + (6Bc^2 t^2 - B^2) E_1(B/c^2 t^2)] \quad (2.59)$$

where $E_1(x)$ is the exponential integral defined by³¹

$$E_1(x) = \int_x^\infty y^{-1} e^{-y} dy.$$

We also note the expressions for $V_g(t)$ resulting from Eq. (2.54) upon integration at $\omega+v=1/3$ and $2/3$, respectively:

$$V_g(t) = (a/2)V_0 A K^3 c^{3\omega-2} [c^2 t^2 \exp(-B/c^2 t^2) - 2(\pi B)^{1/2} ct \operatorname{erfc}(B^{1/2}/ct) + BE_1(B/c^2 t^2)], \quad (2.60)$$

$$V_g(t) = (a/3)V_0 A K^3 c^{3\omega-3} [(c^3 t^3 - 2Bct) \exp(-B/c^2 t^2) - (\pi B)^{1/2} (3c^2 t^2 - 2B) \operatorname{erfc}(B^{1/2}/ct) + 3Bct E_1(B/c^2 t^2)]. \quad (2.61)$$

Despite the complexity of the above formulae for $V_g(t)$, in the limit of small t -values they

take a simple and physically more transparent form. Indeed, using the known asymptotic expansions of $\text{erfc}(x)$ and $E_1(x)$ (Ref. 31), from the above three equations we find that (Γ is the complete gamma-function)

$$V_g(t) = \Gamma(3\omega + 3\nu + 1) a V_0 A K^3 c^{3\omega} (c^2/2B)^{3(\omega+\nu)+1} t^{9(\omega+\nu)+3} \exp(-B/c^2 t^2) \quad (2.62)$$

provided that t satisfies practically the same condition as Eq. (2.56). Eq. (2.62) is valid not only for $3(\omega+\nu)=1, 2$ and 3 , but also at $\omega+\nu=0$, which means that it can be used as a reliable approximation to the general $V_g(t)$ dependence, Eq. (2.54), for arbitrary positive $3(\omega+\nu) \neq 0, 1, 2, \dots$. In the $\omega+\nu=1$ case Eq. (2.62) reads:

$$V_g(t) = (3a V_0 A K^3 c^{3\omega+8/8B^4}) t^{12} \exp(-B/c^2 t^2). \quad (2.63)$$

Comparison of Eq. (2.62) with Eq. (2.46) shows that the $V_g(t)$ dependence has a much less pronounced threshold character at constant than at steadily increasing supersaturation. The delay time, which has the physical significance of induction time, is more obvious in the latter case because of both the expected exponential term and the extra $t^{9\omega+6\nu+2}$ -factor in Eq. (2.62). Figure 3 illustrates the dependence of V_g on t from Eq. (2.63) at three supersaturating rates c (as indicated). The values of c , A and B are those already used for the $N(t)$ plots in Fig. 2, and the other parameters are $a=2\pi/3$ (hemispherical bubbles), $\omega=1/2$ (e.g. diffusion-controlled bubble growth after Eq. (2.19)) and $K=10^{-9} \text{ m/s}^{1/2} \text{ Pa}^{1/2}$. As seen from Fig. 3, the $V_g(t)$ dependence is similar to that of $N(t)$ in Fig. 2: V_g is strongly affected by changes in c and is considerably retarded, since initially $s=0$.

In the case of gas phase formation by IN, the character of the $V_g(t)$ dependence is the same at both constant and linearly increasing supersaturation. Inspection of Eqs. (2.47) and (2.55) shows that in both cases V_g is a power function of t , somewhat stronger in the latter case, as $\omega > 0$. Also, in the latter case V_g is a power function of the supersaturating rate c .

2. Critical supersaturation

We now have the results needed for determination of the critical supersaturation s_c in the case of gas phase formation at constant supersaturating rate. We shall only use definitions (a) and (c) from Section VI, because definition (b) requires analyzing the mathematically complex Eq. (2.53). Moreover, since τ and t_1 are numerically close to each other (see Fig. 2 in which τ is roughly the moment at which P has an inflection), definition (b) may be expected to yield results practically equivalent to those based on definition (a).

Definition (a) has to be used in conjunction with $N(t)$ from Eq. (2.51). However, since the moment t_1 of appearance of the first nucleus bubble satisfies virtually always the inequality (2.56), the use of Eq. (2.57) is also legitimate. This leads to the following implicit dependence of t_1 (i.e. of the induction time) on the supersaturating rate c :

$$t_1 = [16\pi/3kT \ln(3V_0 AkTc^2 t_1^3 / 32\pi\sigma_{ef}^3)]^{1/2} (\sigma_{ef}^{3/2}/c). \quad (2.64)$$

As expected, increasing rate c and temperature T and/or decreasing effective surface energy σ_{ef} shorten the time needed for the birth of the first nucleus bubble. At the moment t_1 the supersaturation in the liquid assumes its critical value s_c defined by

$$s_c = ct_1. \quad (2.65)$$

Using this relation allows an equivalent, but more convenient presentation of Eq. (2.64) in the form

$$c = (V_0 A / 2B) s_c^3 \exp(-B/s_c^2). \quad (2.66)$$

This equation gives implicitly the sought dependence of the critical supersaturation s_c on the supersaturating rate c in the scope of definition (a). Eq. (2.66) applies to gas phase formation by the mononuclear mechanism and predicts a linear dependence if experimental $s_c(c)$ data are plotted in $\ln(c/s_c^3)$ -vs.- $1/s_c^2$ coordinates:

$$\ln(c/s_c^3) = \ln(3V_0 AkT / 32\pi\sigma_{ef}^3) - (16\pi\sigma_{ef}^3 / 3kT)(1/s_c^2). \quad (2.67)$$

From the intercept and the slope, the kinetic parameter A of bubble nucleation and the effective specific surface energy of the nucleus bubble can be calculated, respectively.

Figure 4 depicts the $s_c(c)$ dependence from Eq. (2.66) at two different values, 2.5 and 2.7 mJ/m², of σ_{ef} (as indicated) when $V_0 = 10^{-3}$ m³, $A = 10^{25}$ m⁻³s⁻¹ and $T = 300$ K. As seen, at a given supersaturating rate, s_c is smaller when σ_{ef} is lower (approximately, $s_c \sim \sigma_{ef}^{3/2}$). Also, s_c is much less sensitive to changes in the rate of pressure decline than in σ_{ef} .

Following the above procedure, let us now use definition (c) to determine the critical supersaturation s_c corresponding to the moment t_v at which the total volume of the gas phase becomes equal to $V_d = \alpha V_0$. Since t_v satisfies practically always the condition given by inequality (2.56), it is justified to use Eq. (2.62) instead of the complicated Eqs. (2.59)-(2.61).

Thus, we find implicitly the dependence of t_v (i.e. of the induction time) on c in the form

$$t_v = \{B / [\Gamma(3\omega + 3\nu + 1) a A K^3 c^{9\omega + 6\nu + 2} t_v^{9(\omega + \nu) + 3} / \alpha (2B)^{3(\omega + \nu) + 1}]\}^{1/2} c^{-1}. \quad (2.68)$$

In the case of $\omega + \nu = 1$ (time-independent growth rate) this equation becomes

$$t_v = [16\pi/3kT \ln(\lambda a A K^3 k^4 T^4 c^{3\omega + 8} t_v^{12} / \alpha \sigma_{ef}^{12})]^{1/2} (\sigma_{ef}^{3/2}/c) \quad (2.69)$$

where $\lambda = (2/\pi^4)(3/16)^5 \approx 5 \times 10^{-6}$.

Comparison of the above equations with Eq. (2.64) leads to the conclusion that c , σ_{ef} and T have essentially the same effect on both t_v and t_1 because of the relative constancy of the logarithmic terms. Defining the critical supersaturation s_c by

$$s_c = ct_v, \quad (2.70)$$

we can represent Eq. (2.68) by the following more convenient formula

$$c^{3\nu + 1} = [\Gamma(3\omega + 3\nu + 1) a A K^3 / \alpha (2B)^{3(\omega + \nu) + 1}] s_c^{9(\omega + \nu) + 3} \exp(-B/s_c^2) \quad (2.71)$$

which in the $\omega + \nu = 1$ case reads:

$$c^{4-3\omega} = (3aAK^3/8\alpha B^4)s_c^{12}\exp(-B/s_c^2). \quad (2.72)$$

Equations (2.71) and (2.72) represent the sought dependence of the critical supersaturation on the supersaturating rate in the scope of definition (c). They give only implicitly s_c as a function of c and are applicable to gas phase formation by the polynuclear mechanism in the case of PN. When $\omega+v=1$, according to Eq. (2.72), a suitable representation of experimental $s_c(c)$ data is a plot of $\ln(c^{4-3\omega}/s_c^{12})$ vs. $1/s_c^2$. The resulting linear dependence reads

$$\ln(c^{4-3\omega}/s_c^{12}) = \ln(\lambda aAK^3k^4T^4/\alpha\sigma_{ef}^{12}) - (16\pi\sigma_{ef}^3/3kT)(1/s_c^2) \quad (2.73)$$

and has a slope from which σ_{ef} can be calculated. The intercept, however, does not allow a separate determination of the nucleation and growth kinetic parameters A and K : as already noted, it yields only the product AK^3 . From the quality of the linear regression, a best-fit value of ω may be determined which could give information about the bubble growth mechanism.

Comparison of Eqs. (2.66) and (2.71) shows that the two $s_c(c)$ dependences are similar. This is also seen in Fig.4 which displays s_c from Eq. (2.72) as a function of c at the two σ_{ef} values used to calculate s_c from Eq. (2.66). The values of A and T are also the same, and $a=2\pi/3$ (hemispherical bubbles), $\alpha=0.001$, $\omega=1/2$ (e.g. diffusion-controlled bubble growth) and $K=10^{-9} \text{ m/s}^{1/2}\text{Pa}^{1/2}$. As seen from Fig.4, s_c from Eq. (2.72) is considerably higher than s_c from Eq. (2.66) and, as before, it decreases with decreasing c and σ_{ef} . Again, s_c is much less sensitive to changes in c than in σ_{ef} .

Finally, we can also determine the critical supersaturation s_c for gas phase formation by the polynuclear mechanism in the case of IN. Applying definition (c) to Eq. (2.55) and using Eq. (2.70), we find explicitly the corresponding dependences of t_v and s_c on the supersaturating rate c :

$$t_v(c) = (\alpha/aN_0K^3)^{1/3}(\omega+v)c^{-\omega/(\omega+v)} \quad (2.74)$$

$$s_c(c) = (\alpha/aN_0K^3)^{1/3}(\omega+v)c^{v/(\omega+v)}. \quad (2.75)$$

Comparison of these equations with Eqs. (2.68) and (2.71) reveals that while in this case t_v changes with c essentially as in the case of PN, the dependence of s_c on c is much stronger than for PN. For example, for diffusion-controlled growth with $v=1/2$ and $\omega=1/2$, it follows from Eq. (2.75) that $s_c \sim c^{1/2}$, i.e. a 100 times higher rate c should result in a ten-fold increase in s_c . Similar strong effect of c on s_c has been observed in experiments on bubble formation in porous media [1,5]. Eq. (2.75) shows also that in the case of heterogeneous IN, s_c increases when any of the three specific surface energies (i.e. the wetting angle θ) is changed in such a way that the shape factor a diminishes. For instance, if the bubbles are of equilibrium shape, $a=4\pi\phi(\theta)/3$ so that $s_c \sim \phi^{-1/3(\omega+v)}$. This is in contrast with the case of PN in which s_c increases with ϕ approximately as $s_c \sim \sigma_{ef}^{3/2} \sim \phi^{1/2}$ (see Eqs. (2.71) and (2.72) and Fig.4). It is important to note as well that using Eq. (2.75) to analyze experimental $s_c(c)$ data (e.g., for gas phase

formation by seeded bubble nucleation) allows a separate determination of the bubble growth parameter K if N_0 is independently known.

VIII. Conclusions

The present study leads to the following conclusions concerning gas phase formation by progressive nucleation or instantaneous nucleation in the cases of constant and steadily increasing supersaturation.

Constant supersaturation.

1. The volume of the newly formed gas phase is a stronger function of time and especially of supersaturation for progressive than for instantaneous nucleation.
2. The induction time is weakly dependent on supersaturation for instantaneous nucleation, while it is a strong function of supersaturation for progressive nucleation.

Steadily increasing supersaturation.

1. In gas phase formation by progressive nucleation the number of bubbles nucleated increases sharply with time after a certain time delay.
2. The volume of the newly formed gas phase increases sharply with time for progressive nucleation, while for instantaneous nucleation the volume increase is gradual.
3. The volume of the gas phase also shows higher sensitivity to the supersaturating rate in the case of progressive nucleation than in the case of instantaneous nucleation.
4. For progressive nucleation the critical supersaturation is highly sensitive to the effective surface tension and weakly dependent on the supersaturating rate.
5. In contrast to progressive nucleation, the critical supersaturation for gas phase formation by instantaneous nucleation shows sensitivity to the supersaturating rate. Also, unlike in the case of progressive nucleation, the critical supersaturation is not strongly affected by the effective surface tension when the gas phase forms by instantaneous nucleation.

REFERENCES

- 1 A.Firoozabadi, B.Ottesen, and M.Mikklesen, Paper SPE 19694, (1989).
- 2 T.F.M.Kortekaas and F.V.Poelgeest, SPE Reservoir Eng., 329 (Aug.1991).
- 3 Y.C.Yortsos and M.Parlar, Paper SPE 19697, (1989).
- 4 X.Li and Y.C.Yortsos, Paper SPE 22589, (1991).
- 5 J.C.Moulu, J.Pet.Sci.Eng. **2**, 379 (1989).
- 6 A.El Yousfi, C.Zarcone, S.Bories, and R.Lenormand, C.R.Acad. Sci.(Paris) **313**,7 (1991).
- 7 M.Volmer, *Kinetik der Phasenbildung* (Steinkopff, Dresden, 1939).
- 8 R.Kaischew and B.Mutaftschiew, Comm.Inst.Phys.Chem.Bulg.Acad.Scis. **2**, 75 10.
- 9 J.P.Hirth and G.M.Pound, *Condensation and Evaporation* (Pergamon, Oxford, 1963).
- 10 E.I.Nesis, *Kipenie Zhidkostey* (Nauka, Moscow, 1973).
- 11 M.Blander and J.L.Katz, AIChE J. **21**, 833 (1975).
- 12 M.Blander, Adv.Colloid Interface Sci. **10**, 1 (1979).
- 13 T.Theofanous, L.Biasi, H.S.Isbin, and H.Fauske, Chem.Eng.Sci. **24**, 885 (1969).
- 14 J.Szekely and G.P.Martins, Chem.Eng.Sci. **26**, 147 (1971).
- 15 J.Szekely and S.D.Fang, Chem.Eng.Sci. **28**, 2127 (1973).
- 16 M.S.Plesset and S.A.Zwick, J.Appl.Phys. **25**, 493 (1954).
- 17 D.E.Rosner and M.Epstein, Chem.Eng.Sci. **27**, 69 (1972).
- 18 K.E.Perepelkin and V.S.Matveev, *Gazovye Emulsii* (Khimiya, Leningrad, 1979).
- 19 A.N.Kolmogorov, Bull.Acad.Sci.URSS (Sci.Math.Nat.) **3**, 355 (1937).
- 20 M.Avrami, J.Chem.Phys. **7**, 1103 (1939); **8**, 212 (1940); **9**, 177 (1941).
- 21 A.E.Nielsen, *Kinetics of Precipitation* (Pergamon, Oxford, 1964).
- 22 D.Kashchiev, Surface Sci. **18**, 293 (1969).
- 23 D.Kashchiev, Surface Sci. **22**, 319 (1970).
- 24 D.Kashchiev, Cryst.Res.Technol. **19**, 1413 (1984).
- 25 D.Kashchiev, Surface Sci. **14**, 209 (1969).
- 26 S.Toshev, A.Milchev, and S.Stoyanov, J.Cryst.Growth **13/14**, 123 (1972).
- 27 D.Kashchiev, D.Verdoes, and G.M. van Rosmalen, J.Cryst.Growth **110**, 373 (1991).
- 28 M.C. van der Leeden, D.Verdoes, D.Kashchiev, and G.M. van Rosmalen, in *Advances in Industrial Crystallization*, edited by J.Garside, R.J.Davey, and A.G.Jones (Butterworth, London, 1991), p.31.
- 29 D.Verdoes, D.Kashchiev, and G.M. van Rosmalen, J.Cryst.Growth, **118**, 401 (1992).
- 30 M.C. van der Leeden, D.Kashchiev, and G.M. van Rosmalen, J.Colloid Interface Sci., **152**, 338 (1992).

- 31 M.Abramowitz and I.E.Stegun (editors), *Handbook of Mathematical Functions* (Dover, New York, 1965).

Fig. 2.1

Dependence of the critical supersaturation on the effective specific surface energy for progressive nucleation: dotted curve - Eq. (2.48), dashed and solid curves - Eq. (2.49) with diffusion growth constant $K=10^{-7}$ and 10^{-9} $\text{m/s}^{1/2}\text{Pa}^{1/2}$, respectively.

Fig. 2.2

Time dependence of the number N of bubbles nucleated and of the probability P to form at least one nucleus bubble: solid, dashed and dotted curves - Eqs. (2.57) and (58) with supersaturating rate $c=1, 5$ and 25 Pa/s, respectively.

Fig. 2.3

Time dependence of the total volume of the gas in the case of progressive nucleation: solid, dashed and dotted curves - Eq. (2.63) with supersaturating rate $c=1$, 5 and 25 Pa/s, respectively.

Fig. 2.4

Dependence of the critical supersaturation on the supersaturating rate for progressive nucleation: solid and dash-dotted curves - Eq. (2.66) with effective specific surface energy $\sigma_{ef}=2.7$ and 2.5 mJ/m^2 , respectively; dashed and dotted curves - Eq. (2.72) with $\sigma_{ef}=2.7$ and 2.5 mJ/m^2 , respectively.

2.2 PRESSURE AND VOLUME EVOLUTION DURING GAS PHASE FORMATION IN POROUS MEDIA

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SUMMARY

Literature data and the theory (to be presented in this paper) show that the formation of the gas phase in porous media is an instantaneous nucleation process. Therefore, the classical nucleation theory (both heterogeneous and homogeneous) cannot describe the solution gas drive process.

Our theoretical analysis reveals that supersaturation for gas bubble formation (i.e., non-equilibrium) in porous media is strongly affected by the pressure decline rate or the volume expansion rate. As such, laboratory recovery measurements could be different from oil reservoirs due to rate effect. Also, material balance expression based on the equilibrium assumption may not be used in laboratory experiments to calculate the amount of gas phase in a core. Supersaturation is expected to be less pronounced in low permeability than in high permeability porous media. The number of gas bubbles formed as a result of pressure decline or volume expansion rates is orders of magnitude less than the number of pores. This conclusion is in line with the early bubble density measurements reported in the petroleum literature.

INTRODUCTION

The solution gas-drive process is classified as one of the five main mechanisms of oil recovery from fractured and unfractured reservoirs (the other four are gravity drainage, capillary imbibition, viscous displacement, and compressibility effects both from rock and fluid). This process is also the least understood mechanism due to its complexity. The formation of a gas phase, when the pressure of an undersaturated oil in a porous medium is lowered below the bubblepoint pressure, is a non-equilibrium process. The oil normally goes from an undersaturated to a supersaturated state prior to formation of gas bubbles. The number of gas bubbles, their growth, and their connectivity to provide a mobile gas

phase affect the oil recovery performance of both unfractured and especially fractured reservoirs.

In fractured reservoirs, as the new gas bubbles expand, the oil phase is pushed from the matrix to the fractures. There is a meaning to a capillary pressure between the continuous oil phase (bulk liquid phase) and the growing gas bubbles. This capillary pressure is related to the curvature and surface tension. But there is no meaning to a capillary pressure/saturation relationship which is commonly used in Darcy's equation for oil flow in the matrix towards the fracture network. In other words, the oil flow prior to the mobilization of the gas phase follows the single phase Darcy equation. In the two-phase gas-oil flow (when both gas and oil phases are continuous), reinfiltration adversely affects the oil flow rate, due to capillary and gravity effects.¹ When gas bubbles are growing but have not mobilized, the process of reinfiltration is absent. Therefore, solution gas drive process with a high critical gas saturation is an efficient process in fractured reservoirs due to providing the conditions for oil flow through the fractures.

The formation of the new gas phase, when the pressure is lowered below the bubble-point pressure, is a kinetic process.² Therefore, the rate effect (i.e., the rate of pressure decline) will influence the pressure volume relationship especially at laboratory conditions. As an example, consider an undersaturated oil with a bubblepoint pressure of say 2500 psi at an initial pressure of 3000 psi. The amount of oil recovery for constant volume depletion process at isothermal conditions will be a function of the rate of depletion. The recovery of oil at a final pressure of say 50 psi could be 15 percent for a low depletion rate and 30 percent at a high depletion rate. Rate dependent recovery by a factor of two at the same final pressure has been observed in the laboratory³ (15 percent recovery at low rate, 30 percent recovery at high rate). The rate effect is so strong that at laboratory conditions, it may be difficult to eliminate its effect on the critical gas saturation and recovery. Due to kinetic effects² (i.e., supersaturation), one may not use the material balance expressions to calculate the amount of oil and gas phases at early stages of a new gas phase formation.

The petroleum literature contains experimental and theoretical studies on gravity drainage, capillary imbibition, and viscous displacement processes. The study of the solution gas drive process is, however, limited to a few investigations. Ref. 4 provides a review of the two main issues of the solution gas-drive process: supersaturation and critical gas saturation. Recent works not reviewed in Ref. 4 will be discussed briefly next.

In 1991, Kortekaas and van Poelgest reported critical gas saturation measurements in the presence of connate water for 6" long cores.⁵ Based on several experiments, these authors concluded that critical gas saturation is decreased by lowering the pressure decline rate. Yortsos and Parlari⁶ modelled the appearance of the gas phase in the solution gas drive process. Nucleation, supersaturation and bubble growth in porous media were included in their model. They analyzed the onset of nucleation under both homogeneous and heterogeneous nucleation conditions. These authors concluded that the supersaturation needed for the appearance of gas bubbles should be largely independent of the rate of pressure decline. This conclusion is not in accord with the results of Refs. 4 and 5. In a more recent study, Li and Yortsos⁷ based on a network model concluded that the critical gas saturation decreases with an increase in the pressure decline rate. This conclusion is reached on the assumption that the nucleation occurs from specific active sites, each of which is activated at a specified local supersaturation. This conclusion is also different from the experimental observations.^{4,5,8}

The goal of this work is to provide a theoretical framework to understand more comprehensively the evolution of the gas phase in porous media. In this paper, we will first formulate the problem of the gas phase formation in porous media. The theoretical model will then be used to describe both the constant expansion and pressure decline rate experiments reported in the literature. The paper will conclude with our comments on the description of the gas phase formation in porous media.

PROBLEM FORMULATION

Consider a single liquid phase of volume V^o and pressure p^o at constant temperature T at time $t = 0$. At the initial time, $t = 0$, the liquid is either at the bubblepoint or at a

supersaturated state. At a later time, $t > 0$, depending on the values of p^o and V^o and the change of p and V , the system may contain a certain volume $V_g(t)$ of a new gas phase formed by bubble nucleation and growth.

The volume balance equation can be written as

$$V(t) = V_l(t) + V_g(t) \quad (2.76)$$

In Eq. 2.76, $V(t)$ is the total fluid volume, and $V_l(t)$, and $V_g(t)$ are the volume of liquid and gas phases, respectively. These volumes are at time t , when the system has gone through either a pressure decline or a volume expansion process. Neglecting the shrinkage of the liquid phase due to mass transfer, one may write

$$V_l(t) = V^o + \kappa[p^o - p(t)] \quad (2.77)$$

where κ is a positive factor related to the isothermal compressibility c_T of the liquid by $\kappa = c_T V^o$

The determination of the volume $V_g(t)$ of the gas phase is the major issue of this paper. Under non-equilibrium conditions, $V_g(t)$ is essentially a kinetic quantity which may depend on the nucleation rate J (bubbles per unit time per unit volume) and the growth rate G (length per unit time) of the bubbles in the liquid. Let us make a distinction between two important cases of nucleation: 1) progressive nucleation, and 2) instantaneous nucleation. In progressive nucleation (PN), new nucleus bubbles appear continuously between the growing supernucleus bubbles. In instantaneous nucleation (IN), all nucleus bubbles form at once and then they only grow. Recently, we have shown that,²

$$V_g(t) = V^o \int_0^t J(t') v_b(t, t') dt' \quad (2.78)$$

$$V_g(t) = V^o N^o v_b(t, 0) \quad (2.79)$$

for PN, and IN, respectively. In the above expressions, the compressibility of the liquid and the volume of the nucleus bubbles are neglected. (Note that a nucleus is defined as

a stable bubble at the time of formation. Also note that the assumption of zero liquid compressibility in Eqs. 2.78 and 2.79 is justified in most cases due to small liquid volume change.) In Eq. 2.79, N^0 is the number of nucleus bubbles per unit volume formed instantaneously at time $t = 0$, and $v_b(t, t')$ is the volume at time t of a bubble nucleated at time t' ($0 \leq t' \leq t$). For growth of bubbles in three dimensions, v_b is represented as

$$v_b(t, t') = ar^3(t - t') = a \left[\int_{t'}^t G(t'' - t') dt'' \right]^3 \quad (2.80)$$

In Eq. 2.80, $r(t - t')$ is the effective radius at time t of a bubble formed at time t' , and $G(t'' - t')$ is the growth rate of a bubble at any given time t'' which was formed at time t' ($t' \leq t'' \leq t$). Parameter "a" is a shape factor which is the ratio of the true volume of the bubble to the third power of effective radius of the bubble. (Shape factors for needles, disks and squares can be found in Ref. 23.)

Kinetic considerations⁹⁻¹⁴ show that bubble nucleation rate, J , and growth, G , can be different functions of time and the supersaturation in the system. Nevertheless, they can be represented by the following general expressions:²

$$J(t) = A \exp \left[-B/s^2(t) \right] \quad (2.81)$$

$$G(t) = (\omega + \nu) K s^\omega(t) t^{\nu-1} \quad (2.82)$$

In Eq. 2.81, A and B are the kinetic and thermodynamic constants of bubble nucleation. In Eq. 2.82, K is the kinetic constant of bubble growth, and ω and ν are positive numbers of the order of unity. (We will later, in detail, derive the expression for the bubble growth in a diffusion-controlled process.) The supersaturation, s , in the above two expressions is defined as:

$$s(t) = p_e(t) - p(t) \quad s \geq 0 \quad (2.83)$$

Here, p_e is the pressure at which the liquid and gas phases are at equilibrium. It is also assumed to be practically the same as the pressure inside the nucleus bubble. The

equilibrium pressure in the two-phase gas-liquid region of a multicomponent system is related to the bubblepoint pressure p_b according to the approximate relationship,

$$p_e \approx p_b - \beta(V - V_b) \quad (2.84)$$

In Eq. 2.84, V_b is the volume at the bubblepoint, and $\beta \geq 0$ is the slope of p_e vs. V line. Eq. 2.84 is an excellent approximation for both binary and multicomponent systems (away from the critical point) where p_e is say less than 0.2 to 0.3 p_b (see dashed lines in Figs. 2.7 and 4 and dotted lines in Figs. 2.9 and 2.10). For a single component system $\beta = 0$ and, therefore, $p_e(t) = p_b$.

Combining Eqs. 2.76 to 2.78, and 2.80 to 2.83, and Eqs. 2.76, 2.77, 2.79, 2.80, 2.82 and 2.83, gives the following general expressions for the volume $V(t)$ for PN and IN processes, respectively.

$$V(t) = V^o + \kappa [p^o - p(t)] + Q_1 \int_0^t \exp \left\{ -B / [p_e(V(t')) - p(t')]^2 \right\} \\ \times \left\{ \int_0^{t-t'} [p_e(V(\tau)) - p(\tau)]^\omega \tau^{\nu-1} d\tau \right\}^3 dt' \quad (2.85)$$

$$V(t) = V^o + \kappa [p^o - p(t)] + Q_2 \left\{ \int_0^t [p_e(V(\tau)) - p(\tau)]^\omega \tau^{\nu-1} d\tau \right\}^3 \quad (2.86)$$

The kinetic constants Q_1 , and Q_2 are also different in the two cases of nucleation:

$$Q_1 = a(\omega + \nu)^3 V^o A K^3 \quad (2.87)$$

for PN and

$$Q_2 = a(\omega + \nu)^3 V^o N^o K^3 \quad (2.88)$$

for IN.

Eqs. 2.85 and 2.86 govern the dependence of p and V on t in the two cases of bubble nucleation. Each equation, however, is one equation in two unknowns, p and V . One more independent relation between p or V and t is required to define the problem. The

additional relationship comes from the way either p or V is varied with time. In the following, we consider two cases that have been used to conduct the experiments.

CONSTANT PRESSURE DECLINE RATE - For this case, the pressure in the system is decreased linearly with time according to

$$p(t) = p^o - ct \quad (2.89)$$

where $c > 0$ is the pressure decline rate. We seek the relation between volume and time. When p^o is selected to be the bubblepoint pressure p_b (then also $V^o = V_b$) and if it is assumed that $\beta = 0$ (see Eq. 2.84), the liquid will be under a state of steadily increasing supersaturation $s(t) = p_b - p(t) = ct$. Eqs. 2.85 and 2.86, then, simplify to

$$V(t) = V_b + \kappa ct + \left\{ Q_1 / [(\omega + \nu)^3 c^{3\nu+1}] \right\} \int_0^{ct} (ct - s)^{3(\omega+\nu)} \exp(-B/s^2) ds \quad (2.90)$$

for PN process, and to

$$V(t) = V_b + \kappa ct + \left[Q_2 c^{3\omega} / (\omega + \nu)^3 \right] t^{3(\omega+\nu)} \quad (2.91)$$

for IN process.

Eqs. 2.90 and 2.91 give the dependence of V on t for the two models of bubble nucleation. The last term of these two equations represent the volume of the newly formed gas phase and are identical to the corresponding equations of Ref. 2. A good approximation to Eq. 2.90 for arbitrary positive values of $(\omega + \nu)$, and small t -values ($ct \ll B^{1/2}$) is,²

$$V(t) = V_b + \kappa ct + \Gamma [3(\omega + \nu) + 1] (\omega + \nu)^{-3} Q_1 c^{3\omega} (c^2/2B)^{3(\omega+\nu)+1} \times t^{9(\omega+\nu)+3} \exp \left[-B/(c^2 t^2) \right] \quad (2.92)$$

In Eq. 2.92, Γ is the complete gamma-function.

Eqs. 2.91 and 2.92 express explicitly, V as a function of t for the IN and PN cases, respectively. Using Eq. 2.89, the dependence of V on p for the PN and IN are given by the following expressions.

$$V(p) = V_b + \kappa s + \Gamma [3(\omega + \nu) + 1] (\omega + \nu)^{-3} \left[\frac{Q_1}{(2B)^{3(\omega+\nu)+1} c^{3\nu+1}} \right] \times$$

$$s^{9(\omega+\nu)+3} \exp(-B/s^2) \quad (2.93)$$

$$V(p) = V_b + \kappa s + \left[\frac{Q_2}{(\omega + \nu)^3 c^{3\nu}} \right] s^{3(\omega+\nu)} \quad (2.94)$$

In Eqs. 2.93 and 2.94, $s = p_b - p$. These two equations reveal a much stronger (exponential) increase of V with decreasing p for the PN than for the IN process. A similar power-type increase of V with increasing rate of pressure decline, c , is given for both cases of bubble nucleation. The following two examples will illustrate the difference between these two important processes.

For the first example, the effect of rate of pressure decline on the volume of the new gas phase is studied—high pressure decline rate of 5 psi/hr, and low pressure decline rate of 0.5 psi/hr. Parameters common to both PN and IN are: $a = \frac{4\pi}{3}$, $K = 1.6 \times 10^{-5}$, $\frac{ft}{psi^{1/2} \cdot hr^{1/2}}$, and $p_b = 1450$ psia, $\omega = 1/2$, and $\nu = 1/2$. Parameters of the PN are: $A = 10^{27} \frac{1}{ft^3 \cdot hr}$, and $B = 2.1 \times 10^6$ psi². In order to have an appreciation of the meaning of various parameters and a feeling of their magnitude, the reader can read Ref. 2. Bubble density of the IN is $N^o = 2.8 \times 10^6$ bubbles/ft³. Using the last term on the right hand side of Eqs. 2.93 and 2.94, and the above values, Fig. 2.5 was constructed. This figure shows clearly the strong influence of pressure decline rate on the critical supersaturation and supersaturation for the IN, and a weak effect on the PN process. (We define critical supersaturation the supersaturation at which a detectable gas phase is formed). The second example is designed to illustrate both the effect of interfacial tension σ and rate c on supersaturation vs. gas volume in a PN process. For this case, three values of interfacial tension and two rates are considered: $\sigma = 2, 6$ and 8 dynes/cm, and $c = 1$ and 1000 psi/hr. Also assume $Q_1 = 10^{-9} \frac{ft^3}{psi^{3/2} s^{5/2}}$. The thermodynamic parameter B of the nucleation rate is assumed to be 2×10^4 psi² at an interfacial tension of 2 dynes/cm. For an interfacial tension of 6 dynes/cm, the value of $B = 57 \times 10^4$ psi² [$B = (16\pi\sigma^3)/(3kT)$, see Ref. 2]. The initial volume at the bubblepoint is assumed to be 30 cm³. Fig. 2.6 shows the calculated supersaturation vs. V_g/V^o for the conditions of the second example. This figure reveals an important point—that an increase in interfacial tension by a factor of 3 has a much

more pronounced effect on supersaturation than an increase in rate of pressure decline by a factor of 1000. Figs. 2.5 and 2.6 lead us to suspect that in porous media the progressive nucleation process may not realize and the classical nucleation rate expression given by Eq. 2.81 may not be used. The early experimental data of Kennedy and Olsen¹⁵ show a strong effect of rate of pressure decline on critical supersaturation. The recent data presented in Refs. 4 and 5 do not show a drastic influence of interfacial tension on critical supersaturation. In addition to the above evidences, the early work of Kennedy and Olsen shows that virtually all bubbles are formed instantaneously. The recent experiments by El Yousfi et al.¹⁶ also support the occurrence of the process of instantaneous nucleation in porous media (their Fig. 2). As a result, it is expected that the nucleation process in porous media occurs by IN. The porous media offer nucleation-active sites on which all growing bubbles may be nucleated practically instantaneously at an initial supersaturation s° . Therefore, the use of the classical nucleation rate expression (Eq. 2.81) may not be permissible to study solution gas drive process. Use of the classical rate expression in Ref. 6 led the authors to conclude that the supersaturation needed for the appearance of the first bubble (critical supersaturation) should be independent of the rate of pressure decline. In the following, all the derivations will be based on the simple instantaneous nucleation model of Eq. 2.79.

To obtain a relationship between the volume of the new formed gas phase and time (or pressure), the time scale is chosen such that at $t = 0$, the gas phase begins to form. This implies that at $t = 0$, there could be an initial supersaturation s° . We assume that

$$p_e(V) \approx p_e^\circ - \beta V_g \quad (2.95)$$

where p_e° is the equilibrium pressure at time $t = 0$. Eq. 2.95 is based on the assumption that volume of the liquid phase remains constant during the pressure decline rate at the initial value of V° . This simplifying assumption is made to be able to use the model for simulation of experiments from Refs. 5 and 8. The authors of Refs. 5 and 8 apparently made this assumption to estimate the volume of the gas phase. These references report measured pressure, p , and gas volume, V_g . Substitution of Eqs. 2.89 and 2.95 into the

general expression for the gas phase volume (last term on the right hand side of Eq. 2.86) gives:

$$V_g(t) = Q \left\{ \int_0^t [s^o - \beta V_g(\tau) + c\tau]^\omega \tau^{\nu-1} d\tau \right\}^3 \quad (2.96)$$

In Eq. 2.96, $s^o = p_e^o - p^o$ is the initial supersaturation. Differentiation of Eq. 2.96 with respect to t leads to the following first order nonlinear differential equation

$$\frac{dV_g}{dt} = 3Q_2^{1/3} t^{\nu-1} V_g^{2/3} (s^o + ct - \beta V_g)^\omega \quad (2.97)$$

The initial condition is,

$$V_g(0) = 0 \quad (2.98)$$

Eq. 2.97 takes the following form

$$g_1^{1/3} \frac{d(V_g/V_o)}{dx} = 3C_1 x^{\nu-1} (V_g/V_o)^{2/3} [s^o + x - g_1(V_g/V_o)]^\omega \quad (2.99)$$

for parameters

$$x = ct = p^o - p(t) \quad (2.100)$$

$$g_1 = \beta V^o \quad (2.101)$$

$$C_1 = \frac{\beta^{1/3} Q_2^{1/3}}{c^\nu} \quad (2.102)$$

The constant g_1 is a known parameter. Parameters ω and ν are chosen according to a given model of bubble growth (to be discussed later). Parameter C_1 can be determined by fitting the solution of Eq. 2.99 to experimental V_g vs. p data. Therefore, Eq. 2.99 contains one fitting parameter, C_1 , which controls the evolution of gas phase through the product $\beta^{1/3} Q_2^{1/3} / c^\nu$ in which the pressure decline rate, c , and the equilibrium parameter, β , are known. The only unknown is Q_2 , which is related to the number of bubbles, N^o .

The problem is to solve Eq. 2.99 together with the initial condition given by Eq. 2.98. Eq. 2.99 has no obvious analytical solution and, therefore, it could be solved numerically using the Runge-Kutta method.¹⁷ The results of the solution to Eq. 2.99 will be presented in the section on experiments and theory.

CONSTANT VOLUME EXPANSION RATE - In this process, the volume of the system is increased linearly with time according to

$$V(t) = V^o + qt \quad (2.103)$$

where $q > 0$ is the rate of volume expansion. We seek to find the $p(t)$ dependence. Combining Eqs. 2.83, 2.84, 2.86, and 2.103 results in

$$(1 - \kappa\beta)qt = \kappa [s(t) - s^o] + Q_2 \left[\int_0^t s^\omega(\tau) \tau^{\nu-1} d\tau \right]^3 \quad (2.104)$$

for the IN process. Note that $s^o = p_e^o - p^o$ is the initial supersaturation. The solution to Eq. 2.104 in terms of supersaturation $s(t)$ can lead to $p(t)$ determination from the expression

$$p(t) = p^o + s^o - \beta qt - s(t) \quad (2.105)$$

which follows from Eqs. 2.83, 2.84, and 2.103. It should be emphasized again that when gas is formed in porous media, it is expected that the process occurs by IN for the case of constant volume expansion rate. Therefore, the description of the bubble nucleation process will be restricted to IN.

Going back to Eq. 2.104, differentiating it with respect to t , the following first-order differential equation is obtained,

$$g_2 \frac{ds}{dy} = 1 - 3C_2 x^{\nu-1} s^\omega [y - g_2(s - s_o)]^{2/3} \quad (2.106)$$

The boundary condition is

$$s(o) = s^o \quad (2.107)$$

Here, y is scaled to time,

$$y = qt \quad (2.108)$$

Due to Eq. 2.103, y is the expanded volume $V - V^o$. The positive constants g_2 and C_2 are given by

$$g_2 = \kappa / (1 - \kappa\beta) \quad (2.109)$$

$$C_2 = Q_2^{1/3} / [(1 - \kappa\beta)^{1/3} q^\nu] \quad (2.110)$$

Substitution of Q_2 from Eq. 2.88 in Eq. 2.110 gives,

$$C_2 = (\omega + \nu)(aV)^{1/3} N^{1/3} K / [(1 - \kappa\beta)^{1/3} q^\nu] \quad (2.111)$$

Similar to the case of constant pressure decline rate, the constant g_2 is known and the parameters ω and ν are selected according to a given model of bubble growth. Parameter C_2 , therefore, controls the evolution of solution gas through the product $N^{1/3} K/q^\nu$, in which the number of bubbles is unknown.

The Runge-Kutta method¹⁷ is used to solve the first order non-linear differential equation given by Eq. 2.106. The results are presented later on in this paper. The growth of a bubble is discussed next.

BUBBLE GROWTH

The application of the above results requires knowledge of parameters ω , ν , and K of bubble growth. The growth of an isolated bubble is controlled by mass, momentum and heat transfer across the bubble/liquid interface,¹²⁻¹⁴ and by compression or expansion. Mass transfer occurs by evaporation and condensation at the interface for single component systems. Mass transfer also occurs by diffusion in the liquid phase for multicomponent systems. Momentum transfer is governed by hydrodynamic forces in connection to the capillary pressure, liquid inertia, and viscosity. Heat transfer takes place by the flow of heat from the liquid towards the bubble. Change of pressure in the liquid is accompanied by expansion and compression of the gas bubble.

In general, the dependence of the bubble growth, G , on s and t , is complicated. However, G can be approximated for a number of cases by Eq. 2.82, and thus ω , ν , and K can be determined. For example, if bubble growth is assumed to be controlled only by the inertia of the liquid, it can be shown² that for small enough ds/dt ,

$$K = [8/(27\rho)]^{1/2} \quad \omega = 1/2, \nu = 1 \quad (2.112)$$

where ρ is the mass density of the liquid. The bubble growth in porous media at isothermal conditions for solution gas drive is believed to be mainly controlled by diffusion. In this case, known expressions of the diffusion flux, j , across the bubble/liquid interface can be

used to determine ω , ν , and K . Assuming ideal gas behavior for the bubble, the number of moles in the bubble is given by $n_b = p_e v_b / RT$ provided the capillary pressure is negligible. (This assumption is justified as it will be shown in the next section.) The surface area, s_b , of the bubble/liquid interface is related to the bubble volume, v_b , by $dv_b = s_b dr$. The mass balance equation for the bubble is given by

$$\frac{dn_b}{dt} = j s_b \quad (2.113)$$

Differentiation of $n_b = p_e v_b / RT$ with respect to time leads to the expression for growth G ,

$$G = \frac{RT}{p_e} j - \frac{1}{3} r \frac{dp_e}{dt} \quad (2.114)$$

In Eq. 2.114, the first and second terms represent the contribution of diffusion and expansion/compression to growth, respectively. According to the equation, initially the growth is diffusion-controlled, but at late stages, expansion could become important at high pressures. In order to keep the simplicity of the model, the effect of the second term will be neglected in this study.

The diffusion flux, j , for a one-dimensional (1D) diffusion in semi-infinite space, and stationary three-dimensional (3D) diffusion from a finite space with no flux at the outer boundary towards a spherical bubble with radius, r , are given by Refs. 8, 18

$$j = (D/\pi t)^{1/2} K_s (p_e - p) \quad (2.115)$$

$$j = DK_s (p_e - p) / r \quad (2.116)$$

respectively. In Eqs. 2.115 and 2.116, D is the effective diffusion coefficient of the dissolved volatile component in the liquid in porous media. K_s is the solubility constant (of the volatile component) relating concentration to pressure. Its value is nearly independent of pressure over a wide pressure range in the course of supersaturating the liquid. The above expressions for diffusion flux are then introduced into Eq. 2.114 and the second term on the right hand side of this equation is neglected. Recalling that $s = p_e - p$ and comparing the results with Eq. 2.82, we find

$$K = 2RTD^{1/2} K_s / 3\pi^{1/2} p_b, \quad \omega = 1, \quad \nu = 1/2 \quad (2.117)$$

for 1-D diffusion, and

$$K = (RTDK_s/2p_b)^{1/2} \quad , \quad \omega = 1/2, \quad \nu = 1/2 \quad (2.118)$$

for 3-D diffusion.

In Eqs. 2.117 and 2.118, p_e is replaced by p_b . This assumption is justified as long as $s \ll p_b$. Eq. 2.118 is obtained by approximating $\left(\int_0^t s(t')dt'\right)^{1/2}$ by $\int_0^t s^{1/2}(t')dt'^{1/2}$, which has no error when s is constant. Comparison of the measured bubble growth to a 3-D diffusion model was made recently by El Yousfi, et al.¹⁶ The agreement between data and the model is an indication of the appropriateness of the 3-D growth model. Next we will use the 3-D diffusion model to apply the above general results to study gas phase formation in a solution gas-drive process.

THEORY AND EXPERIMENTS

The process of gas phase formation in porous media is theorised to be initiated by $N^{\circ}V^{\circ}$ bubbles nucleated instantaneously on randomly located active sites on the surface of the grains at an initial supersaturation s° . The bubbles have an irregular shape, and it is assumed that they all grow in three dimensions as spheres of effective radius r . The bubble volume is represented by Eq. 2.80 in which the shape factor " a " is regarded as a constant (independent of time). It accounts for the size and geometry of the grains and the pores of the porous media. It does not account for possible changes in the shape of the bubbles during growth. A given bubble grows by diffusion of dissolved gas in the liquid towards the bubble/liquid interface. Due to the presence of the grains, and the nature of porous media, establishment of spherically symmetrical concentration field around a bubble may not be realistic. Even each portion of the bubble/liquid interface could advance as a result of 1-D diffusion of dissolved gas along a channel at a given time. In other words, growth in 3-D could occur as a result of mass transfer in one dimension. However, it is judged that 3-D diffusion with effective radius, r , could represent an approximation to bubble growth in porous media. In the following, a description of the experimental data of Refs. 4, 5, and 8 will be made with the theory. First, the results on constant pressure decline rate of Refs. 8 and 5 will be presented.

CONSTANT PRESSURE DECLINE RATE EXPERIMENTS - Ref. 8 provides the results of solution gas drive experiments on a St. Maximin limestone core ($L=5$ cm, $d=5$ cm, $\phi=29.3\%$, and $k=211$ md). A ternary hydrocarbon mixture of $C_1/C_3/C_{10}$ for which $p_b = 28.44$ bar [404.4 psia] at 37.8°C [100°F] was used in the experiments. At the bubblepoint, the interfacial tension is reported as 11.4 dynes/cm. Depletion experiments were performed by decreasing the pressure of a slightly undersaturated fluid at rates of 0.03, 0.22, 1.9, and 5 bar/day. Fig. 2.7 shows the data of Ref. 8. In this figure, the dashed line represents the equilibrium state. Fig. 2.7 also shows the calculated values from Eqs. 2.98 and 2.99 with $\omega = 1/2$ and $\nu = 1/2$. s° values are read from the experimental data of Fig. 2.7. The value of β can be obtained from the slope of the equilibrium line— $\beta = 0.87$ bar/cm³. With $V^\circ = 28.7$ cm³, $g_1 = 25$ bar (see Eq. 2.101). Table 1 gives C_1 values for various pressure decline rate experiments. The interesting step is the calculation of the number of bubbles. Parameter C_1 is related to the number of bubbles $N^\circ V^\circ$ according to the following relationship:

$$C_1 = \frac{\beta^{1/3}(aV^\circ N^\circ)^{1/3}}{c^{1/2}} \left(\frac{RTDK_s}{2p_b} \right)^{1/2} \quad (2.119)$$

Eq. 2.119 is derived by combining Eqs. 2.87, 2.102, and 2.118. Substitution of $D = 3 \times 10^{-5}$ cm²/s (from Ref. 19), and $K_s = 2.6 \times 10^{-6} \frac{\text{gmole } C_1}{\text{psi} \cdot \text{cm}^3}$ (calculated from the Peng-Robinson Equation of State²⁰ (PR-EOS)) and the remaining parameters into Eq. 2.119 gives $aV^\circ N^\circ$ values. These values are listed in Table 1. Assuming a value of $a = 0.1$ for the shape factor, the number of bubbles, $N^\circ V^\circ$, are then readily calculated. These values are also listed in Table 1. The number of bubbles, $N^\circ V^\circ$, should be considered correct only as to the order of magnitude due to the numerical value of the assigned shape factor. Table 1 shows that as the rate of pressure decline increases, the number of bubbles also increases. One may also conclude that at field rates of pressure decline, the bubble density, N° , would be very small. Our calculations are in line with the conclusions of Kennedy and Olsen that only one bubble per several order of magnitude pores of the porous media nucleates in the solution gas drive process. Due to low bubble density in comparison to the number of pores, the neglect of coalescence in the early phase of gas phase formation

is justified. Prior to gas flow, we assume that the bubbles are not connected.

Moulu and Longeron⁸ used the classical nucleation rate expression to calculate the number of bubbles. They had to use the rate expression with very low nucleation rate to compensate for model deficiency. The number of bubbles calculated by these authors is less than values of Table 1. As was pointed out earlier, the classical nucleation rate expression cannot provide the observed effect of rate on the critical supersaturation.

Ref. 5 reports gas saturation as a function of pressure for various binary hydrocarbon systems. The solution-gas drive experiments were conducted on a 550 md core from the North Sea. A connate water saturation was established in the core prior to hydrocarbon saturation. Fig. 2.8 shows the data of Ref. 5 for two mixtures of C_1/C_3 system. The bubblepoint pressure of 1 and 5 dynes/cm mixtures are 980, and 420 psia, respectively.²¹ Other data for these two mixtures are:²¹

1. $\sigma = 1$ dyne/cm mixture, $T = 86^\circ\text{F}$, $x_{c_1} = 35.8$ mole %, and
2. $\sigma = 5$ dynes/cm mixture, $T = 77^\circ\text{F}$, $x_{c_1} = 87.5$ mole %.

The pressure decline rate for the two mixtures are 11.7 psi/hr, and 5 psi/hr. The values of β , and K_s for the 1 and 5 dynes/cm mixtures are $\beta = 5.8$, and 2.9 psi/cm³, and $K_s = 6.22$ and 5.34×10^{-6} gmole of C_1 /[psi · cm³], respectively. The PR-EOS was used to calculate the solubility data, and β -values were estimated from Fig. 3 of Ref. 5. The diffusion coefficient of C_1 in the C_1/C_3 liquid phase was estimated from Fig. 4-12 of Ref. 24 as $D = 14 \times 10^{-8}$ ft²/s. Fig. 2.8 shows the calculated results and data. The C_1 values are listed in Table 1. The point where the gas saturation data start to deviate to the right shows the flow of gas out of the core. Beyond this point, the theory of this work does not apply. The dashed lines in Fig. 2.8 represent the equilibrium gas saturation data—the dashed line giving higher gas saturation at a given $(1 - p/p_b)$ value belongs to the 1 dyne/cm mixture. Both the theory and data show high supersaturation for the 5 dynes/cm mixture. Table 1 lists the number of computed bubbles (assuming $a = 0.1$). The rate of pressure decline is different only by a factor of 2.3 for the two mixtures. Since

the effect of rate on supersaturation is proportional to its square root, the main parameter to influence the supersaturation is the bubble density. The description of these two experiments by the theory shows that at low pressures, the supersaturation will increase significantly. This is an important finding and may be of practical importance for some low pressure oil reservoirs.

CONSTANT VOLUME EXPANSION RATE EXPERIMENTS - Solution gas-drive experiments by constant expansion rate are reported in Ref. 4. Both Berea and Chalk core samples were used. Binary mixtures of C_1/nC_{10} were employed in the measurement.

Fig. 2.9 shows the pressure vs. the expanded volume for two of the experiments on the Berea sandstone. The dashed line represents the single phase liquid both at the undersaturated and supersaturated conditions. The dotted line represents the equilibrium pressure/total volume (gas and liquid) expansion. The solid lines show the calculated pressure/expansion behavior according to Eqs. 2.106 and 2.107, with $\omega = 1/2$, and $\nu = 1/2$. Table 1 lists the C_2 parameter used to match the calculated and measured non-equilibrium pressure/volume values. Similar to the experiments for which the pressure decline rate was held constant, the number of bubbles increases by increasing the rate of volume expansion. The combined effect of rate and bubble density, however, results in a higher supersaturation at higher expansion rates.

Fig. 2.10 shows the pressure vs. the expanded volume on the Chalk sample. The fluid system and the rate of volume expansion is about the same as the high volume expansion rate of the Berea sample. Fig. 2.10 shows that after the initial supersaturation, the supersaturation decreases very fast. The main parameter to influence the pronounced decrease of the supersaturation is the high bubble density (see Table 1). The high bubble density for the Chalk sample is due to the large number of active sites for the low permeability Chalk. The value of the parameters used in the calculations for Figs. 2.9 and 2.10 are: $D = 6.5 \times 10^{-8}$ ft²/sec., $K_s = 1.75 \times 10^{-6}$ gmole of C_1 /(psi · cm³), and $\kappa = 0.0023$ cm³/psi, and $\beta = 14$ psi/cm³. Diffusion coefficient was obtained from Ref. 19, and the PR-EOS was used to calculate K_s , κ , and β . The value of shape factor was assumed to

be 0.1. The diffusion coefficient in the Chalk was also reduced by a factor of 4 to account for the porous media effect. The calculated values shown in parenthesis (see Table 1) are based on the effective diffusion coefficient.

We have also calculated the number of bubbles for the high pressure experiments of Ref. 4. The bubble density is much higher at high pressure than the low pressure tests.

CONCLUDING REMARKS

We have made a number of simplifying assumptions to introduce a simple theory for gas phase formation in porous media. The assumptions include neglect of: 1) capillary pressure, 2) compression or expansion on the growth of bubble, and 3) substitution of bubblepoint pressure for the pressure in the bubble growth expression. These assumptions are justified in most cases. As an example, the supersaturation is often of the order of 10-100 psi whereas capillary pressure is of the order of 1 psi or less. Here the issue of capillary pressure is different from the issue of interfacial tension. The interfacial tension affects the number of bubble nucleated at initial supersaturation s^o (see Table 1). Also at high pressures, the contribution of expansion on the growth of bubbles is small. This is mainly due to the fact that prior to gas mobilization, a pressure drop of say 100 to 300 psi will be small compared to a gas bubble pressure of say 2000 to 3000 psi.

In this study we have learned that solution gas drive process is initiated by gas phase formation through instantaneous nucleation. The bubble density measurements in Refs. 15 and 16 support the instantaneous nature of nucleation in porous media. The weak effect of the rate on the critical supersaturation, and supersaturation in general, for the progressive nucleation process (whether homogeneous or heterogeneous) is not in line with laboratory measurements. The instantaneous nucleation process accommodates the rate effect on supersaturation. The theory presented in this paper shows that supersaturation in porous media is related to $\frac{c^{1/2}}{No^{1/3}} \left(\frac{p_b}{TDK_s} \right)^{1/2}$ and $\frac{q^{1/2}}{No^{1/3}} \left(\frac{p_b}{TDK_s} \right)^{1/2}$ for constant pressure decline, and constant expansion rates, respectively. The supersaturation, therefore, increases with the square root of the rate if the number of active sites were the same. The supersaturation decreases with the cubic root of the number of bubbles. Low permeabil-

ity rocks generally provide more active sites, and therefore, should provide the condition for low supersaturation. In instantaneous nucleation, the effect of interfacial tension on supersaturation is far less than that of progressive nucleation.

Comparison of the results from the theory of this work with the experiments provides a sound model for the evolution of gas phase in porous media. The next step is to extend the model to critical gas saturation estimation.

Nomenclature

- A = kinetic constant of the classical nucleation rate expression,
 $1/(ft^3 \cdot s)$
- a = shape factor of a bubble
- B = thermodynamic constant of the classical bubble nucleation
rate expression, psi^2
- c = pressure decline rate
- c_T = isothermal compressibility
- C_1 = constant defined by Eq. 2.119
- C_2 = constant defined by Eq. 2.111
- d = core diameter
- D = diffusion coefficient of volatile component in the liquid phase
- G = growth rate of bubble, $G = \frac{dr}{dt}$
- g_1 = parameter defined by Eq. 2.101
- g_2 = parameter defined by Eq. 2.109
- J = nucleation rate, bubble/(volume · time)
- j = diffusion flux
- k = permeability
- K_s = solubility constant
- K = kinetic constant of bubble growth given by Eqs. 2.117 and 2.118

N^o	=	density of bubbles nucleated instantaneously at $t = 0$
n_b	=	number of moles
p	=	pressure
p_b	=	bubblepoint pressure
p_e	=	equilibrium pressure
q	=	volume expansion rate
Q_1	=	kinetic constant of progressive nucleation given by Eq. 2.87
Q_2	=	kinetic constant of instantaneous nucleation given by Eq. 2.88
R	=	universal gas constant
r	=	effective radius of a bubble
s	=	supersaturation
s_b	=	surface area of bubble/liquid interface
t	=	time
T	=	temperature
V	=	total volume
V_l	=	volume of liquid phase
V_g	=	volume of gas phase
V_b	=	volume at the bubblepoint
v_b	=	volume of a single bubble
x	=	variable defined by Eq. 2.100
x_{c_1}	=	mole % of C_1 (methane)
y	=	variable defined by Eq. 2.108

Greek Symbols

β	=	slope of the two-phase equilibrium line
κ	=	parameter of the liquid volume expansion defined by Eq. 2.77
ν	=	growth-parameter power
ρ	=	mass density

σ	=	interfacial tension
τ	=	time
ϕ	=	porosity, %
ω	=	growth-parameter power

Superscript

o	=	initial condition
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References

1. Barkve, T., and Firoozabadi, A.: "Analysis of Reinfiltration in Fractured Porous Media," SPE 24900, paper presented at the 67th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Washington D.C., Oct. 4-7, 1992, also Chapter 5.
2. Kashchiev, D., and Firoozabadi, A.: "Kinetics of the Initial Stage of Isothermal Gas Phase Formation," J. of Chemical Physics (March 15, 1993) 98, 6, 4690-4699, also Chapter 2.
3. Krous, W.: Amoco Production Company, private communications, March 1993.
4. Firoozabadi, A., Ottensen, B., and Mikklesen, M.: "Measurement of Supersaturation and Critical Gas Saturation," SPEFE Journal (Dec. 1992), 337-344.
5. Kortekaas, T.F.M., and van Poelgest, F.: "Liberation of Solution Gas During Pressure Depletion of Virgin Watered-Out Oil Reservoirs," SPERE Journal (Aug. 1991), 329-335.
6. Yortsos, Y., and Parlar, M.: "Phase Change in Binary Systems in Porous Media: Application to Solution Gas Drive," SPE 19697, paper presented at the 64th Annual

Technical Conference and Exhibition of the Society of Petroleum Engineers, San Antonio, TX, Oct. 8-11, 1989.

7. Li, X., and Yortsos, Y.: "Visualization and Numerical Studies of Bubble Growth During Pressure Depletion," SPE 22589, paper presented at the 66th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Dallas, TX, Oct. 6-9, 1991.
8. Moulu, J.C., and Longeron, D.: "Solution-Gas Drive: Experiments and Simulation," paper presented at the Fifth European Symposium on Improved Oil Recovery, April 1989, Budapest, Hungary.
9. Hirth, J.P., and Pound, G.M.: Condensation and Evaporation, Pergamon, Oxford, 1963.
10. Blander, M., and Katz, J.: "Bubble Nucleation in Liquids," A.I.Ch.E. J. (Sept. 1975) 21, 833-848.
11. Blander, M.: "Bubble Nucleation in Liquids," Adv. Colloid Interface Sci. (1979) 10, 1-32.
12. Theofanous, T., Biasi, L., Isbin, H.S., and Fauske, H.: "A Theoretical Study on Bubble Growth in Constant and Time-Dependent Pressure Fields," Chem. Eng. Sci. (1969) 24, 885-97.
13. Szekely, J., and Martins, G.P.: "Non-Equilibrium Effects in the Growth of Spherical Gas Bubbles Due to Solute Diffusion," Chem. Eng. Sci. (1971) 26, 147-59.
14. Szekely, J., and Fang, S.D.: "Non-Equilibrium Effects in the Growth of Spherical Gas Bubbles Due to Solute Diffusion - II: The Combined Effects of Viscosity, Liquid Inertia, Surface Tension and Surface Kinetics," Chem. Eng. Sci. (1973) 28, 2127-40.
15. Kennedy, H.T., and Olsen, R.: "Bubble formation in supersaturated hydrocarbon mixtures," Petroleum Transactions of AIME (1952) 195, 271-278.

16. El-Yousfi, A., Zarcone, C., Bories, S., and Lenormand, R.: "Mechanisms de formation d'une phase gaseuse per detente d'un liquide en milieu poreux," C.R. Acad. Sci. t.313, Paris (1991) ser. II, 1093-1098.
17. Press, W.H., Flannery, B.P., Teuksolsky, S.A., and Vetterling, W.T.: Numerical Recipes, Cambridge University Press, Cambridge (1989).
18. Cussler, E.L.: Diffusion-Mass Transfer in Fluid Systems, Cambridge University Press, Cambridge, U.K. (1984) Chapter 2, 44.
19. Reamer, H.H., Opfell, J.B., and Sage, B.H.: Diffusion Coefficients in Hydrocarbon Systems - Methane in Liquid Phase," Ind. and Eng. Chem. (Feb. 1956), 275-282.
20. Peng, D.Y., and Robinson, D.B.: "A New Two-Constant Equation of State," Ind. and Eng. Chem. (1976) No. 1, 59-64.
21. Van Engen, H.: Shell International, private communications, July 1992.
22. Katz, D.L., et al.: Handbook of Natural Gas Engineering, McGraw-Hill Book Co., New York, NY (1959) Chapter 4, 103.
23. Kaschiev, D., Verdoes, D., and Rosmaalen, G.M.: "Induction Time and Metastability Limit in New Phase Formation," J.of Crystal Growth (1991) 10, 373-380.

Table 2.1: Calculated bubble density (bubbles/cm³) for various solution gas drive experiments.

Rate	$aV^o N^o$	$V^o N^o$	$N^o, \frac{\text{bubbles}}{\text{cm}^3}$	Supersaturation Coefficient	Ref.
0.03 bar/day	0.75	7	0.2	$C_1 p_b^{2/3} = 9.8$	8
0.22 bar/day	19	190	0.7	$C_1 p_b^{2/3} = 11.25$	8
1.9 bar/day	45	450	1.5	$C_1 p_b^{2/3} = 4.85$	8
5 bar/day	137	1370	5	$C_1 p_b^{2/3} = 4.47$	8
11.7 psi/hr	290	2900	60*	$C_1 p_b^{2/3} = 7.7$	5
5 psi/hr	11	110	2*	$C_1 p_b^{2/3} = 2.3$	5
1.44 cm ³ /day	25	250	2	$C_2 p_b^{1/2} V_o^{1/6} = 6.95$	4
7.20 cm ³ /day	86	860	7	$C_2 p_b^{1/2} V_o^{1/6} = 4.73$	4
7.20 cm ³ /day	2200 (18,000)	22,000 (180,000)	98 (800)	$C_2 p_b^{1/2} V_o^{1/6} = 15.8$	4

*Estimated volume, $V^o = 50 \text{ cm}^3$

Fig. 2.5: Effect of rate on gas volume vs. pressure in PN and IN processes.

Fig. 2.6: Effect of pressure decline rate and interfacial tension on supersaturation in PN process.

Fig. 2.7: Measured and calculated non-equilibrium gas saturation/ P for constant pressure decline rate data of Ref.8.

Fig. 2.8: Measured and calculated non-equilibrium gas saturation/ P for constant pressure decline rate data of Ref.5.

Fig. 2.9: Measured and calculated non-equilibrium P/V for constant volume expansion rate data of the Berea sample from Ref.4.

Fig. 2.10: Measured and calculated non-equilibrium P/V for constant volume expansion rate of the Chalk sample from Ref.4.

CHAPTER 3

MISCIBLE DISPLACEMENT IN FRACTURED POROUS MEDIA

OVERVIEW

Two factors slow down gravity drainage rate in fractured porous media: 1) the capillary pressure contrast between the matrix medium and the fracture medium, and 2) the reinfiltration of the produced oil from the upper to the lower matrix blocks. Miscible displacement eliminates the capillary pressure contrast and reduces the reinfiltration. It also provides the conditions for a pronounced crossflow process. As a result, miscible displacement becomes a very efficient displacement process. In the following, we will show that concerns about production of large quantities of the injected fluid are ill-founded.

In this chapter, the first two subsections deal with the theoretical analysis of miscible displacement in fractured porous media. The experimental results are presented next. Finally, the data are analyzed.

3.1 THEORETICAL ANALYSIS OF MISCIBLE DISPLACEMENT IN FRACTURED POROUS MEDIA BY A ONE-DIMENSIONAL MODEL: PART II - FEATURES

James C.-T. Tan and Abbas Firoozabadi

INTRODUCTION

In Section 3.1 of this study, a one-dimensional model was formulated to describe miscible displacement in fractured porous media. The basic equations for the fracture (medium 1) and the matrix (medium 2) in dimensionless form are:

$$R_1 \frac{\partial C_1}{\partial t_D} + \frac{\partial (q_{1D} C_1)}{\partial z_D} - \frac{\partial q_{1D}}{\partial z_D} C_u = 0 \quad (3.97)$$

$$R_2 \frac{\partial C_2}{\partial t_D} + \frac{\partial (q_{2D} C_2)}{\partial z_D} + \frac{\partial q_{1D}}{\partial z_D} C_u = 0 \quad (3.98)$$

In Eqs. 3.97 and 3.98, q_{1D} and q_{2D} are the rates (volume/time) in the fracture and the matrix, respectively. These rates can be obtained from the expressions provided in Section 3.1 for both no crossflow (NC) and crossflow equilibrium (CE) cases.

For a single block system, the initial and boundary conditions are:

$$C_1 = C_2 = 1, \text{ and } q_{tD} = 0 \quad \text{at } t_D = 0, \text{ for all } z_D \quad (3.99)$$

$$P_{1D} = P_{2D} = \frac{DN_{gx}}{(D-1)} z_D \quad \text{at } t = 0 \quad (3.100)$$

$$q_{1D} = 1 \quad \text{at } t_D > 0, \text{ for all } z_D \quad (3.101)$$

$$C_1 = C_2 = 0, \text{ and } P_{1D} = P_{2D} = 0 \quad \text{at } t_D > 0, z_D = 0 \quad (3.102)$$

$$P_{1D} = P_{2D} \quad \text{at } t_D > 0, z_D = 1 \quad (3.103)$$

which, if it can be combined with the increase in gas-oil gravity drainage and the decrease in reinfiltration, results in a very efficient oil recovery process. Miscible displacement in fractured porous media encompasses all three.

There is very little work in the literature on miscible displacement in fractured porous media. Theoretical work is limited to a few papers published in the Russian literature. These papers center around the same formulation but do not address the basic issues of miscible displacement in fractured porous media. An early paper by Kats, et al.⁴ considers only the effect of gravity on crossflow between the matrix and the fractures in the context of a dual-porosity model. The authors compared the calculated effluent concentration with the measured data of Thompson and Mungan⁵. The lack of agreement between measurements and calculations leads the authors to conclude that convection currents which were not accounted for in their model were the main reason. A general model by Bedrikovetskii, et al.⁶ and Basniev and Bedrikovetskii⁷ is described in Refs. 6 and 7. The model of Refs. 6 and 7 includes the effect of gravity, diffusion, and convection. However, the assumptions which are required to proceed with the solution of the flow equations in a 3-D space are many and some unjustified. Comparison of the calculated effluent concentration and measured data of Ref. 5 shows lack of agreement. Results shown in Ref. 7 indicate that the recovery at 1 PV injection is sensitive to rate for low values, but at high rates it becomes insensitive to rate. This reveals that above a certain flow rate, the crossflow of the injected fluid from the fracture to the matrix becomes independent of flow rate. Such a behavior, if true, makes miscible fluid injection in fracture porous media unattractive. Our experimental work (currently in progress and to be published later) has shown clearly that miscible displacement in fractured porous media is an efficient process. Injected fluid is produced at low concentrations even at high injection rates. This implies that the magnitude of crossflow is very significant.

The purpose of this work is to provide a theoretical analysis of the miscible displacement in fractured porous media. Physical concepts related to the process will be emphasized in this work. A main goal is to examine the significance of gravity and viscous forces on the crossflow process towards the efficiency of miscible displacement in fracture porous media. Another objective is to study the effect of fracture density on miscible displacement efficiency. In this study, a one-dimensional model will be used. As will be seen later, the one-

dimensional model can capture the main features of miscible displacement in fractured porous media.

FORMULATION

Let us simplify our analysis by treating the fracture-matrix as a two-media system shown in Fig. 3.1. The displacement direction is the same as the gravity field, and z direction is positive downward. We also assume that (a) each medium is homogeneous with permeability k_i , (b) the displacing and displaced fluids are incompressible and first contact miscible (FCM), (c) dispersive mixing is negligible comparing with the convection effect by crossflow, (d) viscous fingering is negligible, (e) there is no volume change on mixing, and (f) the displacement is strictly one-dimensional. The governing equations can be written as:

$$\phi_1 A_1 \frac{\partial C_1}{\partial t} + \frac{\partial (q_1 C_1)}{\partial z} - \frac{\partial q_1}{\partial z} C_u = 0 \quad (3.1)$$

$$\phi_2 A_2 \frac{\partial C_2}{\partial t} + \frac{\partial (q_2 C_2)}{\partial z} + \frac{\partial q_1}{\partial z} C_u = 0 \quad (3.2)$$

$$\frac{\partial P_1}{\partial z} = - \frac{\mu_1 q_1}{k_1 A_1} + \rho_1 g \quad (3.3)$$

$$\frac{\partial P_2}{\partial z} = - \frac{\mu_2 q_2}{k_2 A_2} + \rho_2 g \quad (3.4)$$

$$\rho_i = \rho_i(C_i) \quad i = 1, 2 \quad (3.5)$$

$$\mu_i = \mu_i(C_i) \quad i = 1, 2 \quad (3.6)$$

where C_i is the solute concentration in media i (i.e., $C_i=1$ for solute and $C_i=0$ for solvent). Other symbols are defined in the Nomenclature. Subscripts 1, 2 and u refer to fracture, matrix, and upstream direction, respectively.

Eqs. 3.1 and 3.2 are mass conservation in the fracture (medium 1) and the medium (layer 2). The last term in these two equations represents the crossflow between the two media. Eqs. 3.3 and 3.4 are Darcy's law, and Eqs. 3.5 and 3.6 are density-concentration and viscosity-concentration relations, respectively. For the case of no volume change on mixing, density changes linearly with concentration. We will also assume that viscosity changes linearly with concentration. Therefore,

$$\rho_i = \rho_b + C_i \Delta \rho \quad i = 1, 2 \quad (3.7)$$

$$\mu_i = \mu_b + C_i \Delta \mu \quad i = 1, 2 \quad (3.8)$$

where $\Delta \rho = \rho_a - \rho_b$ and $\Delta \mu = \mu_a - \mu_b$, and ρ_a , μ_a , and ρ_b , μ_b are densities and viscosities of the solute (displaced fluid) and solvent (displacing fluid), respectively. Other models for viscosity-concentration relationship will be studied in the Chapter 3.2.

The above equations have to be solved with proper initial and boundary conditions. The initial and boundary conditions are:

$$C_1 = C_2 = 1, \text{ and } q_t = 0 \quad \text{at } t = 0, \text{ for all } z \quad (3.9)$$

$$P_1 = P_2 = \rho_a g z \quad \text{at } t = 0 \quad (3.10)$$

$$q_t = \text{constant} \quad \text{at } t > 0, \text{ for all } z \quad (3.11)$$

$$C_1 = C_2 = 0, \text{ and } P_1 = P_2 = 0 \quad \text{at } t > 0, z = 0 \quad (3.12)$$

$$P_1 = P_2 \quad \text{at } t > 0, z = L \quad (3.13)$$

The above initial condition states that the system is initially saturated with solute ($C_i=1$). At $t > 0$, solvent is injected at $z=0$ at a constant rate, inlet and outlet pressures are the same for fracture and matrix media, and inlet pressure is held constant at zero.

We next define dimensionless quantities,

$$\begin{aligned} Q_{iD} &= \frac{Q_i}{Q_c}, \quad z_D = \frac{z}{L}, \quad t_D = \frac{Q_c t}{\bar{\phi} A_c L}, \quad \mu_{iD} = \frac{\mu_i}{\mu_a}, \\ \rho_{iD} &= \frac{\rho_i}{\rho_a}, \quad P_{iD} = \frac{P_i k_2 A_c}{\mu_a Q_c L} \end{aligned} \quad (3.14)$$

and parameters

$$R_1 = \frac{\phi_1 A_1}{\bar{\phi} A_c}, \quad R_2 = \frac{\phi_2 A_2}{\bar{\phi} A_c} \quad (3.15)$$

$$S_1 = \frac{A_1}{A_c}, \quad S_2 = \frac{A_2}{A_c} \quad (3.16)$$

$$K = \frac{k_1}{k_2} \quad (3.17)$$

$$B = \frac{k_1 A_1}{k_2 A_2} \quad (3.18)$$

$$M = \frac{\mu_a}{\mu_b} \quad (3.19)$$

$$D = \frac{\rho_a}{\rho_b} \quad (3.20)$$

$$N_{gr} = \frac{\Delta \rho g k_2 A_c}{\mu_a Q_c} \quad (3.21)$$

where $A_c = A_1 + A_2$ and $\bar{\phi} = (\phi_1 A_1 + \phi_2 A_2) / A_c$. In the above equations, R_i is normalized medium capacity, S_i normalized medium area, K permeability ratio, B productivity capacity ratio, M viscosity ratio, D density ratio, and N_{gr} gravity number (parameters R_i and S_i are of the same

order). Note that, for fractured porous media, $K \gg 1$, but the productivity capacity ratio B can still be an order of 1 quantity. Also notice that there are two driving mechanisms that cause the crossflow: the viscous force that is characterized by M and the gravity force that is characterized by N_{gr} . We shall only investigate the case of $M > 1$ in this study.

Equations 3.1 through 3.6 in dimensionless form become

$$R_1 \frac{\partial C_1}{\partial t_D} + \frac{\partial (q_{1D} C_1)}{\partial z_D} - \frac{\partial q_{1D}}{\partial z_D} C_u = 0 \quad (3.22)$$

$$R_2 \frac{\partial C_2}{\partial t_D} + \frac{\partial (q_{2D} C_2)}{\partial z_D} + \frac{\partial q_{1D}}{\partial z_D} C_u = 0 \quad (3.23)$$

$$\frac{\partial P_{1D}}{\partial z_D} = - \frac{\mu_{1D} q_{1D}}{K S_1} + \frac{N_{gr} D \rho_{1D}}{(D-1)} \quad (3.24)$$

$$\frac{\partial P_{2D}}{\partial z_D} = - \frac{\mu_{2D} q_{2D}}{S_2} + \frac{N_{gr} D \rho_{2D}}{(D-1)} \quad (3.25)$$

$$\rho_{iD} = \rho_{iD}(C_i) = \frac{1 + (D-1) C_i}{D} \quad (3.26)$$

$$\mu_{iD} = \mu_{iD}(C_i) = \frac{1 + (M-1) C_i}{M} \quad (3.27)$$

with initial and boundary conditions:

$$C_1 = C_2 = 1, \text{ and } q_{uD} = 0 \quad \text{at } t_D = 0, \text{ for all } z_D, \quad (3.28)$$

$$P_{1D} = P_{2D} = \frac{D N_{gr}}{(D-1)} z_D \quad \text{at } t = 0 \quad (3.29)$$

$$q_{1D} = 1 \quad \text{at } t_D > 0, \text{ for all } z_D \quad (3.30)$$

$$C_1 - C_2 = 0, \text{ and } P_{1D} - P_{2D} = 0 \quad \text{at } t_D > 0, z_D = 0 \quad (3.31)$$

$$P_{1D} - P_{2D} \quad \text{at } t_D > 0, z_D = 1 \quad (3.32)$$

Furthermore, if the crossflow is from medium 1 (fracture) to medium 2 (matrix),

$$C_u = C_1 \quad (3.33)$$

and if the crossflow is from medium 2 (matrix) to medium 1 (fracture),

$$C_u = C_2 \quad (3.34)$$

From the total material balance,

$$q_{1D} + q_{2D} = 1 \quad (3.35)$$

Substitution of Eqs. 3.33 through 3.35 into Eqs. 3.22 and 3.23 results in:

Region I, crossflow from medium 1 to medium 2,

$$R_1 \frac{\partial C_1}{\partial t_D} + q_{1D} \frac{\partial C_1}{\partial z_D} = 0 \quad (3.36)$$

$$R_2 \frac{\partial C_2}{\partial t_D} + q_{2D} \frac{\partial C_2}{\partial z_D} + \frac{\partial q_{2D}}{\partial z_D} (C_2 - C_1) = 0 \quad (3.37)$$

Region II, crossflow from medium 2 to medium 1,

$$R_1 \frac{\partial C_1}{\partial t_D} + q_{1D} \frac{\partial C_1}{\partial z_D} + \frac{\partial q_{1D}}{\partial z_D} (C_1 - C_2) = 0 \quad (3.38)$$

$$R_2 \frac{\partial C_2}{\partial t_D} + q_{2D} \frac{\partial C_2}{\partial z_D} = 0 \quad (3.39)$$

Equations 3.36 through 3.39 combined with Eqs. 3.24 through 3.32 are partial differential equations that can be solved to determine the amount of crossflow. However, the above system of partial differential equations are difficult to solve, even numerically, when R_1 is small.

In order to simplify the problem, we now consider two limiting cases: Case I - no crossflow (NC), and Case II - crossflow equilibrium (CE). The former assumes that there is no mass exchange between fracture and matrix, while the latter assumes an instantaneous mass transfer between the two media. The CE is often referred to as the vertical equilibrium assumption for horizontal displacement (Coats et al.⁸, and Martin⁹). It was employed by Zapata and Lake¹⁰ and Pande and Orr¹¹ to study viscous crossflow problem in layered media. Neither NC nor CE is strictly valid for miscible displacement in fractured or layered media. The analysis of the two cases, however, will help to gain a better understanding of the problem, as well as to learn the dependence of the problem on different parameters.

The two cases we study are:

Case I - No crossflow (NC)

For this case,

$$\frac{\partial q_{1D}}{\partial z_D} = 0 \quad (3.40)$$

Case II - Crossflow equilibrium (CE)

With CE assumption, we assume the pressure in both media is equal at any point along the displacement length at all times, i.e.,

$$P_{1D}(t_D, z_D) = P_{2D}(t_D, z_D) \quad (3.41)$$

Combining Eqs. 3.24 through 3.27 and Eq. 3.41 leads to

$$Q_{1D}(C_1, C_2) = \frac{\mu_{2D}B - N_{gr}(C_2 - C_1)BS_2}{\mu_{1D} + \mu_{2D}B} \quad (3.42)$$

and

$$Q_{2D}(C_1, C_2) = \frac{\mu_{1D} + N_{gr}(C_2 - C_1)BS_2}{\mu_{1D} + \mu_{2D}B} \quad (3.43)$$

The formulation is now complete. Notice that Eqs. 3.24 and 3.25 are valid for equal density solute and solvent (i.e., $D=1$), since singularity can be removed from the second term on the right hand side of these two equations.

With the above formulation, the system is governed by the five parameters, R_1 , S_1 , B , M and N_{gr} . Since R_1 and S_1 are of the same order, four parameters R_1 , B , M and N_{gr} characterize the miscible displacement in fractured porous media. Next, the solution method will be provided.

SOLUTION METHOD

Case I - No Crossflow (NC)

In this case, since it is assumed that there is no dispersion or mixing between the injecting and displaced fluids, we anticipate piston-like fronts propagating in both the fracture and the matrix as shown in Fig. 3.2. The solution can be derived as follows:

(i) before breakthrough time, i.e., $t_D \leq t_{bD}$

Let z_{1D} and z_{2D} be the positions of interface between the solute and solvent for media 1 and 2, respectively. Integrating Eqs. 3.24 and 3.25 from $z_D=0$ to $z_D=1$ gives,

$$P_{1D}(t_D, 1) = -\frac{Q_{1D}}{MKS_1} [z_{1D} + M(1 - z_{1D})] + \frac{N_{gr}D(1 - z_{1D})}{D-1} + \frac{N_{gr}z_{1D}}{D-1} \quad (3.44)$$

$$P_{2D}(t_D, 1) = -\frac{Q_{2D}}{MS_2} [z_{2D} + M(1 - z_{2D})] - \frac{N_{gr}D(1 - z_{2D})}{D-1} + \frac{N_{gr}z_{2D}}{D-1} \quad (3.45)$$

Combining the above two equations with Eqs. 3.32 and 3.35, an expression for q_{1D} is obtained:

$$Q_{1D} = \frac{B [z_{2D} + M(1 - z_{2D}) - N_{gr} M (z_{1D} - z_{2D}) S_2]}{[z_{1D} + M(1 - z_{1D}) + B [z_{2D} + M(1 - z_{2D})]]} \quad (3.46)$$

Since $q_{1D} = R_1(dz_{1D}/dt_D)$ and $q_{2D} = R_2(dz_{2D}/dt_D)$, therefore

$$\frac{dz_{1D}}{dt_D} = \frac{B [z_{2D} + M(1 - z_{2D}) - N_{gr} M (z_{1D} - z_{2D}) S_2]}{R_1 [z_{1D} + M(1 - z_{1D}) + B [z_{2D} + M(1 - z_{2D})]]} \quad (3.47)$$

and

$$\frac{dz_{2D}}{dt_D} = \frac{1}{R_2} (1 - R_1 \frac{dz_{1D}}{dt_D}) \quad (3.48)$$

Eqs. 3.47 and 3.48 are integrated with respect to t_D and initial conditions $z_{1D} = z_{2D} = 0$, at $t_D = 0$ are applied to solve for z_{1D} and z_{2D} versus time. The breakthrough time t_{bD} is acquired from

$$z_{1D} = 1, \quad \text{at } t_D = t_{bD} \quad (3.49)$$

The total recovery R in this period is $R = t_D$, and solvent cut is $S_c = 0$.

(ii) after breakthrough time, i.e, $t_D > t_{bD}$

For $t_D > t_{bD}$, the fracture is filled with solvent so that

$$P_{1D}(t_D, 1) = - \frac{Q_{1D}}{MKS_1} + \frac{N_{gr}}{D-1} \quad (3.50)$$

Elimination of P_{1D} from Eqs. 3.45 and 3.50 gives

$$\frac{dz_{2D}}{dt_D} = \frac{Q_{2D}}{R_2} = \frac{1 + N_{gr} MB (1 - z_{2D}) S_2}{R_2 [1 + B [z_{2D} + M(1 - z_{2D})]]} \quad (3.51)$$

The total recovery is obtained by integrating Eq. 3.51 from t_{bD} to t_D :

$$R = t_{bD} + \int_{t_{bD}}^{t_D} Q_{2D} dt_D \quad (3.52)$$

and the solvent cut is $S_c = q_{1D}$.

Case II - Crossflow Equilibrium (CE)

In this case, the system of quasi-linear hyperbolic equations, Eqs. 36 through 39, can be solved by utilizing the method of characteristics (MOC)^{12,13}. The scheme is summarized below:

Equations 36 through 39 are first formulated as an eigenvalue problem. The system of equations in matrix form is:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial C_1}{\partial z_D} \\ \frac{\partial C_2}{\partial z_D} \end{bmatrix} - \lambda \begin{bmatrix} \frac{\partial C_1}{\partial z_D} \\ \frac{\partial C_2}{\partial z_D} \end{bmatrix} \quad (3.53)$$

The elements of the above matrix for Regions I and II are defined in Appendix A.

For both regions, the eigenvalues are

$$\lambda_1 = a_{11} \quad (3.54)$$

$$\lambda_2 = a_{22} \quad (3.55)$$

Stated simply, the eigenvalues are the characteristic velocities of a given concentration in the two media (i.e., $\lambda = (dz_D/dt_D)_{C=\text{const}}$). In each medium, only one eigenvalue can be selected to be the characteristic velocity at a given time. Since Eqs. 3.54 and 3.55 give two eigenvalues for each medium, the selection of the eigenvalue will depend on the nature of the crossflow. In the following, three branches of solution will be identified.

Dominant Viscous Crossflow

The criterion for a dominant viscous crossflow is $N_{gr} \leq (M-1)/[MS_2(B+1)]$. The derivation of

this criterion will be discussed shortly.

Let us take the eigenvalue $\lambda_1^I = a_{11} = q_{1D}/R_1$ for Region I and $\lambda_2^{II} = a_{22} = q_{2D}/R_2$ for Region II. The solution along the characteristic for each region is:

(a) Region I (medium 1 $\rightarrow$ medium 2)

$$\left(\frac{\partial C_2}{\partial C_1} \right)_{\lambda_1^I} = \frac{\frac{R_1}{R_2} \left[\frac{\partial q_{2D}}{\partial C_1} (C_2 - C_1) \right]}{q_{1D} - \frac{R_1}{R_2} \left[q_{2D} + \frac{\partial q_{2D}}{\partial C_2} (C_2 - C_1) \right]} \quad (3.56)$$

(b) Region II (medium 2 $\rightarrow$ medium 1)

$$\left(\frac{\partial C_1}{\partial C_2} \right)_{\lambda_2^{II}} = \frac{\frac{\partial q_{1D}}{\partial C_2} (C_1 - C_2)}{q_{1D} - \frac{\partial q_{1D}}{\partial C_1} (C_2 - C_1) - \frac{R_1}{R_2} q_{2D}} \quad (3.57)$$

As illustrated in Fig. 3.3, the crossflow direction is from fracture (medium 1) to matrix (medium 2) at the leading front of the displacement and vice versa at the trailing front. In order to determine composition paths, expressions for q_{1D} and q_{2D} from Eqs. 3.42 and 3.43 are used. Then the final form of Eq. 3.56 is integrated from the initial composition of $C_1 = C_2 = 1$ (i.e., point A) to the final composition $C_1 = 0$ and $C_2 = C_{2B}$ (i.e., point B) in Region I. Similarly, the composition path in region II is obtained by integrating Eq. 3.57 from the composition $C_1 = C_2 = 0$ (i.e., point C) to the final composition $C_1 = C_{1D}$ and $C_2 = 1$ (i.e., point D). Fig. 3.4 shows a typical C_2 versus C_1 plot for Region I (from point A to B) and Region II (from point C to D), respectively. The two curves intersect at point E with composition (C_{1E}, C_{2E}) . The complete composition path, therefore, goes from point A to E to C. The parameters that were used to prepare this plot are shown in Fig. 3.4. The characteristic velocity for any given C_1 and C_2 is calculated from $\lambda_1^I = q_{1D}/R_1$, and $\lambda_2^{II} = q_{2D}/R_2$, respectively, for the two regions. Plots of C_1 and C_2 versus z_D (shown as dashed lines) in Figs. 3.5 and 3.6 indicate that multiple concentrations exist at a given point. This is physically unrealistic and suggests concentration shocks. Since it is also required that solutions from the two regions match, we expect two

shocks (a leading shock and a trailing shock) and a zone of constant concentration (C_1^* , C_2^*) between the two shocks.

For the leading shock, the shock velocity λ_s^I is given by:

$$\lambda_s^I = \frac{Q_{1D}(1, 1) + Q_{2D}(1, 1) - C_1^* Q_{1D}(C_1^*, C_2^*) - C_2^* Q_{2D}(C_1^*, C_2^*)}{R_1(1 - C_1^*) + R_2(1 - C_2^*)} \quad (3.58)$$

For the trailing shock, the shock velocity λ_s^{II} is obtained from:

$$\lambda_s^{II} = \frac{C_1^* Q_{1D}(C_1^*, C_2^*) + C_2^* Q_{2D}(C_1^*, C_2^*)}{R_1 C_1^* + R_2 C_2^*} \quad (3.59)$$

In Eqs. 3.58 and 3.59, C_1^* and C_2^* are concentrations in the zone between the two shocks in media 1 and 2, respectively. These two equations are derived by using mass conservation principle across the shocks.

We also require the amount of solvent in each medium at any given time to be the same from both the concentration profile of the shock approach and from the dashed line profile of Figs. 3.5 and 3.6, i. e.,

$$\int_0^{C_{1s}} \frac{Q_{2D}}{R_2} dC_1 + \int_{C_{1E}}^1 \frac{Q_{1D}}{R_1} dC_1 = C_1^* \lambda_s^I + (1 - C_1^*) \lambda_s^{II} \quad (3.60)$$

and

$$\int_0^{C_{2s}} \frac{Q_{2D}}{R_2} dC_2 + \int_{C_{2E}}^1 \frac{Q_{1D}}{R_1} dC_2 = C_2^* \lambda_s^I + (1 - C_2^*) \lambda_s^{II} \quad (3.61)$$

Eqs. 3.60 and 3.61 can be viewed graphically in Figs. 3.5 and 3.6 as the area from the left hand side of the dashed line is equal to the area from the left hand side of the solid line. This kind of treatment is equivalent to the Buckley-Leverett theory for two media and two shocks

system.

We therefore solve Eqs. 3.58 through 3.61 for four unknowns: λ_s^I , λ_s^{II} , C_1^* and C_2^* . The solid line in Figs. 3.5 and 3.6 can then be plotted by knowing these values.

The initial breakthrough time is

$$t_{bD} = \frac{1}{\lambda_s^I} \quad (3.62)$$

and solvent cut is

$$S_c = (1 - C_1^*) q_{1D}(C_1^*, C_2^*) + (1 - C_2^*) q_{2D}(C_1^*, C_2^*) \quad (3.63)$$

Displacement ends (100% recovery) with the secondary breakthrough in medium 2 at which

$$t_{sD} = \frac{1}{\lambda_s^{II}} \quad (3.64)$$

Fig. 3.7 shows the flow rate in medium 1, which demonstrates that crossflow only occurs at two points: the leading shock and the trailing shock. The total flux of solute is

$$F^I = C_{1s} [q_{1D}(C_1^*, C_2^*) - q_{1D}(1, 1)] = C_1^* [q_{1D}(C_1^*, C_2^*) - \lambda_s^I] + \lambda_s^I - q_{1D}(1, 1) \quad (3.65)$$

at the leading shock, from fracture to matrix, and

$$F^{II} = C_{2s} [q_{2D}(0, 0) - q_{2D}(C_1^*, C_2^*)] = C_2^* [\lambda_s^{II} - q_{2D}(C_1^*, C_2^*)] \quad (3.66)$$

at the trailing shock, from matrix to fracture. In Eqs. 3.65 and 3.66, C_{1s} and C_{2s} are leading shock and trailing shock concentrations in media 1 and 2, respectively. Eqs. 3.65 and 3.66 can be derived through the mass balance of solute from upstream and downstream of the shock in the two media. For a fractured system, after a quick breakthrough of the leading shock, we

will have a constant total flux of solute, Eq. 3.66, from matrix to fracture, at any time before the secondary breakthrough.

Note that one has to use the conservation of mass, Eqs. 3.58 through 3.61, to solve for shock locations and concentrations. The assumption that the fractional flow at the shock is equal to the average of fractional flow from the upstream and downstream of the shock^{11,14} leads to $C_{1s}=C_1^*/2$ and $C_{2s}=(1+C_2^*)/2$, which is different from the results of Eqs. 3.65 and 3.66, and leads to erroneous solutions of the shock position.

In dominant viscous crossflow case, crossflow is driven by viscous force while gravity force opposes the process. Therefore, the amount of crossflow and thus C_1^* decreases by increasing N_{gr} . When $\partial C_2/\partial C_1=0$ for Region I and $\partial C_1/\partial C_2=0$ for Region II, $C_2^*=1$ and $C_1^*=0$, which implies that there is no crossflow (i.e., gravity crossflow completely counterbalances the viscous crossflow). This occurs when $N_{gr}=(M-1)/[MS_2(B+1)]$, which is obtained by setting the numerator of Eqs. 3.56 and 3.57 to zero. Thus the composition path of Fig. 3.4 could realize when $N_{gr} \leq (M-1)/[MS_2(B+1)]$.

Moderate Gravity Crossflow

The moderate crossflow is characterized by crossflow from medium 2 (matrix) to medium 1 (fracture) at the leading front and from medium 1 to medium 2 at the trailing front (see Fig. 3.8). The criterion for the moderate gravity crossflow is :

$$\frac{(M-1)}{MS_2(B+1)} < N_{gr} \leq \frac{(B+1) \left(M^2 B - \frac{R_1}{R_2} \right)}{BMS_2 \left[2MB + M + 1 + \frac{R_1}{R_2} (MB + B + 2) \right]} \quad (3.67)$$

The derivation of the above criterion will be discussed later.

For the moderate gravity case, the eigenvalues and solutions along the characteristic are:

(i) Region I (medium 1 $\rightarrow$ medium 2)

$$\lambda_2^I = a_{22} = \frac{q_{2D}}{R_2} + \frac{\partial q_{1D}}{\partial C_2} \frac{C_1 - C_2}{R_2}, \text{ and } C_1 = 0 \quad (6.68)$$

(ii) Region II (medium 2 $\rightarrow$ medium 1)

$$\lambda_1^{II} = a_{11} = \frac{q_{1D}}{R_1} + \frac{\partial q_{1D}}{\partial C_1} \frac{C_1 - C_2}{R_1}, \text{ and } C_2 = 1 \quad (3.69)$$

The composition path for this case is illustrated in Fig. 3.9, from point A to B along $C_1=0$ for Region I, and from point C to D along $C_2=1$ for Region II. In both regions, the high concentration fronts always move faster than the low concentration fronts. A continuous spectrum of characteristics, therefore, exists. A plot of C_1 and C_2 versus z_D at $t_D=0.5$ is shown in Fig. 3.10.

Let us define the following dimensionless times which correspond to the arrival time of points A, B, C and D at the outlet (see Fig. 3.10):

$$t_{aD} = \frac{R_1}{q_{1D}(1, 1)} \quad (3.70)$$

$$t_{bD} = \frac{R_1}{q_{1D}(0, 1) - \frac{\partial q_{1D}}{\partial C_1}(0, 1)} \quad (3.71)$$

$$t_{cD} = \frac{R_2}{q_{2D}(0, 1) - \frac{\partial q_{1D}}{\partial C_2}(0, 1)} \quad (3.72)$$

and

$$t_{dD} = \frac{R_2}{q_{2D}(0, 0)} \quad (3.73)$$

The solvent cut for various intervals can be expressed as:

$$(a) \quad t_D \leq t_{aD}, \quad S_c = 0 \quad (3.74)$$

$$(b) \quad t_{aD} < t_D \leq t_{bD}, \quad S_c = (1 - C_1) \alpha_{1D}(C_1, 1) \quad (3.75)$$

where C_1 is acquired by solving the following equation:

$$\alpha_{1D}(C_1, 1) - (1 - C_1) \frac{\partial \alpha_{1D}}{\partial C_1}(C_1, 1) = \frac{R_1}{t_D} \quad (3.76)$$

$$(c) \quad t_{bD} < t_D \leq t_{cD}, \quad S_c = \alpha_{1D}(0, 1) \quad (3.77)$$

$$(d) \quad t_{cD} < t_D \leq t_{dD}, \quad S_c = \alpha_{1D}(0, C_2) + (1 - C_2) \alpha_{2D}(0, C_2) \quad (3.78)$$

where C_2 is acquired by solving the following equation:

$$\alpha_{2D}(0, C_2) - C_2 \frac{\partial \alpha_{2D}}{\partial C_2}(0, C_2) = \frac{R_2}{t_D} \quad (3.79)$$

Fig. 3.11 shows the flow rate in medium 1 for a moderate gravity crossflow case at $t_D=0.5$, which indicates that crossflow occurs from point A to B and point C to D. At the leading front (C to D), pure solute flows from medium 2 (matrix) to medium 1 (fracture), while at the trailing front (A to B), pure solvent flows from medium 1 to medium 2. Both processes should help the recovery performance.

In this branch of the solution, the difference of $(t_{bD}-t_{cD})$ decreases as N_{gr} increases. When $t_{bD}=t_{cD}$, $N_{gr}=N_{gr}^*$, and from Eqs. 3.71 and 3.72,

$$N_{gr}^* = \frac{(B+1) \left(M^2 B - \frac{R_1}{R_2} \right)}{BMS_2 \left[2MB + M + 1 + \frac{R_1}{R_2} (MB + B + 2) \right]} \quad (3.80)$$

We now, move into the next branch of the solution.

Strong Gravity Crossflow

The criterion for a strong gravity crossflow is $N_{gr} > N_{gr}^*$, where N_{gr}^* is given by Eq. 3.80. For this case, the crossflow direction is the same as that in the moderate gravity crossflow case shown in Fig. 3.8. The eigenvalues and solutions along the characteristic are:

(i) Region I (medium 1 $\rightarrow$ medium 2)

As shown in Fig. 3.12, the composition path is from point A to B to C, and the eigenvalues and solutions along the characteristic are:

(a) For $0 \leq C_2 < C_{2B}$

$$\lambda_2^I = a_{22} = \frac{q_{2D}}{R_2} + \frac{\partial q_{1D}}{\partial C_2} \frac{C_1 - C_2}{R_2}, \text{ and } C_1 = 0 \quad (3.81)$$

(b) For $C_2 \geq C_{2B}$

$$\lambda_1^I = a_{11} = \frac{q_{1D}}{R_1} \quad (3.82)$$

and

$$\left(\frac{\partial C_2}{\partial C_1} \right)_{\lambda_1^I} = \frac{\frac{R_1}{R_2} \left[\frac{\partial q_{2D}}{\partial C_1} (C_2 - C_1) \right]}{q_{1D} - \frac{R_1}{R_2} \left[q_{2D} + \frac{\partial q_{2D}}{\partial C_2} (C_2 - C_1) \right]} \quad (3.83)$$

C_{2B} is the concentration at which the eigenvalues given by Eqs. 3.81 and 3.82 are equal,

$$\frac{q_{2D}(0, C_{2B})}{R_2} - \frac{\partial q_{1D}}{\partial C_2} \frac{C_{2B}}{R_2} = \frac{q_{1D}(0, C_{2B})}{R_1} \quad (3.84)$$

(ii) Region II (medium 2 $\rightarrow$ medium 1)

There is also a change of characteristic in this region:

(a) For $C_{1E} < C_1 \leq 1$

$$\lambda_1^{II} = a_{22} = \frac{q_{1D}}{R_1} + \frac{\partial q_{1D}}{\partial C_1} \frac{C_1 - C_2}{R_1}, \text{ and } C_2 = 1 \quad (3.85)$$

(b) For $C_1 \leq C_{1E}$

$$\lambda_2^{II} = a_{22} = \frac{q_{2D}}{R_2} \quad (3.86)$$

and

$$\left(\frac{\partial C_1}{\partial C_2} \right)_{\lambda_2^{II}} = \frac{\frac{\partial q_{1D}}{\partial C_2} (C_2 - C_1)}{q_{1D} - \frac{\partial q_{1D}}{\partial C_1} (C_2 - C_1) - \frac{R_1}{R_2} q_{2D}} \quad (3.87)$$

C_{1E} is the concentration at which the eigenvalues given by Eqs. 3.85 and 3.86 are equal,

$$\frac{q_{1D}(C_{1E}, 1)}{R_1} + \frac{\partial q_{1D}(C_{1E}, 1)}{\partial C_1} \frac{C_{1E} - 1}{R_1} = \frac{q_{2D}(C_{1E}, 1)}{R_2} \quad (3.88)$$

The composition path for region II is shown in Fig. 3.12 from point D to E to F. It intersects Region I at point G.

We next calculate the characteristic velocity λ along the composition path for both regions. Between points B and G, and points E and G (shown as dash lines in Figs. 3.13 and 3.14) multiple values of concentration exists, which again suggests a concentration shock. The shock velocity is, given by the jump condition (derivation given in Appendix B):

$$\lambda_s = \frac{q_{1D}(C_1^*, 1) C_1^* + q_{2D}(C_1^*, 1) - q_{2D}(0, C_2^*) C_2^*}{(R_1 C_1^* + R_2 - R_2 C_2^*)} \quad (3.89)$$

In Eq. 3.89, C_1^* and C_2^* are the concentrations upstream and downstream of the shock, in the fracture and matrix, respectively. Due to the fact that the solution is continuous elsewhere, we can approach the shock velocity from either sides of the shock, so that

$$\lambda_s = \lambda_s^I = \frac{q_{2D}(0, C_2^*)}{R_2} - \frac{\partial q_{1D}}{\partial C_2}(0, C_2^*) \frac{C_2^*}{R_2} \quad (3.90)$$

and

$$\lambda_s = \lambda_s^{II} = \frac{q_{1D}(C_1^*, 1)}{R_1} - \frac{\partial q_{1D}}{\partial C_1}(C_1^*, 1) \frac{(1 - C_1^*)}{R_1} \quad (3.91)$$

Equations 3.89 through 3.91 can be solved for λ_s , C_1^* and C_2^* . The position of the shock at any given time, t_D , is $\lambda_s t_D$. The solid lines in Figs. 3.13 and 3.14 are drawn by a knowledge of C_1^* , C_2^* and $\lambda_s t_D$.

The solvent cut for the dominant gravity crossflow case at various time intervals is

$$(a) \quad t_D \leq t_{ad}, \quad S_c = 0 \quad (3.92)$$

$$(b) \quad t_{ad} < t_D \leq \frac{1}{\lambda_s}, \quad S_c = (1 - C_1) q_{1D}(C_1, 1) \quad (3.93)$$

where C_1 is acquired by solving the following equation:

$$q_{1D}(C_1, 1) - \frac{\partial q_{1D}}{\partial C_1}(C_1, 1) = \frac{R_1}{t_D} \quad (3.94)$$

$$(c) \quad \frac{1}{\lambda_s} < t_D \leq t_{ad}, \quad S_c = q_{1D}(0, C_2) + (1 - C_2) q_{2D}(0, C_2) \quad (3.95)$$

where C_2 is acquired by solving the following equation:

$$q_{2D}(0, C_2) - \frac{\partial q_{1D}}{\partial C_2}(0, C_2) = \frac{R_2}{\tau_D} \quad (3.96)$$

Parameters λ_s , t_{1D} and t_{2D} are defined in Eqs. 3.89, 3.70 and 3.73, respectively.

Figure 3.15 shows flow rate in medium 1 for a strong gravity crossflow case at time $t_D=0.5$. This figure indicates that the crossflow occurs between the two fronts. From the leading front to the shock position, pure solute flows from medium 2 (matrix) to medium 1 (fracture), while from the trailing front to the shock, pure solvent flows from medium 1 to medium 2. Both processes should, again, help the recovery performance.

CONCLUDING REMARKS

A one-dimensional model has been formulated to study miscible displacement in fractured porous media comprised of a matrix and a fractured medium. The crossflow between the two media is characterized by the viscosity ratio M , and the gravity number N_{gr} . These two dimensionless numbers represent viscous and gravity crossflow mechanisms. Depending on the magnitude of viscosity ratio and gravity number, crossflow direction changes along the length of the matrix-fracture surface.

Three branches of solution are identified: 1) dominant viscous crossflow, 2) moderate gravity crossflow, and 3) strong gravity crossflow cases. The criteria for these cases have been established. In part 3.2, by using the methodology of this study, features of miscible displacement in fractured porous media will be presented. The predicted result from theory will be compared with the data of Thompson and Mungan⁵. The assumptions which were made to develop the model will also be justified in part 3.2.

NOMENCLATURE

A = cross sectional area

B = productivity capacity ratio, defined by Eq. 3.18

C = volumetric concentration per unit volume

D = density ratio, defined by Eq. 3.20
 F = total flux of solute
 k = permeability
 K = permeability ratio, defined by Eq. 3.17
 L = total length
 M = viscosity ratio, defined by Eq. 3.19
 N_{gr} = gravity number, defined by Eq. 3.21
 N_{gr}^* = a gravity number defined by Eq. 3.80
 P = pressure
 q = volumetric flow rate
 R = total recovery
 R_i = normalized medium capacity, defined by Eq. 3.15
 S_i = normalized medium area, defined by Eq. 3.16
 S_c = solvent cut
 t = time
 t_b = breakthrough time
 W = width
 z = distance in vertical direction
 ρ = mass density
 ϕ = porosity
 μ = viscosity
 λ = eigenvalue
 λ_1 = first eigenvalue
 λ_2 = second eigenvalue

Subscripts

$1,2$ = medium index (fracture, matrix)
 a = displaced fluid (solute)
 b = displacing fluid (solvent)
 D = dimensionless
 s = shock

t = total

u = upstream

Superscripts

I = Region I

II = Region II

REFERENCES

- [1] Firoozabadi, A., and Ishimoto, K.: "Theory of Reinfiltration in Fractured Porous Media: Part I - One-Dimensional Model," SPE 21796, paper presented at the 1991 Western Regional Meeting, Long Beach, CA, March 20-22, also SPE Advanced Technology Series (to appear).
- [2] Firoozabadi, A., Ishimoto, K., and Dindoruk, B.: "Theory of Reinfiltration in Fractured Porous Media: Part II - Two-Dimensional Model," SPE 21798, paper presented at the 1991 Western Regional Meeting, Long Beach, CA, March 20-22.
- [3] Firoozabadi, A. and Markeset, T.: "An Experimental Study of the Gas-Liquid Transmissibility in Fractured Porous Media," SPE 24919, paper presented at the 67th Technical Conference and Exhibition of the Society of Petroleum Engineers, Washington D.C., October 4-7, 1992, also SPE Reservoir Engineering (to appear), also Chapter 4.
- [4] Kats, R. M., Kurbanov, A. K., and Yusupova, Z. S.: "Flow of Two Miscible Fluids in a Fractured Porous Medium in a Gravity Field," Izvestiya Akademicheskikh Nauk SSSR, Mekh. Zhidk. Gaza, (March-April 1973) No. 2, 166-168.
- [5] Thompson, J. L., and Mungan, N.: "A Laboratory Study of Gravity Drainage in Fractured Systems Under Miscible Conditions," SPEJ (June 1969) 247-254.
- [6] Bedrikovetskii, P. G., Istomin, G. D., and Knyazeva, M. B.: "Miscible Displacement from Fractured Porous Media," Izvestiya Akademicheskikh Nauk SSSR, Mekh. Zhidk. Gaza, (Nov.-Dec. 1989) No. 6, 100-110.
- [7] Basneiv, K., and Bedrikovetskii, P. G.: "Miscible Displacement in Fractured Porous Media (Theory and Experiments)," paper presented at the 6th European IOR Symposium, Stavanger, Norway, May 21-23, 1991.
- [8] Coats, K. H., Dempsey, J. R., and Henderson, J. H.: "The Use of Vertical Equilibrium

in Two-Dimensional, Simulation of Three-Dimensional Reservoir Performance," Trans., AIME, 251, 63-71 (1971).

[9] Martin, J. C.: "Partial Integration of Equations of Multiphase Flow," Trans., AIME 243, 370-380 (1968).

[10] Zapata, V. J. and Lake, L. W.: "A Theoretical Analysis of Viscous Crossflow," SPE 10111, paper presented at 56th Annual Fall Technical Conference and Exhibition of the Society of Petroleum Engineers, San Antonio, Texas, October 5-7, 1981.

[11] Pande, K. K., and Orr, F. M., Jr.: "Interaction of Phase Behavior, Reservoir Heterogeneity, and Crossflow in CO₂ Floods," SPE 19668, paper presented at the 64th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, San Antonio, Texas, October 8-11, 1989.

[12] Rhee, H., Aris, R., and Amundson, N. R.: "First-Order Partial Differential Equations," Volume I, Prentice-Hall, Englewood Cliffs, N.J. (1986).

[13] Rhee, H., Aris, R., and Amundson, N. R.: "First-Order Partial Differential Equations," Volume II, Prentice-Hall, Englewood Cliffs, N.J. (1989).

[14] Pande, K. K. : "Interaction of Phase Behavior with Nonuniform Flow," PhD Dissertation, Stanford University, Stanford, CA (1989).

Appendix A: Elements of Matrix in Eq. 3.53

The elements of the matrix in Eq. 3.53 are:

(i) Region I (crossflow from medium 1 to medium 2)

$$a_{11} = \frac{q_{1D}}{R_1} \quad (A-1)$$

$$a_{12} = 0 \quad (A-2)$$

$$a_{21} = \frac{\partial q_{1D}}{\partial C_1} \frac{C_1 - C_2}{R_2} \quad (A-3)$$

$$a_{22} = \frac{q_{2D}}{R_2} + \frac{\partial q_{1D}}{\partial C_2} \frac{C_1 - C_2}{R_2} \quad (A-4)$$

(ii) Region II (crossflow from medium 2 to medium 1)

$$a_{11} = \frac{q_{1D}}{R_1} + \frac{\partial q_{1D}}{\partial C_1} \frac{C_1 - C_2}{R_1} \quad (A-5)$$

$$a_{12} = \frac{\partial q_{1D}}{\partial C_2} \frac{C_1 - C_2}{R_1} \quad (A-6)$$

$$a_{21} = 0 \quad (A-7)$$

$$a_{22} = \frac{q_{2D}}{R_2} \quad (A-8)$$

Appendix B: Derivation of Eq. 3.89

Addition of Eqs. 3.22 and 3.23 results in:

$$\frac{\partial(R_1 C_1 + R_2 C_2)}{\partial t_D} + \frac{\partial(q_{1D} C_1 + q_{2D} C_2)}{\partial z_D} = 0 \quad (B-1)$$

The shock condition for Eq. B-1 is¹²:

$$\lambda = \frac{(q_{1D} C_1 + q_{2D} C_2)^{II} - (q_{1D} C_1 + q_{2D} C_2)^I}{(R_1 C_1 + R_2 C_2)^{II} - (R_1 C_1 + R_2 C_2)^I} \quad (B-2)$$

where subscripts II and I refer to upstream and downstream conditions at the shock.

Substitution of $C_1^{II}=C_1^*$, $C_2^{II}=1$, $C_1^I=0$, $C_2^I=C_2^*$ results in Eq. 3.89.

Fig. 3.1 Schematic of miscible displacement in a fractured system.

Fig. 3.2 Schematic of no-crossflow displacement.

Fig. 3.3 Crossflow direction for the dominant viscous crossflow case.

Fig. 3.4 Compositional path for the dominant viscous crossflow case.

Fig. 3.5 C_1 vs. z_D for the dominant viscous crossflow case.

Fig. 3.6 C_2 vs. z_D for the dominant viscous crossflow case.

Fig. 3.7 q_{1D} vs. z_D for the dominant viscous crossflow case.

Fig. 3.8 Crossflow direction for the gravity crossflow case.

Fig. 3.9 Compositional path for the moderate gravity crossflow case.

Fig. 3.10 C_1 and C_2 vs. z_D for the moderate gravity crossflow case.

Fig. 3.11 q_{1D} vs. z_D for the moderate gravity crossflow case

Fig. 3.12 Compositional path for the strong gravity crossflow case.

Fig. 3.13 C_1 vs. z_D for the strong gravity crossflow case.

Fig. 3.14 C_2 vs. z_D for the strong gravity crossflow case.

Fig. 3.15 q_{1D} vs. z_D for the strong gravity crossflow case.

3.1 THEORETICAL ANALYSIS OF MISCIBLE DISPLACEMENT IN FRACTURED POROUS MEDIA BY A ONE-DIMENSIONAL MODEL: PART II - FEATURES

James C.-T. Tan and Abbas Firoozabadi

INTRODUCTION

In Section 3.1 of this study, a one-dimensional model was formulated to describe miscible displacement in fractured porous media. The basic equations for the fracture (medium 1) and the matrix (medium 2) in dimensionless form are:

$$R_1 \frac{\partial C_1}{\partial t_D} + \frac{\partial (q_{1D} C_1)}{\partial z_D} - \frac{\partial q_{1D}}{\partial z_D} C_u = 0 \quad (3.97)$$

$$R_2 \frac{\partial C_2}{\partial t_D} + \frac{\partial (q_{2D} C_2)}{\partial z_D} + \frac{\partial q_{1D}}{\partial z_D} C_u = 0 \quad (3.98)$$

In Eqs. 3.97 and 3.98, q_{1D} and q_{2D} are the rates (volume/time) in the fracture and the matrix, respectively. These rates can be obtained from the expressions provided in Section 3.1 for both no crossflow (NC) and crossflow equilibrium (CE) cases.

For a single block system, the initial and boundary conditions are:

$$C_1 = C_2 = 1, \text{ and } q_{tD} = 0 \quad \text{at } t_D = 0, \text{ for all } z_D \quad (3.99)$$

$$P_{1D} = P_{2D} = \frac{DN_{gr}}{(D-1)} z_D \quad \text{at } t = 0 \quad (3.100)$$

$$q_{1D} = 1 \quad \text{at } t_D > 0, \text{ for all } z_D \quad (3.101)$$

$$C_1 = C_2 = 0, \text{ and } P_{1D} = P_{2D} = 0 \quad \text{at } t_D > 0, z_D = 0 \quad (3.102)$$

$$P_{1D} = P_{2D} \quad \text{at } t_D > 0, z_D = 1 \quad (3.103)$$

All symbols are defined in the Nomenclature. The above system was solved analytically in Section 3.1.

Four parameters characterize miscible displacement in fractured porous media. These are: normalized fracture capacity R_1 ($R_1=(\phi_1 A_1)/(\phi A_J)$), productivity capacity ratio B ($B=(k_1 A_1)/(k_2 A_2)$), viscosity ratio M ($M=\mu_a/\mu_b$), and gravity number N_{gr} ($N_{gr}=(\Delta \rho g k_2 A_1)/(\mu_a q_1)$). The objectives of this study are to: 1) compare the result from theory to the experimental data of Thompson and Mungan¹, 2) examine the effect of different parameters on the displacement efficiency, and 3) study the effect of fracture density on the displacement performance. In the following, we will first study the features of miscible displacement in fractured media composed of a fracture and a matrix. The effect of fracture density on miscible displacement will be presented next. Finally, we will provide justification for some of the assumptions that we made in our model.

SINGLE BLOCK FEATURES

Thompson and Mungan¹ have conducted experiments to study miscible displacement in fractured porous media. In their experiments, liquid soltrol 130 was used as the displaced fluid, and liquid normal butane as the displacing fluid. The viscosity and density of normal butane and soltrol 130 at the condition of the experiments are 0.163 and 1.02 cp and 0.575 and 0.753 g/cm³, respectively. Let us select core/fracture system 11F of Ref. 5 for which $R_1=0.012$, $S_1=0.0023$, and $B=4.03$ ($k_1=1066$ darcy and $k_2=0.6445$ darcy). The viscosity ratio of soltrol 130 and normal butane is 6.3, and N_{gr} values are 0.42, 0.70, 1.05, 1.68, and 8.41. The five N_{gr} values are calculated from the V/V_c of Thompson and Mungan that equal to 2.0, 1.2, 0.8, 0.5 and 0.1, respectively. V is the actual displacement rate and V_c is the critical rate of instability (viscous fingering) defined by Slobod and Howlet² for miscible displacement in vertical homogeneous media,

$$V_c = k g \frac{\Delta \rho}{\Delta \mu} \quad (3.104)$$

The relationship between V_c/V and N_{gr} is:

$$N_{gr} = \frac{V_c}{V} \frac{M-1}{M} \quad (3.105)$$

Thus N_{gr} approaches V_c/V for large M .

The five flowrates for the core/fracture system 11F of Thompson and Mungan are in the gravity crossflow region. The criterion for strong gravity crossflow and moderate gravity crossflow are $N_{gr} > 0.543$ and $0.168 < N_{gr} \leq 0.543$, respectively (see Eqs. 3.67 and 3.80). Of the five different flow rates with five sets of N_{gr} , according to our criterion, one is characterized as the moderate gravity crossflow ($N_{gr}=0.42$), and the other four as the strong gravity crossflow case.

Fig. 3.16 shows the computed solvent cut as a function of time for various flow rates (i.e., gravity numbers) when there is no crossflow (i.e., NC case). Fig. 3.17 presents the same results for the CE condition. These two figures clearly indicate that there is a substantial difference in the main characteristics of NC and CE cases. The former shows that at low N_{gr} , there is a quick primary breakthrough, followed by a gradual increase in solvent cut, then a secondary breakthrough. But at high N_{gr} , the primary breakthrough time can be as late as 1 PV, while the latter always exhibits an early primary breakthrough, a plateau of constant solvent cut, followed by a secondary breakthrough with a long tail. It is also observed that CE is more efficient than NC at low N_{gr} but less efficient at high N_{gr} . This suggests that at low rates crossflow does not increase the miscible displacement process efficiency.

We next compare the theory with experiment 11F by Thompson and Mungan. As shown in Fig. 3.18 (Fig. 4 in Ref. 1), the data of Ref. 1 display a fast primary breakthrough, small increase in solvent cut for a long period of time, and secondary breakthrough with a long tail. All these features are captured by our CE model. We observe that there is good quantitative agreement between experiments and CE predictions as well. However, experimental data show a gradual increase of solvent cut instead of a sharp primary breakthrough for an initial period of time from the CE theory. This difference may be due to dispersive, nonuniform fracture aperture, or two-dimensional effects which are not included in our model. Nevertheless, we can conclude that the one-dimensional CE model is adequate to apprehend essential features

of miscible displacement in fractured porous media. It should be noted that results in Fig. 3.17 are predictions which are based on the displacement rate and the fluid and rock properties. We would also like to point out that the no-crossflow (NC) data of Thompson and Mungan for which the fracture surface was sealed have a very different trend than the predicted results in Fig. 3.16. The theory for the NC case is simple and is not expected to deviate from the data to a large extent. An effort is under way to repeat the measurements and results will be published later. In what follows, we will focus our attention on the effect of different parameters on the CE solutions.

Effect of R_1 ($=(\phi_1 A_1)/(\phi A_1)$) - For fractured porous media, the normalized fracture capacity $R_1 \ll 1$. Therefore we are only interested in small values of R_1 . Figure 3.19 shows solvent cut versus time for $M=6.3$, $N_{gr}=0.421$, $B=4.03$, $S_1=0.001$, and $R_1=0.1, 0.05, 0.01, 0.001$, respectively. As expected, R_1 determines the breakthrough time, but the effect on recovery is small. Since R_1 is generally less than 0.01 for fractured porous media, we can conclude that its influence on the recovery efficiency is small.

Effect of N_{gr} ($=(\Delta \rho g k_2 A_1)/(\mu_a q_1)$), M ($=\mu_a/\mu_b$) and B ($=(k_1 A_1)/(k_2 A_2)$) - As mentioned in Section 3.1, these three parameters are closely related to one another, and are the most important parameters in determining the crossflow direction of miscible displacement in fracture porous media. Therefore we shall discuss them at the same time.

Fig. 3.20 shows the effect of N_{gr} on recovery performance, which illustrates that solvent cut increases with decreasing N_{gr} . Since N_{gr} is the ratio of gravity to viscous forces, it relates the competition between the two forces on the efficiency of the displacement process. The competition between the two forces also determines the direction of crossflow. According to the analysis in Section 3.1, displacement is viscous crossflow dominant if $N_{gr} < (M-1)/[MS_2(B+1)]$, and gravity crossflow dominant if $N_{gr} > (M-1)/[MS_2(B+1)]$. The direction of the crossflow at the leading and trailing fronts reverses for the two cases. We can distinguish the two cases by examining the solvent cut plots of Fig. 3.20. For the dominant viscous crossflow case ($N_{gr}=0.1$, and 0.15 in Fig. 3.20), the solvent cut at the secondary breakthrough rises sharply to 1, while for the gravity crossflow case (all other cases), the solvent cut increases to 1 gradually after the secondary breakthrough. According to the CE theory of Section 3.1, there are two gravity crossflow cases: moderate gravity crossflow and strong

gravity crossflow. Yet we do not observe a significant difference between the characteristics of the solvent cut plots for the two cases in Fig. 3.20 ($N_{gr}=0.42$ for moderate gravity crossflow, and all others for strong gravity crossflow cases).

Fig. 3.21 presents solvent cut history for different values of the productivity capacity ratio B . We observe a shift from viscous dominant crossflow to gravity dominant crossflow as B increases. The effect of B on recovery is strong when B is small (dominant viscous crossflow case), but is weak when B is large (gravity crossflow case). Since B is usually in the range of 10 to 100 for fractured reservoirs, the effect of B can be small when the displacement rate is low (large N_{gr}).

Fig. 3.22 depicts solvent cut at different values of viscosity ratio M . The figure reveals that the effect of M on the recovery performance is small, especially for large M values. Since M is generally greater than 10 for field scale miscible flooding operations, and we are only interested in recovery up to 1 PV injection, the effect of M on the recovery performance can be small, provided the gravity number does not change when M is varied (i.e., only displacing fluid viscosity is changed).

MULTIPLE BLOCK MISCIBLE DISPLACEMENT FORMULATION

Fractured porous media are comprised of matrix blocks and fracture network. In addition to vertical and sub-vertical fractures, there is often horizontal and sub-horizontal fractures. The presence of fractures in the direction normal to flow (fracture density) causes mixing of fluids produced from upper matrix blocks and fracture fluids. In order to study the effect of fracture density on miscible displacement performance, a simple multiple block model shown in Fig. 3.23 is included in our analysis. The two media, fracture (medium 1) and matrix (medium 2) act exactly the same as the sketch shown in Fig. 3.1. We assume complete mixing between fluids coming out of the fracture and matrix of each row in Fig. 3.23, before entering into the next row of the matrix/fracture media (although partial mixing could also be assumed). Complete or partial mixing can provide valuable information on the importance of mixing of effluent from the upper matrix blocks and fractures.

Stacked Block Formulation and Numerical Scheme

The governing equations for the multiple block system are the same as that for the single block

system, Eqs. 3.97 and 3.98, and initial and boundary conditions, Eqs. 3.99 through 3.100. In addition, there is a boundary condition that describes mixing between fluids produced from the upper fracture and matrix of row n to the inlet of the matrix and fracture of row $n+1$:

$$C_1^{n+1} - C_2^{n+1} = \alpha_1^n C_1^n + \alpha_2^n C_2^n \quad n = 1, 2, \dots, N-1 \quad (3.106)$$

In Eq. 3.106, C_1^{n+1} and C_1^n are the inlet and outlet concentrations for the matrix and the fracture of rows $n+1$ and n , respectively, and N is the total number of stack rows in the system (Eq. 3.106 assumes complete mixing).

We are interested in the CE solution of the above problem. There may be no analytical solution and we have to solve the problem numerically. Thus, we need to develop a reliable numerical scheme and validate it with the method of characteristics (MOC) of Section 3.1. Solution of Eqs. 3.97 and 3.98 when R_1 is small can create substantial numerical difficulties and generate large material balance error, due to the stiffness of the system. Therefore, an alternative form of the material balance equations will be used to reduce the error. Addition of Eq. 3.97 to Eq. 3.98 yields a total material balance equation that has no crossflow term:

$$\frac{\partial (R_1 C_1 + R_2 C_2)}{\partial t} + \frac{\partial (\alpha_1 C_1 + \alpha_2 C_2)}{\partial x} = 0 \quad (3.107)$$

Defining $C_3 = R_1 C_1 + R_2 C_2$ results in:

$$\frac{\partial C_3}{\partial t} + \frac{\partial (\alpha_1 C_1 + \alpha_2 C_2)}{\partial x} = 0 \quad (3.108)$$

Eqs. 3.97 and 3.108 are solved instead of Eqs. 3.97 and 3.98 by using the finite difference method. An explicit first order upstream finite difference scheme was employed, and C_2 was evaluated from $C_2 = (C_3 - R_1 C_1) / R_2$. We used 200 grids in the z direction for most of our simulations, and 400 grids for the 16-block case. Also Δt was kept small to assure numerical stability.

Fig. 3.24 compares the finite difference (FD) solution with the analytical (i.e, MOC) results

for different N_{gr} values for the single block problem. The agreement between the two is excellent. The FD solution shows some numerical dispersion around the secondary breakthrough. This has a negligible effect on the total recovery performance. Thus we feel confident to apply the same numerical scheme to the multiple block problem. The objective of the following presentation is to investigate the effect of the number of blocks, N , on miscible displacement performance.

MULTIPLE BLOCK FEATURES

Figs. 3.25 through 3.27 compare the single block and four block-stack solutions for $M=6.3$, $R_1=0.012$, $S_1=0.0023$, $B=4.03$ and $N_{gr}=0.421$, 0.842 , and 1.683 ($V/V_c=2$, 1 , and 0.5), respectively. The total length of the system is the same for both the single block and the block-stack cases. It is observed that the solvent cut is significantly lower at early times for the four block-stack system than that of the single block system. The four-block solution exhibits an early primary breakthrough, similar to the characteristics of the single block system. But the solvent cut in the four block stack system, after a while, increases gradually with time. Figs. 3.25 through 3.27 demonstrate that miscible displacement in a multiple block-stack is more efficient than in the single block system, especially at high displacement rates. Figure 3.28 compares the solvent cut for different stack block numbers, N . As expected, the initial solvent cut decreases with increasing the block number. We therefore achieve a more efficient recovery at $PV=1$ when the fracture density increases. Thompson and Mungan¹ performed experiments in a core with one vertical fracture and several fractures with a 45 degrees angle in the direction of flow. They found that the angled fractures enhance the recovery performance, in accordance with our model prediction.

JUSTIFICATION OF ASSUMPTIONS

We have made certain assumptions to develop a model for the study of miscible displacement in fractured porous media. These assumptions have to be justified. In what follows, we shall investigate and evaluate some of these assumptions.

(i) Viscous Fingering Effect

Viscous fingering could arise when a more viscous fluid is displaced by a less viscous one.

For miscible displacement in fractured porous media, since the velocity and the dimension of the flow path is significantly different in the fracture and in the matrix, we need to investigate the viscous fingering effect in both media.

Fracture aperture (of the order of 0.1 mm or less) in comparison to matrix width is extremely small. Therefore only tiny fingers could form in it. These small fingers, according to the linear theory³, are damped out by the transverse dispersion before they can grow. On the other hand, for flow in the matrix, the process can be treated as miscible displacement in homogeneous porous media. According to the criterion in Ref. 2 for frontal instability in vertical displacements, viscous fingering could occur when the velocity in the matrix, V_2 , is larger than the critical velocity V_c , above which the displacement is unstable. In Fig. 3.29, we have plotted V_2/V_c at $z_D=1$ versus time for the case $R_1=0.012$, $S_1=0.0023$, $B=4.03$, $M=6.3$, and $N_{gr}=0.421$ ($V/V_c=2$). The above set of parameters represent Thompson and Mungan's experiment for the highest injection/production rate. Figure 3.29 shows that $V_2/V_c < 1$ at all times during the displacement (the existence of a transition zone between the displacing and displaced fluids can cause the displacement to be partially unstable at a rate lower than the critical rate⁴). A sharp secondary breakthrough instead of a smearing front in all the experiments supports that the viscous fingering effect is negligible. We therefore conclude that all of Thompson and Mungan's experiments were performed at the condition that the viscous fingering, if even occurred locally, should not have affected the recovery process. However, caution has to be taken if one wishes to apply the CE model to very high rates (i.e., very small N_{gr}).

(ii) Viscosity-Concentration Relation

Viscous force is one of the main driving mechanisms that governs miscible displacement in fractured porous media. Therefore, one may expect the viscosity dependence on concentration to be important. Let us examine the influence of the following three μ -C relationships on recovery performance:

(a) linear:

$$\mu_D = 1 + (M - 1) C \quad (3.109)$$

(b) quadratic:

$$\mu_D = 1 + (M - 1) C^2 \quad (3.110)$$

(c) quarter mixing:

$$\mu_D = \frac{M}{[(1 - C) + CM^{1/4}]^4} \quad (3.111)$$

The viscosity versus concentration for the above three cases for $M=6.3$ is shown in Fig. 3.30. From this figure, it is observed that the viscosity profile for the quarter mixing model is similar to that of the quadratic model and both show a slower variation of viscosity at low concentrations and a faster variation at high concentrations than that of the linear model. In Fig. 3.31, the calculated solvent cut is plotted versus time for the above three μ -C relations. Parameters used in the calculation are shown as a legend. Results are based on the finite difference method that was discussed in previous section. In addition to some 10% difference in solvent cut, the secondary breakthrough profile is also sharper for the quadratic and quarter mixing model. Both the quadratic and quarter mixing models result in a more efficient displacement than the linear model. Nevertheless, general features of the solvent cut are not affected by the μ -C relations. In our previous work in applying the CE theory, we used the linear μ -C relation for mathematical convenience. The quarter mixing model is a more common correlation and better describes the mixing behavior between hydrocarbons. This relation has also been widely applied in the simulation of miscible displacement in porous media⁵. We shall use quarter mixing rule in the rest of this study.

(iii) Dispersion

Dispersion could achieve an additional mixing between fluids. Due to small liquid-liquid diffusion coefficients, the molecular contribution to dispersion is negligible, especially at early times. The velocity is also low enough to expect a small contribution from flow to dispersion. The comparison of the measured effluent concentrations of Thompson and Mungan shown in Fig. 3.18 and results from theory shown in Fig. 3.17 suggests that the transverse dispersion

mechanism may not be negligible in the fracture, even at low velocity. Nevertheless, qualitative agreement between Figs. 3.17 and 3.18 is an indication that the dispersion effect cannot be pronounced.

(iv) Crossflow Equilibrium

Crossflow equilibrium is a key assumption of our model. It allows to eliminate the pressure dependency of the formulated problem and to solve the system of partial differential equations analytically. This assumption is justified if the resistance to flow (i.e, inverse of the linking permeability) between the fracture and matrix is small. Consequently, the pressure in the fracture and in the matrix at any given height will be the same and crossflow would occur instantaneously. However, both at laboratory and field conditions, the flow resistance between the fracture and matrix always exists. Thus, it is important to investigate the appropriateness of the CE assumption by comparing it with a true situation. Zapata and Lake⁶ suggested that in a horizontal displacement, under the condition of negligible gravity and capillary effects, the CE assumption (which they referred to it as vertical equilibrium) is valid for reservoirs with R_L values greater than ten. (Parameter R_L is given by $R_L = (L/H)(k_v/k_h)^{1/2}$, where k_v is the thickness-weighted harmonic vertical permeability, k_h the thickness-weighted arithmetic average horizontal permeability, L the length of the system, and H the thickness.) To our knowledge, no study has been performed for a vertical displacement process where the gravity effect is important. In order to investigate the issue of crossflow equilibrium, ECLIPSE reservoir simulator⁷ was used. The miscible displacement option in ECLIPSE with mixing parameter⁸ $\omega=1$ achieves complete mixing between the solvent and solute. This simulator assumes a quarter mixing viscosity-concentration relation.

The diameter of the cores used by Thompson and Mungan¹ was 5.08 cm (2'') and the length 30.48 cm (1'). For numerical simulation convenience, the cylindrical core is converted to a rectangular geometry with the same aspect ratio and the total pore volume. Due to the symmetry of the system, the displacement in one half of the experimental setup can be simulated.

First, the ECLIPSE results are validated by simulating a zero crossflow case (i.e, NC case). This can be achieved by assigning the linking permeability k_{12} between the fracture and the matrix to be zero. Comparison between the analytical and numerical solutions of a case is

shown in Fig. 3.32 (parameters are shown in the legend). This figure indicates that the simulation result is generally in good agreement with the analytical solution except at the location near the secondary breakthrough, where the simulation result shows a spreading front. The spreading is mainly due to the numerical dispersion effect and the stiffness of the problem (i.e., high aspect ratio and high permeability contrast). Nevertheless, the difference is not pronounced. Next, we increase the linking permeability to a value of $100k_2$ to look into results at the displacement rate $V=2V_c$ ($N_{gr}=0.42$), where k_2 is the matrix permeability. Fig. 3.33 gives the solvent cut versus time. Simulation results show a certain grid number effect. But the numerical result approaches the CE prediction when the number of grids in the Z direction, N_z , is increased to 500. Therefore, when the linking permeability equals to $100k_2$, the crossflow equilibrium assumption is justified. Simulation results with different linking permeability values at two different rates: $V=2V_c$ ($N_{gr}=0.42$) and $V=0.5V_c$ ($N_{gr}=1.68$) are given in Figs. 3.34 and 3.35, respectively. It is indicated that at high displacement rate ($V=2V_c$), the solvent cut decreases by increasing the linking permeability, and the CE and NC solutions are the lower and upper bounds of the solvent cut curves. This suggests that crossflow helps the miscible displacement at the high displacement rate. On the other hand, at the low displacement rate of $V=0.5V_c$, we observe a reverse trend: the recovery is higher for a smaller k_{12} value, and the CE and NC solutions are the upper and lower bounds of the solvent cut curves. This suggests that crossflow hinders the displacement at the low rate. An especially interesting scenario is when $k_{12}=k_2$, a case where the linking permeability achieves its lowest possible value (generally $k_2 < k_{12} < k_1$). From Figs. 3.34 and 3.35, we observe that the solvent cut for this case is about 8% higher than the CE solution for $V=2V_c$, and 2% lower than the CE solution for $V=0.5V_c$. This difference is not significant. Furthermore, we observe general features of the solvent cut from $k_{12}=k_2$ and the CE solution are similar. Therefore, it is concluded that the CE assumption might be a reasonable assumption.

DISCUSSION AND CONCLUSIONS

One main difference in feature between our theoretical prediction and Thompson and Mungan's experiments is that the experimental data show a gradual increase of solvent cut at early time, while our theory predicts a sharp primary breakthrough. This difference cannot be explained

by our one-dimensional model and is currently under investigation.

In this study, we have provided a multiple block fracture model to account for the fracture density. Complete mixing of produced fluids from the upper matrix blocks and fractures is assumed in our model. This assumption might predict a too optimistic recovery performance and can be replaced by a partial mixing model.

The model presented in this work is a simple tool which captures the important features of miscible displacement in fractured porous media. Our theoretical description leads to the following conclusions:

- 1 - The gravity number, N_{gr} is the most important parameter which affects miscible displacement performance in fractured porous media. This conclusion is in line with the experimental observations of Ref. 1.
- 2 - Fracture density has an important effect on miscible displacement efficiency, especially at high rates (i.e., low N_{gr}). The recovery efficiency improves as the fracture density increases. This conclusion is also in line with the experimental data of Ref. 1.
- 3 - There is an early breakthrough of the injected solvent but the solvent cut can be small in densely fractured porous media even at high rates of injection/production.
- 4 - The crossflow of injected solvent from the fracture to the matrix, and the crossflow of the solute from the matrix to the fracture is an important feature of miscible displacement in a fractured system. The magnitude of the crossflow, however, depends on the rate of displacement.
- 5 - The crossflow equilibrium is a reasonable assumption in studying miscible displacement in fractured porous media.

NOMENCLATURE

A = cross sectional area

B = productivity capacity ratio

C = concentration of solute, dimensionless

C_3 = total concentration defined in Eq. 3.108

D = density ratio

k = permeability

M = viscosity ratio

N = total number of blocks in a stack

N_z = total number of grids in the Z direction

N_{gr} = gravity number

N_{gr}^* = a gravity number above which strong gravity crossflow realizes

P = pressure

q = volumetric flow rate

R_i = normalized medium capacity

S_i = normalized medium area

t = time

V = velocity

V_c = critical velocity defined in Eq. 3.104

z = distance in vertical direction

ϕ = porosity

μ = viscosity

Subscripts

1,2 = medium (fracture, matrix)

a = displaced fluid (solute)

b = displacing fluid (solvent)

D = dimensionless

t = total

u = upstream

Superscripts

n = block index in a stack

REFERENCES

- [1] Thompson, J. L., and Mungan, N.: "A Laboratory Study of Gravity Drainage in Fractured Systems Under Miscible Conditions," SPEJ (June 1969) 247-254.
- [2] Slobod, R. L. and Hoelett, W. E.: "The Effects of Gravity Segregation in Studies of Miscible Displacement in Vertical Unconsolidated Porous Media," SPEJ (March, 1964) 1-8.

- [3] Gardner, J. W. and Ypma, J. G. J.: "An Investigation of Phase Behavior-Macroscopic Bypassing Interaction in CO₂ Flooding," SPE 10686, paper presented at the 1982 SPE/DOE Enhanced Oil Recovery Symposium, Tulsa, April 4-7.
- [4] Dumore, J. M.: "Stability Consideration in Downward Miscible Displacement," SPEJ (Dec. 1964) 356-62.
- [5] Koval, E. J.: "A Method for Predicting the Performance of Unstable Miscible Displacements in Heterogeneous Media," Trans., AIME (1963) 228, 145-154.
- [6] Zapata, V. J. and Lake, L. W.: "A Theoretical Analysis of Viscous Crossflow," SPE 10111, paper presented at 56th Annual Fall Technical Conference and Exhibition of the Society of Petroleum Engineers, San Antonio, Texas, October 5-7, 1981.
- [7] Eclipse Reference Manual, Version 92A; Exploration Consultants Ltd. (1992).
- [8] Todd, M. R. and Longstaff, W. J.: "The Development, Testing and Application of a Numerical Simulator for Predicting Miscible Flood Performance," Trans., AIME (1972) 253, 874-882.

Fig. 3.16 No crossflow solutions for a fractured system.

Fig. 3.17 Crossflow equilibrium solutions for a fractured system.

Fig. 3.18 Effect of displacement rate on butane cuts for core 11F containing a vertical fracture (taken from Fig. 4 of Ref. 1)

Fig. 3.19 Effect of R_1 on solvent cut.

Fig. 3.20 Effect of N_{gr} on solvent cut.

Fig. 3.21 Effect of B on solvent cut.

Fig. 3.22 Effect of M on solvent cut.

Fig. 3.23 Schematic of the multiple block system.

Fig. 3.24 Comparison between FD and MOC solutions.

Fig. 3.25 Comparison between single block and four block-stack solutions ($N_{gr}=0.42$).

Fig. 3.26 Comparison between single block and four block-stack solutions ($N_{gr}=0.84$).

Fig. 3.27 Comparison between single block and four block-stack solutions ($N_{gr}=1.68$).

Fig. 3.28 Effect of fracture density on solvent cut.

Fig. 3.29 Matrix production rate history at high injection rate ($N_{gr}=0.42$).

Fig. 3.30 Viscosity-concentration profile for the three relationships.
($M=6.3$ for all cases)

Fig. 3.31 Solvent cut versus time for different viscosity-concentration models.

Fig. 3.32 Comparison between simulation and analytical results for no-crossflow case.

Fig. 3.33 Simulation results for $k_{12}=100k_2$ as a function of N_Z .

Fig. 3.34 Simulation results of a displacement at $V=2V_c$.

Fig. 3.35 Simulation results of a displacement at $V=0.5V_c$.

3.3 MISCIBLE DISPLACEMENT IN FRACTURED POROUS MEDIA-PART I: EXPERIMENTS

Abbas Firoozabadi, and Tore Markeset

INTRODUCTION

Fractured reservoirs, similar to unfractured reservoirs may contain volatile oil. Some unfractured volatile oil reservoirs are generally excellent candidates for miscible gas injection¹. In the case of fractured volatile oil reservoirs, a major concern for miscible fluid injection is the early breakthrough and production of large quantities of the injected fluid. Capillary pressure contrast of the fracture and the matrix is a major parameter which causes low recovery efficiency of fractured reservoirs². Capillary pressure contrast of layered reservoirs also influences the gravity drainage performance³. Reduction or elimination of capillary pressure (miscibility or near-miscibility) is expected to improve recovery performance of both fractured and layered reservoirs for gravity drainage process^{2,3}. Miscible displacement could also be considered for fractured reservoirs which do not contain volatile oil. The results of a CO₂ miscible flooding pilot program in a fractured oil reservoir (API=29) in Canada has been promising⁴.

Our understanding of the basic mechanisms of miscible displacement in fractured porous media is very limited. The literature contains little information to address the basic issues of miscible displacement in fractured petroleum reservoirs. In 1969, Thompson and Mungan⁵ reported the results of an experimental study on gravity drainage in fractured porous media under first contact miscible conditions. They used vertically mounted 2-inch diameter fractured and unfractured rocks (mostly Berea). Soltrol 130 was used as the oil (solute) and liquid nC₄ as the displacing fluid (solvent). In these experiments, the fracture was represented by the space between the two half cores. Thompson and Mungan mainly studied the effect of displacement rate on recovery efficiency. They compared the displacement velocity to the critical velocity⁶ given by:

$$V_c = (\Delta\rho / \Delta\mu) g k \quad (3.112)$$

In Eq. 3.112 $\Delta\rho$ and $\Delta\mu$ are the density and viscosity difference between the solute (Soltrol), and solvent (nC₄), respectively. Parameter k is the permeability of the matrix. Based on a number of experiments, the authors concluded that the critical rate is a significant factor in the process efficiency. (Results of the displacement tests were expressed in terms of (V/V_c) where V is the overall velocity in the fracture/matrix system). A major characteristic of miscible displacement data of Thompson and Mungan is the shape of the effluent concentration of the solvent. A quick breakthrough is followed by

a sharp increase in the solvent concentration, then it stays constant for an extended period, followed by another rapid increase in concentration. The results from Ref. 5 show that fracture permeability does not affect the displacement process. On the whole, the data presented in Ref. 5 give recoveries of less than 60 percent at one PV injection for $(V/V_c)=2$. We are not aware of any other published experimental work on miscible displacement in fractured porous media.

EXPERIMENTAL

Fracture and matrix configuration of Ref. 5 covered a limited variety and the matrix blocks used were thin (less than 1 inch). The objective of our study was to study the effect of matrix/fracture configuration, fracture aperture and roughness, and fluid pairs on miscible displacement efficiency.

Apparatus:

Fig. 3.36 shows the schematics of the apparatus used for miscible displacement experiments. The setup consists of 3 distinct parts: 1) glass core-holder, 2) sampling device and gas chromatography (GC), and 3) vacuum and fluid charge. The glass core holder houses the 2-, 3-, and 12-slab assemblies. The glass is sealed using a rubber sheet which is stretched and glued to it in order to prevent fluid flow around the slabs. Pressure is applied around the glass plates to form the rubber to the rock surface.

A displacement pump and a back pressure regulator at the outlet end control the production rate. The effluent is then sampled in Teflon sealed vials using a multi-position electrically activated valve. An electric timer allows the sampling time of the produced fluid accumulation. The samples are weighed and analyzed by a computer controlled gas chromatograph.

The vacuum setup provides the dry slabs to be saturated with either normal decane (nC_{10}) or normal tetradecane (nC_{14}). Normal pentane (nC_5) reservoir at the top of the glass core holder provides the displacing fluid. The level in the nC_5 reservoir is held within 5 cm during an experiment.

Rock and Fluid Properties:

Table 3.1 gives the combined matrix/fracture pore volume (PV), matrix permeability, effective matrix/fracture permeability, fracture permeability, and fracture aperture of the various matrix/fracture configurations used in this study (see Fig. 3.37). Fracture PV is small (around 1% or less) and, therefore, matrix PV is close to the PV data of Table 3.1.

Matrix permeability was measured by a mini-permeameter. The two slabs comprising the two-slab configuration (see Fig. 3.37a) have different vertical permeabilities. The average permeability of one slab is 720 md, and the other slab is 898 md. The arithmetic average permeability of the two slabs is 809 md. Arithmetic average permeability of the three and twelve slabs is close to 700 md. Matrix porosity of the Berea sandstone slabs of Table 3.1 is around 22 percent. In all the experiments to be presented later, nC₁₀ (solute) and nC₁₄ (solute) were displaced by nC₅ (solvent). Table 3.2 shows the fluid properties of the three fluids at 76°F and 1 atmosphere. The density difference between nC₁₀ and nC₅, and nC₁₄ and nC₅ is 0.106, and 0.142 g/cm³, respectively. The viscosity ratio of the fluid pairs is 3.84, and 9.11.

Test Procedure:

Prior to every test, the slabs inside the core holder were dried under vacuum. Then, they were saturated with either nC₁₀ or nC₁₄. Next, nC₅ was injected to displace the in-situ fluid at a constant rate. Normal C₅ was provided from a reservoir to the inlet of the core holder. The reservoir was open to the atmosphere. At the test conditions of atmospheric pressure and a room temperature of 76°F (controlled to within $\pm 0.3^\circ\text{F}$), nC₅, nC₁₀, and nC₁₄ are in the liquid state. The amount of the produced fluid was recorded and weighed at regular intervals and was analyzed by the GC.

RESULTS

A number of experiments were carried out to examine the effect of, 1) fracture aperture, 2) fracture orientation, 3) fracture density, and 4) viscosity ratio and density difference of displaced and displacing fluids on miscible displacement in fractured porous media. The results will be presented in the following sections.

Two-Slab Experiments:

Four tests (S2A, S2B, S2C, and S2D) were carried out on the two slab assembly of Fig. 2a to examine the effect of, 1) fracture aperture, and 2) rate. The displaced fluid was nC₁₄ in all of the tests. In Tests S2A, and S2B, the faces of the matrix blocks (slabs) comprising the fracture surface were not machined. The fractures are, therefore, designated as unsmooth. When the two slabs were assembled, the opening on some locations at the inlet and outlet side of the two slab assembly was as large as 0.5 mm. Nevertheless, the two fracture surfaces were in contact in few spots. Two small (1 × 0.5 cm) shims of 0.4 mm thickness were inserted in the openings — one at the inlet face, the other at the outlet face

to avoid fracture configuration change after the core holder was assembled. Single phase flow of nC_{14} was used to measure the hydraulic aperture of the fracture between the two slabs. The fracture hydraulic aperture is about 400 microns. This aperture corresponds to a permeability of 13,170 darcy.

For Tests S2C, and S2D, the two faces of the slabs comprising the fracture were machined to decrease fracture surface waviness. When the two slabs were assembled, no openings could be detected in the fracture between the two slabs. The measured hydraulic aperture using liquid nC_{14} is 205 microns and the corresponding fracture permeability is 3500 darcy. After completion of Tests S2C and S2D, the fracture surfaces were examined. There were 8 areas of contact on each face. These contact areas were about 4 cm^2 each. The location of contact area was the same on each fracture surface. Unlike the fracture surfaces of the slab assemblies of Figs. 2b and 2c, where machining nearly eliminated fracture surface waviness, machining did not eliminate waviness of the fracture surface of the two slabs for Tests S2C and S2D.

The displacement results of Tests S2A, S2B, S2C, and S2D are depicted in Figs. 3 and 4. Figure 3.38 shows the amount of nC_5 (in terms of volume percent) in the effluent vs. the pore volume (PV) of the injected nC_5 . Fig. 3.39 shows the recovery of nC_{14} vs. the injected PV of nC_5 . The concentration profiles of Fig. 3.38 reveal that fracture aperture has a small effect at a low rate of $0.75\text{ cm}^3/\text{min}$ [$(V/V_c)=1$]. At a high rate of $1.5\text{ cm}^3/\text{min}$ [$(V/V_c)=2$] the effluent concentration profile becomes sensitive to the fracture aperture. The contact area between the two faces of the fracture promotes fluid crossflow between the matrix and the fracture. This point will be discussed in the companion report. The main characteristics of the effluent concentration profiles of the four tests are similar to the data of Thompson and Mungan⁵. These tests, however, reveal a more efficient displacement than the results in Ref. 5 and our own theoretical model^{7,8}. The analysis of the tests will be presented in the companion report.

Three-Slab Experiments:

A total of five tests were carried out with the slab arrangement of Fig. 3.37b. In Tests S3A, S3B, and S3C, nC_{10} was used as the displacing fluid, while in Tests S3D and S3E, nC_{14} was used. The slab faces comprising the fractures were machined to eliminate surface waviness. The measured fracture hydraulic aperture using single phase flow of both pure nC_{10} , and nC_{14} is about 150 microns. The corresponding fracture permeability is 2,055 darcy. Examination of the slab faces comprising the fracture after the completion of the tests revealed complete contact between the fracture surfaces all over.

Figs. 3.40 and 3.41 show the concentration and recovery plots for Tests S3A, S3B, and S3C. It is interesting to notice that the effluent concentration profile has a very different characteristics than the previous tests (S2A, S2B, S2C, and S2D) and the data reported by Thompson and Mungan⁵. These three tests (S3A, S3B, and S3C) and the two tests to be presented next, show a strong influence of fracture characteristics on miscible displacement efficiency. The effluent concentration profile increases gradually and then there is a sharp increase (especially for displacement rates of 0.56 and 1.4 cm³/min ($V/V_c=0.4$, and 1)). The steepness of the concentration profiles indicates that the concentration shock front in the matrix has reached the outlet boundary. The combination of gradual increase of the effluent concentration and the presence of a concentration shock in the matrix make the process of miscible displacement in fractured porous media very efficient. (This point will be discussed in detail in the companion report).

Figs. 3.42 and 3.43 present results of the displacement of nC₁₄ by nC₅ at two rates. The effluent concentration profiles and the recovery data indicate an efficient displacement process.

Twelve-Slab Assembly:

A set of comprehensive tests on the twelve-slab assembly of Fig. 3.37c were carried out to examine the influence of fracture density, fracture orientation, and rate on miscible displacement efficiency in fractured porous media. The slab faces similar to the slabs of Tests S3A, S3B, S3C, S3D, and S3E were machined to reduce surface waviness. After assembly, the effective single phase permeability was measured by flowing both nC₁₀, and nC₁₄. An effective permeability of $k_e = 8$ darcy was measured for the combined matrix/fracture system (see Table 3.3). The fracture hydraulic aperture based on the effective permeability is 180 microns, and the fracture permeability is 3,400 darcy. Test results for the twelve-slab assembly will be presented in the following order.

Vertical 12-Slab Assembly:

Figs. 3.44 and 3.45 depict the effluent concentration profile and recovery data for Tests S12A, S12B, S12C, S12D, and S12E. These tests conducted at different rates. For the fracture/matrix configuration of Fig. 3.37c, the miscible displacement efficiency exceeds the results of single row slabs presented earlier and the data of Thompson and Mungan. Note that at $[(V/V_c) = 2]$, the recovery factor at 1 PV injection is close to 80%, whereas the single row three-slab recovery is around 68%. In order to appreciate the importance of the results presented in Figs. 3.44 and 3.45, one may compare the immiscible gravity drainage performance of the same system presented in Fig. 3.46. The matrix/fracture

system was initially saturated with nC_{10} and then, gravity drainage took place due to invasion atmospheric air. As expected, the initial rate is high but decreases to $0.5 \text{ cm}^3/\text{min}$ in one day. After 14 days the recovery is 40 percent. On the other hand, the miscible displacement recovery is over 98 percent at 1 PV injection after 11 days for tests S12A (see Fig. 3.46). For Test S12E, at 1 PV injection (2.2 days) the recovery factor is close to 80%. It should be noted that the fluid density difference for the immiscible case is 0.724 g/cm^3 , and for the miscible case is only 0.104 g/cm^3 . Therefore, miscible displacement efficiency of the same setup would have been higher for $\Delta\rho=0.724 \text{ g/cm}^3$.

In order to study the influence of a different fluid system, one test (S12F) was carried out in which nC_{14} was displaced by nC_5 at a high displacement rate of $1.3 \text{ cm}^3/\text{min}$. $[(V/V_C)=2]$. The results are depicted in Fig. 3.46. Note that the recovery at 1 PV injection is about 82%. Except sharper effluent concentration at the secondary breakthrough (at $PV=1.3$) than the corresponding test with nC_5/nC_{10} fluid system (Test S12VE), the results of Tests S12F and S12E are similar [both have a $(V/V_C)=2$].

Forty-five Degree Tilted 12-Slab Assembly:

Three tests at a tilt angle of 45 degrees were carried out to study the effect of fracture orientation on the performance of miscible displacement process. In the tests, the core holder assembly was rotated along the edge. Figs. 3.48 and 3.49 present the results for Tests S12 T1, S12T2, and S12T3. These tests were carried out at the rates of 0.56, 1.4, and $2.8 \text{ cm}^3/\text{min}$ $[(V/V_C)=0.4, 1, \text{ and } 2]$. Both effluent concentration profiles and recovery plots show an extremely high displacement efficiency even at high rates. Note that the critical rates shown in Figs. 3.48 and 3.49 are based on vertical direction. Comparison of Figs. 3.45 and 3.49 demonstrates that the three tests at a 45 degree tilt angle are as efficient as the corresponding vertical tests. The high rate test (Test T12T3) even outperforms the corresponding vertical tests (T12VE). An efficient crossflow process and gravity drainage over a wide surface area are primary reasons for the high displacement efficiency (analysis will be made in the companion report). In the following two tests, we have further reduced the gravity substantially by increasing the tilt angle to 75 degrees.

Seventy-five Degree Tilted 12-Slab Assembly:

Two tests at a tilt angle of seventy-five degrees were conducted at flow rates of 1.4 and $2.8 \text{ cm}^3/\text{min}$ $[(V/V_C)=1, \text{ and } 2]$. Similar to the previous case, the rectangular core holder was rotated along the edge. Figs. 3.50 and 3.51 show the results of these two tests (T12TD, and T12TE). It is surprising that at a rate equal to the vertical critical rate [i.e., $(V/V_C)=1$] the concentration increases slowly to about 0.8 PV injection. It, then, increases at a fast rate to about 1.3 PV injection. A recovery factor of over 80 percent at 1 PV

injection is a sign of high displacement efficiency at $(V/V_C)=1$ (see Fig. 3.51). The crossflow between the fractures and the slabs improves the efficiency of displacement process. Nearly identical results indicating excellent reproducibility were obtained when duplicate runs were made for three of the tests presented above.

DISCUSSIONS AND CONCLUSIONS

Intuitively one would expect that for a miscible process when a fluid is injected in fractured porous media, the fractures would provide the flow path for the injected fluid. In other words, naturally fractured reservoirs with a very high fracture permeability ($k_f > 10^4 - 10^5 k_m$) are believed to be poor candidates for miscible fluid injection. The experimental work presented in this report suggests that first contact miscibility is a very efficient process. Due to a pronounced fluid crossflow process between the matrix and the fractures, a significant amount of the injected fluid flows from the fracture to the matrix. As a result, a significant amount of the matrix oil flows to the fracture. The fluid crossflow which is even higher than the results predicted from our theoretical model^{7,8} makes miscible displacement a very efficient process for fractured reservoirs.

This study has been limited to first contact miscibility with no volume change upon mixing of the fluids. The first contact miscibility allows to focus on the study of crossflow in a laboratory scale. Since miscible injection would be likely to be of vaporizing and/or condensing processes, this research work has to be extended to fluid systems which are not miscible in first contact. A comprehensive study of various miscible fluid injection processes in fractured porous media is a challenging task. Nevertheless, this study has revealed that the efficiency of miscible fluid injection for first contact miscibility in fractured porous media is strongly enhanced by, 1) a small fracture aperture and the degree of contact between the two faces comprising a fracture, and 2) fracture density. The results on the multi-block (i.e., 12-slab arrangement) fractured porous media reveal that miscible displacement is an extremely efficient process. The breakthrough is quick but the concentration of the injected fluid in the effluent increases very slowly to 1 PV injection (Analysis of the results of various experiments is presented in the companion report). Fracture aperture in a naturally fractured reservoir is generally less than 100 microns and the rates are often less than critical ($V < V_C$). The combination of these two factors could make miscible displacement in fractured porous media an extremely efficient process. On the whole, the miscible displacement experiments presented in this report are much more efficient than the results from Ref. 5.

NOMENCLATURE

- k — permeability
 k_e — effective permeability of the matrix/fracture system
 k_f — fracture permeability
 k_m — matrix permeability
 t_f — fracture aperture
 V — Darcy velocity
 V_c — critical velocity
 $\Delta\mu$ — viscosity difference of solute and solvent
 $\Delta\rho$ — density difference of solute and solvent

REFERENCES

- [1] Stalkup, F.: "Miscible Displacement," Chapter 7, Monograph, Society of Petroleum Engineers of AIME (1984).
- [2] Dindoruk B., and Firoozabadi, A.: "Computation of Gas-Liquid Drainage in Fractured Porous Media," Chapter 5.
- [3] Correa, A., and Firoozabadi, A.: "Concept of Gravity Drainage in Layered Porous Media," SPE 26299 (paper in review), also chapter 6.
- [4] Beliveau, D., and Payre, D.A.: "Analysis of a Tertiary CO₂ Flood in a Naturally Fractured Reservoir," SPE 22947, Paper presented at the 66th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers.
- [5] Thompson, J., and Mungan, N.: "A Laboratory Study of Gravity Drainage in Fractured Systems Under Miscible Conditions," SPEJ (June, 1969) 247–234.
- [6] Slobod, R.L., and Howlett, W.E.: "The Effect of Gravity Segregation in Studies of Miscible Displacement in Vertical Unconsolidated Porous Media," SPEJ (March 1964) 1–8.
- [7] Tan, C.T., and Firoozabadi, A.: "Theoretical Analysis of Miscible Displacement in Fractured Porous Media—Part I: Theory," Chapter 3.
- [8] Tan, C.T., and Firoozabadi, A.: "Theoretical Analysis of Miscible Displacement in Fractured Porous Media—Part II: Features," Chapter 3.
- [9] Tan, C.T., and Firoozabadi, A.: "Miscible Displacement in Fractured Porous Media—Part II—Analysis," Chapter 3.

Table 3.1 - PV, permeability, and aperture data.

Slab configuration	PV cm ³	k _m , vertical md	k _m , horizontal md	k _e darcy	t _f μm
Two-slab (unsmooth fracture)	2402	809	620	13,170	395
Two-slab (smooth fracture)	2338	809	620	3,500	204
Three-slab	2340	694	464	2,055	150
Twelve-slab	9020	734	634	3,400	180

Table 3.2 - Fluid Properties.

Fluid	Density, gr/cc	Viscosity, cp
normal Pentane (n-C ₅)	0.618	0.226
normal Decane (n-C ₁₀)	0.724	0.868
normal Tetradecane (n-C ₁₄)	0.760	2.06

Table 3.3 - Summary of Tests.

Test	Displaced Fluid	Tilt Angle, degree
S2A	nC ₁₄	0
S2B	nC ₁₄	0
S2C	nC ₁₄	0
S2D	nC ₁₄	0
S3A	nC ₁₀	0
S3B	nC ₁₀	0
S3C	nC ₁₀	0
S3D	nC ₁₄	0
S3E	nC ₁₄	0
S12A	nC ₁₀	0
S12B	nC ₁₀	0
S12C	nC ₁₀	0
S12D	nC ₁₀	0
S12E	nC ₁₀	0
S12F	nC ₁₄	0
S12TA	nC ₁₀	45
S12TB	nC ₁₀	45
S12TC	nC ₁₀	45
S12TD	nC ₁₀	75
S12TE	nC ₁₀	75

Fig. 3.36 - Sketch of the apparatus used in the experiments.

Fig. 3.37 - Matrix slab arrangements.

Fig. 3.38 - n-C5 Effluent Concentration (Vol%) vs. PV of Injected n-C5.

Fig. 3.39 - n-C14 Recovery (PV) vs. PV Injected n-C5.

Fig. 3.40 - n-C5 Effluent Concentration (Vol %) vs. PV of Injected n-C5.

Fig. 3.41 - n-C10 Recovery (PV) vs. PV Injected n-C5

Fig. 3.42 - n-C5 Effluent Concentration (Vol%) vs. PV Injected n-C5.

Fig. 3.43 - n-C10 or n-C14 Recovery (PV) vs. PV Injected n-C5.

Fig. 3.44 - n-C5 Effluent Concentration (Vol %) vs. PV of Injected n-C5.

Fig. 3.45 - n-C10 Recovery (PV) vs. PV Injected n-C5

Fig. 3.46 - n-C10 Recovery from Immisible Displacement.

Fig. 3.47 - n-C5 Effluent Concentration (Vol%) vs. PV Injected n-C5.

Fig. 3.48 - n-C5 Effluent Concentration (Vol %) vs. PV of Injected n-C5.

Fig. 3.49 - n-C10 Recovery (PV) vs. PV Injected n-C5

Fig. 3.50 - n-C5 Effluent Concentration (Vol %) vs. PV of Injected n-C5.

Fig. 3.51 - n-C10 Recovery (PV) vs. PV Injected n-C5

3.4 MISCIBLE DISPLACEMENT IN FRACTURED POROUS MEDIA - PART 2 ANALYSIS

James C.-T. Tan and Abbas Firoozabadi

INTRODUCTION

The initial (also the maximum) gravity drainage rate, q_o , of a homogeneous rock matrix for an immiscible process is¹

$$q_o = \frac{k_o}{\mu_o} \left(\Delta \rho g - \frac{P_c^{th}}{L} \right) \quad (3.113)$$

In Eq. 3.113, P_c^{th} is the threshold capillary pressure and L is the height. Other symbols are defined in the Nomenclature. Assumptions made in the derivation of Eq. 3.113 include infinite gas mobility. In a fractured porous media comprised of matrix blocks and fracture network, the initial (also the maximum) gravity drainage rate does not exceed the unfractured porous media, provided the fracture storage is negligible². After the initial period, due to the contrast in capillary pressures of the matrix and the fracture, the gravity drainage rate of the fractured media is less than the unfractured media³. For zero capillary pressure, Eq. 3.113 reduces to $q_o = (k_o/\mu_o)\Delta\rho g$. The critical rate q_{oc} for frontal instability (viscous fingering) in a homogeneous medium is given by⁴ $q_{oc} = (k_o/\Delta\mu_o)\Delta\rho g$. Therefore, the maximum drainage rate, $q_o \leq q_{oc}$, provided the displacing fluid has a negligible viscosity. In the experiments presented in Ref. 5, even when the rate of injection or production exceeded the critical rate, an efficient displacement was achieved. The main reason for such high recovery efficiency is a pronounced crossflow mechanism between the fracture medium and the matrix blocks.

The primary objective of this work is to analyze the laboratory measurements reported in Ref. 5 to investigate the mechanism which results in a very pronounced crossflow of fluids between the matrix and the fractures.

EFFECT OF FRACTURE HETEROGENEITY

Ref. 5 provides data on miscible displacement tests for different fluids at various injection rates (different V/V_c , V_c is the critical velocity, and $q_o = V$, $q_{oc} = V_c$ for a unit cross sectional area) in the two-, three- and twelve-slab assembly. The data reveal that the recovery performance in these tests are more efficient than the corresponding Thompson and Mungan's experiments⁶ performed at the same V/V_c . The effluent concentration is small after the primary breakthrough, then gradually increases to about 1 PV of injection. This characteristic is

completely different from the data of Thompson and Mungan, which show a sharp primary breakthrough, plateau of solvent cut for a long period of time, then a sharp secondary breakthrough. The use of both analytical crossflow equilibrium theory⁷ and numerical simulation in our previous study⁸ could predict the behavior of the experiments of Ref. 6, but gave a very different trend of solvent cut profile from the data of Ref. 5. As an example, Fig. 3.52 shows the simulation result and the measured solvent cut for Test S3C of Ref. 5. In the test, n-C₁₀ was displaced by n-C₅ in the three-slab assembly (see Fig. 2b of Ref. 5). The injection rate was 2.8 cm³/min ($V/V_c=2$). The numerical simulations were performed by using the Eclipse reservoir simulator⁹. The miscible displacement option in Eclipse with mixing parameter¹⁰ $\omega=1$ achieves complete mixing between the solvent and solute. This simulator assumes a quarter mixing viscosity-concentration relation. We assigned a constant permeability of $k_f=2,055$ darcy to the fracture medium. Other relevant data of matrix, fluid properties and fracture aperture were used in the simulation. The permeability for flow between fracture and matrix (linking permeability) is assumed to be 7 darcy. To reduce the number of simulation grids, we converted the three-slab assembly to an equivalent single matrix/fracture geometry with the same total pore volume (schematic is shown in Fig. 3.53). The total number of grids used in the simulation study is $2 \times 1 \times 100$. In Fig. 3.54, we compare the solvent cut profiles for displacement in the single matrix/fracture geometry and the three-slab assembly, which indicates that the results from the two geometries are equivalent. We also refined grids in the x and z directions and found little effect. Therefore, the difference between the simulation results and the data shown in Fig. 3.52 can be attributed to some mechanisms which are not included in the simulation model.

After an examination of the effect of various parameters, we were led to postulate that the difference in the characteristic of fractures in Refs. 5 and 6 is the reason for the difference in the solvent cut profiles. In the simulation results shown in Fig. 3.52, a constant permeability of 2,055 darcy and an aperture of 0.150 mm was assigned to the fracture medium. This may imply that the two faces of the matrix blocks comprising the fracture provide a gap that are not in direct contact. Due to a procedure used in cutting the cylindrical cores of Ref. 6, the fractures faces may have had small contact area. The two faces of the fractures in the three-slab assembly of Ref. 5, however, have a very high contact area. Therefore, one may not assign a constant fracture permeability to the entire fracture medium to simulate the displacements of Ref. 5. In the following, the effect of non-uniform fracture permeability on miscible flow performance will be examined.

Consider fracture permeability maps shown in Fig. 3.55 for the y-z plane. The permeability in the white region is 2,055 darcy and in the black region is 25 darcy. For Case 1, the fracture

permeability of 25 darcy is assigned to two grids, one located at $Z_D=0.2$ and the other at $Z_D=0.3$. Note that Z_D is the normalized vertical distance from the top. The total number of grids is $2 \times 2 \times 100$. The dimensions and matrix properties of the fractured porous media corresponding to Case 1 and the other cases shown in Fig. 3.55 are the same as the case shown in Fig. 3.53. The simulation results for the heterogeneous fracture medium of Case 1 as well as the homogeneous fracture medium case are shown in Fig. 3.56. Note that there is a substantial increase in recovery performance as a result of fracture heterogeneity. In order to examine the reasons beyond the improvement in recovery, the simulated results of the solute concentrations in the matrix and fracture are investigated. Fig. 3.57 depicts the solute concentration profile versus Z_D at $t_D=0.089$ PV for both homogeneous and heterogeneous fracture cases. In the figure, the concentration, both in the matrix and fracture is averaged over two grids in the y direction. Fig. 3.57a reveals that for the homogeneous fracture case, the solute concentration stays low in fracture and high in matrix as a result of low crossflow. On the other hand, for the heterogeneous fracture case, Fig. 3.57b reveals a sharp increase of the solute concentration in the fracture from $Z_D=0.2$, and $Z_D=0.3$, where the fracture heterogeneities are located. This implies high crossflow of solute from matrix to fracture. The crossflow of solvent from fracture to matrix takes place just before heterogeneity locations, as can be illustrated by a sharp decrease of concentration in the matrix. The combination of these two crossflow processes results in a stripping of the solute from matrix to fracture. We shall refer to this phenomenon, which is first reported in the literature, as the "stripping" effect. The stripping of solute from matrix to fracture, therefore, could significantly increase the recovery performance of miscible displacements in a fractured porous media. In what follows, we shall investigate the effect of the location and the magnitude of the fracture permeability heterogeneity on the recovery performance.

Effect of heterogeneity location

In this part of study, we will perform numerical simulation of miscible displacement for the fractured porous media of Fig. 3.53, where one of the six heterogeneous fractures of Fig. 3.55 represents the fracture medium. For all cases, two permeability values will be assigned to the fracture medium: 2,055 darcy to the white regions, and 25 darcy to the black regions. Matrix permeability and the linking permeability between the fracture and matrix are 0.7 darcy and 7 darcy (similar to the Case 1 described above), respectively. Note that for Cases 1 to 5, the two grids with permeability of 25 darcy change locations. For Case 6, the entire left hand side of fracture has a permeability 25 darcy. The injection rate for all cases was $2.8 \text{ cm}^3/\text{min}$.

Fig. 3.58 shows the solvent cut profiles for Cases 1 through 4. The solvent cut for the corresponding homogeneous fracture case is also shown in the same figure. Fig. 3.58 indicates that the recovery efficiency is always higher if the fracture is heterogeneous. The effect of the location of the fracture permeability heterogeneity is also important. Recovery seems to be more efficient when the location of fracture heterogeneity is in the middle. This can be explained by examining the solute concentration profiles at $t_D=0.754$ PV shown in Fig. 3.59. If permeability heterogeneity is located at an upper location (Case 1, Fig. 3.59a), once the displacement front in the matrix passes through the location of heterogeneity, the crossflow will not be much enhanced. On the other hand, if the location of heterogeneity is at the lower part of the fracture medium (Case 4, Fig. 3.59d), the stripping of solute is restricted to the limited end area. It is only when the location of permeability heterogeneity is in the middle part of the fracture (Cases 2 and 3, Figs. 3.59b and 3.59c) that one can achieve a prolonged and persistent crossflow of solute from matrix to fracture to maximize the stripping effect.

In Fig. 3.60a, we compare the solvent cut plots between Cases 2 and 5. For both cases, the location of the low permeability grids is the same ($Z_D=0.4$ and 0.5). But in Case 2, the low permeability grids are at the opposing sides of the fracture plane, while in Case 5 the grids are at the same side. Fig. 3.60a indicates that the recovery performance from Case 2 is more efficient than Case 5. This suggests that by alternating the location of fracture heterogeneity between the opposing sides, the effective area of stripping, and, therefore, the recovery performance increases.

Fig. 3.60b shows the solvent cut profiles for Cases 2 and 6. In Case 6, we assigned the permeability value of 2,055 darcy to the right hand side, and 25 darcy to the left hand side of the fracture. The total fracture kA value for Case 6 is about one half of the homogeneous fracture case with a permeability of 2,055 darcy. The kA value of Case 2 is nearly the same as for the homogeneous case. Fig. 9b indicates that, due to the additional crossflow caused by a larger variation of fracture permeability, Case 2 is more efficient than Case 6. This suggests that it is not the fracture kA value (i.e., fracture hydraulic permeability), but the location of the fracture permeability heterogeneity, that determines the recovery performance of miscible displacement in fractured porous media.

From this part of the study, we conclude that: 1) fracture heterogeneity can cause a stripping mechanism that significantly enhances the miscible recovery efficiency, and 2) the location of the fracture permeability heterogeneity is an important factor in the determination of the recovery performance.

Effect of magnitude of permeability heterogeneity

In this section, our objective is to investigate the effect of the magnitude of fracture permeability heterogeneity. A fracture medium with the permeability map shown in Fig. 3.61 was used in the study. A permeability of 2,055 darcy is assigned to the white region ($k_{\text{white}}=2,055$ darcy). The six grids shown in black in Fig. 10 (located at $Z_D=0.2, 0.3, 0.6, 0.7, 0.8,$ and 0.9) will be assigned different permeabilities in different simulation runs. The permeability at those six grids will be represented by k_{black} . Fig. 3.62 shows the effect of k_{black} on the simulated solvent cut profiles. Results indicate that the recovery performance to 1 PV increases with the decrease of k_{black} . As we increase the magnitude of permeability contrast, a larger pressure gradient between the fracture and the matrix at those locations increases the amount of crossflow to enhance the stripping effect. The solvent cut is approximately constant for a long period of time when $k_{\text{black}}=k_{\text{white}}$ (displacement in a homogeneous fracture medium), and increases gradually with time when k_{black} is small (displacement in a more heterogeneous fracture medium). The experiments by Thompson and Mungan show a trend similar to the homogeneous fracture permeability case (i.e., $k_{\text{black}}=2,055$ darcy). The experiments of Ref. 5 are represented by a fracture which has a heterogeneous permeability distribution. We can conclude, therefore, that the displacement performance in fractured porous media is enhanced in the presence of pronounced permeability variation of the fracture medium.

ANALYSIS OF EXPERIMENTS

From the above study, it is realized that the difference between the simulation results and the experimental data of Ref. 5 (see Fig. 3.52) is mainly due to the lack of fracture permeability heterogeneity description in our simulation model. We therefore, adjust the fracture permeability profile in our numerical simulations to match the experimental data. There is no unique fracture permeability profile in which we can achieve the match (inverse problem), and there is no known method to be used in the estimation of permeability variation in a given fracture. Perhaps, miscible displacement experiments could provide guidance of the degree of fracture permeability heterogeneity distribution. What we wish to establish in this paper is to illustrate that, once we can characterize the fracture permeability to match one experiment, by using the same permeability profile, we will be able to predict the results of all other experiments that were performed at different rates and with different fluids.

In what follows, we will simulate the experiments of Ref. 5 in the order of three-slab, twelve-slab, and two-slab assemblies (see Fig. 2 of Ref. 5 for system schematics).

Three-slab assembly

Previously we have demonstrated that the miscible displacement process in the three-slab assembly is the same as in the single matrix/fracture geometry of Fig. 3.53. To reduce the simulation time, we therefore used the single matrix/fracture geometry. In all the simulation runs of this section, $2 \times 2 \times 100$ grids were used.

Our first attempt was to match the solvent cut history for the displacement of $n\text{-C}_{10}$ by $n\text{-C}_5$, at the injection rate of $2.8 \text{ cm}^3/\text{min}$ (Test S3C in Ref. 5, $V/V_c=2$). By using the fracture permeability profile shown in Fig. 3.61, $k_{\text{white}}=2,055$ darcy and $k_{\text{black}}=25$ darcy, we were able to obtain an excellent match. Comparison between the simulation result and the measured data is shown in Fig. 3.63. We next simulated the displacement of $n\text{-C}_{10}$ by $n\text{-C}_5$ by using the same fracture permeability profile for the two other injection rates, $0.56 \text{ cm}^3/\text{min}$ (Test S3A, $V/V_c=0.4$) and $1.4 \text{ cm}^3/\text{min}$ (Test S3B, $V/V_c=1$). Comparison between the simulation results and data is shown in Fig. 3.64. The figure reveals that up to about 0.8 PV, there is an excellent agreement between data and simulation results for both tests. We observe that, even though the simulation results exhibit a certain amount of numerical dispersion, secondary breakthrough time is predicted satisfactorily. We also simulated the displacement of $n\text{-C}_{14}$ by $n\text{-C}_5$, at the injection rates of $0.65 \text{ cm}^3/\text{min}$ (Test S3D, $V/V_c=1$) and $1.3 \text{ cm}^3/\text{min}$ (Test S3E, $V/V_c=2$), respectively. Results shown in Fig. 3.65 indicate excellent agreements between simulation results and experimental data. Therefore, by using the same permeability profile for the fracture medium, we were able to predict all miscible displacement experiments that were performed in the 3-slab assembly at different rates and with different fluids. Notice that, in the simulation, we used two grids in the y-direction and a very simple fracture permeability profile. This suggests that there might be a simple way to characterize a fracture medium.

Twelve-slab assembly

The setup of the 12-slab assembly consists of four stacked-blocks of the 3-slab assembly, where each 3-slab row is separated from the top and bottom rows by horizontal fractures. All properties of the 12-slab system are similar to those of the 3-slab assembly (slightly different, see data in Table 1 of Ref. 5). Since the 12 slabs were machined in the same manner as the slabs of the 3-slab arrangement, we expect the fracture characteristic of the 12-slab assembly and the 3-slab assembly to be similar. Therefore, in numerical simulations, we used the fracture permeability map that we acquired from the match of the 3-slab experiment (shown in Fig. 3.61) as the fracture permeability profile for each of the four-stacked blocks. Matrix properties are given in Table 1 of Ref. 5. For fracture properties, k_{white} and k_{black} are 3,400 and 41 darcy (the same $k_{\text{black}}/k_{\text{white}}$ ratio as that for the 3-slab assembly). The permeability of horizontal

fractures between each block row is assumed to be 3,400 darcy. The number of grids used in the vertical 12-slab simulations is $2 \times 2 \times 403$.

In Fig. 3.66a, we compare the simulation prediction and experimental data for the displacement of $n\text{-C}_{10}$ by $n\text{-C}_5$, at an injection rate of $2.8 \text{ cm}^3/\text{min}$ (Test S12E, $V/V_c=2$). There is a reasonable agreement between the predicted results and data. The solute concentration profiles in both matrix and fracture at $t_D=0.089$ and 0.421 PV are shown in Fig. 3.67 (at each location Z_D , the concentration is the average of two grid point concentrations). Results show that there is a change of concentration in the matrix throughout, due to the stripping effect and the mixing in the horizontal fractures, which cause the displacement process to be very efficient. We observe that the concentration in matrix approaches to the concentration in fracture at $Z_D=0.25, 0.5$ and 0.75 , respectively (locations of horizontal fractures). The dashed lines in Fig. 3.67 designate the location of horizontal fractures. Fluids leaving the fracture and matrix system of the upper row mix in the horizontal fracture before entering the next row. In our previous report⁸, we developed a multiple stacked-block model to account for the mixing in the horizontal fractures. This model was based on the assumption of complete mixing. It appears from the simulation results in Fig. 3.67 that a partial mixing model is more appropriate to describe the mixing phenomenon. The concentration profiles of Fig. 3.67 provide a clear indication that the density of horizontal fractures enhances miscible displacement process efficiency.

Fig. 3.66b shows the result for the displacement of $n\text{-C}_{14}$ by $n\text{-C}_5$, at the injection rate of $1.3 \text{ cm}^3/\text{min}$ (Test S12F, $V/V_c=2$). We observe a good agreement between the simulation result and data. Notice that, all the simulation results in the 12-slab assembly are predictions. The only information we acquired was the fracture permeability profile that we obtained from the match of one experiment in the 3-slab system. Therefore, it seems that if one can characterize the fracture permeability heterogeneity for a small part of the media, it is possible to generalize the description to simulate the performance of miscible displacement in the entire fractured porous media.

We also attempted to simulate displacement in the tilted 12-slab assembly. Two cases were studied. Both cases are the displacement of $n\text{-C}_{10}$ by $n\text{-C}_5$ at the injection rate of $2.8 \text{ cm}^3/\text{min}$, and at inclined angles of 45° and 75° (Tests S12TC and S12TE) from the vertical axis. In the simulation, we increased the number of grids in x-direction to 3 (two matrix blocks and a fracture in between) to account for the gravity segregation effect. All the media properties and the fracture permeability profile are the same as those used in the vertical case. The total number of grids is $3 \times 2 \times 403$.

Figs. 3.68 shows the comparison between simulation results and data, which indicates less than a satisfactory agreement. We did not have adequate number of grids in the x- and y-directions to accurately model the gravity segregation effect. This issue is currently under investigation. Nevertheless, we can still observe in Fig. 3.68 that, due to the stripping caused by the fracture permeability heterogeneity and the mixing of fluids caused by the inclined fractures, miscible displacement in fractured porous media can be efficient, even for low inclined angles.

Two-slab assembly

All the experiments for the 2-slab assembly show a solvent cut characteristic that is similar to the data of Thompson and Mungan (sharp breakthrough and constant solvent cut for an extended period). They differ from the results of displacement in the 3-slab assembly (gradual increase of solvent cut). This observation could mislead one to believe that geometry might be important. In other words, 2-slab and 3-slab arrangements could result in different solvent cut profiles. After careful simulation work, we concluded that with identical fracture permeability, the 2- and 3-slab assemblies would give similar solvent cut profiles. The main difference between the matrix blocks used in the 2-slab and 3-slab assemblies is that, in the 2-slab case, the faces of the matrix blocks comprising the fracture surface were not initially machined. The fractures in the 2-slab case before machining are, designated as unsmooth. When the two slabs were assembled, the two fracture surfaces were in contact in limited contact areas, and the average hydraulic aperture between the slabs were larger than the 3-slab case. We used appropriate fracture properties and matrix data in numerical simulations ($k_m=0.809$ darcy, $k_{white}=13,170$ darcy, and $k_{black}=10$ darcy), and found that we were able to match the data by assigning all the fracture heterogeneities in the same side of y axis. The fracture permeability profile that was used to simulate the 2-slab arrangement is shown in Fig. 3.69. Two experiments were simulated. Both are the displacement of n-C₁₄ by n-C₅, at the injection rates of 0.75 cm³/min (Test S2A, $V/V_c=1$) and 1.5 cm³/min (Test S2B, $V/V_c=2$). Comparison between simulation results and data are shown in Fig. 3.70. The agreement between data and simulation results is very good - both the slowly increasing solvent cut segment of the profile and the secondary breakthrough time.

DISCUSSION AND CONCLUDING REMARKS

The fluid crossflow between matrix blocks and fracture medium exceeds the theoretical limit of crossflow equilibrium concept^{7,8,11}. In the crossflow equilibrium theory⁷, a constant fracture permeability and uniform aperture were assumed for the fracture medium. However, the fracture medium, which is composed of the domain between the matrix blocks may have a non-

uniform hydraulic aperture, and the two faces of a fracture may have limited or extensive contact area. The degree of contact could vary from fractures in one reservoir to another, or in one laboratory configuration to another. What we have described in the report, through analyzing laboratory miscible displacement data, is that fracture permeability heterogeneity strongly influences the miscible displacement process. We examined a variety of factors before arriving at our conclusion. For example, initially, it was speculated that the cubic law for the fracture aperture determination may not be valid¹². The use of nitrogen gas and liquids in flow measurements gave the same effective permeability of the matrix/fracture system. The measurements demonstrated the adequacy of cubic law.

The analysis of all the experiments reveals that the miscible fluid injection in various laboratory configurations of fractured porous media is a very efficient process. Crossflow between the fracture and matrix due to: 1) fracture permeability heterogeneity, and 2) fracture density (horizontal fractures) is the main reason for high miscible displacement efficiency.

NOMENCLATURE

A = cross sectional area

g = gravity

k = permeability

k_o = single phase oil permeability

L = height

P_c^{th} = threshold capillary pressure

q = volumetric flow rate

q_c = critical volumetric flow rate

t_D = time, PV

V = injection velocity

V_c = critical velocity for instability

Z_D = dimensionless distance in vertical direction

μ = viscosity

ρ = density

ω = mixing parameter

Subscript

o = oil

f = fracture

m = matrix

REFERENCES

- [1] Barkve, T., and Firoozabadi, A., "Analysis of Reinfiltration in Fractured Porous Media," SPE 24900, paper presented at the 67th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Washington D.C., October 4-7, 1992.
- [2] Firoozabadi, A., and Markeset, T., "An Experimental Study of the Gas-Liquid Transmissibility," SPE 24919, paper presented at the 67th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Washington D.C., October 4-7, 1992, also SPE Reservoir Engineering (to appear), also Chapter 4.
- [3] Dindoruk, B., and Firoozabadi, A., "Computation of gas-liquid drainage in fractured porous media," Chapter 5.
- [4] Slobod, R. L., and Howlett, W. E., "The Effects of Gravity Segregation in Studies of Miscible Displacement in Vertical Unconsolidated Porous Media," SPEJ (March, 1964) 1-8.
- [5] Firoozabadi, A., and Markeset, T., "Miscible Displacement in Fracture Porous Media - Part 1 Experiments," Chapter 3.
- [6] Thompson, J., and Mungan, N., "A Laboratory Study of Gravity Drainage in Fractured System Under Miscible Conditions," SPEJ (June, 1969) 247-254.
- [7] Tan, C. T., and Firoozabadi, A., "Theoretical Analysis of Miscible Displacement in Fractured Porous Media - Part I Theory," Chapter 3.
- [8] Tan, C. T., and Firoozabadi, A., "Theoretical Analysis of Miscible Displacement in Fractured Porous Media - Part II Features," Chapter 3.
- [9] Eclipse Reference Manual, Version 92A, Exploration Consultants Ltd. (1992).
- [10] Todd, M. R. and Longstaff, W. J., "The Development, Testing and Application of a Numerical Simulator for Miscible Flood Performance," Trans., AIME (1972) 253, 874-882.
- [11] Zapata, V. J., and Lake, L. W., "A Theoretical Analysis of Viscous Crossflow," SPE 10111, paper presented at the 64th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, San Antonio, Texas, October 5-7, 1981.
- [12] Iwai, K., "Fundamental Studies of Fluid Flow Through a Single Fracture," Ph.D. thesis, University of California, Berkeley, 1976.

Fig. 3.52 Comparison of the measured solvent cut and simulation result of Test S3C (assumed $k_f=2,055$ darcy).

Fig. 3.53 Geometry used in the 3-slab assembly simulation.

Fig. 3.54 Comparison between simulation results in the 3-slab and the single matrix/fracture geometry.

y-z Plane

Fig. 3.55 Sketches of the fracture permeability profiles used in the study, (a) Case 1, (b) Case 2, (c) Case 3, (d) Case 4, (e) Case 5, and (f) Case 6.

Fig. 3.56 Comparison between simulation results of homogeneous and heterogeneous fracture medium.

Fig. 3.57 Solute concentration profiles at $t_D=0.089$ PV for (a) homogeneous fracture, and (b) heterogeneous fracture.

Fig. 3.58 Solvent cut profiles for Cases 1 through 4.

Fig. 3.59 Solute concentration profiles at $t_D = 0.754$ PV for (a) Case 1, (b) Case 2, (c) Case 3, (d) Case 4, and (e) homogeneous fracture case.

Fig. 3.60 Comparison of solvent cut profiles between (a) Cases 2 and 5, and (b) Cases 2 and 6.

y-z plane

Fig. 3.61 Fracture Permeability map on the y-z plane used in the study of the effect of magnitude of permeability heterogeneity.

Fig. 3.62 Effect of the magnitude of fracture permeability heterogeneity on the solvent cut profile.

Fig. 3.63 Comparison between simulation result and data of Test S3C.

Fig. 3.64 Comparison between predicted simulation results and data of (a) Test S3A, and (b) Test S3B.

Fig. 3.65 Comparison between predicted simulation results and data of (a) Test S3D, and (b) Test S3E.

Fig. 3.66 Comparison between predicted simulation results and data of (a) Test S12E, and (b) Test S12F.

Fig. 3.67 Simulated solute concentration profile for the displacement in the 12-slab assembly (Test S12E) at the time, (a) 0.089 PV, and (b) 0.421 PV.

Fig. 3.68 Comparison between predicted simulation results and data for tilted 12-slab assembly, (a) 45° inclined angle, Test SC12TC, and (b) 75° inclined angle, Test SC12TE.

y-z plane

Fig. 3.69 Fracture permeability profile on y-z pane in the study of Tests S2A and S2B.

Fig. 3.70 Comparison between simulation results and data in the 2-slab assembly for (a) Test S2A, and (b) Test S2B.

CHAPTER 4

EXPERIMENTAL STUDIES OF GAS-OIL GRAVITY DRAINAGE IN FRACTURED POROUS MEDIA

OVERVIEW

Gas-oil gravity drainage in fractured porous media is strongly affected by, 1) liquid flow in the fractures, and 2) reinfiltration of the produced oil to the matrix. The flow of oil phase in the fractures is a key element to production performance of fractured reservoirs for which gravity drainage is the dominant mechanism. In this chapter, we will address various issues of immiscible gas-oil displacement in fractured porous media by conducting appropriate laboratory experiments. First the issue of reinfiltration will be presented. Then the effect of fracture aperture on liquid phase transmissibility will be studied. The topic of capillary crossflow across a fracture will be discussed last.

4.1 LABORATORY STUDY OF REINFILTRATION FOR GAS-LIQUID SYSTEM IN FRACTURED POROUS MEDIA.

Abbas Firoozabadi, and Tore Markeset

INTRODUCTION

Numerical simulation¹⁻⁶ and analytical studies^{7,8} have shown that reinfiltration is an important process for gas-oil gravity drainage in fractured porous media. However, there is no quantitative experimental work in the literature to examine the extent of its significance. There is a need to provide experimental data which can be used to validate simulation models.

Some of the fractures in naturally fractured reservoirs are cemented. The question of capillary continuity across cemented fractures is also an unresolved issue. In this paper, the question of capillary continuity in cemented fractures is also addressed.

EXPERIMENTAL

Apparatus — Experiments were conducted using two test systems for two basic experiments: 1) single block drainage, and 2) stacked block drainage. Each of the two test systems differed only in height (to accommodate either a single block or a stack of blocks). The apparatus consisted of a glass-walled case with metal top and bottom plates supported by a steel framing system. Design allows the stack of blocks to be tilted through 240° about a central horizontal axis.

Case walls of the apparatus were constructed of 0.95 cm thick glass plates. Top and bottom plates were constructed of 1.27 cm thick aluminum plates and provided with connections for vacuum, ventilation, fluid loading and drainage.

Peripheral accessory equipment consisted of a vacuum pump, vacuum gauge and a balance interfaced to a computer for recording fluid production. Fig. 4.1 shows the apparatus schematic.

The laboratory room housing the apparatus was provided with a temperature control unit to control temperature within a tolerance of $\pm 0.3^\circ\text{F}$. This type of temperature control is essential to reduce the effect of temperature variation on capillary pressure (i.e., through surface tension) when studying the late stage drainage performance of a stack of blocks. All the experiments were conducted at a temperature of 76°F .

Core Preparation and Properties — Original Berea sandstone blocks were supplied with dimensional tolerance of ± 1.6 mm. Local machining provided final finishing to a dimensional tolerance of ± 25 microns. The four matrix blocks listed in Table 4.1 have dimensions of 14.71 x

14.71 x 45.5 cm. The three 60.5 cm matrix blocks (see Table 4.1) were cut from the 15.16 x 15.16 x 182.9 cm Tall block. No finishing job was done on the Tall matrix block of Table 4.1. Permeability of these blocks were measured by the use of a minipermeameter^{9,10}. Several permeabilities in the horizontal and the vertical directions were measured. Table 4.1 provides the arithmetic average for these measurements.

Porosity of the blocks of Table 4.1 was measured by saturating them with normal decane in the core holder and producing the liquid first by drainage and then by applying a vacuum. The production by vacuum was collected at -90°F. Table 4.1 lists the porosities.

Test Preparation — Blocks were mounted in the case with four nylon spacers (1.3 x 2.4 x 0.1 cm) beneath the bottom block and three nylon spacers (0.1 cm thick and 1.3 cm wide) vertically along each window. A single additional spacer (1.3 x 2.4 x 0.1 cm) was also placed at the top center between each face of the top matrix block and the case wall. The glass case was sealed with a fuel resistant room temperature vulcanizing fluorosilicone rubber (FRV 1106-General Electric). The case and the blocks were then evacuated to remove adsorbed fluid and then saturated with normal decane. A period of two days was allowed to achieve full saturation. The case top lid was then opened and the spacers in the fractures withdrawn and the top again closed. This procedure gives a fracture spacing between the matrix block and the glass wall of 1.0 mm.

Test Procedure — Liquid normal decane was first rapidly drained to a standing level at approximately 1.0 cm above the bottom face of the lower matrix block. Subsequent production was accumulated in a separate container and recorded by the balance and computer. The duration of the rapid production was about half a minute. In a typical experiment, it took two to three minutes for the fracture liquid level to be stabilized. The liquid of the space between the glass and the matrix was produced in the first minute of the experiment and was excluded from the data.

RESULTS

Single block drainage experiments

In the experiments to be presented in this section, the drainage performance of single blocks of three distinct dimensions were measured. The measurements included drainage of, 1) 182.90 cm long (Tall) block, 2) four identical size 45.5 cm long blocks, and 3) three identical size 60.5 cm long blocks. Drainage data of the Tall block provides basic information such as capillary pressure, relative permeability, and porosity. Data of the Tall block also serves as a reference for the stacked block experiments which will be discussed in this paper and Ref. 10. Drainage data of the 45.5 and 60.5 cm long blocks will be useful in analyzing reinfiltration experiments.

Drainage of four single 45.5 cm long blocks — Drainage data of each of the four matrix blocks SA, SB, SC, and SD was measured. Figs. 4.2, 4.3 and 4.4 show the measured production and drainage rate data. The position of the liquid level in the fractures are also given in these figures. Liquid level position is measured from the bottom face. It has a significant effect on final recovery due to a high threshold height (35 to 41 cm).

The measured drainage rate for matrix block SA for two liquid positions (1.35 cm and 0.25 cm) was about $1.5 \text{ cm}^3/\text{min}$ at time 3 minutes. For this block, it took about 8 hours to reach gravity-capillary equilibrium. The final recovery for block SA when the liquid level is held at 0.25 cm is 4.8 percent of the pore volume(PV). The corresponding recovery when the liquid level is held at 1.6 cm is 3.5 percent of PV.

The final recoveries of block SB for liquid level at 0.6 and 1.5 cm are 4.2 and 3.5 percent of PV, respectively. The drainage rate at time 3 minutes is slightly less than block SA.

For block SC, the liquid level is held at 0.3 cm. The final recovery for this block is 4.5 percent of PV. The drainage rate at time 3 minutes is about the same as for block SB. Finally, the ultimate recovery for block SD with liquid level position at 0.8 cm is 2.9 percent of PV and the rate at time 3 minutes is around $1.3 \text{ cm}^3/\text{min}$. Fig. 4.4 shows that gravity-capillary equilibrium is established for all the above blocks in less than 14 hours. Due to uncertainty in recovery and rate measurement in the first two minutes, the data of Figs. 4.2, 4.3, and 4.4 are based on extrapolation of rate data at time 3 minutes.

Drainage of three single 60.5 cm long blocks — The drainage data on 45.5 cm blocks reveal that due to high threshold height, the recovery is low. However, for 60.5 cm long blocks, considerable recovery might be expected.

Figs. 4.5, 4.6, and 4.7 show the measured drainage performance and rate data of the three 60.5 cm matrix blocks on a linear and semi-log plots. These figures also provide the liquid level in the fracture space from the bottom face of the matrix block. It is interesting to note that the production rate for the first 3 minutes is about $10 \text{ cm}^3/\text{min}$ for all three blocks. The rate, then, drops sharply. At time 10 minutes, the drainage rate reduces to 1.8, 2.5, and $2.3 \text{ cm}^3/\text{min}$ for blocks LA, LB, and LC, respectively. For a while (up to 300 minutes), the rate of decline is slow, and then increases. Fig. 4.7 shows that all three blocks stop draining oil after 20 hours. The figure also show that blocks LB and LC have a similar drainage behavior. A duplicate run for block LC (not shown) revealed very similar results.

The high production rate in the first three minutes results from film flow along the vertical matrix faces. Measured liquid saturation data at the end of the tests are listed in Table 4.2.

Drainage of the Tall block — As a reference case, the drainage performance of a single block with a height equal to the stacked blocks will be useful. The relative permeability curve can also

be inferred from the drainage curve. A single block with dimensions of 182.9 x 15.14 x 15.16 cm was used for this purpose. The average porosity of the block is 21.6 % and, therefore, $PV=9043 \text{ cm}^3$.

The drainage performance and rate data of the Tall block are shown in Fig. 4.8 for a linear time scale. The same data are shown in Fig. 4.9 on a semi-log scale. During the test, liquid level in the fracture space around the block was kept at 1.45 cm above the bottom face of the block. Fig. 4.9 gives the production rate for the early period of the test. There is a rate change around 8 minutes time. Prior to that, the rate is $12 \text{ cm}^3/\text{min}$, and, after a short transient period, it decreases to $6.5 \text{ cm}^3/\text{min}$. Visual inspection of the side faces of the block showed that surface wetting changed rapidly. After 6 minutes, surface wetting fell to about 45 cm above the bottom face; at 9 minutes, it fell to some 15 cm; and at 14 minutes, the wetting surface was 5 cm above the bottom face of the block. The initial rate of $12 \text{ cm}^3/\text{min}$ is about twice the rate computed from the following equation (general derivation is given in Ref. 11):

$$q_0 = \frac{k}{\mu_0} \Delta \rho g (1 - h_{th}/h_b) \quad (4.1)$$

Symbols are defined in the nomenclature. The measured high initial rate includes the film flow along the side faces. The film thickness is, therefore, about 50 microns. Fig. 4.9 shows that even after 39 days, the production continues at a rate of around $8 \text{ cm}^3/\text{day}$. Later in this paper, drainage data will be used to infer the liquid relative permeability.

Capillary Pressure — The drainage capillary pressure is an important parameter in the interpretation and understanding of the experiments of this paper and Ref. 10. Special effort was, therefore, made in its measurement. Instead of measuring capillary pressure of a small core sample by the porous-plate technique, it was decided to measure drainage capillary pressure of the Tall block by the following approach.

The Tall block and the surrounding fracture space were initially saturated with normal decane. The production from the block was measured when the liquid level in the fracture space was set to a series of increasingly lower levels. A total of 12 liquid level steps were selected from the top to the bottom of the block. The procedure for the capillary pressure measurement included a rapid liquid production in the fracture space to the desired level. It took 30 to 60 seconds for the liquid level to stabilize. Table 4.3 shows the liquid level (measured from the block top), duration for which the liquid level was held constant, and the amount of production at each level. Table 4.3 also gives the average liquid saturation for the portion of the block above the liquid level. In the first two steps, no liquid production from the block was observed (see Table 4.3) — indicating a threshold height of more than 37 cm. By the end of steps 3 to 9, there was

no production from the block. However, steps 10, 11, and 12 were terminated while still a small production was going on. The production rate at the termination of steps 11 and 12 was about 1.5 cm³/day. The production at late stages of drainage is believed to be from film drainage^{12, 13} (in the matrix) which could continue for months at progressively lower rates. For practical purposes, the termination of the production at each of the three last steps is justified because P_c data will be used in the interpretation of stacked block experiments. These experiments usually take less than a month.

In order to obtain the capillary pressure curve, we assume that the following model would represent the P_c curve¹⁴,

$$P_c = -\sigma \ln \left(\frac{S_o - S_{or}}{1 - S_{or}} \right) + P_{th} \quad (4.2)$$

The parameter σ incorporates the effects of surface tension, wettability and pore size distribution. Expressing P_c and P_{th} in terms of distance from the liquid level height, h , and threshold height, h_{th} , respectively.

$$P_c = \Delta \rho g h \quad (4.3)$$

and

$$P_{th} = \Delta \rho g h_{th} \quad (4.4)$$

Combining Eqs. 4.2 to 4.4:

$$S_o = S_{or} + [1 - S_{or}] \exp[\Delta \rho g (h_{th} - h)/\sigma], \quad h \geq h_{th} \quad (4.5)$$

The average saturation in the top portion of the block (to liquid level) is given by

$$h \bar{S}_o = h_{th} + \int_{h_{th}}^h S_o dh \quad (4.6)$$

Combining Eqs. 4.5 and 4.6 one obtains

$$\bar{S}_o = h_{th}/h + S_{or}[1 - h_{th}/h] + \frac{\sigma}{\Delta \rho g h} [1 - S_{or}] [1 - \exp\{\Delta \rho g (h_{th} - h)/\sigma\}] \quad (4.7)$$

Table 4.3 provides $\bar{S}_o$ and h values. The liquid density of normal decane at 76°F is 0.724 gr/cm³. Given these data, Eq. 4.7 is used to estimate h_{th} , S_{or} and σ parameters of the capillary model. The estimated parameters are: $h_{th}=40.93$ cm, $\sigma=0.1702$ psi and $S_{or}=0.26$. The average absolute deviation between the measured and calculated $\bar{S}_o$ values (from Eq. 4.7) is 0.14 percent which shows excellent fit of the data to the model. Fig. 4.10 shows the capillary

pressure curve of the Berea sandstone. The shape of the capillary pressure curve indicates a uniform pore size distribution.

Relative Permeability — The drainage data of the Tall block was used to compute relative permeability. Since capillary pressure curve and absolute permeability data are available, a numerical simulator can be used to compute the relative permeability. The Eclipse model¹⁵ was used for this purpose.

It was pointed out earlier that the measured initial rate of drainage is higher than the rate computed from Eq. 4.1. For an absolute permeability of 750 md, Eq. 4.1 gives an initial rate of 7.75 cm³/min., when $h_{th}=0$, but the computed rate for $h_{th}=41$ cm is 6 cm³/min. The initial rate of drainage using the Eclipse model ($k=750$ md, and measured capillary pressure, P_c) is about 6.5 cm³/min. After 8 minutes from the start of the experiment, the measured rate is 6.5 cm³/min. It should be pointed out that minipermeameter measurements revealed that the Tall block has a uniform horizontal as well as vertical permeability (top and bottom faces), except a narrow region in the top portion of the block. This observation leads us to believe that permeability variation cannot be the cause of the high initial rate. The measured initial high rate is due to, 1) film flow along the side faces, and 2) matrix drainage.

Fig. 4.11 shows the measured drainage data and the results from simulation (note that in the interval 1 to 8 minutes, the rate is assumed to be 6 cm³/min to exclude the film drainage contribution). Gas phase relative permeability is assumed to be given by:

$$k_{rg} = (1 - S_o^*)^3 \quad (4.8)$$

where
$$S_o^* = \frac{S_o - S_{or}}{1 - S_{or}}$$

Oil phase relative permeability given by $k_{ro} = S_o^{*3}$ was found to give a good match of the data and calculations. A better recovery match is possible by further adjustment of the oil phase relative permeability exponent. This is unnecessary in light of possible variation in permeability and other rock data.

Stacked Block Experiments

The main purpose of the experiments was to study reinfiltration and to observe the process visually. The experiments were also designed in such a way that capillary continuity across cemented fractures could be studied. A total of 5 tests were conducted. Metallic plates were inserted in the fracture space between the matrix blocks to avoid direct physical contact. The

plates covered the entire block face. The thickness of the plates varied from 1.2 mm to 0.025 mm.

In Tests 1, 2, and 3, the four 45.5 cm blocks were stacked in the order of SA, SB, SC, and SD from top to bottom. In these tests, three metallic plates (aluminum) with areal dimensions of 14.6 x 14.6 cm were inserted in the three fractures; one in each fracture. The thickness of the plates was 1.2, 0.1, and 0.025 cm (1200, 100, and 25 microns) for Tests 1, 2, and 3, respectively. In Tests 4 and 5, the three 60.5 cm blocks were stacked in the order LC, LB, and LA from top to bottom. For these two Tests, 1.2 mm thick plates were used. Test 5 is a duplication of Test 4. In the following, the four-block stack will be described first.

Four-block stack

Test 1 (1.2 mm plates) — The first test on the four block stack is with 1.2 mm plates. The use of thick plates was believed to result in capillary discontinuity. Figs. 4.12 and 4.13 show drainage data on linear and semi-log plots. Rate data are plotted in Figs. 4.14 and 4.15. These figures show that after one day, the production rate decreases to about 7 cm³/day, while by the end of the third day, production rate becomes less than 2 cm³/day (a variation in room temperature of $\pm 0.3^{\circ}\text{F}$ causes rate variation due to change in surface tension). Similar to the Tall block, the initial production rate is high — about 11 cm³/min. After seven minutes, the rate drops rapidly. During the 10 to 20 minute period, the rate is about 2 cm³/min. Note that this rate is substantially less than the combined drainage of the individual matrix blocks by a factor of 3. Reinfiltration of drained liquid from upper to lower blocks is the reason for this large rate reduction. After 150 minutes, a rapid decline in rate is observed. Visual inspection of the vertical fracture space surrounding the blocks showed that liquid leaving the upper blocks reinfiltrates directly into the vertical face at the top of the next lower block. The zone of reinfiltration was approximately 1 to 2 cm. During the first minute of the experiment, it could not be determined whether the liquid produced from upper blocks reinfiltrates to the lower blocks.

Table 4.4 gives the measured liquid saturation at the end of the test. Comparison of the single block drainage test and the data of Test 4 suggests that the top block has desaturated beyond its capillary-gravity saturation.

Test 2 (0.1 mm plates) — With exception of top block saturation, it was hard to judge capillary continuity in the first test. The capillary continuity question lead us to use 0.1 mm plates in the space between the matrix blocks. Figs. 4.12, 4.13, 4.14 and 4.15 show the drainage and rate data for Test 2. There is continued drainage even after 8000 minutes, implying capillary continuity. For Test 2, the rate of drainage in the 1 to 6 minute interval is 12 cm³/min,

which is close to Test 1 and the Tall block. There is a sharp drop in the rate in the 7 to 8 minute interval, and then the rate drops slowly. In the 10 to 20 minute interval the rate of drainage is $2 \text{ cm}^3/\text{min}$. At the end, the rate of drainage was about $8 \text{ cm}^3/\text{day}$, whereas for Test 1, it was less than $2 \text{ cm}^3/\text{day}$.

The liquid saturation of the blocks at the end of the test are shown in Table 4.4. For the top 2 blocks, liquid saturations are less than the values of Table 4.2, which is the result of capillary continuity across the 100 micron plates.

Test 3 (0.025 mm plates) — In this test, plate thickness was 25 microns. A problem encountered with these plates was the damage, as a result of heavy weight of the blocks. Damage was in the form of several small holes. In the test which is presented here, two very fine holes in two plates were observed.

The data for Test 3 are also plotted in Figs. 4.12 to 4.15, where the data for Tests 1 and 2 are also shown. Up to 300 minutes, the difference in the drainage rates for Tests 1, 2, and 3 is not substantial. There is very little difference between Tests 2 and 3 to about time 1000 minutes. However, at $t = 8000$ minutes, the rate of drainage for Test 3 is about $24 \text{ cm}^3/\text{day}$, which is 3 times higher than Test 2. From this time to the end of Test 3, the rate of drainage decreases very slowly. At the end ($t = 22$ days), the rate is about $8 \text{ cm}^3/\text{day}$. Table 4.5 provides measured block saturations at the termination of the tests. Drainage from the top two matrix blocks is much higher than corresponding values of single block drainage. The results from Test 3 are very important in relation to capillary continuity and high drainage rate with impermeable barriers between the matrix blocks.

Three-block stack

Due to high capillary threshold height of the Berea in relation to block height, the following experiments were performed on three longer blocks to study the reinfiltration process when the initial drainage rate is high. Tests 4 and 5 were identical and the purpose was to check reproducibility.

Tests 4 and 5 (1.2 mm plates) — The three 60.5 cm blocks were stacked in the order LC, LB, and LA from top to bottom. They were separated by 1.2 mm plates which covered the top and bottom faces of the blocks. In these two tests, the flow of the liquid produced from the upper blocks to the blocks below was recorded by a video camera for replay. The video showed that the produced liquid from blocks LC and LB reinfiltrated very quickly into the blocks below. In the first 8 minutes, when the liquid film was flowing along the side faces of the blocks, the reinfiltration fully realized as soon as the liquid film passed the fracture space between the matrix blocks. It was also observed that the liquid production from blocks LC and LB formed,

first, at the bottom of these blocks. When the volume of the liquid reached to about 0.1 cm^3 , then it flowed to the block below, and reinfilted quickly. Lack of continuous flow is believed to be due to low liquid transmissibility of the plates (see Ref. 10). For three hours, this type of fluid flow was observed. No video recording was continued after time 3 hours.

Figs. 4.16 and 4.17 show the production and rate data for Tests 4 and 5. In the first 8.5 minutes, the rate of drainage is nearly constant — $10 \text{ cm}^3/\text{min}$. After a transient period of 3 minutes, it drops to $4 \text{ cm}^3/\text{min}$, and the rate of decrease is gradual to about 500 minutes. The period of high decline follows to the end of Tests 4 and 5. At the end ($t=4100$ minutes), the rate was around $2 \text{ cm}^3/\text{day}$. This implies that even for 1.2 mm spacers, the liquid phase continuity is a reality. Liquid saturation of block LC at the end of the test (see Table 4.5) reconfirms capillary continuity for a 1.2 mm thick plate. Should we have kept the liquid level at the bottom face of this block in the single block test, the final saturation would have been about 85.1 percent. (The final saturation of 85.1 percent is computed by using numerical simulation). The difference between 85.1 and 82.4 percent is due to capillary continuity across the 1.2 mm plate. Fig. 4.18 gives a comparison of the production performance of Tests 4 and 5 with the combined production of matrix blocks LC, LB, and LA. The early film drainage is included since its contribution is the same for the stacked block and the combined data of single blocks. Considering that only two blocks, LC and LB are participants of the reinfiltration process, reinfiltration has a significant effect on drainage behavior.

CONCLUDING REMARKS

The laboratory data presented in this paper could be used to examine the validity of fractured reservoir simulators. In addition to individual matrix block drainage data, the drainage data across a stack of blocks with impermeable barriers are also provided. Visual inspection of the drainage process in this work reveals that all the oil produced from an upper matrix block reinfiltrates into the block below. The reinfiltration takes place over a short distance and, therefore, it is a localized phenomenon. This confirms our previous analytical work on the topic of reinfiltration^{7,8,9}. (The analytical work was based on certain assumptions). The experimental work also clearly shows that capillary continuity is a reality in fractured porous media with an aperture as high as 1.2 mm. Even for cemented fractures, there may be phase continuity. When the fracture aperture of the cemented fracture is small, drainage of the oil could be substantial.

NOMENCLATURE

- A — Cross sectional area
g — gravitational acceleration
h — liquid level, measured from the top
 h_b — block height
 h_{th} — threshold capillary height
k — absolute permeability
 k_h — horizontal permeability
 k_v — vertical permeability
 k_{ro} — oil relative permeability
 k_{rg} — gas relative permeability
 P_{th} — threshold pressure
 P_c — capillary pressure
 q_o — oil flow rate
 S_o — oil saturation
 S_{or} — residual oil saturation
 $\bar{S}_o$ — average oil saturation
 S_o^* — normalized oil saturation
 μ_o — oil viscosity
 $\Delta\rho$ — oil and gas density difference

REFERENCES

- [1] Lefebvre du Prey, E. J. L.: "Drainage en cascade de blocs d'un reservoir fissure," Revue Institute Francais du Petrole, (Jan-Feb., 1976) 173-178.
- [2] Saidi, A. M., Tehrani, D. H. and Wit, K.: "Mathematical simulation of fractured reservoir performance, based on physical model experiments," Proc. 10th World Petroleum Conference, Bucharest (1979).
- [3] Festoy, S. and van Golf Racht, T. D.: "Gas gravity drainage in fractured reservoirs through new dual continuum approach", SPE Reservoir Engineering (Aug. 1989) 271-278.
- [4] Por, P. J., Boerrigter, P., Maas, J. G. and deVries, A.: "A fractured reservoir simulator capable of modeling block-block interaction. Paper 19807, Proc. SPE 64th Annual Technical Conference and Exhibition, San Antonio (1979) 373-331.

- [5] Fung, L.: "Simulation of block-to-block processes in the naturally fractured reservoirs," SPE Reservoir Engineering (Nov. 1991) 477-484.
- [6] daSilva, F. V. and Meyer, B.: "Analysis of gravity-capillary equilibrium in chalk fractured reservoirs," Third North Sea Chalk Symposium, Copenhagen (1990).
- [7] Firoozabadi, A. and Ishimoto, K.: "Theory of reinfiltration in fractured porous media. Part I: One dimensional model," Paper SPE 21796, SPE Western Regional Meeting, Long Beach, CA, March 20-22, 1991, Paper to be published in SPE Advanced Technology Series.
- [8] Firoozabadi, A., Ishimoto, K. and Dindoruk, B.: "Theory of reinfiltration in fractured porous media. Part II: Two dimensional model," Paper SPE 21798, SPE Western Regional Meeting, Long Beach, CA, March 20-22, 1991.
- [9] Guggins, D. J., Thrasher, R. L. and Lake, L. W.: "A Theoretical and Experimental Analysis of Minipermeameter Response Including Gas Slippage and High Velocity Flow Effect," In Situ (1988) 79-116.
- [10] Firoozabadi, A., and Markeset, T.: "Laboratory Study of Gas-Liquid Transmissibility in Fractured Porous Media," SPE 24919, paper presented at the 1992 SPE Annual Technical Conference and Exhibition, Washington D.C., Oct. 4-7, 1992, to be published by SPE Reservoir Engineering Journal.
- [11] Barkve, T., and Firoozabadi, A.: "An Analysis of Reinfiltration in Fractured Porous Media," SPE 24900, paper presented at the 1992 SPE Annual Technical Conference and Exhibition, Washington D.C., Oct. 4-7, 1992.
- [12] Pawone, D., Bruzzi, P., and Verre, R.: "Gravity Drainage at Low Interfacial Tension," paper presented at the Fifth European Symposium on Improved Oil Recovery, Budapest, April 25-27, 1989.
- [13] Firoozabadi, A., and Aziz, K.: "Relative Permeabilities from Centrifuge Data," Journal of Canadian Petroleum Technology (Sept.-Oct. 1991) 30, 33-42.
- [14] Bentsen, R.G., and Anli, J.: "A New Displacement Capillary Pressure Model," Journal of Canadian Petroleum Technology (July - Sept. 1976) 75-79.
- [15] Eclipse Reference Manual, Version 90A, Exploration Consultants Ltd (1990)

Table 4.1 - Permeability of Berea Sandstone Blocks.

Block	Average Permeability, md			Dimension, cm x cm x cm	Porosity, %
	k_h	k_v	k_v		
	Side Faces	Top Face	Bottom Face		
SA	275	1035	865	14.71 x 14.71 x 45.49	22.3
SB	799	468	786	14.71 x 14.71 x 45.49	21.6
SC	346	811	642	14.71 x 14.71 x 45.49	22.2
SD	698	695	574	14.71 x 14.71 x 45.49	21.9
LC	643	653	653	14.71 x 14.71 x 60.48	21.9
LB	594	751	603	14.71 x 14.71 x 60.44	21.8
LA	608	597	666	14.71 x 14.71 x 60.48	21.4
TALL	416	406	754	15.16 x 15.14 x 182.9	21.6

Table 4.2 - Saturation of Single Block Experiments at the End of Drainage Test.

Block	Pore Volume, cm ³	Liquid Saturation, % (Liquid Level)
SA	2198	95.2 (0.25 cm)
		96.5 (1.60 cm)
SB	2132	95.8 (0.60 cm)
		96.5 (1.50 cm)
SC	2188	95.5 (0.30 cm)
SD	2152	97.1 (0.80 cm)
LC	2874	86.5 (1.20 cm)
LB	2857	85.5 (1.20 cm)
LC	2812	90.3 (1.20 cm)
TALL	9043	49.8 (1.45 cm)

Table 4.3 - Block Saturation at the End of Tests 1, 2, and 3.

Block	Liquid Saturation, %		
	Test 1	Test 2	Test 3
SA	90.0	91.0	69.7
SB	95.0	92.0	80.3
SC	96.5	94.6	84.6
SD	97.6	97.9	97.9
Liquid Level	0.3 cm	0.7 cm	1.2 cm
Duration	4,202 min.	8,184 min.	32,759 min.

Table 4.4 - Liquid Level, Duration, Production and Liquid Saturation for the Pc Experiment.

Step	h, cm	Duration, hour	Incremental Production, cm ³	$\bar{S}_o$, Fraction
1	20.25	2.5	0.0	1.0000
2	37.00	18.0	0.0	1.0000
3	45.00	8.5	9.2	0.9943
4	55.05	20.5	112.4	0.9383
5	65.05	51.0	187.2	0.8674
6	75.05	72.5	207.0	0.8081
7	85.00	48.0	240.0	0.7517
8	94.00	160.0	232.9	0.7063
9	110.05	1450.0	415.5	0.6437
10	129.65	360.0	514.8	0.5866
11	164.8	480.0	919.0	0.5189
12	180.60	475.0	430.0	0.4947

Table 4.5 - Block Saturation at the End of Tests 4, and 5.

Block	Liquid Saturation, %	
	Test 4	Test 5
LC	82.4	82.5
LB	84.1	84.3
LA	90.5	90.3
Liquid Level	1.3 cm	1.1 cm
Duration	4,483 min.	4,381 min.

Fig. 4.1-Apparatus schematic

Fig. 4.2 - Drainage performance vs. time (linear scale) for matrix blocks SA, SB, SC, and SD.

Fig. 4.3 - Drainage performance vs. time (log scale) for matrix blocks SA, SB, SC, and SD.

Fig. 4.4 - Drainage rate vs. time (log scale) for matrix blocks SA, SB, SC and SD.

Fig. 4.5 - Drainage performance vs. time (linear scale) for matrix blocks LA, LB, and LC.

Fig. 4.6 - Drainage performance vs. time (log scale) for matrix blocks LA, LB, and LC.

Fig. 4.7 - Production rate vs. time (log scale) for matrix blocks LA, LB, and LC.

Fig. 4.8 - Production and rate data vs. time (linear scale) for the Tall matrix block.

Fig. 4.9 - Production and rate data vs. time (log scale) for the Tall matrix block.

Fig. 4.10 - Capillary pressure of the Berea sandstone.

Fig. 4.11 - Comparison of the measured drainage data and the simulation results for the Tall block.

Fig. 4.12 - Production performance vs. time (linear scale) for Tests 1, 2, and 3.

Fig. 4.13 - Production performance vs. time (log scale) for Tests 1, 2, and 3.

Fig. 4.14 - Rate data vs. time (linear scale) for Tests 1, 2, and 3.

Fig. 4.15 - Rate data vs. time (log scale) for Tests 1, 2, and 3.

Fig. 4.16 - Production and rate data vs. time (linear scale) for Tests 4, and 5.

Fig. 4.17 - Production and rate data vs. time (log scale) for Tests 4, and 5.

Fig. 4.18 - Comparison of the measured stack block and combined single blocks production.

4.2 LABORATORY STUDY OF FRACTURE LIQUID TRANSMISSIBILITY IN FRACTURED POROUS MEDIA.

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ABSTRACT

The flow of oil phase in the gas-oil two-phase region in the fractures is a key element to production performance of fractured petroleum reservoirs for which gravity drainage is the dominant mechanism. In this study, data on a set of experiments which center around flow of oil phase in the fractures are presented.

In a number of experiments, the rate of drainage across a stack of matrix blocks was measured. In addition to rate, average matrix block saturation was also measured. In a typical experiment, the fracture aperture was fixed by inserting several thin metallic spacers in the fracture space between the matrix blocks. The contact area in the fracture was varied by changing the number of spacers. Fracture aperture was changed by varying the spacer thickness. The experiments covered fracture aperture in the range of 50 to 1200 microns and the contact area was varied by a factor of 16. The data show that oil phase flow across fractures is a key parameter.

INTRODUCTION

Fundamental forces (capillary, gravity, viscous and diffusive) which govern multiphase flow have been extensively investigated for unfractured porous media. However, the research on various forces for flow in fractured porous media is limited. These limited studies address mainly drainage and imbibition behavior of a single matrix block which sometimes has little relevancy to multiphase flow in fractured porous media. For gas-oil systems, the flow of oil phase in the fracture space between the matrix blocks is a key element to production performance of fractured petroleum reservoirs. Few efforts to address this topic has been reported in the literature.

Firoozabadi and Hauge¹ in the study of fracture capillary pressure demonstrated that the rate of drainage across a stack of three blocks could not be calculated using fracture capillary pressure derived from the Young-Laplace equation of capillarity. Some apparent anomaly in block saturation was also observed in their data. In an attempt to calculate the drainage performance of a two-block stack, Labastie² concluded that a fracture permeability, taking into account the flow through liquid films, should be developed. The authors of Refs. 1 and 2 conclude that there is a need for a basic understanding of gas-oil flow in the fracture medium. Both experimental and theoretical work will be required to resolve the issue of gas-oil gravity

drainage in fracture porous media. This paper presents the result of our experimental research effort. The data are expected to provide a basis for theoretical research in this area.

EXPERIMENTAL SETUP

In the following, after a brief description of apparatus and the experimental procedure, the results of various tests that were performed to study oil flow across fractures will be presented.

Apparatus

The apparatus used to measure the drainage performance of the single matrix block, and the stack of matrix blocks consisted of a glass-walled case with metal top and bottom plates. The block(s)-holder assembly was supported by a steel framing system. Fig. 4.19 shows the apparatus schematic. Peripheral accessory equipment consisted of a vacuum pump, vacuum gauge and a balance interfaced to a computer for recording liquid production. The laboratory room housing the apparatus was provided with a temperature control unit to control temperature within a tolerance of ± 0.3 °F. All the experiments were conducted at a temperature of 76 °F.

Core Preparation and Properties

The Berea sandstone blocks were machined to a dimensional tolerance of ± 25 microns. Table 4.6 provides the dimensions, permeabilities and porosity of the Berea sandstone blocks which were used in the experiments. No finishing job was done on the Tall matrix block of Table 4.6. Permeability were measured by the use of a minipermeameter. Several permeabilities on the same face of each matrix block were measured. Table 4.6 provides the arithmetic average for these measurements. Porosity was measured by saturating a single matrix block with normal decane in the coreholder.

Test Preparation

Blocks were mounted in the case with four nylon spacers (1.3 x 2.4 x 0.1 cm) beneath the bottom block and three nylon spacers (0.1 cm thick and 1.3 cm wide) vertically along each glass window. A single additional spacer (1.3 x 2.4 x 0.1) was placed at the top center between each face of the top matrix block and the case wall. The case and the blocks were then evacuated to remove adsorbed fluid and then saturated with normal decane. The case top lid was then removed and the spacers in the fractures between the matrix block(s) and the glass case withdrawn and the top again closed. This procedure gives fracture spacing between the matrix block and the glass wall of 1.0 mm.

Test Procedure

Liquid normal decane was first rapidly drained to a standing level of approximately 1.0 cm above the bottom of the lower matrix block. Subsequent production was accumulated in a separate container and recorded by the balance and computer. The duration of the rapid production was about half a minute. The liquid of the space between the glass and the matrix was produced in the first minute of the experiment and was excluded from the data. Air from the top lid replaced the drained normal decane in all the experiments.

Prior to conducting the experiments to be described next, the drainage capillary pressure of the Tall block for the air/normal decane fluid system was measured. To measure the capillary pressure, the Tall block and the fracture space surrounding it were initially saturated with normal decane. The production from the Tall block was measured when the liquid level in the fracture space was set to a series of increasingly lower levels. The procedure for the capillary pressure measurement included a rapid liquid production in the fracture space to the desired level. Then the average saturation in the block for each level was established. The average saturation could be converted to capillary pressure³. The expression for the capillary pressure from the above procedure is

$$P_c = P_{th} - \sigma \ln(S_o - S_{or}) / (1 - S_{or}) \quad (4.9)$$

with $S_{or} = 0.26$, $P_{th} = 0.42$ psi, and $\sigma = 0.17$ psi. In Eq. 4.9, P_c is in psi, P_{th} is the threshold pressure and S_{or} is the residual saturation. Note that since $P_{th} = \rho g h_{th}$ (ρ is the density of normal decane, 0.724 g/cm^3 at 76°F , and h_{th} is the threshold height), then $h_{th} \approx 41 \text{ cm}$.

RESULTS

Two sets of experiments were performed by stacking, 1) four 45.5 cm long blocks, and 2) three 60.5 cm long blocks on top of each other. In the four-block stack, the order was SA, SB, SC, and SD from top to bottom (see Table 4.6). In the three-block stack, block LC was on the top, block LB in the middle and block LA at the bottom of the stack (see Table 4.6). In the first six tests, the four-block stack and in the last three tests, the three-block stack were used. In each set of experiments, blocks were separated by using metallic shims (spacers). These spacers were wetted easily with normal decane.

Four 45.5 cm Long Block Experiments

Test 1 (blocks in direct contact) — Figs. 4.20 and 4.21 show the drainage performance of the stack of four blocks in direct contact vs. time in linear and log scales. Reproducibility is

excellent for the three tests 1A, 1B, and 1C. Figs. 4.20 and 4.21 show also the drainage data for the Tall block. The rate data are shown in Fig. 4.22.

In the 1 to 7 minute interval, the production rate for Tests 1A, 1B, and 1C is nearly constant — close to $690 \text{ cm}^3/\text{hr}$. After a transient period of 8 to 12 minutes, it decreases to $270 \text{ cm}^3/\text{hr}$. Visual inspection of the fracture space between the matrix blocks showed that the liquid in the fracture space bridging between the matrix blocks broke down after about 2.5, 4.5, and 7.5 minutes for the top, middle, and lower fractures, respectively (the values are for Test 1A, but the data for Tests 1B and 1C are very similar). Because the blocks were in direct contact, the state of liquid saturation in the fracture space (between the blocks) could not be observed. The high production rate of about $700 \text{ cm}^3/\text{hr}$ in the first seven to ten minutes is the combined result of the drainage from the matrix and film flow from the outside vertical faces. After this initial period, film flow from the vertical faces stops. The film flow from the vertical faces realized because the fractures surrounding the matrix block(s) were initially filled with liquid. The rapid production of liquid in the vertical fractures left a thin layer of liquid film on the vertical faces. This liquid film traveled at a velocity of about $20 \text{ cm}/\text{min}$ (visual observation).

Table 4.7 provides the liquid saturation of the blocks at the termination of Tests 1A, 1B, and 1C, and the time at which these experiments were stopped. The same table also lists the computed saturation of these blocks when capillary/gravity equilibrium is established for the whole stack. The computed values are based on the measured capillary pressure without any adjustment. The computed equilibrium saturation for block SC reflects the sensitivity to the capillary pressure in the region where saturation changes rapidly with a small change in capillary pressure. The last row of Table 4.7 and subsequent tables give the position at which the liquid level was held from the bottom face of the stack during the tests.

Figs. 4.21 and 4.22 reveal that capillary continuity exists to the end of experiments, where stacked blocks, and the single Tall block give the same recovery. For both the stacked blocks, and the Tall block, the production rate at the end ($t=39$ days) is about $0.3 \text{ cm}^3/\text{hr}$. Figs. 4.21 and 4.22 show that the major difference between the stacked blocks and the Tall block is the rate of drainage. In the 10 to 1000 minute interval there is an appreciable difference between the Tall block and the stack block drainage rate. This difference is caused simply by the high liquid pressure drop across the fractures. Two factors influence the slowdown of the liquid flow across a fracture, 1) fracture gas-oil capillary pressure, and 2) fracture liquid relative permeability. The combined effect of these two factors is called fracture liquid transmissibility.

Test 2 (three 1.2 mm spacers) — In this test, three 1.2 mm thick ($2.5 \times 2.5 \times 1.2 \text{ mm}$ each) metallic shims were inserted in the fracture space between the matrix blocks. Figs. 4.23 to 4.26 show the drainage performance and the production rate of the stack of four blocks vs. time.

Table 4.8 provides the final oil saturation in blocks SA, SB, SC, and SD at the termination of this run. An important observation in this test is that production continues beyond 8 days. During the first 14 minutes, the drainage rate is very high and constant — about $660 \text{ cm}^3/\text{hr}$. After about 15 minutes, the drainage rate decreases to about $120 \text{ cm}^3/\text{hr}$. (see Fig. 4.26). Because the duration of early drainage behavior appeared suspect, this test was repeated two additional times. Identical early production in all three runs validated the data. Visual inspection of the fracture space between the blocks revealed the cause of the change in slope of the drainage data. In the first 4 minutes of the test, all fractures between the matrix blocks were full of liquid. At about 4 minutes, the liquid bridging between blocks SA and SB started to shrink. The liquid bridging between the two top blocks was completely broken at $4 \frac{1}{2}$ minutes. The liquid bridge between blocks SB and SC started to shrink at around 8 minutes and was completely broken at $9 \frac{1}{2}$ minutes. The last liquid bridge in the fracture between blocks SC and SD started to shrink at 11 minutes and was completely broken at about 14 minutes. It is interesting to note that the time at which the liquid bridge between blocks SC and SD breaks down is the time for which the slope of the production data changes (see Fig. 4.26). Once liquid bridging all fractures has ended, production for upper to lower blocks still continues. Fig. 4.27 shows a schematic of the process of liquid bridging forming and breakdown which was visually observed. During the first two hours of the test, the process of liquid bridging and breakdown was observed in all three fractures. After two hours, the formation and breakdown of liquid bridges continued in the lower two fractures only. After five hours, the process of bridge formation and breakdown was only observed in the fracture space between blocks SC and SD. Observation ended after 7 hours at which time the process of liquid formation and breakdown for the last fracture was very slow.

Fig. 4.25 also illustrates that after 8 days, there is still some production. In the last four days of the run, the production rate was nearly constant at around $0.1 \text{ cm}^3/\text{hr}$. Therefore, one may conclude that the existence of the three spacers (each with a dimension of $2.5 \times 2.5 \times 1.2 \text{ mm}$) causes capillary continuity, but a low fracture liquid transmissibility precludes significant flow. The low transmissibility in the fracture is, therefore, the parameter that lowers the liquid saturation of the top matrix block to about 89 percent. This value is substantially less than the 95 percent saturation of the gravity/capillary equilibrium (see Table 4.8).

In all of the stack block tests of this study, except Test 2, the duration of the initial high production is less than 10 minutes. The reason for the 14 minute duration for Test 2 is the extra liquid volume of some 60 cm^3 in the fracture space between the matrix blocks. Most of this extra liquid volume had to flow down as a film flow across the vertical face of the matrix blocks.

Test 3 (three 0.25 mm spacers) — In this test, the blocks were separated by three spacers (2.5 x 2.5 x 0.25 mm each). The measurement of the fracture thickness by a feeler gauge showed that the middle and lower fractures were not uniform. While the aperture of the upper fracture was 0.23 mm and uniform, the thickness varied from 0.14 to 0.20 mm in the middle fracture and from 0.13 to 0.23 mm in the lower fracture. As we will see shortly, the variation has a significant influence in the saturation distribution across the stack of matrix blocks. Figs. 4.23 to 4.26 show the production and rate data for Test 3 over the total test duration of 21 days. Table 4.8 gives the liquid saturation of the blocks at the end of the test. The measured block saturation indicates the influence of fracture aperture on fracture liquid transmissibility.

Fig. 4.26 shows the early drainage of the block stack. Prior to 8 minutes, the production rate is about 660 cm³/hr. After 10 minutes, it reduces to 140 cm³/hr. The sharp rate reduction is similar to Tests 1 and 2 and in line with the following observation.

The liquid bridging the fracture space between the matrix blocks was observed to break at about 3, 5 and 7.5 minutes for the top, middle, and the lower fractures, respectively. Due to small fracture thickness between the matrix blocks, we could not observe the formation of liquid droplets by naked eye. However, the passage of light through the fracture space revealed that all fractures were occupied by gas.

Fig. 4.26 shows that the production rate toward the end of the experiment is about 1.0 cm³/hr. This is a clear indication that capillary continuity between the matrix blocks exists for a large fracture. However, low fracture liquid transmissibility prevents high rate of drainage from the stack.

Test 4 (forty-nine 0.25 mm spacers) — The purpose of the test was to investigate the effect of contact area on liquid transmissibility between the matrix blocks. An array of 49 spacers (2.5 x 2.5 x 0.25 mm) was installed in the fractures between the matrix blocks (see Fig. 4.28). Measurement of the fracture thickness revealed a nearly uniform thickness for the top two fractures: upper fracture, 0.25 to 0.28 mm; middle fracture 0.25 to 0.28 mm; and the lower fracture 0.23 to 0.28 mm.

Figs. 4.23 to 4.26 show the drainage performance and rate data of the block stack for this test. It is interesting to note that a 16 fold increase in the number of spacers lowers the fracture liquid transmissibility. Comparison of Tests 3 and 4 shows that there is a significant influence of fracture aperture on the drainage behavior of a stack of blocks. One major effect of increasing the number of spacers in Test 4, is an increase in fracture aperture by making the aperture even. Table 4.8 provides the measured block saturation at the end of this test. Total test duration was 15 days. One may notice that liquid saturation of block SC is lower than those of blocks SA and

SB. (Fracture aperture of the lower fracture was less than the other two fractures.) The saturation distribution across the stack of blocks is, therefore, influenced by fracture aperture. Fig. 4.26 shows that the production data in the early period is the same as for Test 3. Visual examination of lower fracture showed low liquid saturation at 8 minutes and empty at 9 minutes. A film of liquid covered a spacer for 14 minutes, but the liquid film could not be seen at 15 hours. Toward the late stage of production, drainage rate, although small, decreased slowly. After some 15 days of drainage, the rate as shown in Fig. 4.26 is around $0.5 \text{ cm}^3/\text{hr}$. The late stage drainage data of Fig. 4.25 is another indication of capillary continuity through 0.25 mm spacers between the matrix blocks. Similar conclusion was drawn by Labastie in Ref. 2.

Test 5 (five 0.1 mm spacers) — The purpose of this test is to continue to investigate the effect of fracture aperture on fluid transmissibility between the matrix blocks. In the test, five 0.1 mm spacers with dimensions of about $2.5 \times 2.5 \times 0.1 \text{ mm}$ were placed in the fracture space between the matrix blocks. One spacer was placed at the center and the other four were placed in the middle of each side — 2.5 cm from the edge in the fracture plane. Measurement of the fracture aperture revealed that there was considerable variation in each of the three fractures. The upper fracture aperture varied from .07 to 0.10 mm [70 to 100 microns], the middle fracture from 0.06 to 0.08 mm [60 to 80 microns], and the lower fracture from 0.04 to 0.08 mm [40 to 80 microns]. These measurements were made from two neighboring vertical faces. Measurements at the end of the test from the other two faces showed that in some locations, the aperture of the middle and lower fractures were less than 0.04 mm. A fiber optic light source and a video camera equipped with a zoom lens were used to visually observe the fractures. The use of the light source revealed that the faces of the blocks comprising the upper fracture did not touch each other — and the gap was observed to exist over the entire fracture system. However, the light source could not pass through some segments of the middle and lower fractures which could be an indication of possible contact.

Figs. 4.29, 4.30 and 4.31 depict the production and rate data vs. time and Table 4.8 lists the measured liquid saturation of the matrix blocks at the end of the test. A recovery of 43.3 percent (974 cm^3) from the top matrix block is a clear indication that a substantial amount of liquid production occurs from liquid film flow along the spacers. Fig. 4.29 reveals that after a period of more than a month, the difference in recovery performance of the direct contact blocks (Test 1B) and the block stack of Test 5 remains significant. The rate data of Fig. 4.31, however, shows that at late stages, the rate of drainage of the stacked blocks of Test 5 is more than that of Test 1B. Fracture liquid transmissibility along the spacers is a key parameter which influences such a behavior.

Early data from this test, similar to other tests, include film flow along the side faces. Prior to 9 minutes, the production rate is about $680 \text{ cm}^3/\text{hr}$, and after a short transient period, it decreases to $210 \text{ cm}^3/\text{hr}$. Visual observation of the fractures showed that the liquid bridge in the fracture space broke down at 3, 5, and 8 minutes in the upper, middle and lower fractures, respectively. We observed that the portions of the fracture space which had an aperture of more than 0.04 mm became empty at about 7, 9, and 10 minutes, respectively.

Fig. 4.31 implies that towards the end of the test, the rate decreases slowly. The experiment was terminated after 35 days. At the time of termination, the drainage rate was about $0.7 \text{ cm}^3/\text{hr}$.

Test 6 (nine 0.1 mm spacers) — This test is similar to Test 5 with the exception of the number of spacers. In Test 5, five 0.1 mm spacers were used for every fracture space between the blocks whereas in this case, nine spacers (of the same dimensions) were used. One spacer was placed at the center, four in the corners, and four in the middle of each side of the fracture plane. The distance of the corner and side spacers from the vertical block faces was about 2.5 cm. Considerable effort was made to measure and examine the fracture aperture between the matrix blocks.

Fig. 4.34 shows a sketch of fracture aperture for the fractures between the matrix blocks. These measurements were made by inserting a feeler gauge in the fracture space before saturating the matrix blocks. For the upper fracture, the aperture varied from a minimum of 0.06 mm to a maximum of 0.10 mm. A large part of this fracture had an aperture of 0.08 mm. The middle fracture had a minimum aperture of 0.04 mm and a maximum of 0.08 mm. The aperture of the lower fracture varied considerably — from 0.025 mm to 0.10 mm. Three days after the start of this test, a fiber optic light source and a video camera were used to examine visually the fractures. The use of the light source revealed that the faces of the blocks comprising the middle fracture did not touch each other. However, the light source could not pass through a small segment of the upper fracture and a large segment (about 1/2) of the lower fracture. One may suspect that the opposing faces of the blocks comprising the lower fracture had developed a physical contact.

Figs. 4.29, 4.30, and 4.31 display the total production and rate data and Table 4.8 gives the matrix block liquid saturations at the end. The figures also show the drainage performance of Tests 1B and 5. The measured block saturations are compatible with the aperture data. The production from blocks SA and SB is apparently from film flow across the spacers while physical contact might be responsible for high recovery for Block SC. Fig. 4.31 shows that the drainage rate from stack of blocks of Test 6 is less than that of Test 5 after the first week. This

implies that the fracture aperture could be the most critical parameter to influence the rate of drainage across a stack of blocks.

Early drainage data of Tests 5 and 6 are nearly identical (see Fig. 4.31). Visual observation of the fractures showed that the upper, middle, and lower fractures were empty at approximately 5, 8, and 10 minutes, respectively. Toward the end of the test, the drainage rate reduced to about $0.5 \text{ cm}^3/\text{hr}$. The rate of drainage for Test 5 at the same time was about $0.7 \text{ cm}^3/\text{hr}$. It could, therefore, be concluded that the film flow along the spacers is mainly a function of fracture aperture.

Three 60.5 cm Long Block Experiments

The use of small spacers caused rock surface scratching which resulted in unevenness of the fracture surface and variations in the fracture aperture. The above blocks could not, therefore, be used for thin fractures (say 50 microns). It was decided to conduct the following experiments with wide spacers to avoid fracture surface scratching. New 60.5 cm blocks were used. Careful machining technique resulted in very uniform fracture aperture for the new blocks.

Test 7 (four 50 micron spacers) — In this test, four spacers each with a dimension of $20 \times 20 \times 0.05 \text{ mm}$ were used in the two fractures between the matrix blocks. Total area of each spacer is about 100 times higher than the spacers used in the tests described above. These spacers were placed in each corner of the fracture surface (about 2 cm from the edge). Measurement of the fracture aperture by a feeler gauge showed uniform thickness — about 40 to 50 microns. The entire fracture space was viewed by the use of a light source and camera.

Figs. 4.33 to 4.35 show the drainage performance and rate data of Test 7 and two other tests to be described next. At about 3500 minutes time, the top block (block LC) was removed from the stack and weighed to measure the liquid saturation. The process of dismounting, weighing and remounting took 10 minutes. At time of 17600 minutes, the saturation of the two top blocks, LC and LB, was again measured, and the blocks were assembled. It is likely that the liquid film covering the spacers is either disconnected or evaporated in the saturation measurement process. We did not try to wet them in this test. Similar to other tests, the saturation of the blocks was also measured at the end of the test. Saturation data are listed in Table 4.9.

The initial rate of production for Test 7 in the 1 - 9 minute period (also for Tests 8 and 9) is about $550 \text{ cm}^3/\text{hr}$. This is somewhat less than initial rate of production for the four block stack — which was about $660 \text{ cm}^3/\text{hr}$. The surface roughness could influence the film drainage along the side surfaces. After the transient period of 10 to 15 minutes, drainage rate decreases to about $240 \text{ cm}^3/\text{hr}$. The sharp change in the slope of the rate data at 3500 and 17,500 minutes is

the result of matrix block(s) removal for saturation measurement. At time 3500 minutes, the rate of drainage was $7 \text{ cm}^3/\text{hr}$ before and $3.5 \text{ cm}^3/\text{hr}$ after saturation measurement of the top block. At time 17,500 minutes, the rate of drainage dropped from $1.6 \text{ cm}^3/\text{hr}$ to $0.75 \text{ cm}^3/\text{hr}$ when matrix blocks LC and LB were removed and assembled. At the end of the test ($t=49,000$ minutes) the rate of drainage was $0.44 \text{ cm}^3/\text{hr}$.

At time 3500 minutes, the measured liquid saturation of matrix block LC is 65.3 percent. Production from this block is, therefore, around 1000 cm^3 . Total production from the stack including the initial film flow across the side matrix faces is 2787 cm^3 . At time 17,500 minutes, the measured liquid saturation of the top block is 53.1 percent, and that of the middle block is 64.7 percent. Therefore, during the time period of 3500 to 17,500 minutes, the top block has produced 350 cm^3 . The film drainage along the 50 micron spacers has allowed production of this sizable amount of oil. Use of a video camera which enlarged the fracture space between blocks showed that the top fracture was empty after the initial period of about 4 minutes. Except in the early part of the test (11 to 12 minutes), production from blocks LC and LB is from film drainage across the spacers. This is an important finding and it will guide us to search for a theoretical model for flow across the fractures.

Test 8 (sixteen 50 micron spacers) — The difference between this test and Test 7 is that instead of four spacers, sixteen spacers with the same dimensions were used. These spacers were arranged on the fracture surface in a row of 4 by 4 and the distance between them was about 0.9 cm. Use of a feeler gauge showed uniform fracture thickness — 40 to 50 microns. Second fracture had a slightly less thickness than the first one.

Figs. 4.33 to 4.36 show the measured data for Test 8. These figures show that up to the time $t=3500$ minutes, both the production and rate data for Tests 7 and 8 are nearly the same. Since there was no interruption of the test for saturation measurement, the drainage rate declines gradually. However, apparently, the rate of drainage of Tests 7 catches up with Test 8 after each interruption. The rate of drainage at the end ($t=33$ days) is about $0.44 \text{ cm}^3/\text{hr}$, which is the same as Test 7.

Saturation data at the end are shown in Table 4.9. The top two blocks have nearly the same saturation which could be the result of smaller aperture of the second fracture. From comparison of Tests 7 and 8, one may conclude that increasing the contact area has a marginal effect on fracture liquid transmissibility across the spacers.

Test 9 (1: four 100 micron spacers, 2: four 50 micron spacers, 3: direct contact)—This is the last and also the most comprehensive experiment of this series to further examine the effect of spacer thickness on fracture liquid transmissibility. Total test duration was about 8 1/2 weeks. In the first three weeks, the spacer dimensions were $2 \times 2 \times 0.001 \text{ cm}$ (i.e., 100 microns

thick). They were, then, replaced with 50 microns thick spacers in the second period. In the last part of the test, the blocks were in direct contact (DC). For the first two periods, the spacers were arranged on the face of the blocks similar to Test 7.

In the first period, saturation measurements were made at three periods — 3200, 18,300, and 29,800 minutes. (At the time of these saturation measurements, the spacers were removed from the matrix blocks. They were placed in the fracture space without wetting them at the time of stack block assembly.) The flow in the fracture space between the matrix blocks was also recorded (with a 15 X magnification) on a video camera for visual inspection. Replay of the images revealed that at time 3 minutes, the upper fracture started to desaturate. By time 4 minutes, it was completely empty. The average drainage rate from the stack in the time interval of 15 to 30 minutes is $180 \text{ cm}^3/\text{hr}$. Film flow in the upper fracture along the spacers is the only flow mechanism which contributes to liquid flow from the top to the middle block from $t=4$ minutes and beyond. Fracture aperture measurement by a feeler gauge showed uniform fracture thickness — 80 to 100 microns for both fractures.

Measured production and rate data for Test 9 are shown in Figs. 4.33 to 4.36. Saturation data are shown in Table 4.9. Figs. 4.33 and 4.34 demonstrate the strong influence of fracture aperture on liquid transmissibility. As fracture aperture decreases, liquid transmissibility in the fracture increases. Figs 4.35 and 4.36 show that the rate of drainage for the first period of Test 9 is less than those of Tests 7 and 8.

At time 3200 minutes, the measured saturation of the top matrix block is 74.3 percent. This implies that the top matrix block contribution to the total production of 1635 cm^3 (this value includes the early film drainage from the vertical faces of the stack) is about 740 cm^3 . For Test 7, the contribution of this block at time 3500 minutes to production was 1000 cm^3 . At time 18,300 minutes, the measured saturations of matrix blocks LC and LB are 69.3 and 67.1 percent, respectively. During the time interval of 3200 to 18,300 minutes, matrix block LC has, therefore, produced 143 cm^3 , which is less than half of the corresponding production for the same block in Test 7. Film flow along the spacers is, therefore, strongly influenced by fracture thickness. At time 29,800 minutes, the saturation of matrix blocks LC and LB were again measured. Of the 50 cm^3 production from the stack from 18,300 to 29,800 minutes, the contributions of matrix blocks LC, LB, and LA are 16, 31, and 3 cm^3 , respectively. As Figs. 4.35 and 4.36 show, due to rupture or evaporation of the liquid film on the spacers at the time of saturation measurement, there is a sudden decrease of drainage rate. The sharp break was avoided in the remaining period of the test by wetting the spacers with few droplets of the liquid at the time of remounting the matrix blocks.

At time 29,850 minutes, the 100 micron spacers were replaced with 50 micron spacers. The spacers were wetted by pouring a few drops of normal decane on their surface. The 50 micron spacers were removed at time $t=70,270$ minutes and the blocks were put in direct contact. Therefore, in the 30,000 to 70,000 time period the blocks were separated by 50 micron spacers. Figs. 4.35 and 4.36 show the sharp increase in the drainage rate at the beginning of the second period ($t=30,000$ minutes) — from 0.2 to 5 cm^3/hr . At $t=52,900$ minutes, the saturation of the top two blocks, LC and LB, were measured. The data are shown in Table 4.9. The measured saturations reveal that most of the production in the 30,000 to 53,000 minute period is from desaturation of the top block. Note that at the time of saturation measurement ($t=53,000$ minutes), there is no sharp drop in the drainage rate (see Figs. 4.35 and 4.36).

At time $t=70,270$ minutes, the spacers were removed from the fractures and saturation of blocks LC, LB, and LA were measured. Table 4.9 shows the measured saturations. A sharp increase in rate is the result of direct contact assembly. For this period of the test, the saturation of blocks LC, and LB were measured only once at time $t=80,200$ minutes. The results shown in Table 4.9 reveal a substantial production from the middle block LB. At the termination of the test, the saturation of all the three blocks were measured (see Table 4.9). The saturation data demonstrate that the stack block drainage is moving in the direction of gravity/capillary equilibrium.

Various measurements of Test 9 demonstrate the strong influence of fracture aperture on two-phase fracture liquid transmissibility. Visual inspection of flow in the fractures also show that film flow across the spacers basically contributes to flow.

DISCUSSION

The experimental work performed in this study shows that when the fracture aperture is large, the film flow cannot provide adequate liquid transmissibility for flow across a fracture between the matrix blocks. In Test 2, where 1.2 mm spacers were used, the process of liquid bridge forming and breakdown mostly contributed to production from the stack. This process can only continue to the point of gravity/capillary equilibrium of a single block. Desaturation beyond the capillary/gravity equilibrium can be only achieved by film flow across spacers. Prior to performing the above experiments, our understanding was that the Young-Laplace equation of capillarity, $P_c=2\sigma/t$, provides the framework for fracture capillary pressure. Continued drainage from the stack of blocks of Tests 7, 8, 9 and other tests demonstrates clearly that capillary continuity is preserved beyond the Young-Laplace equation. As an example for the first part of Test 9, for a spacer thickness of 100 microns, the computed capillary pressure from

$P_c = 2\sigma/t$ is about 0.1 psi. The measured block saturation of about 70 percent requires a capillary pressure in excess 0.6 psi in the fractures (based on measured matrix capillary pressure). The metallic spacers used in our work are wetted easily with normal decane, and, therefore, we assume $\cos\theta=1$.

In contrast to Test 2, visual inspection of the fracture space between the matrix blocks of Test 8 with 50 micron spaces (fracture aperture in the range of 40 to 50 microns) showed that film flow along the spacers accounts for nearly all the liquid flow from one block to another. In a number of other tests, visual observations lead us to the same conclusion. Theoretical modeling of fracture liquid flow and analysis of data presented in this paper is a real challenge. Nevertheless, the data of this paper leads to the following conclusions.

CONCLUSIONS

1. For the gas-oil gravity drainage process, the fracture liquid flow is mainly film flow.
2. The rate of drainage across a stack of matrix blocks is governed by the fracture liquid transmissibility. This parameter is very sensitive to fracture aperture.
3. The fracture liquid transmissibility may not be sensitive to the number of contact points or contact area (at least for a given range).

NOMENCLATURE

- h_{th} – Threshold capillary pressure height
- k_h – Horizontal permeability
- k_v – Vertical permeability
- P_c – Capillary pressure
- P_{th} – Threshold capillary pressure
- S_o – Oil saturation
- S_{or} – Residual oil saturation
- t – Fracture thickness, also time
- σ – A coefficient in Eq. 4.9
- ρ – Density

REFERENCES

1. Firoozabadi, A., and Hauge, J.: "Capillary pressure in fractured porous media," JPT (June, 1990) 784-791.
2. Labastie, A.: "Capillary continuity between blocks of a fractured reservoirs," SPE 20515, paper presented at the SPE 65th Annual Technical Conference and Exhibition, New Orleans, LA, Sept. 23-26, 1990.
3. Firoozabadi, A. Sorooch, H. and Hassanpour, G.H.: "Drainage Performance and Capillary Pressure Curves Utilizing a New Centrifuge," JPT (July (1988) 913-919.

Table 4.6 - Permeability of Berea Sandstone Blocks.

Block	Average Permeability, md			Dimension, cm x cm x cm	Porosity, %
	k_h	k_v	k_v		
	Side Faces	Top Face	Bottom Face		
SA	275	1035	865	14.71 x 14.71 x 45.49	22.3
SB	799	468	786	14.71 x 14.71 x 45.49	21.6
SC	346	811	642	14.71 x 14.71 x 45.49	22.2
SD	698	695	574	14.71 x 14.71 x 45.49	21.9
LC	643	653	653	14.71 x 14.71 x 60.48	21.9
LB	594	751	603	14.71 x 14.71 x 60.44	21.8
LA	608	597	666	14.71 x 14.71 x 60.48	21.4
TALL	416	406	754	15.16 x 15.14 x 182.9	21.6

Table 4.7 - Block Saturation at the End of the Tests 1A, 1B, and 1C.

Block	Liquid Saturation, %			Calculated Equilibrium Saturation, %
	Test 1A	Test 1B	Test 1C	
SA	34.7	28.8	39.8	26.3
SB	40.1	31.3	43.5	28.5
SC	44.8	39.6	47.1	44.8
SD	98.1	98.1	98.1	98.5
Duration, min.	21,368	56,190	15,382	
Liquid Level	0.9 cm	0.6 cm	0.9 cm	

Table 4.8 - Block Saturation at the End of Tests 2 to 6.

Block	Liquid Saturation, %					Single Block Equilibrium Saturation, %
	Test 2	Test 3	Test 4	Test 5	Test 6	
SA	88.9	74.5	84.7	56.7	61.3	95.0
SB	93.7	64.9	87.6	49.7	61.6	95.0
SC	95.5	59.0	80.7	48.2	44.3	95.2
SD	97.3	97.3	97.1	97.7	97.8	96.8
Duration, min.	11,747	30,489	21,379	50,469	56,469	
Liquid Level	0.3 cm	0.55 cm	0.45 cm	0.5 cm	0.85 cm	

Table 4.9 - Block Saturations at the End of Tests 7 to 9.

Block	Liquid Saturation, % (Measurement Time, min)			Single Block Equilibrium Saturation, %
	Test 7	Test 8	Test 9	
LC	47.7 (47,527) 53.1 (17,600) 65.3 (3,250)	52.9	30.6 (86,475) 31.9 (80,200) 37.0 (70,270) 37.3 (66,208) 39.2 (52,909) 68.7 (29,863) 69.3 (18,349) 74.3 (3,243)	86.5
LB	59.8 (47,527) 64.7 (17,600)	53.4	35.0 (86,475) 36.1 (80,200) 52.1 (70,270) 60.9 (52,909) 66.1 (29,863) 67.1 (18,349)	85.5
LA	90.4 (47,527)	90.5	90.3 (86,475) 90.6 (70,270)	90.3
Duration, min.	47,527 min.	47,850 min.	86,475 min.	
Liquid Level	1.1 cm	0.9 cm	1.15 cm	

Fig. 4.19-Apparatus schematic

Fig. 4.20 - Production data vs. time (linear scale) for Tests 1A, 1B, 1C, and the Tall block.

Fig. 4.21 - Production data vs. time (log scale) for Tests 1A, 1B, 1C, and the Tall Block.

Fig. 4.22 - Rate data vs. time (log scale) for Tests 1A, 1B, 1C, and the Tall block.

Fig. 4.23 - Production data vs. time (linear scale) for Tests 2, 3, and 4.

Fig. 4.24 - Production data vs. time (log scale) for Tests 2, 3, and 4.

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Fig. 4.25 - Rate data vs. time (linear scale) for Tests 2, 3, and 4.

Fig. 4.26 - Rate data vs. time (log scale) for Tests 2, 3, and 4.

Fig. 4.27 - Schematic of liquid bridge formation and breaking in the fractures of Test 2.

Fig. 4.28 - Spacer array in the fracture space of Tests 2, 3, and 4.

A) Test 4.

B) Tests 2, and 3.

Fig. 4.29 - Production data vs. time (linear scale) for Tests 5, 6, and 1B.

Fig. 4.30 - Production data vs. time (log scale) for Tests 5, 6, and 1B.

Fig. 4.31 - Rate data vs. time (log scale) for Tests 5, 6, and 1B.

Fig. 4.32 - Fracture aperture for Test 6.

Upper fracture.

Middle fracture.

Lower fracture.

Fig. 4.33 - Production data vs. time (linear scale) for Tests 1, 7, 8, 9, and the Tall block.

Fig. 4.34 - Production data vs. time (log scale) for Tests 1, 7, 8, 9, and the Tall block.

Fig. 4.35 - Rate data vs. time (linear scale) for Tests 1, 7, 8, 9, and the Tall block.

Fig. 4.36 - Rate data vs. time (log scale) for Tests 1, 7, 8, 9, and the Tall block

4.3 LABORATORY STUDY OF CAPILLARY CROSSFLOW IN FRACTURED POROUS MEDIA.

Abbas Firoozabadi, and Tore Markeset

INTRODUCTION

Fractured reservoirs are comprised of matrix blocks and fractured networks. One may view these reservoirs as the extreme of heterogeneous reservoirs. Physical phenomena such as capillary, gravity, and viscous crossflow are believed to govern the multiphase flow process. The studies in the literature center around viscous crossflow in layered porous media^{1,2}. Only a few studies have touched upon the subjects of capillary and gravity crossflow in heterogeneous porous media^{3,4}. To our knowledge, there is no published report on capillary and gravity crossflow in fractured porous media.

Fracture orientation varies from one reservoir to another. Therefore, the significance of capillary and gravity crossflow is reservoir dependent. For naturally fractured reservoirs with heterogeneous rock matrix, capillary and gravity crossflow is believed to play a dominant role in the reservoir performance, even for vertical fractures. In this paper, our experimental work on the drainage performance of an aggregate of matrix blocks (see Fig. 4.37) to study capillary crossflow is presented. The results provide a basis for future theoretical studies.

EXPERIMENTAL.

Apparatus — The experimental setup used in this work is similar to the one described in Refs. 5 and 6. In the experiments on the stack of 12 matrix slabs, there was no space between the rock matrix and the glass wall of the core holder.

Core Preparation and Properties — In addition to matrix blocks SA, SB, SC, and SD described in Refs. 5, tests with new matrix slabs were carried out in this project. In some of the tests, twelve Berea sandstone slabs were used. The slabs were grinded to a dimensional tolerance of ± 0.025 mm. The dimension of each slab is 45.53 x 14.95 x 4.89 cm.

A minipermeameter was built to measure permeability. A ball and tube flowmeter and pressure transducer were used for measuring conditions of injection of a gas (such as nitrogen) into the face of a porous sandstone block. The injector tip seal consisted of a closed cell-0.159 cm thick self-adhesive Buna washer with an inside diameter of 0.556 cm and outside diameter of 1.270 cm. The minipermeameter was, first, used to measure the permeability of the flat faces of two Berea sandstone samples (1 1/2" diameter and 2.0" long). The permeability of the same cores was also measured in a core holder setup. A minipermeameter measures the permeability around the tip, while an average permeability is given by a core holder. The arithmetic average

of the permeability of one of the core samples by the minipermeameter was 709 md (the measurements were made at two different flow rates at the two flat faces). The permeability of the same core sample measured in the core holder is 715 md. The measured permeability of the second core sample measured by the two techniques was nearly the same. The minipermeameter was then used to measure the permeability of various Berea sandstone blocks and slabs. The results for the slabs are listed in Table 4.10. Each data is the arithmetic average permeability of several measurements for each face. The measurements are made at the same tip location for different flow rates, and by changing the location of the tip. The data of Table 4.10 indicate that the Berea sandstone used in our work is homogeneous.

In order to establish the pore volume and porosity of the slabs, four of them were saturated and drained individually. Fig. 4.38 shows the drainage performance of these four slabs. Due to a high threshold height, the production is a small percentage of the pore volume. Table 4.11 provides the porosity and the measured liquid saturation at the end of drainage tests. The position of the liquid level in the vertical fracture space (from the bottom face of the block) is also listed in Table 4.11. It should be noted that data of Fig. 4.38 exclude the initial film flow along slab side faces. The average porosity of the sandstone slabs of Table 4.11 is 21.8 percent. This value was assigned to the porosity of the rest of the slabs. Table 4.13 gives the slab pore volume. These values are based on, 1) porosity of 21.8 percent, 2) grain density of 2.64 g/cm^3 , and 3) dry weight measurement.

Test Procedures — Two types of tests were performed in this work. In the first group, four blocks, each with dimensions of $14.71 \times 14.71 \times 45.49 \text{ cm}$ (see Ref. 5), were stacked and tilted. In order to keep the fracture thickness of 1 mm between the side face of the blocks and glass wall of the container, one small 1 mm thick spacer was provided on each side face of all the blocks. In the second group of tests with the slabs, they were arranged according to Fig. 4.37. The slabs were in direct contact with the glass. Saturating the slab assembly and saturation measurement procedure at the end of the tests are similar to the tests that are described in Refs. 5 and 6. Normal decane was used in all the tests.

RESULTS

Two set of experiments were performed: 1) drainage of the tilted four block stack, and 2) drainage of the twelve slab assembly of Fig. 4.37. In the first set, there is no crossflow along the side faces. In the second set, capillary crossflow along the side face of the matrix slabs could take place. The experiments of the first set would provide a frame of reference for the stacked slab tests.

Stacked Block Experiments

Test 1 (four 0.25 mm spacers, $\Theta=30^\circ$) — In Test 1, four 250 micron spacers (with areal dimensions of 0.25 x 0.25 cm) were inserted in the fracture space between the matrix blocks. After saturating the stack, the block assembly was tilted 30° (from the vertical axis). The tilt was made by rotating the stack along the side edge rather than block side face. In this test, the bottom face of matrix block SD was fully immersed in liquid, and the liquid level was held 9 mm above the top corner of the matrix block SD. Fracture aperture was measured by a feeler gauge which showed a variation of 0.22 to 0.25 mm for the upper fracture, 0.1 to 0.27 mm for the middle fracture, and 0.04 to 0.35 mm for the lower fracture. Use of a video camera was helpful to observe that there was no physical contact between the matrix blocks.

Figs. 4.39 to 4.42 show the drainage performance and rate data for Test 1, and Test 3 of Ref. 6 (the corresponding vertical stacked block test). Table 4.12 provides the measured block saturation at the end of the test ($t=21$ days). Fig. 4.42 gives the early rate data of about $10 \text{ cm}^3/\text{min}$ in the 1 to 6 minute interval. The rate drops to $2 \text{ cm}^3/\text{min}$ at time of about 8 minutes, and decreases thereafter. Visual inspection showed that the upper, middle, and lower fractures were empty at 2, 3, and 6.5 minutes, respectively. In the early production period of 1-8 minutes, the drainage of the top three matrix blocks is due to the liquid film flow along the spacers. Toward the end of this test, the drainage rate from the stack was about $18 \text{ cm}^3/\text{day}$. Fig. 4.42 shows a more gradual rate reduction during the initial period of this test than Test 3 of Ref. 6. The spacers between the matrix blocks and the glass wall help to drain part of the liquid film on the side faces of the matrix blocks.

Saturation data of Test 1 (see Table 4.12) and Test 3 of Ref. 6 (see Table 4.6 of Ref. 6) at the end of these two tests show the same trends. Comparison of the drainage performance of these two tests reveals a significant effect of tilt angle on recovery. Towards the end, at $t=21$ days, the recovery of tilted stack is about $2/3$ of the vertical stack.

Test 2 (Direct Contact, $\Theta=45^\circ$) — In this test, the blocks were stacked in direct contact and after saturating, the setup was tilted 45° . The liquid level was held 10 mm above the lowest corner of the bottom face of matrix block SD.

Figs. 4.43, 4.44, and 4.45 show the measured data for Test 2. These figures also show the drainage data of the blocks in direct contact in the vertical position (Test 1B Ref. 6). In the 1 to 5 minute interval, the rate of production (from Fig. 4.45) is about $7 \text{ cm}^3/\text{min}$. After 11 minutes, the rate reduces to about $2.7 \text{ cm}^3/\text{min}$. Toward the end ($t=34$ days), the drainage rate reduces to $15 \text{ cm}^3/\text{day}$.

Measured saturation data at the end of the test are listed in Table 4.12. The saturation data follow the same trend as saturation data of Test 1B of Ref. 6. Both the saturation and production performance data show a pronounced effect of tilt angle of 45° . The production from the vertical stack at $t=9$ days is about 3500 cm^3 . The same amount of production from the tilted stack is realized at $t=32$ days.

Stacked Slab Experiments

The purpose of the set of experiments with an array of slabs was to investigate the effect of capillary crossflow and gravity on flow in fractured porous media. In the experiments to be described next, the tilt angles were 0° , 30° , and 45° . Tilting promotes capillary crossflow and reduces gravity effect.

Test 3 (Direct contact, $\Theta=0^\circ$) — In this test, the 12 slab arrangement of Fig. 4.37, unlike the tests with the matrix blocks, was in direct contact with the glass walls. This reduces the volume of liquid in the fracture space between the rock and glass wall significantly.

Figs. 4.46, 4.47, and 4.48 show the production and drainage data for Test 3 and Test 1B of Ref. 6 (blocks in direct contact, $\Theta=0^\circ$). In the 1 to 15 minute period, part of the production is from the fracture space between the matrix blocks, and the blocks and the glass. Film flow along the matrix faces could also contribute to production during this early period. Drainage from the slabs also contributes to production during this period. Unlike the tests of stacked blocks (with a 1 mm fracture between rock matrix and glass), there is no sharp break in the early drainage period (see Figs. 4.47 and 4.48). The average rate of production in the 1 to 5 minute period was $22.5 \text{ cm}^3/\text{min}$. In the 5 to 16 minute period, the rate decreased from $18.7 \text{ cm}^3/\text{min}$ to $4.8 \text{ cm}^3/\text{min}$. Thereafter, the rate decreased slowly. The rate at 16 minutes is close to the corresponding value for the stack blocks of Test 1B of Ref. 6. The rates for these two tests remain nearly the same to the end.

In the first minute of Test 3 part of production was due to the liquid from the four corners of the glass plates (about 16 cm^3). This amount is excluded from the production data of Figs. 4.46, 4.47, and 4.48. Therefore, the early production rate has a small effect on the overall recovery from the rock matrix. Comparison of the total recovery from the stack of slabs and the stack of blocks shows a higher liquid recovery for the stack of blocks. This difference may be partly attributed to minor variations in rock property of these two tests and the fracture aperture.

Table 4.13 shows the measured slab saturations at the end of the test. It is interesting to note that the liquid saturation of the slabs from the same row (except the bottom row) is nearly the same. This is an indication that the slabs are homogeneous. The variation of the liquid

saturation of the bottom row slabs is in relation to the sensitivity of the recovery to the threshold height. There is a good agreement between the slab saturations of this test with Test 1A of Ref.6, although test durations differed by 8 days.

Test 4 (Direct contact, $\Theta=30^\circ$) — In the second test on the twelve Berea sandstone slabs, the setup was tilted 30° and similar to Test 3, the rotation was along the side edge. The slabs were in direct contact and, therefore, there was no empty space between the rock matrix and the glass wall. The total height of the stack of slabs (in the vertical position) is 182.1 cm. The vertical distance from the uppermost corner of the top row of the slabs (slab A1) to the lowest corner of the bottom row slabs (slab D3) is 168.2 cm. Therefore, the difference in the vertical height of the slab arrangement of Tests 3 and 4 is only 14 cm.

Figs. 4.49 to 4.52 show the production performance and rate data of the stack of slabs for the entire period of the test (43 days). Table 4.13 lists the measured saturations of the slabs at the end. Figs. 4.49 to 4.52 also show the production performance and rate data of Test 3 and Test 5 (to be described next). An interesting observation is that the 30° tilt does not seem to influence the production of the stack of slabs significantly. In the case of the stack of blocks (see Fig. 4.43), a significant effect of tilt angle on the recovery performance was observed. Capillary crossflow between the neighboring slabs is the only mechanism which could enhance the production performance of the stack of tilted slabs. In the case of tilted blocks, there is no capillary crossflow across the side faces.

In the first 18 minutes of the test, the production rate is high and has a rapid decreasing trend (see Fig. 4.52). After that, the rate of change declines for a period. The production of the early period is influenced from the fracture volume contribution. Toward the end, the rate of drainage is about $8 \text{ cm}^3/\text{day}$, which is comparable to the Tall block drainage rate at the same time (see Ref. 5).

Table 4.13 lists the measured saturation data. The saturation of the slabs in the first row are close. The difference in saturation of the second row slabs is also small. The saturation of the bottom row slabs, although high, varies due to sensitivity to the capillary threshold height.

Test 5 (Direct Contact, $\Theta=45^\circ$) — In this experiment, the slab setup was tilted 45° . The tilting procedure was similar to Tests 3 and 4. For this test, the vertical distance from the uppermost corner of the top row of the slabs (slab A1) to the lowest corner of the bottom slabs (slab D3) is 143.5 cm. The vertical height is, therefore, 38.6 cm shorter than that of the vertical slabs of Test 3.

Comparison of the production and rate data of Tests 3, 4 and 5 shows that even a tilt angle of 45° does not have a significant influence on drainage characteristics of the stack of slabs (see

Figs. 4.49, 4.50, and 4.51). Capillary crossflow across the side faces is the main mechanism to reduce the effect of tilting on the drainage performance of the stack.

Fig. 4.52 shows that the early rate data follow a similar trend to Tests 3 and 4. The change in this period is more gradual. Table 4.13 lists the measured slab saturation at the end of the test ($t=23$ days). It is interesting to note that saturation of the third row slabs reflect their respective positions. Similar behavior is also seen in saturation data of first and second row slabs-to a lesser degree. In Table 4.13, the slabs of each row are listed according to their positions. As an example, slab B1 is at a higher position than slab B2. Slab B3 is at a lower position than slab B2.

We used Eclipse model⁷ to simulate the drainage performance of the Tall block of Ref. 5 for tilt angles of 0° , 30° , and 45° . In performing the numerical simulation, The Tall block geometry was rotated along the side edge; the same manner that the experiments were conducted. Permeability, capillary pressure and relative permeability of Ref. 5 were used to perform the simulation. Due to the geometry of the system, Cartesian coordinates were used. Gridding included the fractures surrounding the block. The use of Cartesian coordinates slowed the time step size. The Newton iteration did not converge even after 30 iterations for most of the time steps. Various efforts to increase time step size did not help. Figs. 4.53 and 4.54 show the simulation results for the Tall block. (The simulation results give the production and rate information to only 120 hours). The production behavior of the Tall block shows a pronounced effect of tilting (as expected). Had we simulated to time of 50,000 minutes, the same trend of production would have been observed. There is a major difference between the production trends in Figs. 4.51 and 4.53. The production trend in Figs. 4.44 and 4.53 is, however, similar. Capillary crossflow across the side faces is the mechanism which affects the drainage characteristics of the tilted slabs.

DISCUSSION AND CONCLUDING REMARKS

Throughout this paper, we have used the term capillary crossflow (across a fracture medium between two matrix blocks). A simple sketch will perhaps help to define what we mean by capillary crossflow. Consider two matrix blocks, M1 and M2 depicted in Fig. 4.55. Points A and B are located on opposite faces of the fracture within the matrix blocks M1 and M2, respectively. As a result of drainage in some stage, $S_{oA} < S_{oB}$, although points A and B are very close; consequently $P_{cA} < P_{cB}$. (Note that capillary continuity does not mean $P_{cA} = P_{cB}$.) Since $P_{cA} = P_{gA} - P_{oA}$ and $P_{cB} = P_{gB} - P_{oB}$ and if we assume infinite gas mobility $P_{gA} = P_{gB}$, then $P_{oA} > P_{oB}$. This implies that the flow of oil is from point A to point B. The flow of oil from point A to B is due to capillary pressure difference between points A and B, therefore, capillary

crossflow term is used. For the single phase oil, P_{OA} is nearly the same as P_{OB} , but in the two-phase gas-oil flow P_{OA} could be as much as 0.5 psi higher than P_{OB} over a distance of 10-100 microns.

The contribution of the capillary crossflow from the side faces of the matrix rock increases as the tilt angle increases. Comparison of the drainage performance of the tilted blocks with the vertical stacked blocks shows a considerable effect of tilt angle on recovery rate. In the case of stacked slabs, the effect of tilt angle on rate was not pronounced. Capillary crossflow across the side faces of the matrix slabs provides the condition for enhanced drainage rate. Realistic study of fractured petroleum reservoirs under drainage should include capillary crossflow from side face of matrix blocks. The data presented in this paper will help to guide future theoretical and computational efforts on this topic.

NOMENCLATURE

P_c — capillary pressure

P — pressure

S — saturation

Subscripts:

A — point A

B — point B

g — gas phase

o — oil phase

REFERENCES

- [1] Goddin, C.S., Jr., Craig, F.F., Jr., Wilkes, J.O., and Tek, M.R.: "A Numerical Study of Waterflood Performance in a Stratified System with Viscous Cross Flow." JPT (1966), 765-771.
- [2] Zapata, V.J., and Lake, L.W.: "A Theoretical Analysis of Viscous Crossflow," SPE 10111, paper presented at the SPE 56th Annual Technical Conference, San Antonio, TX, Oct. 5 -7, 1981.
- [3] Yokoyama, Y., and Lake, L.W.: "The Effects of Capillary Pressure on Displacement in Stratified Porous Media," SPE 10109, paper presented at the SPE 56th Annual Meeting, Dallas, TX, Oct. 5-7, 1981.
- [4] Sorbie, K.S., Sheb, M., Hosseini, A., and Wat, R.M.S.: "Scaled Miscible Floods in Layered Beadpacks Investigating Viscous Crossflow, the Effects of Gravity and the Dynamics of Viscous Slug Breakdown, " SPE 20520, paper presented at the SPE Annual Technical Conference and Exhibition, New Orleans, Sept. 23-26, 1990.
- [5] Firoozabadi, A., and Markeset, T.: "Laboratory Study of Reinfiltration for Gas Liquid System in Fractured Porous Media," Chapter 4.
- [6] Firoozabadi, A., and Markeset, T.: "Laboratory Study of Gas-Liquid Transmissibility in Fractured Porous Media," SPE 24919, paper presented at the 1992 SPE Annual Technical Conference and Exhibition, Washington D.C., Oct. 4-7, 1992, to be published by SPE Reservoir Engineering Journal.
- [7] Eclipse Reference Manual, Version 90A, Exploration Consultants Ltd. (1990).

Table 4.10 - Permeability of the Berea Sandstone Slabs.

Slab	Permeability, md		
	k_h	k_v	k_v
	Side Faces	Top Face	Bottom Face
A1	461	818	591
A2	567	813	699
A3	717	871	684
B1	702	801	844
B2	736	932	725
B3	811	724	913
C1	645	668	701
C2	619	734	649
C3	570	731	694
D1	570	705	751
D2	580	632	675
D3	631	578	673

Table 4.11 - Porosity and Equilibrium Liquid Saturation of Selected Slabs.

Slab	Porosity, %	Liquid Saturation, %	Liquid Level Position, cm
A2	21.91	97.6	0.70
C1	21.95	98.1	0.95
C2	21.76	97.7	1.20
D3	21.68	98.7	0.90

Table 4.12 - Block Saturation at the End of Tests 1 and 2.

Block	Liquid Saturation, %	
	Test 1	Test 2
SA	80.2	36.6
SB	79.3	37.8
SC	66.1	54.8
SD	99.7	100.0
Test Duration	29,891 min.	48,861 min.

Table 4.13 - Slab Saturation at the End of Tests 3 to 5.

Location	Slab	Liquid Saturation, %			Pore Volume, cm ³
		Test 3	Test 4	Test 5	
First Row (from top)	A1	35.4	30.9	34.5	732.8
	A2	34.6	30.6	34.3	735.1
	A3	35.3	31.3	35.7	733.1
Second Row	B1	38.5	34.4	37.5	726.9
	B2	38.8	35.3	39.0	727.5
	B3	38.0	35.4	39.5	732.7
Third Row	C1	44.4	41.8	50.3	737.4
	C2	44.1	43.3	53.8	731.8
	C3	43.2	44.1	57.7	730.6
Bottom Row	D1	94.8	93.1	95.5	734.2
	D2	98.3	98.2	99.2	727.4
	D3	97.1	97.3	98.4	726.2
Test Duration		32,706 min.	61,466 min.	32,991 min.	

Fig. 4.37 - Matrix slab arrangement used to study capillary and gravity crossflow.

Fig. 4.38 - Drainage Performance of the Berea slabs.

Fig. 4.39 - Production data vs. time (linear scale) for Test 1,
and Test 3 of Ref. 6.

Fig. 4.40 - Rate data vs. time (linear scale) for Test 1, and for Test 3 of Ref. 6.

Fig. 4.41 - Production data vs. time (log scale) for Test 1, and
for Test 3 of Ref. 6.

Fig. 4.42 - Rate data vs. time (log scale) for Test 1, and for
Test 3 of Ref. 6.

Fig. 4.43 - Production and rate data vs time (linear scales) for
Test 2, and Test 1B of Ref. 6.

Fig. 4.44 - Production and rate data vs. time (log scale) for Test 2, and for Test 1B of Ref. 6.

Fig. 4.45 - Rate data (linear scale) vs. time (log scale) for Test 2, and Test 1B of Ref. 6.

Fig. 4.46 - Production and rate data vs. time (linear scale) for Test 3, and for Test 1B of Ref. 6.

Fig. 4.47 - Production and rate data vs. time (log scale) for
Test 3, and Test 1B of Ref. 6.

Fig. 4.48 - Rate data vs. time (log scale) for Test 3, and Test 1B of Ref. 6.

Fig. 4.49 - Production data vs. time (linear scale) for Tests 3, 4, and 5.

Fig. 4.50 - Rate data vs. time (linear scale) for Tests 3, 4, and 5.

Fig. 4.51 - Production and rate data vs. time (log scale) for Tests 3, 4, and 5.

Fig. 4.52 - Rate data (linear scale) vs. time (log scale) for Tests 3, 4, and 5.

Fig. 4.53 - Simulation results of production vs. time (log scale)
for tilt angles of 0, 30, and 45 degrees for the Tall block.

Fig. 4.54 - Simulation results of drainage rate vs. time (log scale) for tilt angles of 0, 30, and 45 degrees for the Tall block.

Fig. 5.55 - Capillary crossflow across fractures
in gas-oil two-phase flow.

CHAPTER 5

THEORETICAL AND NUMERICAL STUDIES OF IMMISCIBLE GAS-OIL FLOW IN FRACTURED POROUS MEDIA

OVERVIEW

Oil flow across fractures between two matrix blocks in a gas-oil two-phase region is mainly driven by pressure. As a result, over a distance of 10 to 100 microns across a fracture, capillary pressure could vary as much as 1 psi. Since oil flow is governed by the capillary pressure difference of the neighboring matrix block faces, fracture capillary pressure becomes the most significant parameter of flow. In this chapter, first a detailed analysis of oil flow in a single liquid bridge between two matrix blocks is analyzed. A simple expression for the liquid transmissibility in the fracture is derived. Our expression replaces both the fracture permeability and fracture relative permeability. Then the numerical simulation of gas-oil flow in fractured porous media is carried out. At the end, the reinfiltration process will be analyzed.

5.1 LIQUID FILM FLOW IN A FRACTURE BETWEEN TWO POROUS BLOCKS

Birol Dindoruk, and Abbas Firoozabadi

INTRODUCTION

Two phase gas-oil flow in fractured porous media differs significantly from flow in porous media without fractures. In fractured porous media, flow is affected by: 1) matrix flow, and 2) fracture flow. In this report, we will limit ourselves to fracture flow for gas-liquid systems. Particularly, the flow between two matrix blocks through a fracture will be focused. The two matrix blocks are assumed to be separated by spacers which create a fracture between matrix blocks. This type of geometry has been used for the study of fracture flow by Firoozabadi and Markeset [15]. In the experiments, saturated liquid flow was observed initially. Saturated flow in the fractures lasted only a short time (in terms of total recovery). Then, film flow along the spacers was observed. Film flow lasted to the end of the experiments. The spacer provided capillary continuity between the two matrix blocks. Unlike the conventional analysis, overall flow is perpendicular to the fracture plane.

In this report, our main purpose is to investigate the flow for an isolated liquid bridge (see Fig. 5.1) seeping through a single spacer. This type of detailed analysis is expected to yield effective fracture liquid transmissibility. The liquid transmissibility can help us to understand the flow between matrix blocks and to quantify the flow in the fractures. Therefore, we can investigate how to increase fracture liquid transmissibility.

A brief analysis of the variables which affect the film flow indicates that the shape of the film is an important parameter. However, the shape of the film (free boundary) is also an unknown. The free boundary which contains the flow is the fundamental difference between this study and the previous one by Arbabi et al. [2]. The free boundary must be studied under dynamic rather than static conditions. Under static conditions (no-flow) the shape of the free surface may be determined easily. In general, the shape will be

circular when there is no flow, and gravity effects are negligible.

PROBLEM FORMULATION

Flow through a liquid bridge between two matrix blocks can be modeled by a two-dimensional free-boundary problem. Previously, the same problem has been solved by Arbabi, Firoozabadi, and Barkve [2] for a fixed circular shape free boundary using a special form of stream function adopted from lubrication theory [3]. Their solution has introduced an error term due to the assumed circular boundary. The error term was insignificant for a range of characteristic length ratios and total flux. Formulation of the flow for a liquid bridge is based on the following assumptions:

1. Incompressible fluids.
2. Negligible gravity.
3. Laminar flow and negligible inertia effects.
4. Steady state.
5. Two dimensional geometry.
6. Matrix dimensions and the distance between the spacers are large with respect to fracture aperture.
7. Shear stress in the gas phase is negligible (zero gas viscosity).

Under the assumptions above Navier-Stokes equation of motion becomes the Stokes equation [4, 17],

$$\vec{\nabla} P = \mu \vec{\nabla}^2 \vec{V} , \quad (5.1)$$

coupled with the continuity equation,

$$\vec{\nabla} \cdot \vec{V} = 0 . \quad (5.2)$$

Boundary Conditions

Solid Wall (AD) - Solid wall (spacer) does not permit any flow through, therefore,

$$\text{B.C.1: } \vec{V} \cdot \vec{n} = 0 \quad (5.3)$$

where $\vec{n}$ is a unit surface normal vector. Liquid particles on the wall, also, do not move.

The no-slip condition is stated as,

$$\text{B.C.2: } \vec{V} \cdot \vec{t} = 0 \quad (5.4)$$

where $\vec{t}$ is a unit tangent vector. This indicates that there is no relative motion between the fluid and the solid.

Top and Bottom Porous Rocks (AB and CD) - Flow distribution and the rotation tendency of the flow (vorticity or rotation) can be specified. In porous media, the flow is assumed to be irrotational, or potential flow ($\vec{\nabla} \times \vec{V} = \vec{0}$) [25, 5]. Complex structure of the pores can locally cause rotation, but by the averaging procedure the rotations occurring within the individual pores balance out. Therefore, the average flow is irrotational and a velocity potential exists.

$$\text{B.C.3 - 4: } \omega = 0 \quad (5.5)$$

In the above boundary condition, ω is the vorticity. It will be defined later in detail. The porous medium acts like a porous wall and it can permit fluid(s) to pass through. Therefore, the no-flow condition can be relaxed on the velocity component normal to the wall [28]. The proper conditions are complicated due to porosity of the wall. The flow condition is not exactly known. In our formulation, streamlines along the porous matrix boundary are distributed according to

$$\text{B.C.5 - 6: } \psi = f(x) \quad (5.6)$$

where ψ is stream function.

Free Surface (BC) - Free surface (gas-liquid interface) poses three boundary conditions instead of two. The extra boundary condition determines the shape of the free surface.

One boundary condition defines the velocity normal to the interface to be zero. This condition requires the free surface to be a streamline (a line such that at any point, the local instantaneous velocity is tangent to it [26]) since no fluid particles may cross the surface.

$$\text{B.C.7: } \vec{V} \cdot \vec{n} = \vec{0} \quad (5.7)$$

The tangential force balance or the shear stress condition is given by [4],

$$\text{B.C.8: } \vec{t} \cdot \vec{e} \cdot \vec{n} = \vec{0} \quad (5.8)$$

where $\vec{e}$ is rate of stress tensor. The normal force balance for the free surface is somewhat more complex due to the surface tension. The normal stress acting on the two sides of a surface element must be balanced by an amount depending on the surface tension, σ , and curvature, κ , given by [9, 11, 4, 13, 12]

$$\text{B.C.9: } [-P + \vec{n} \cdot \vec{e} \cdot \vec{n}]_+ - [-P + \vec{n} \cdot \vec{e} \cdot \vec{n}]_- = -\sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (5.9)$$

The rate of strain tensor, $\vec{e}$, due to viscosity is given by [4]

$$e_{ij} = \mu \left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i} \right) \quad (5.10)$$

In Eq. 5.10, w_i and w_j are the components of velocity in the direction of x_i and x_j respectively. In Eq. 5.9 $\vec{n}$ is unit outward normal vector and the signs of R_1 and R_2 (principal radii of curvature of a surface element) are considered to be positive if the corresponding centers of curvature lie on the side of surface element to which the (+) sign refers. The end point conditions (contact point) for the free surface come from the solid-liquid contact angle such that the free surface is connected to the solid with a wetting angle at points B and C (Fig. 5.1). The interface defined by Eqs. 5.7, 5.8 and 5.9 is a two-dimensional continuum coupled to the three-dimensional body of fluids in which it exists. The dividing surface or the interface (or free surface) has zero thickness. Therefore, interface force balance serves as a jump condition between two different fluids. So far the governing equations and the boundary conditions are stated independent of coordinate

systems. Later on, the problem will be converted to an appropriate coordinate system to achieve the solutions.

Two-Dimensional Flow

For steady state two-dimensional flow, flow equations, Eqs. 5.1 and 5.2, are stated in scalar form (see Appendix A). Stokes equation, Eq. 5.1 is

$$\frac{\partial P}{\partial x} = \mu \nabla^2 u \quad , \quad (5.11)$$

$$\frac{\partial P}{\partial y} = \mu \nabla^2 v \quad ,$$

and the continuity equation, Eq. 5.2 is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad . \quad (5.12)$$

Resulting boundary conditions become the following,

Solid Wall (AD) - The normal flow condition is

$$\text{B.C.1: } u = 0 \quad . \quad (5.13)$$

Eq. 5.13 indicates that there is no flow in x -direction. The no-slip condition is

$$\text{B.C.2: } v = 0 \quad . \quad (5.14)$$

Eqs. 5.13 and 5.14 state that both components, u and v , of the velocity vector, $\vec{V}$, is zero.

Top and Bottom Porous Rocks (AB and CD) - The boundary conditions for the liquid/porous rock interface will later be expressed in terms of solution variables. Irrotationality of the flow at the porous media interface is characterized by

$$\text{B.C.3 - 4 } \omega = 0 \quad . \quad (5.15)$$

This is because flow is assumed to be irrotational in homogeneous porous media. The seepage from/into the top and bottom porous rocks is defined by the distributed stream function along the top and bottom boundaries.

$$\text{B.C.5 - 6: } \psi = f(x) \quad (5.16)$$

Amount of fluid flowing for the bottom face of the bridge, **AB**,

$$Q \frac{1}{\tau} = \int_0^{x_b} v \, dx \quad (5.17)$$

where Q is total flow and x_b is the x -coordinate of point **B**. Because the flow is incompressible, Eq. 5.17 can also be written for the face of the top porous block.

Free Surface (BC) - Free surface in two-dimensions can be represented in y -explicit or x -explicit form. Here, we will adopt x -explicit form since our problem is multivalued in y -explicit form. However, for the solution of the half problem any form is acceptable. For $x = h(y)$, the unit normal vector is [27]

$$\vec{n} = \frac{1}{\sqrt{1 + h'^2(y)}} = \begin{bmatrix} -1 \\ h'(y) \end{bmatrix}, \quad (5.18)$$

and the unit tangent vector is [27]

$$\vec{t} = \frac{1}{\sqrt{1 + h'^2(y)}} = \begin{bmatrix} h'(y) \\ 1 \end{bmatrix}. \quad (5.19)$$

The tangent angle is the angle between the tangent to the free surface and the x axis counter-clockwise.

$$\phi = \tan^{-1} \frac{dy}{dx} = \tan^{-1} \frac{1}{h'(y)} \quad (5.20)$$

We will leave the kinematic condition, Eq. 5.7, and the tangential force balance, Eq. 5.8, as they are, since the products can easily be expressed once the unit normal vector and the unit tangent vector are known. The normal force balance, Eq. 5.9 can still be simplified for the problem considered. The (+) side of the curved interface is selected to be the liquid phase. Since the gas phase is assumed to have zero viscosity the viscous effects denoted by $\vec{n} \cdot \vec{e} \cdot \vec{n}$ term vanish for the gas side of the interface. The second radius of curvature is taken to be zero due to infinite film in z -direction (two dimensional approach with a unit depth in z -direction). Therefore, Eq. 5.9 becomes

$$\text{B.C.9:} \quad -P_l + \vec{n} \cdot \vec{e} \cdot \vec{n} - P_g = -\sigma\kappa \quad (5.21)$$

where P_g and P_l are gas and liquid pressures, respectively. The curvature, κ is given by [27]

$$\kappa = \frac{h''(y)}{[1 + h'^2(y)]^{\frac{3}{2}}} \quad (5.22)$$

The rate of strain tensor is

$$e_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (5.23)$$

Stream Function Formulation

The Stokes equation, Eq. 5.11, and the continuity equation, Eq. 5.12, are coupled and they need to be solved simultaneously. As it is, the problem is already complex and can be simplified one step further. To do that, we introduce utilization of an arbitrary stream function, ψ where we select x and y components of velocity as

$$u = \frac{\partial \psi}{\partial y} \quad (5.24)$$

$$v = -\frac{\partial \psi}{\partial x}$$

respectively. Substitution of the definition of the x and y components of the velocity into the continuity equation, Eq. 5.12, automatically satisfies that equation. Therefore, there is no need to solve the continuity equation. The creeping flow relationships, Eq. 5.11, become

$$\frac{\partial P}{\partial x} = \mu \frac{\partial}{\partial y} (\nabla^2 \psi) \quad (5.25)$$

$$\frac{\partial P}{\partial y} = -\mu \frac{\partial}{\partial x} (\nabla^2 \psi)$$

The pressure dependency of Eqs. 5.25 cancels out when the first equation is differentiated with respect to y and the second equation with respect to x then subtracted from each other. The resulting equation contains only stream function as a dependent variable.

$$\frac{\partial^2 (\nabla^2 \psi)}{\partial x^2} + \frac{\partial^2 (\nabla^2 \psi)}{\partial y^2} = 0 \quad (5.26)$$

In short notation,

$$\nabla^4 \psi = 0 \quad (5.27)$$

Eq. 5.27 is the biharmonic equation. Although biharmonic equation is linear, in general, it cannot be solved using separation of variables due to cross derivatives.

Numerical Solution

When a free surface similar to Fig. 5.1 exists, the flow equations are no longer linear due to boundary interfacial conditions. The ellipticity of the Stokes equation ensures the existence of fundamental solutions which can be used to convert the equation into an integral equation involving boundary integrals (boundary integral, or boundary element method). As a result, only boundaries need to be discretized in contrast to finite difference and finite element methods (domain methods) which require discretization of the domain of interest. This will allow us to reduce the dimensions of the problem by one. In two space dimensions, the discretization on the boundary is in one space dimension. Therefore, the number of unknowns are usually reduced in boundary element method (BEM). The main advantages of this method are:

- There are no grid orientation effects, since the interior of domain is not discretized.
- The analytic character of the solution is preserved due to use of Green's function of the problem.
- The boundary integral method (BIM) (or boundary element method - BEM), allows calculation of dependent variables for any point at the interior of the domain directly.
- BEM is an efficient method for moving boundary problems and it also allows free surfaces with widely varying curvatures.
- BEM is computationally more efficient than the domain methods.

The main disadvantages of the BEM method are:

- The resulting matrices are generally full.
- There is a lack of good methods for estimating errors in solutions.

- The use of the Green's function restricts the application of the method to linear problems with constant coefficients [18, 21, 16].

The advantages of BEM outweigh its disadvantages for certain class of problems, and in recent years, many authors have used BEM to solve elliptic boundary value problems. The early development of this technique took place in potential theory (solution of Laplace equation). Recently, various type of biharmonic problems using BEM are collected in the notes of Ingham and Kelmanson [18]. In the literature, several biharmonic viscous flow problems are solved. Kelmanson [20] solved incompressible liquid flow from a semi infinite channel bounded by two no-slip planes in the absence of gravity. In that study, the shape of the expanding jet was solved iteratively using normal force balance, and the singularities along the contact line [12] was not considered. The initial guess was taken to be the first two terms of the perturbation solution for a large surface tension. Another version of the same problem was also studied by Bush and Tanner [7]. Later, Lu and Chang [23] solved various channel flow problems using the same technique employed by Kelmanson [20]. The iteration procedure, however, was slightly different. They also used parametric continuation to increase the range of the parameters. In a study by Goodwin and Homsy [16], two-dimensional viscous coating problem was solved. In this study, the stability of the solution was found to be extremely sensitive to the initial estimate of the free surface shape. Therefore, parametric continuation and relaxation techniques were used to overcome these difficulties. Contact line singularity was also resolved in the same study. So far, the systems mentioned were steady state. Lasseux et al. [22] solved the problem of gas-liquid drainage in a capillary cavity using BEM. They calculated the location of the free surface as drainage progresses. The main difficulty in the solution process was the convergence for small capillary numbers, $\frac{Q\mu}{\sigma l}$, ($< 2 \times 10^{-3}$) similar to what Lu and Chang [23] observed for a channel flow problem with a smaller order ($< 5 \times 10^{-4}$). Furthermore, the coefficient matrix became ill-conditioned as the discretization is refined. That type of computational difficulty is not unusual when large fully populated and nonsymmetric matrices are involved.

In all the above biharmonic viscous systems at least one of the two boundaries (inlet or outlet of the domain of interest) was extended to infinity. Therefore, the boundary conditions at infinity basically come from Poiseuille flow. In our problem, however, the domain of interest (liquid domain or film) is completely bounded as shown in Fig. 5.1. Thus, the flow conditions at the inlet or outlet (top or bottom porous rocks) are not known accurately. As a general thought, the flow equations in the porous matrix and the liquid film must be solved simultaneously using compatibility conditions at the contact surfaces. Yet, a solution is sought for the *isolated* liquid film. The significance of the boundary conditions in terms of BEM variables will be stressed shortly.

In order to solve the biharmonic equation, Eq. 5.27, for the stream function, ψ , in a domain, Ω , enclosed by a boundary, $\partial\Omega$, we split the biharmonic equation into its coupled form by introducing an intermediate dependent variable, ω , known as vorticity. This variable can also be related to other flow variables using standard flow equations (see Robertson [26], Milne-Thompson [24], White [28]). With vorticity variable, ω , the coupled form of biharmonic equation, Eq. 5.27, is

$$\nabla^2\psi = \omega , \quad (5.28)$$

and

$$\nabla^2\omega = 0 . \quad (5.29)$$

The standard solution methodology for Eqs. 5.28 and 5.29 is mainly adopted from Fairweather et al. [14] and Kelmanson [20]. Eq. 5.28 is Poisson equation, and Eq. 5.29 is Laplace equation. The fundamental solution (Green's function) of Laplace equation is well known through potential theory applications. The fundamental solutions of both equations, Eqs. 5.28 and 5.29, come from the solution of the following equations.

$$\nabla^2 G_1(p, q) = \delta(|p - q|) \quad (5.30)$$

and

$$\nabla^4 G_2(p, q) = \delta(|p - q|) \quad (5.31)$$

where $|p - q|$ is the distance between points $p \in \Omega \cup \partial\Omega$ and $q \in \partial\Omega$ defined as

$$|p - q| = r = \sqrt{(x - x_0)^2 + (y - y_0)^2} \quad (5.32)$$

and δ is Dirac delta function. The solutions of Eqs. 5.30 and 5.31; G_1 and G_2 , are the responses to a source of strength unity (denoted with delta function). The fundamental solutions in two-dimension are [14]

$$G_1(p, q) = \ln r \quad (5.33)$$

and

$$G_2(p, q) = r^2 (\ln r - 1) \quad (5.34)$$

Using Green's second identity for functions, v and w [10],

$$\begin{aligned} \eta(p)\psi(p) = & \int_{\partial\Omega} \left(\psi(q) \frac{\partial G_1}{\partial n} - \frac{\partial \psi(q)}{\partial n} G_1 \right) dS(q) + \\ & \frac{1}{4} \int_{\partial\Omega} \left(\omega(q) \frac{\partial G_2}{\partial n} - \frac{\partial \omega(q)}{\partial n} G_2 \right) dS(q) \end{aligned} \quad (5.35)$$

and

$$\eta(p)\omega(p) = \int_{\partial\Omega} \left(\omega(q) \frac{\partial G_1}{\partial n} - \frac{\partial \omega(q)}{\partial n} G_1 \right) dS(q) \quad (5.36)$$

where, $\eta(p)$ is a factor related to the location of p with respect to boundary, $\partial\Omega$

$$\eta(p) = \begin{cases} 2\pi, & p \in \Omega \\ \theta, & p \in \partial\Omega \\ 0, & p \notin \Omega \cap \partial\Omega \end{cases} \quad (5.37)$$

In Eqs. 5.35 and 5.36, $dS(q)$ denotes the differential increment of $\partial\Omega$ at q . Parameter θ is the internal angle at point p between the tangents to $\partial\Omega$ on either sides of p [19, 18]. For constant elements, the boundary is always "smooth", hence θ is identically equal to π [6]. In general, it may be impossible to solve the boundary integral equations, Eqs. 5.35 and 5.36, analytically, and therefore, some form of numerical approximation is necessary. The boundary, $\partial\Omega$, is divided into N straight-line segments, $\partial\Omega_j$, where $j = 1, \dots, N$

(Fig. 5.2). Over each interval $\partial\Omega_j$, ψ , $\frac{\partial\psi}{\partial n}$, ω and $\frac{\partial\omega}{\partial n}$ in the integral equations, Eqs. 5.35 and 5.36, are represented by piecewise-constant discrete functions, ψ_j , $\left(\frac{\partial\psi}{\partial n}\right)_j$, ω_j and $\left(\frac{\partial\omega}{\partial n}\right)_j$. In other words, we use constant elements [6] where ψ_j , $\left(\frac{\partial\psi}{\partial n}\right)_j$, ω_j and $\left(\frac{\partial\omega}{\partial n}\right)_j$ are constant (over each interval or boundary element, $\partial\Omega_j$ and have a constant value equal to the value at the central node). Hence, Eqs. 5.35 and 5.36 as summation of discrete integrals at the midpoint of each boundary element, $p = q_i$ yield

$$\begin{aligned} \theta_i \psi_i = & \sum_{j=1}^N \left[\psi_j \int_{\partial\Omega_j} \frac{\partial G_1}{\partial n} dS(q) - \left(\frac{\partial\psi}{\partial n} \right)_j \int_{\partial\Omega_j} G_1 dS(q) \right. \\ & \left. + \frac{1}{4} \omega_j \int_{\partial\Omega_j} \frac{\partial G_2}{\partial n} dS(q) - \frac{1}{4} \left(\frac{\partial\omega}{\partial n} \right)_j \int_{\partial\Omega_j} G_2 dS(q) \right] \quad i = 1, \dots, N. \end{aligned} \quad (5.38)$$

and

$$\theta \omega_i = \sum_{j=1}^N \left[\omega_j \int_{\partial\Omega_j} \frac{\partial G_1}{\partial n} dS(q) - \left(\frac{\partial\omega}{\partial n} \right)_j \int_{\partial\Omega_j} G_1 dS(q) \right] \quad i = 1, \dots, N \quad (5.39)$$

The choice of constant elements is justified by the fact that this method is easy to implement, and flow singularities due to sharp corners in the flow field can be avoided. Eqs. 5.38 and 5.39 can be applied to N nodes for $i = 1, \dots, N$. This results in $2N$ equations and $4N$ unknowns. Therefore, on every segment of the boundary, $\partial\Omega_j$, at least two boundary conditions must be specified. The boundary conditions must be expressed in terms of BEM unknowns, ψ_j , $\left(\frac{\partial\psi}{\partial n}\right)_j$, ω_j and $\left(\frac{\partial\omega}{\partial n}\right)_j$. In order to solve the equation system defined by Eqs. 5.38 and 5.39, the integrals involving Green's functions and their normal derivatives must be evaluated. Those integrals (weight factors) are performed analytically using local coordinate system (Fig. B-1). The details of the calculations are given in Appendix C. As described in this Appendix, Eqs. 5.38 and 5.39 become

$$\sum_{j=1}^N \left[\psi_j (W_1^{ij} - \delta_{ij}\theta) + \left(\frac{\partial\psi}{\partial n} \right)_j W_2^{ij} + \omega_j W_3^{ij} + \left(\frac{\partial\omega}{\partial n} \right)_j W_4^{ij} \right] = 0 \quad i = 1, \dots, N \quad (5.40)$$

and

$$\sum_{j=1}^N \left[\omega_j (W_5^{ij} - \delta_{ij}\theta) + \left(\frac{\partial\omega}{\partial n} \right)_j W_6^{ij} \right] = 0 \quad i = 1, \dots, N \quad (5.41)$$

where δ_{ij} is Kronocker delta. Using two boundary conditions per segment of boundary, Eqs. 5.39 and 5.40 constitute a well determined $2N \times 2N$ equation system. By inspection of equation system defined by Eqs. 5.39 and 5.40, the coefficient matrix is a full matrix unless special boundary conditions and/or geometries are imposed.

So far, we have just tried to write down the boundary conditions without constraining them to a specific form. However, we must specify or transform the boundary conditions in terms of BEM variables. The velocity vector in terms of local normal and tangential coordinates is

$$\vec{V} = u\vec{t} + v\vec{n} \quad (5.42)$$

where u and v are defined as

$$u = \frac{\partial \psi}{\partial n} \quad (5.43)$$

$$v = - \frac{\partial \psi}{\partial t}$$

The boundary conditions in terms of BEM variables are:

Solid Wall (AD) - The normal flow condition imposes the following

$$\text{B.C.1: } \frac{\partial \psi}{\partial t} = 0, \quad (5.44)$$

and the tangential or no-slip condition is

$$\text{B.C.2: } \frac{\partial \psi}{\partial n} = 0. \quad (5.45)$$

Alternatively,

$$\text{B.C.1: } \psi = \text{constant} . \quad (5.46)$$

The boundary condition defined by Eq. 5.44 is not in terms of BEM variables. Hence, the conditions described by Eqs. 5.45 and 5.46 will be used.

Top and Bottom Porous Rocks (AB and CD) - Distributed flux along the porous rock is

$$\text{B.C.3 - 4: } \psi = f(x) . \quad (5.47)$$

Eq. 5.47 must also satisfy Eqs. 5.46 and 5.7. Irrotational nature of the porous media flow is characterized by

$$\text{B.C.5-6: } \omega = 0 \quad (5.48)$$

The cumulative flow over the bottom (or top) block, $\text{AB}(y = 0)$, is

$$Q = - \int_0^{x_b} v \, dx = \int_0^{x_b} \frac{\partial \psi}{\partial t} dx \quad (5.49)$$

Since we align the porous boundaries with the x -axis (Fig. 5.1)

$$Q = \int_0^{x_b} \frac{\partial \psi}{\partial x} dx = [\psi|_{x=x_b} - \psi|_{x=0}] \quad (5.50)$$

For a linear flux distribution, combination of Eq. 5.47 and Eq. 5.50 yields

$$\psi = \psi|_{x=0} + \frac{Q}{x_b} x \quad (5.51)$$

The total flux condition is the same for any cross-section of the liquid film, because the flow is considered to be incompressible. Therefore, the flux condition can be written not only for top and bottom boundaries, it can also be written for any cross-section of the liquid film. Furthermore, there seems to be too many boundary conditions over the porous rock boundaries. However, the flux condition is not a boundary condition *over* the top or bottom boundaries. Instead, it describes the stream function difference between two *points*, no-slip wall and free surface BC. In other words, if the value of the stream function along the no-slip wall is taken to be a certain value, say zero, according to Eq. 5.46 the value of the stream function at the other side, free surface, can be determined. The main point here is that the difference of stream functions is important not its point value.

Free Surface (BC) - The kinematic condition basically requires the free surface to be a streamline.

$$\vec{V} \cdot \vec{n} = - \frac{\partial \psi}{\partial t} = 0 \quad (5.52)$$

This implies that

$$\psi = \psi_f = \text{constant} = \psi|_{x=x_b} \quad (5.53)$$

on the free surface. This condition rigorously restates the kinematic condition for the free surface where stream function, $\psi = \psi_0$, is constant. The tangential force balance is somewhat more complex. The tangential force balance is

$$\vec{t} \cdot \vec{e} \cdot \vec{n} = \frac{\partial u}{\partial n} + \frac{\partial v}{\partial t} = 0 \quad (5.54)$$

Substitution of Eq. 5.43 into Eq. 5.54 yields

$$\frac{\partial^2 \psi}{\partial n^2} - \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (5.55)$$

The form of Eq. 5.55 is not useful as it is not in terms of BEM variables. Therefore, further manipulations are needed to extract a usable form. Eq. 5.28 can be represented in terms of local coordinates. The $(\vec{t}, \vec{n})$ axes form a right-handed coordinate system, $\nabla_{x,y}^2 = \nabla_{t,n}^2$ [20]. Accordingly, Eq. 5.28 can be written as

$$\frac{\partial^2 \psi}{\partial n^2} + \frac{\partial^2 \psi}{\partial t^2} = \omega \quad (5.56)$$

$\frac{\partial^2 \psi}{\partial n^2}$ can be eliminated using Eqs. 5.55 and 5.56. Thus,

$$\omega = 2 \frac{\partial^2 \psi}{\partial n^2} = 2 \frac{\partial^2 \psi}{\partial t^2} \quad (5.57)$$

Furthermore, Coyne and Elrod [11] showed that

$$\left. \frac{\partial}{\partial t} \right|_{\partial \Omega} = \frac{d}{dS} = \cos \phi \frac{d}{dx} \quad (5.58)$$

$$\left. \frac{\partial^2}{\partial t^2} \right|_{\partial \Omega} = \frac{d^2}{dS^2} - \kappa \left. \frac{\partial}{\partial n} \right|_{\partial \Omega}$$

Substitution of the expression for the second order tangential derivative from Eq. 5.58 into Eq. 5.57 yields

$$\omega = -2\kappa \frac{\partial \psi}{\partial n} \quad (5.59)$$

since $\frac{d^2 \psi}{dS^2} = 0$ along a streamline. This is basically the result of kinematic condition stating that stream function, ψ , is constant along a streamline. Furthermore, its directional

derivatives along the same streamline are zero. Eq. 5.59 restates the tangential force balance in terms of BEM variables.

Normal Stress Balance

Next, the normal stress balance along the free surface, Eq. 5.21, needs to be put in a usable form. Gas phase pressure, P_g , is assumed to be constant and it can be taken as zero. This is basically due to zero shear stress assumption in the gas phase [4]. We can also drop the subscript l for the liquid pressure. Substitution of the expression for the rate of strain tensor into Eq. 5.21 yields

$$-P + 2\mu \frac{\partial v}{\partial n} = -\sigma \kappa \quad (5.60)$$

$\frac{\partial v}{\partial n}$ can be calculated from the continuity equation, Eq. 5.12, in local coordinates. Therefore,

$$\frac{\partial v}{\partial n} = -\frac{\partial u}{\partial t} \quad (5.61)$$

Substitution of $\frac{\partial v}{\partial n}$ from Eq. 5.61 into Eq. 5.60 results in

$$-P - 2\mu \frac{\partial u}{\partial t} = -\sigma \kappa \quad (5.62)$$

Combining Eqs. 5.43 and 5.62

$$P + 2\mu \frac{\partial^2 \psi}{\partial t \partial n} = \sigma \kappa \quad (5.63)$$

Pressure, P , is not a BEM variable, and hence, it should be eliminated for the sake of simplicity. Differentiation of both sides of Eq. 5.63 in tangential direction yields

$$\frac{\partial P}{\partial t} + 2\mu \frac{\partial^3 \psi}{\partial t^2 \partial n} = \sigma \frac{\partial \kappa}{\partial t} \quad (5.64)$$

Tangential derivative of pressure term in Eq. 5.64 can be obtained from Stokes equation, Eq. 5.25, coupled with Eq. 5.28

$$\frac{\partial P}{\partial t} = \mu \frac{\partial}{\partial n} (\nabla^2 \psi) = \mu \frac{\partial \omega}{\partial n} \quad (5.65)$$

Thus, Eq. 5.64 becomes

$$\mu \frac{\partial \omega}{\partial n} + 2\mu \frac{\partial^2}{\partial t^2} \left(\frac{\partial \psi}{\partial n} \right) = \sigma \frac{\partial \kappa}{\partial t} \quad (5.66)$$

Eq. 5.57 combined with Eq. 5.58 suggests that

$$\frac{\partial^2}{\partial t^2} \left(\frac{\partial \psi}{\partial n} \right) = \frac{d^2}{dS^2} \left(\frac{\partial \psi}{\partial n} \right) - \frac{1}{2} \kappa \omega \quad (5.67)$$

Substitution of Eq. 5.67 into Eq. 5.66 yields

$$\mu \frac{\partial \omega}{\partial n} + 2\mu \left[\frac{d^2}{dS^2} \left(\frac{\partial \psi}{\partial n} \right) - \frac{1}{2} \kappa \omega \right] = \sigma \frac{\partial \kappa}{\partial t} \quad (5.68)$$

Finally, the normal force balance becomes,

$$\frac{\partial \kappa}{\partial t} = \frac{\mu}{\sigma} \left[\frac{\partial \omega}{\partial n} - \kappa \omega + 2 \frac{d^2}{dS^2} \left(\frac{\partial \psi}{\partial n} \right) \right] \quad (5.69)$$

The end points of the free surface are defined by the wetting angle condition (see Fig. 5.1).

$$\begin{aligned} \phi|_B &= \pi - \theta_w \\ \phi|_C &= \theta_w \end{aligned} \quad (5.70)$$

$\frac{\partial \kappa}{\partial t}$ and $\frac{d^2}{dS^2} \left(\frac{\partial \psi}{\partial n} \right)$ could be obtained numerically from the BEM variables.

On each element there are two independent boundary conditions and two coupled equations, Eqs. 5.40 and 5.41, with two remaining unknown quantities available. However, there are three independent boundary conditions along the free surface. One boundary condition along the free surface is used to determine the unknown location of the free surface. This results in an unconventional nonlinear problem such that flow variables occur as explicit terms in the matrix equations (Eqs. 5.41 and 5.42). However, the linear-looking flow variables themselves are function of the unknown location of the free surface. Hence, the shape of the free surface is determined iteratively.

In order to solve a well-determined system of equations, one of the boundary conditions along the free surface is left during the iterations and the position of the free surface is fixed over each iteration level. We choose to drop the normal stress balance, Eq. 5.69, because the second order arc-length derivative of the normal derivative of stream function,

$\frac{d^2}{dS^2} \left(\frac{\partial \psi}{\partial n} \right)$, appears only in the normal stress balance equation. Although the selection of the iteration boundary condition is arbitrary, such choice simplifies the solution procedure. After each iteration, the position of the free surface is updated using normal stress balance.

For a known domain, the algebraic equations, Eqs. 5.40 and 5.41 are solved, using the boundary conditions defined by Eqs. 5.45, 5.46, 5.47, 5.48, 5.53 and 5.59. One of the constants for the stream function can be taken arbitrary, say zero on the wall (Eq. 5.46), and the other one, say for the free surface (Eq. 5.53), is obtained from the total flux condition as described in Eq. 5.50.

Updating the Shape of Free Boundary

So far, the normal stress condition is converted into a first order ordinary differential equation of curvature in terms of BEM variables. The normal stress condition, Eq. 5.69, along with the end-point boundary conditions, Eqs. 5.70, poses a two-point boundary value problem. The iterative construction of the free surface with the normal stress condition, Eq. 5.69, can be achieved by standard shooting method for two point boundary value problems for ordinary differential equations. As suggested by Lu and Chang [23] for steep profiles, the arclength is used as the independent variable in the integration of the differential equations rising from the normal stress balance.

Again, Eq. 5.69 can be integrated to obtain the shape of the free boundary after each iteration. The curvature, κ , is updated directly as a function of arclength.

$$\kappa^{m+1}(S) = \kappa^{m+1}|_{S=0} + \frac{\mu}{\sigma} \left[\int_{S=0}^S \left(\frac{\partial \omega}{\partial n} - \kappa \omega + 2 \frac{d^2}{dS^2} \left(\frac{\partial \psi}{\partial n} \right) \right) dS \right]^m \quad (5.71)$$

where m is the iteration level and $\kappa^{m+1}|_{S=0}$ may or may not be known depending on the boundary conditions given. In our formulation, $\kappa^{m+1}|_{S=0}$ is not known and it must be determined through the shooting iterations. For a specific location defined with the arclength from a reference point on the free surface, the integration in Eq. 5.71, is performed numerically using the BEM variables from the previous iteration. Then arclength, S , versus curvature, κ , can be tabulated. As can be seen from Eq. 5.22, the curvature is still equivalent to second order differentiation. Thus, we need to integrate twice. The form given in Eq. 5.22 is not particularly useful for integration using arclength as an

independent variable. Alternatively, based on the definition of curvature in terms of local tangent angle, ϕ , [27],

$$\kappa = \frac{d\phi}{dS} \quad (5.72)$$

Eq. 5.72 can be integrated to determine the pointwise description of the local tangent angle along the free surface

$$\phi^{m+1}(S) = \phi^{m+1}|_{S=0} + \int_{S=0}^S [\kappa]^m dS \quad (5.73)$$

In our formulation, $\phi^{m+1}|_{S=0}$ is known due to wetting angle conditions defined in Eqs. 5.70. Starting tangent angle, $\phi^{m+1}|_{S=0}$, can be selected from any of the conditions in Eq. 5.70 as long as it is consistent with the reference point of the arclength, $S = 0$. In the following, we choose the bottom tip of the liquid film as the reference point (point **B** in Fig 5.1). In other words,

$$\phi|_{S=0} = \phi|_B = \pi - \theta_w \quad (5.74)$$

The other end condition in Eq. 5.70, $\phi|_C = \theta_w$, will help to determine the curvature of the free surface at the starting point (reference point in the integration where $S = 0$, selected as point **B**) in the following manner. First, we guess a value for the $\kappa|_{S=0}$ to perform the first integration, Eq. 5.71, to tabulate the curvature values. Next, the local tangent angles are determined through the integration defined by Eq. 5.73. If the initial guess for the $\kappa|_{S=0}$ is not correct the tangent angle for the top tip of the free surface, point **C**, will not agree with Eq. 5.70. Then the guess for the curvature at the starting point must be updated until the wetting angle at the top tip of the free surface is honored. In other words, iterations to determine the starting curvature are proceeded until the wetting angle condition ,

$$\phi|_{S=S_C} = \phi|_C = \theta_w \quad (5.75)$$

is satisfied. The tangent angle is defined in Eq. 5.20. Since the fracture aperture is constant and aligned with the y -axis, we prefer to perform the integration for the x -coordinate (x is the dependent variable). Thus,

$$\frac{dx}{dy} = \frac{1}{\tan \phi} \quad (5.76)$$

and,

$$x^{m+1} = x^m|_{y=0} + \int_{y=0}^y \left[\frac{1}{\tan \phi} \right]^{m+1} dy . \quad (5.77)$$

x^{m+1} values yield the new x -coordinates of the free surface for a given set of y . The shooting iterations can be performed using the outcome of the two previous shooting iterations.

$$\kappa^{l+1}|_{S=0} = \kappa^l|_{S=0} + \left(\theta_w - \phi^l|_{S=S_C} \right) \frac{\kappa^l|_{S=0} - \kappa^{l-1}|_{S=0}}{\phi^l|_{S=S_C} - \phi^{l-1}|_{S=S_C}} \quad (5.78)$$

where l is level of the shooting iterations.

Two different convergence criteria are used. One is based on the change in the position of the upper tip of the free surface such that $\frac{|x_c^{m+1} - x_c^m|}{x_c^m} < 10^{-5}$. The other one is based on the total changes of the position of the free surface such that the root-mean-square of change of the free surface position is less than 2×10^{-4} .

Symmetry Boundary Conditions

Due to symmetry assumption we impose alternative boundary conditions for the solution in the half domain. Thus, boundary conditions are imposed along the symmetry axis, $C'D'$, instead of top or bottom of the liquid bridge.

$$\frac{\partial \psi}{\partial n} = 0 \quad (5.79)$$

$$\frac{\partial \omega}{\partial n} = 0$$

Selecting the bottom half portion of the liquid domain for the calculations is more appropriate for clarity, since updating the free surface is explained from bottom to the top of the full liquid bridge. The only difference between the full domain and the half domain calculations is the wetting condition at point C, such that the angle between the liquid bridge and the symmetry axis, $C'D'$, is $\frac{\pi}{2}$. Therefore, the film must be connected to the top boundary $C'D'$, with $\theta = \frac{\pi}{2}$. Accordingly, the curvature at the starting point, $\kappa^{m+1}|_{S=0}$, can be calculated.

Pressure Calculation Along the Free Surface

The pressure along the free boundary can be calculated by integrating the Stokes equation, Eq. 5.65, along the free surface. Expressing Eq. 5.65 in terms of arclength,

$$\frac{\partial P}{\partial S} = \mu \frac{\partial \omega}{\partial n} \quad (5.80)$$

Integration of Eq. 5.80 along the free surface yields

$$P(S) = P(0) + \mu \int_0^S \frac{\partial \omega}{\partial n} dS \quad (5.81)$$

The only unknown in Eq. 5.81 is $P(0)$ which is the pressure at the contact point B (see Fig. 5.1). The rest of the pressure values in the solution domain can be obtained through integration in two spatial dimensions. Here, we use the definition of capillary pressure to estimate the pressure at the contact point B. At this point,

$$P_c = P_g - P_l = -\sigma\kappa \quad (5.82)$$

The gas phase pressure, P_g , is taken to be zero. Thus, the liquid pressure (note that we have already dropped the subscript l) at the contact point is determined by

$$P(0) = \sigma\kappa|_{S=0} \quad (5.83)$$

Thus,

$$P(S) = \sigma\kappa|_{S=0} + \mu \int_0^S \frac{\partial \omega}{\partial n} dS, \quad (5.84)$$

and the integration is performed numerically. Once the pressure along the free boundary is known, the pressures in the solution domain can be obtained through integrations similar to Eq. 5.84. The values of $\frac{\partial \omega}{\partial x}$ and $\frac{\partial \omega}{\partial y}$ in the solution domain should be known with acceptable frequency for the accuracy of numerical integration. The expressions for $\frac{\partial \omega}{\partial x}$ and $\frac{\partial \omega}{\partial y}$ values are provided in Appendix D along with the components of velocity.

Calculation of the liquid transmissibility requires calculation of the average pressure drop across the liquid bridge. The easiest way to perform pressure calculation within the domain is to select a particular y -coordinate. Then, perform the integration away from the free boundary in the x -direction along the path where y -coordinate is constant (a

straight line emanating from the free boundary parallel to x -axis). Repetition of the same procedure for different y -coordinates yields the pressure map of the domain. Only $\frac{\partial \omega}{\partial x}$ is needed for this procedure.

Liquid Transmissibility

The solution of steady creeping flow equations yields the pressure map for a given flow rate, Q . From the pressure map and flow rate the validity of Darcy's law within the film can be investigated. Darcy velocity in y -direction is given by

$$v_y = -\frac{k}{\mu} \frac{\partial P}{\partial y} \quad (5.85)$$

Thus,

$$\frac{Q}{A(y)} = -\frac{k}{\mu} \frac{\partial \bar{P}}{\partial y} \quad (5.86)$$

where, $A(y) = h(y)d$ is the liquid bridge cross section perpendicular to xy -plane per unit depth ($d = 1$) and $\bar{P}$ is the average pressure over $h(y)$. Integration of Eq. 5.86 yields

$$\int_{y=0}^{y_d'} \frac{dy}{h(y)} = -\frac{k}{Q\mu} \int_{y=0}^{y_d'} d\bar{P} \quad (5.87)$$

where y_d' is the y coordinate of the point D' in Fig. 5.1. Permeability, k is calculated from

$$k = \frac{\mathcal{I}\mu Q}{\Delta \bar{P}} \quad (5.88)$$

where $\mathcal{I}$ is the shape factor defined by

$$\mathcal{I} = \int_{y=0}^{y_d'} \frac{dy}{h(y)} \quad (5.89)$$

and

$$\Delta \bar{P} = -(\bar{P}|_{y=0} - \bar{P}|_{y=y_d'}) \quad (5.90)$$

Note that Eq. 5.88 gives directly the liquid permeability of the liquid bridge in the fracture. This equation could be further manipulated to obtain the equivalent Darcy permeability (or transmissibility) of the liquid film.

SAMPLE SOLUTIONS

The solutions are obtained using $12 \times 36 \times 12 \times 12$ mesh, for the porous rock interface, free

surface, symmetry boundary and the solid wall, respectively. This results in 144×144 fully populated coefficient matrix. The arising linear simultaneous equation system is solved by *LU* decomposition. Two different sets of finer meshes, $16 \times 48 \times 16 \times 24$ and $32 \times 96 \times 32 \times 48$, are also used to investigate the effect of mesh refinement. While increasing the accuracy of the results, finer meshes are reported to cause more ill-conditioned coefficient matrices as reported by others [8, 22]. Thus, convergence becomes slower with large N [18]. The comparison of the minimum distance values, x_c , and the curvature at point B (see Fig. 5.1), $\kappa(0)$, for different mesh sizes are shown in Table 5.1. The resulting profiles for those three different mesh sizes are also compared in Fig. 5.3.

At each iteration step, the symmetry boundary and the free boundary are updated. Furthermore, the free boundary is smoothed using cubic splines. Then, both updated boundaries are discretized keeping the element sizes uniform to remedy potential instabilities. We observed, that the solutions were extremely sensitive to the initial guess of the free boundary at high flux values. In order to resolve this difficulty, the calculation is initiated with a parabolic initial estimate for the free surface at a low flux value (or low capillary number). Then, we proceed using the results from the low-flux run as the initial estimate for the high-flux cases.

The code is found to be sensitive to the method of calculation of local discretized curvature. The calculation of local curvature is first performed using cubic splines, since cubic splines can yield continuous first and second derivatives at the data nodes. However, direct determination of the curvature through the use of splines is found to be inaccurate for our purpose. Instead, we have used another direct method exploiting the discrete nature of the problem. Hence, a triangle using three points on the free surface in the vicinity of an element is used. Inverse of the radius of the circle passing from these points is the local curvature of this element.

To investigate the effects of various parameters on the flow along the spacer between the matrix blocks, numerous cases are considered (Table 5.2). Each case in Table 5.2, is thoroughly examined for a wide range of flux, $Q = \psi_f$. For each case there seems to

be a limiting flux, Q_{lim} , which can form the film through the given contact point, x_b . The flux values above the limiting flux, Q_{lim} , cannot be forced through the aperture for a given contact point, x_b . Therefore, the flux must be either reduced or for the same flux the location of the contact point, x_b , must be changed to a larger distance. Note that since we have isolated the film flow in the fracture from the matrix flow, the position x_b is fixed. In a coupled matrix/fracture flow x_b has to be determined. This would be a very complicated and challenging task.

The sensitivity of the gas liquid interface to Q for Case 1 is shown in Fig. 5.4. As the flux increases the top boundary becomes wider (C' moves away from the solid wall) to permit a large amount of liquid to pass across the bridge. The corresponding curvature values become smaller in magnitude (the free-end of the surface becomes flatter).

Similar analysis is also carried out for the same system using a low surface tension of $\sigma = 0.3$ dynes/cm (Case 2) as shown in Table 5.2. The shape of the free boundary for Case 2 and the comparison of average pressure drop versus flux for Case 1 and Case 2 are shown in Figs. 5.5 and 5.6, respectively. The profiles of the free surface in Figs. 5.4 and 5.5 show similar behavior. For example, the profile for $Q = 1 \times 10^{-2}$ cc/cm-sec in Fig. 5.4 is identical to the profile for $Q = 1 \times 10^{-3}$ cc/cm-sec in Fig. 5.5. This means that for a given contact point, film formed for a low surface tension fluid can carry less fluid than a fluid with a higher surface tension. Hence, the contact point, B , moves away from the solid wall to carry excess amount of fluid. At the limit, this argument is compatible with the miscible systems where the interface disappears (here, the interface goes to infinity where the fracture will be filled by only liquid).

For relatively low flow rates, the pressure drop is a linear function of Q as shown in Fig. 5.6. This clearly indicates the validity of the Darcy law as Eq. 5.88 indicates. The permeability value can be calculated from the intercept of the straight line drawn provided that the shape factor $\mathcal{I}$ is known. For relatively high flow rates, pressure drop/ Q relationship deviates from the straight line (which corresponds to Darcy's law). The calculated points in Fig. 5.6 begin to deviate from the straight line around a certain Q

where the liquid film begins to separate from the static film position. Here, static film position is defined as the position when the fluid is not flowing or flowing with a very small rate. The static profile is simply a circle with a radius $\frac{l}{\cos \theta_w}$. The pressure starts to deviate from the broken line about $Q = 5 \times 10^{-3} \frac{\text{cm}^3}{\text{cm-s}}$ for $\sigma = 3 \frac{\text{dynes}}{\text{cm}}$ and, $Q = 5 \times 10^{-4} \frac{\text{cm}^3}{\text{cm-s}}$ for $\sigma = 0.3 \frac{\text{dynes}}{\text{cm}}$. At higher rates, the pressure drop is less than what the Darcy law predicts through the straight line drawn in Fig. 5.6. Also, deviation increases when flux increases. One of the reasons of lower pressure drop is the changes in the flow area due to change in the shape of the free surface.

Pressure drop across different aperture lengths, surface tension, liquid viscosity and characteristic aspect ratios, $R = \frac{|AB|}{|AD|}$, is shown in Fig. 5.7. The following form of Eq. 5.88 is found to scale all the linear portions of the curves to one straight line.

$$\log \Delta \bar{P} = \log \frac{\mathcal{I} l^2 R^2}{k} + \log \frac{Q \mu}{l^2 R^2} \quad (5.91)$$

where characteristic aspect ratio R is defined by

$$R = \frac{|AB|}{|AD|} \quad (5.92)$$

Eq. 5.91 shows a log-log unit slope straight line relationship between $\Delta \bar{P}$ and $\frac{Q \mu}{l^2 R^2}$ where $\frac{\mathcal{I} l^2 R^2}{k}$ term is the intercept. Therefore, permeability, k , can be obtained when the intercept is known ($4.5 \frac{\text{dynes}}{\text{cm}^2}$ or 4.45×10^{-6} atm). Thus,

$$\frac{\mathcal{I} l^2 R^2}{k} = 4.45 \times 10^{-6} \quad (5.93)$$

Rearrangement of Eq. 5.93 yields the permeability in Darcies,

$$k = 2.25 \times 10^5 \mathcal{I} l^2 R^2 \quad (5.94)$$

where $\mathcal{I}$ is calculated to be 1.60 for $R = 1$ for the linear part of the pressure drop curve. In Eq. 5.94, l is in cm. Behavior of $\mathcal{I}$ when pressure drop curve is linear for different R values is shown in Fig. 5.8. Shape factor, $\mathcal{I}$, shows linear behavior for large R as Eq. 5.89 predicts. When R is sufficiently large $\mathcal{I} \rightarrow \frac{1}{R}$ as broken line in Fig. 5.8 suggests.

Based on Eq. 5.91, pressure drop responses are plotted in Fig. 5.9 for various values of σ , μ and l ($R = \frac{|AB|}{|AD|}$ is held equal to 1). In principle, Fig. 5.9 is similar to Fig. 5.6, except that the flux, Q , is scaled with $\frac{\mu}{l^2}$. Again, pressure data in Fig. 5.9 indicate behavior similar to Fig. 5.6. The pressure drop, starts deviating from linearity at high $\frac{Q\mu}{l^2}$ values.

One important aspect of Eq. 5.94 is that the fracture permeability for two phase flow perpendicular to the fracture planes is a function of two dimensions, l and R , rather than aperture, l , alone as Poiseuille flow indicates by [1]

$$k_p = 84.4 \times 10^5 (2l)^2 \quad (5.95)$$

Poiseuille flow assumes fully saturated flow parallel to the fracture planes, and therefore, the permeability is only a function of the fracture aperture given by Eq. 5.95. Another important result deduced from Fig. 5.7 is that the pressure drop estimated from the Darcy law is the upper limit of the fracture pressure drop calculated. As is shown in Fig. 5.7, none of the data points are located above the straight line drawn. For $R = 1$, permeability values from Eq. 5.95 are two order of magnitudes higher than values from Eq. 5.94.

For the sake of completeness, the shape of the free boundary is shown in Fig. 5.10 for $R = 4$ (Case 9). The shape of the free boundary does not change appreciably relative to the distance to the solid wall for low and medium flux values. This is due to a thick liquid film. But, point C' starts moving towards the wall at relatively high fluxes as opposed to the cases with $R = 1$.

Fig. 5.11 shows the capillary pressure variation along the free boundary for Case 4. The difference between the dynamic capillary pressure and the static capillary pressure (represented with the broken line in Fig. 5.11) increases as flux, Q , increases. Note that the capillary pressure at point B (see Fig. 5.1) is $15,000 \frac{\text{dynes}}{\text{cm}^2}$, and at point C (not shown in Fig. 5.11) is $-5,000 \frac{\text{dynes}}{\text{cm}^2}$ for $Q = 10^{-3} \frac{\text{cm}^3}{\text{cm} \cdot \text{s}}$. The driving force is the liquid pressure difference between the DC and AB end planes of the liquid bridge (about 0.30 psi or $20,000 \frac{\text{dynes}}{\text{cm}^2}$). Substantial pressure drop across a 10 micron fracture due to flow can, therefore, realize.

Fig. 5.12 illustrates the location of point C' versus capillary number, $\frac{\mu Q}{\sigma l}$. As shown

in Figs. 5.4 and 5.5, the location of x_c does not change at very low capillary numbers (also implies low flux, Q). At higher capillary numbers, $\frac{x_c}{l}$ starts deviating from the base value of 0.16 then it starts stabilizing around 0.35 at the maximum flux at which the film cannot carry more fluid for $R = 1$. Dimensionless parameter $\frac{x_c}{l}$ can be used to calculate pressure drop for high capillary numbers using Fig. 5.13.

Another interesting plot is the dimensionless curvature at the contact point (i.e., point B in Fig. 5.1) versus capillary number (Fig. 5.14) similar to what Lu et al.[23] presented for the bubble penetration into a channel problem. The dimensionless curvature is defined by

$$\kappa_D(0) = \frac{\kappa(0)}{\frac{\cos \theta_w}{l}} \quad (5.96)$$

The curvature at the contact point is an important parameter to correlate the pressure drop at high capillary numbers (Fig. 5.15) where pressure drop deviates from linearity (Fig. 5.9). Fig. 5.14 shows linear relationship between $1 - \kappa_D(0)$ and capillary number at high capillary numbers where $\kappa_D(0)$ can be determined for a given capillary number. Later, the dimensionless curvature can be used to calculate the pressure drop across the half aperture, l , using Fig. 5.15.

Figs. 5.12, 5.13, 5.14 and 5.15 are important to interpret measurable quantities such as x_c and $\kappa(0)$. Once we know one of those measurable quantities it is possible to calculate the pressure drop and thus the permeability values at high capillary numbers.

DISCUSSION AND CONCLUSIONS

The BEM solution of the film flow in fractured media has been presented for a linear flux distribution at the matrix boundary. The main purpose of this study was to, 1) determine the pressure drop across the liquid film, and 2) obtain the shape of the liquid film. The pressure drop is correlated using $\frac{\mu Q}{l^2 R^2}$ which leads to an expression of equivalent Darcy velocity for the liquid film. Different characteristic aspect ratios, R , are investigated. Small R is shown to have high pressure drops. This is expected, since flow has to pass through a narrow bridge. Thus, the equivalent Darcy permeability is lower for the same aperture as indicated by Eq. 5.94. A similar observation for the pressure drop was also

quoted by Arbabi, Firoozabadi and Barkve [2]. The results are also in agreement with the physical intuition that capillary pressure decreases when the local curvature decreases (magnitude of the local curvature decreases towards the point on the symmetry boundary, x_c').

When $R \rightarrow \infty$, $\mathcal{I} \rightarrow \frac{1}{R}$. Thus, $k \rightarrow \infty$ with R rather than R^2 as Poiseuille law predicts (the aperture length will be R for the Poiseuille law for $R \rightarrow \infty$, when flow is perpendicular to fracture planes).

The nature of BEM formulation requires that the flux to be specified. Then the pressure is calculated. BEM formulation entails appropriate boundary conditions in terms of BEM variables. However, we can interpret the results in such a way that a defined pressure drop can yield the matching rate for a given system. This may be achieved using the graphical solutions presented in this report. It is also possible to obtain flux or pressure drop at high capillary numbers using quantities, such as $\frac{x_c'}{l}$ and $\kappa_D(0)$.

The following conclusions are drawn from this study.

1. The liquid flow is shown to satisfy Darcy's law at relatively low capillary numbers. Thus, a correlation for equivalent Darcy permeability has been developed. At higher capillary numbers, the permeability must be calculated using the graphical correlations for the pressure drop.
2. The shape of the interface is found to be circular at low capillary numbers, whereas it starts to deviate from that base shape at higher capillary numbers (or rates for a given system). Thus minimum film thickness (the length of symmetry boundary) increases with the increasing rate for narrow films ($R = 1$).
3. Fracture permeability goes to infinity with R rather than R^2 as Poiseuille law predicts.
4. The fracture liquid permeability in our analysis is the liquid transmissibility. This means there is no need for fracture liquid relative permeability. In the second part of this study, we will see how one can use the fracture liquid transmissibility.

Nomenclature

$A(y)$	=	cross-section area in Darcy law
Ca	=	capillary number, $Ca = \frac{\mu Q}{\sigma l}$
d	=	depth perpendicular to xy -plane
$\bar{e}$	=	rate of strain tensor
$f(x)$	=	distributed flux along porous rock boundaries
F	=	external force exerted on fluid
$\bar{g}$	=	gravitational force
G_1	=	Green's function of Laplace equation
G_2	=	Green's function of Poisson equation
$h(y)$	=	a smooth function representing the shape of the free surface
$\mathcal{I}$	=	shape factor, integral defined by Eq. 5.89
k	=	Darcy permeability
k_p	=	permeability from Poiseuille law
l	=	half aperture length
$\bar{n}$	=	unit normal vector
N	=	total number of boundary elements
N_{Re}	=	Reynolds number, $\frac{U_P}{\mu}$
P	=	liquid phase pressure
$\bar{P}$	=	average liquid phase pressure over x -direction
P_c	=	capillary pressure
P_g	=	gas phase pressure
P_l	=	liquid phase pressure
$\Delta \bar{P}$	=	average pressure drop across the half liquid bridge
q_y	=	total rate in Darcy law in the direction of y -axis
Q	=	total flux
r	=	distance between two points defined by Eq. 5.32
R	=	characteristic aspect ratio defined by Eq. 5.92

R_1	=	first principal radius of curvature
R_2	=	second principal radius of curvature
S	=	arclength or direction along free boundary
t	=	time
$\vec{t}$	=	unit tangent vector
u	=	component of velocity vector in the direction of x -axis
U	=	characteristic flow velocity
v	=	component of velocity vector in the direction of y -axis
v_y	=	Darcy velocity in the direction of y -axis
$\vec{V}$	=	velocity
w_i	=	velocity in i direction
W_k^{ij}	=	weight factors defined in Appendix C
x	=	apcissa
x_b	=	x -coordinate of point B
$x_{c'}$	=	x -coordinate of point C'
y	=	ordinate
$y_{d'}$	=	y -coordinate of point D'
z	=	axis perpendicular to xy -plane

Greek Symbols

α	=	local rotation angle, Eq. B-2
η	=	local coordinate parallel to an element, Eq. B-1
$\eta(p)$	=	local angle defined by Eq. 5.37 in BEM formulation
κ	=	curvature
$\kappa_D(0)$	=	dimensionless curvature at point B
ψ	=	stream function
ψ_f	=	stream function assigned to free boundary ($\psi_f = Q$)

ϕ	=	local tangent angle
ω	=	vorticity
Ω	=	solution domain
$\partial\Omega$	=	boundary of solution domain
ρ	=	density
σ	=	surface tension
μ	=	viscosity
θ	=	angle defined in Fig. 5.2
θ_w	=	wetting angle
ξ	=	local coordinate, Eq. B-1

Abbreviations

BEM	:	boundary element method
BIM	:	boundary integral method

References

- [1] Amyx, J.W., Bass D.M. Jr. and Whiting, R.L.: *Petroleum Reservoir Engineering, Physical Properties*, McGraw-Hill Book Co. (1960).
- [2] Arbabi, S., Firoozabadi, A. and Barkve, T.: "Fracture Liquid Transmissibility for Two-Phase Flow in Fractured Porous Media," No. 2Q.91, Reservoir Engineering Research Institute (July 1991).
- [3] Ballal, B.Y., and Rivlin, R.S.: "Flow of a Newtonian Fluid Between Eccentric Rotating Cylinders: Inertial Effects," *Arch. Rat. Mech. Analysis* (1976) 6, 237-294.

- [4] Batchelor, G.K.: *An Introduction to Fluid Mechanics*, second edition, Cambridge, Cambridge (1970).
- [5] Bear, J.: *Dynamics of Fluids in Porous Media*, reprint edition, Dover, New York (1988).
- [6] Brebbia, C.A., Telles, J.C.F. and Wrobel, L.C.: *Boundary Element Techniques, Theory and Applications in Engineering*, Springer-Verlag, New York (1984).
- [7] Bush, M.B. and Tanner, R.I.: "Numerical Solution of Viscous Flows Using Integral Equation Methods," *International Journal for Numerical Methods in Fluids* (1983) 3, 71-92.
- [8] Cai, R.Y., Zeng, Z.J. and Chen, F.: "A partitioning Solution of Non-Symmetrical Fully Populated Matrix System in the Boundary Element Method and its Subroutines," presented at the 1990 12th International Conference on Boundary Elements in Engineering, Sapporo, September.
- [9] Chandrasekhar, S.: *Hydrodynamic and Hydromagnetic Stability*, first edition, Oxford University, London (1961).
- [10] Courant, R. and Hilbert, D.: *Methods of Mathematical Physics: Vol. 2. Partial Differential Equations*, Wiley-Interscience, New York (1962).
- [11] Coyne, J.C. and Elrod, H.G. Jr.: "Conditions for the Rupture of a Lubricating Film. Part I: Theoretical Model," *Trans. ASME Journal of Lubrication Technology* (1970) 92, 451-456.
- [12] Davis, S.H.: "Contact-Line Problems in Fluid Mechanics," *Trans. ASME Journal of Applied Mechanics* (1983) 50, 977-982.
- [13] Drazin, P.G. and Reid, W.H.: *Hydrodynamic Stability*, first edition, Cambridge University, New York (1982).

- [14] Fairweather, G., Rizzo, F.J., Shippy, D.J. and Wu, Y.S.: "On the Numerical Solution of Two-Dimensional Potential Problems by an Improved Boundary Integral Equation Method," *Journal of Computational Physics* (1979) **31**, 96-112.
- [15] Firoozabadi, A. and Markeset, T.: "An Experimental Study of the Gas-Liquid Transmissibility," paper SPE 24919 presented at the 1992 67th Annual Technical Conference and Exhibition, Washington, DC, October.
- [16] Goodwin, R. and Homsy, G.M.: "Viscous Flow Down a Slope in the Vicinity of a Contact Line," *Physics of Fluids A* (April 1991) **3**, 515-528.
- [17] Happel, J. and Brenner, H.: *Low Reynolds Number Hydrodynamics*, third edition, Martinus Nijhoff, Boston (1983).
- [18] Ingham, D.B. and Kelmanson, M.A.: *Boundary Integral Equation Analyses of Singular, Potential, and Biharmonic Problems*, Brebbia, C.A. and Orszag, S.A. (ed.), Springer-Verlag, New York (1984).
- [19] Jaswon, M.A. and Symm, G.T.: *Integral Equation Methods in Potential Theory and Electrostatics*, Academic Press, London (1977).
- [20] Kelmanson, M.A.: "Boundary Integral Equation Solution of Viscous Flows With Free Surfaces," *Journal of Engineering Mathematics* (1983) **17**, 329-343.
- [21] Kikani, J.: *Application of Boundary Element Method to Streamline Generation and Pressure Transient Testing*, PhD dissertation, Stanford University (1989).
- [22] Lasseux, D., Fabrie, P. and Quintard, M.: *A Boundary Integral Method Applied to Gas-Liquid Drainage in a Capillary Cavity*, Brebbia, C.A. and Partridge, P.W. (ed.), Computational Mechanics Publications and Elsevier Applied Science, London, New York (1992).

- [23] Lu, W-Q. and Chang, H-C.: "An Extension of the Biharmonic Boundary Integral Method to Free Surface Flow in Channels," *Journal of Computational Physics* (1988) 77, 340-360.
- [24] Milne-Thompson, L.M.: *Theoretical Hydrodynamics*, fifth edition, Macmillan Co., New York (1968).
- [25] Muskat, M.: *The Flow of Homogeneous Fluids Through Porous Media*, reprint edition, IHRDC, Boston (1982).
- [26] Robertson, J.M.: *Hydrodynamics In Theory and Application*, first edition, Prentice-Hall, New Jersey (1965).
- [27] Simmons, G.F.: *Calculus with Analytical Geometry*, first edition, McGraw-Hill, New York (1985).
- [28] White, F.M.: *Viscous Fluid Flow*, second edition, McGraw-Hill, New York (1991).

Appendix A:

Scalar Forms of Flow Equations

Equations of motion, Eqs. 5.1 and 5.2, in 2-D cartesian coordinates are,

$$\frac{\partial P}{\partial x} \vec{i} + \frac{\partial P}{\partial y} \vec{j} = \mu \left[\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \vec{i} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \vec{j} \right] \quad (\text{A} - 1)$$

and the continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\text{A} - 2)$$

The boundary conditions in cartesian conditions are relatively easy for all the boundaries except the free surface. The porous rock boundaries are aligned with the direction of the x -axis and the no-slip wall is aligned with y -axis. Hence, the normal and tangential directions of porous rock boundaries are simply $\vec{j}$ and $\vec{i}$ and the reverse for the no-slip wall. The corresponding directions for the free surface is already defined by Eqs. 5.18 and 5.19 in terms of $\vec{i}$ and $\vec{j}$.

Appendix B:

Local Coordinates

The integrals of Green's functions and the integrals of normal derivatives of Green's functions can be evaluated using local coordinates. Introduction of such coordinate system simplifies analytical evaluation of the integrals substantially. The main advantage of this procedure is the freedom of the choice of two independent coordinates such that one of the axis, ξ , can be aligned with the boundary element (say j^{th} element). Therefore, there will be only one coordinate, $\xi_{j+1} < \xi < \xi_j$, variation along the element where the other, η_j , stays unchanged (Fig. B-1).

The coordinate transformation is achieved in two steps. First, the origin, O , of the original coordinate system, (x, y) , is translated to the point $O'(x_i, y_i)$ where $\eta = 0$ and $\xi = 0$. Second, the new coordinate system, (η, ξ) , is aligned with the element of interest via rotation(α is the rotation angle as shown in Fig. B-1). The resulting coordinate

transformation is

$$\begin{aligned}\xi &= (x - x_i) \cos \alpha + (y - y_i) \sin \alpha \\ \eta &= -(x - x_i) \sin \alpha + (y - y_i) \cos \alpha\end{aligned}\tag{B - 1}$$

The translation coordinates are the old coordinates, (x_i, y_i) , of the origin of the new coordinate system, O' , and the rotation angle is defined by

$$\tan \alpha = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} .\tag{B - 2}$$

Due to alignment of the element with ξ -axis, $\vec{n} \parallel \eta$ and $\frac{\partial \eta}{\partial n}$ is unity. Hence, the normal derivatives become

$$\frac{\partial}{\partial n} = \frac{\partial}{\partial \eta} .\tag{B - 3}$$

The distance, r , from the origin of the new coordinate system to a point on the element is defined by

$$\begin{aligned}r &= \sqrt{(x - x_i)^2 + (y - y_i)^2} \\ &= \sqrt{\eta^2 + \xi^2}\end{aligned}\tag{B - 4}$$

The boundary, $\partial\Omega$ or S , is traced counter clockwise, from j to $j + 1$ in Fig. B-1. However, the local coordinate system is from $j + 1$ to j . Therefore,

$$dS = -d\xi\tag{B - 5}$$

Appendix C:

Calculation of Weight Factors

The weight factors are integrals of Green's function and their derivatives over the element boundaries. First, we convert the integrands into a local coordinate system, then the integration is performed analytically. The integrals are numbered in the order that they appear in the discretized equations, Eqs. 5.38 and 5.39. The integrands will be represented

by T_k and integrals will be represented by I_k . The integrands are

$$\begin{aligned}
 T_1 &= \frac{\partial \ln r}{\partial n} = \frac{\eta}{\eta^2 + \xi^2} \\
 T_2 &= \ln r = \frac{1}{2} \ln(\eta^2 + \xi^2) \\
 T_3 &= \frac{\partial(r^2 \ln r - r^2)}{\partial n} = \eta \ln(\eta^2 + \xi^2) - \eta \\
 T_4 &= (r^2 \ln r - r^2) = (\eta^2 + \xi^2) \left(\frac{1}{2} \ln(\eta^2 + \xi^2) - 1 \right) \\
 T_5 &= T_1 \\
 T_6 &= T_2
 \end{aligned} \tag{C-1}$$

The integrals, I_k , in local coordinates are (see Appendix B)

$$I_k = \int_{\partial\Omega} T_k(x, y) dS(q) = \int_{\xi_j}^{\xi_{j+1}} \bar{T}_k(\eta_j, \xi) (-d\xi) \tag{C-2}$$

Explicitly, the integrals become

$$\begin{aligned}
 I_1 &= - \int_{\xi_j}^{\xi_{j+1}} \frac{\eta_j}{\eta_j^2 + \xi^2} d\xi \\
 I_2 &= - \frac{1}{2} \int_{\xi_j}^{\xi_{j+1}} \ln(\eta_j^2 + \xi^2) d\xi \\
 I_3 &= - \int_{\xi_j}^{\xi_{j+1}} \eta_j (\ln(\eta_j^2 + \xi^2) - 1) d\xi \\
 I_4 &= - \int_{\xi_j}^{\xi_{j+1}} (\eta_j^2 + \xi^2) \left(\frac{1}{2} \ln(\eta_j^2 + \xi^2) - 1 \right) d\xi \\
 I_5 &= I_1 \\
 I_6 &= I_2
 \end{aligned} \tag{C-3}$$

The final results are

$$\begin{aligned}
 I_1 &= \arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \\
 I_2 &= \frac{1}{2} \left(\xi_j \ln(\eta_j^2 + \xi_j^2) - 2\xi_j + 2\eta_j \arctan \frac{\xi_j}{\eta_j} \right) - \\
 &\quad \frac{1}{2} \left(\xi_{j+1} \ln(\eta_j^2 + \xi_{j+1}^2) - 2\xi_{j+1} + 2\eta_j \arctan \frac{\xi_{j+1}}{\eta_j} \right) \\
 I_3 &= \eta_j (\xi_{j+1} - \xi_j) - 2\eta_j I_2 \\
 I_4 &= \left[\frac{1}{2} \left(\eta_j^2 \xi_j + \frac{1}{3} \xi_j^3 \right) \ln(\eta_j^2 + \xi_j^2) - \frac{4}{9} \xi_j^3 - \frac{5}{3} \eta_j^2 \xi_j \right] - \\
 &\quad \left[\frac{1}{2} \left(\eta_j^2 \xi_{j+1} + \frac{1}{3} \xi_{j+1}^3 \right) \ln(\eta_j^2 + \xi_{j+1}^2) - \frac{4}{9} \xi_{j+1}^3 - \frac{5}{3} \eta_j^2 \xi_{j+1} \right] + \frac{2}{3} \eta_j^3 I_1 \\
 I_5 &= I_1 \\
 I_6 &= I_2
 \end{aligned} \tag{C-4}$$

Eqs. 5.38 and 5.39 can be further simplified.

$$\sum_{j=1}^N \left[\psi_j (W_1^{ij} - \delta_{ij}\theta) + \left(\frac{\partial \psi}{\partial n} \right)_j W_2^{ij} + \omega_j W_3^{ij} + \left(\frac{\partial \omega}{\partial n} \right)_j W_4^{ij} \right] = 0 \quad i = 1, \dots, N \quad (C-5)$$

and

$$\sum_{j=1}^N \left[\omega_j (W_5^{ij} - \delta_{ij}\theta) + \left(\frac{\partial \omega}{\partial n} \right)_j W_6^{ij} \right] = 0 \quad i = 1, \dots, N \quad (C-6)$$

where W_k^{ij} 's are

$$\begin{aligned} W_1^{ij} &= I_1 \\ W_2^{ij} &= -I_2 \\ W_3^{ij} &= \frac{1}{4} I_3 \\ W_4^{ij} &= -\frac{1}{4} I_4 \\ W_5^{ij} &= I_1 \\ W_6^{ij} &= I_2 \end{aligned} \quad (C-7)$$

Appendix D:

Calculation of Derivatives of Stream Function and Vorticity

The spatial derivatives of BEM variables are calculated based on Eqs. 5.38 and 5.39. The derivatives are necessary to calculate the components of local flow velocity and pressure.

$$\begin{aligned} u = \frac{\partial \psi}{\partial y} &= \frac{1}{2\pi} \sum_{j=1}^N \frac{\partial}{\partial y} \left[\psi_j \int_{\partial\Omega_j} \frac{\partial G_1}{\partial n} dS(q) - \left(\frac{\partial \psi}{\partial n} \right)_j \int_{\partial\Omega_j} G_1 dS(q) \right. \\ &\quad \left. + \frac{1}{4} \omega_j \int_{\partial\Omega_j} \frac{\partial G_2}{\partial n} dS(q) - \frac{1}{4} \left(\frac{\partial \omega}{\partial n} \right)_j \int_{\partial\Omega_j} G_2 dS(q) \right] \end{aligned} \quad (D-1)$$

and

$$\frac{1}{\mu} \frac{\partial P}{\partial x} = \frac{\partial \omega}{\partial y} = \frac{1}{2\pi} \sum_{j=1}^N \frac{\partial}{\partial y} \left[\omega_j \int_{\partial\Omega_j} \frac{\partial G_1}{\partial n} dS(q) - \left(\frac{\partial \omega}{\partial n} \right)_j \int_{\partial\Omega_j} G_1 dS(q) \right] \quad (D-2)$$

The integrals are numbered in the order that they appear in the discretized equations, Eqs. D-1 and D-2. The integrands will be represented by T_k and integrals will be repre-

sented by I_k . The integrands are

$$\begin{aligned}
T_1 &= \frac{\partial}{\partial y} \left(\frac{\partial \ln r}{\partial n} \right) = \frac{\cos \alpha (\eta^2 + \xi^2) - 2\eta(\eta \cos \alpha + \xi \sin \alpha)}{(\eta^2 + \xi^2)^2} \\
T_2 &= \frac{\partial}{\partial y} (\ln r) = \frac{\eta \cos \alpha + \xi \sin \alpha}{(\eta^2 + \xi^2)} \\
T_3 &= \frac{\partial}{\partial y} \left(\frac{\partial (r^2 \ln r - r^2)}{\partial n} \right) = \cos \alpha (\ln(\eta^2 + \xi^2) - 1) + \\
&\quad 2\eta \frac{\eta \cos \alpha + \xi \sin \alpha}{\eta^2 + \xi^2} \\
T_4 &= \frac{\partial}{\partial y} (r^2 \ln r - r^2) = \\
&\quad (\eta \cos \alpha + \xi \sin \alpha) \ln(\eta^2 + \xi^2) - (\eta \cos \alpha + \xi \sin \alpha) \\
T_5 &= T_1 \\
T_6 &= T_2 .
\end{aligned} \tag{D-3}$$

Again, the integrals, I_k , in local coordinates are (see Appendix B)

$$I_k = \int_{\partial\Omega} T_k(x, y) dS(q) = \int_{\xi_j}^{\xi_{j+1}} \bar{T}_k(\eta_j, \alpha_j, \xi) (-d\xi) . \tag{D-4}$$

Explicitly, the integrals become

$$\begin{aligned}
I_1 &= - \left(\cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{d\xi}{\eta_j^2 + \xi^2} - 2\eta_j^2 \cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{d\xi}{(\eta_j^2 + \xi^2)^2} \right. \\
&\quad \left. - 2\eta_j \sin \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{\xi d\xi}{(\eta_j^2 + \xi^2)^2} \right) \\
I_2 &= - \left(\eta_j \cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{d\xi}{\eta_j^2 + \xi^2} + \sin \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{\xi d\xi}{\eta_j^2 + \xi^2} \right) \\
I_3 &= - \left(\cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} \ln(\eta_j^2 + \xi^2) d\xi - \cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} d\xi \right. \\
&\quad \left. + 2\eta_j^2 \cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{d\xi}{\eta_j^2 + \xi^2} + 2\eta_j \sin \alpha_j \int_{\xi_j}^{\xi_{j+1}} \frac{\xi d\xi}{\eta_j^2 + \xi^2} \right) \\
I_4 &= - \left(\eta_j \cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} \ln(\eta_j^2 + \xi^2) d\xi + \sin \alpha_j \int_{\xi_j}^{\xi_{j+1}} \xi \ln(\eta_j^2 + \xi^2) d\xi \right. \\
&\quad \left. - \eta_j \cos \alpha_j \int_{\xi_j}^{\xi_{j+1}} d\xi - \sin \alpha_j \int_{\xi_j}^{\xi_{j+1}} \xi d\xi \right) \\
I_5 &= I_1 \\
I_6 &= I_2 .
\end{aligned} \tag{D-5}$$

The final results are

$$\begin{aligned}
I_1 &= \cos \alpha_j \frac{1}{\eta_j} \left(\arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \right) - 2\eta_j^2 \cos \alpha_j \left[\frac{\xi_j}{2\eta_j^2(\eta_j^2 + \xi_j^2)^2} \right. \\
&\quad \left. - \frac{\xi_{j+1}}{2\eta_j^2(\eta_j^2 + \xi_{j+1}^2)^2} + \frac{1}{2\eta_j^3} \left(\arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \right) \right] \\
&\quad - 2\eta_j \sin \alpha_j \left(\frac{-1}{2(\eta_j^2 + \xi_j^2)^2} - \frac{-1}{2(\eta_j^2 + \xi_{j+1}^2)^2} \right) \\
I_2 &= \eta_j \cos \alpha_j \frac{1}{\eta_j} \left(\arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \right) + \\
&\quad \frac{1}{2} \sin \alpha_j \left[\ln(\eta_j^2 + \xi_j^2) - \ln(\eta_j^2 + \xi_{j+1}^2) \right] \\
I_3 &= \frac{1}{2} \cos \alpha_j \left[\xi_j \ln(\eta_j^2 + \xi_j^2) - \xi_{j+1} \ln(\eta_j^2 + \xi_{j+1}^2) - 2(\xi_j - \xi_{j+1}) + \right. \\
&\quad \left. 2\eta_j \left(\arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \right) \right] - \cos \alpha_j (\xi_j - \xi_{j+1}) + \\
&\quad 2\eta_j \cos \alpha_j \left(\arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \right) + \\
&\quad \eta_j \sin \alpha_j \left[\ln(\eta_j^2 + \xi_j^2) - \ln(\eta_j^2 + \xi_{j+1}^2) \right] \\
I_4 &= \eta_j \cos \alpha_j \left[\xi_j \ln(\eta_j^2 + \xi_j^2) - \xi_{j+1} \ln(\eta_j^2 + \xi_{j+1}^2) - 2(\xi_j - \xi_{j+1}) + \right. \\
&\quad \left. 2\eta_j \left(\arctan \frac{\xi_j}{\eta_j} - \arctan \frac{\xi_{j+1}}{\eta_j} \right) \right] + \frac{1}{4} \sin \alpha_j \left[(\eta_j^2 + \xi_j^2) \ln(\eta_j^2 + \xi_j^2) - \right. \\
&\quad \left. (\eta_j^2 + \xi_{j+1}^2) \ln(\eta_j^2 + \xi_{j+1}^2) + \xi_j^2 - \xi_{j+1}^2 \right] - \\
&\quad \eta_j \cos \alpha_j (\xi_j - \xi_{j+1}) - \frac{1}{2} \sin \alpha_j (\xi_j^2 - \xi_{j+1}^2) \\
I_5 &= I_1 \\
I_6 &= I_2
\end{aligned} \tag{D-6}$$

Eqs. D-1 and D-2 can be further simplified similar to Appendix C. The x -component of velocity at (x_i, y_i) is

$$u(x_i, y_i) = \left(\frac{\partial \psi}{\partial y} \right)_i = \frac{1}{2\pi} \sum_{j=1}^N \left[\psi_j W_1^{ij} + \left(\frac{\partial \psi}{\partial n} \right)_j W_2^{ij} + \omega_j W_3^{ij} + \left(\frac{\partial \omega}{\partial n} \right)_j W_4^{ij} \right] \tag{D-7}$$

and

$$\frac{1}{\mu} \left(\frac{\partial P}{\partial x} \right)_i = \left(\frac{\partial \omega}{\partial y} \right)_i = \frac{1}{2\pi} \sum_{j=1}^N \left[\omega_j W_5^{ij} + \left(\frac{\partial \omega}{\partial n} \right)_j W_6^{ij} \right] \tag{D-8}$$

where W_k^{ij} 's are defined the same as Appendix C, Eq. C-7.

The expressions for the derivatives with respect to x can be obtained by differentiating Eqs. D-6 with respect to α_j as defined by

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial \alpha_j} \left(\frac{\partial u}{\partial y} \right) \quad (\text{D} - 9)$$

and

$$\frac{1}{\mu} \frac{\partial P}{\partial y} = -\frac{\partial \omega}{\partial x} = -\frac{\partial}{\partial \alpha_j} \left(\frac{\partial \omega}{\partial y} \right) \quad (\text{D} - 10)$$

Table 5.1: Effect of mesh refinement on x_c' and $\kappa(0)$ ($\theta_w = 10$ Degrees).

MESH	Q cm ³ /cm ³ s	x_c' cm	$\kappa(0)$ 1/cm
$12 \times 36 \times 12 \times 12$	1×10^{-5}	0.8122×10^{-4}	-0.1990×10^4
$16 \times 48 \times 16 \times 24$	1×10^{-5}	0.8151×10^{-4}	-0.1993×10^4
$32 \times 96 \times 32 \times 48$	1×10^{-5}	0.8181×10^{-4}	-0.2003×10^4
$12 \times 36 \times 12 \times 12$	1×10^{-4}	0.9090×10^{-4}	-0.2175×10^4
$16 \times 48 \times 16 \times 24$	1×10^{-4}	0.9152×10^{-4}	-0.2209×10^4
$32 \times 96 \times 32 \times 48$	1×10^{-4}	0.9220×10^{-4}	-0.2314×10^4
$12 \times 36 \times 12 \times 12$	1×10^{-3}	0.1348×10^{-3}	-0.4086×10^4
$16 \times 48 \times 16 \times 24$	1×10^{-3}	0.1366×10^{-3}	-0.4600×10^4
$32 \times 96 \times 32 \times 48$	1×10^{-3}	0.1382×10^{-3}	-0.4609×10^4

Table 5.2: Parameters for various cases studied ($\theta_w=10$ Degrees).

CASE #	μ dynes-s/cm ²	σ dynes/cm	ψ_f cm ³ /cm-s	x_b cm	$y_{d'}$ cm	R $\frac{x_b}{y_{d'}}$
1	0.01	3	1×10^{-6} to 1×10^{-1}	5×10^{-2}	5×10^{-2}	1
2	0.01	0.3	1×10^{-6} to 3×10^{-2}	5×10^{-2}	5×10^{-2}	1
3	0.01	3	1×10^{-7} to 3×10^{-2}	5×10^{-3}	5×10^{-3}	1
4	0.01	3	1×10^{-8} to 2×10^{-3}	5×10^{-4}	5×10^{-4}	1
5	0.01	3	1×10^{-8} to 3×10^{-4}	5×10^{-5}	5×10^{-5}	1
6	0.01	3	1×10^{-7} to 1×10^{-2}	1×10^{-3}	5×10^{-4}	2
7	0.01	3	1×10^{-6} to 1×10^{-2}	4×10^{-3}	5×10^{-4}	8
8	0.01	3	1×10^{-8} to 1×10^{-1}	4×10^{-2}	5×10^{-3}	8
9	0.01	3	1×10^{-6} to 1×10^{-3}	2×10^{-3}	5×10^{-4}	4
10	0.01	3	1×10^{-7} to 5×10^{-2}	8×10^{-3}	5×10^{-4}	16
11	0.01	3	1×10^{-8} to 1×10^{-1}	5×10^{-2}	5×10^{-4}	100
12	0.1	3	1×10^{-7} to 2×10^{-4}	5×10^{-4}	5×10^{-4}	1

Figure 5.1: An isolated liquid bridge between two porous matrix blocks.

Figure 5.2: Discretization of the boundary, $\partial\Omega$, and definition of the node angles.

Figure 5.3: Comparison of the free boundary shape for different mesh sizes (Case 4).

Figure 5.4: Rate dependence of the liquid bridge free surface shape for the high surface tension case (Case 1, $\sigma = 3 \frac{\text{dynes}}{\text{cm}}$)

Figure 5.5: Rate dependence of the liquid bridge free surface shape for the low surface tension case (Case 2, $\sigma = 0.3 \frac{\text{dynes}}{\text{cm}}$)

Figure 5.6: Average pressure drop across the half liquid film versus flux for high and low surface tension values (Case 1 and Case 2 with $l = 500$ microns, $R = 1$)

Figure 5.7: Average pressure drop across the half liquid film versus $\frac{Q\mu}{l^2 R^2}$ for various parameters.

Figure 5.8: I versus R .

Figure 5.9: Average pressure drop across the half liquid film versus $\frac{Q\mu}{l^2}$ for various values of σ , μ , and l ($R = 1$).

Figure 5.10: Rate dependence of the liquid bridge free surface shape for Case 9 ($R = 4$).

Figure 5.11: Variation of capillary pressure, P_c , along the free boundary, BC, for various flux values ($\sigma = 3 \frac{\text{dynes}}{\text{cm}}$, $R = 1$ and $l = 5$ microns).

Figure 5.12: Dimensionless minimum film thickness, $\frac{xc'}{l}$, versus capillary number, $\frac{Q\mu}{\sigma l}$, ($R = 1$).

Figure 5.13: Pressure drop correlation for high capillary numbers ($R = 1$).

Figure 5.14: Capillary number, $\frac{Q\mu}{\sigma l}$, dependency of dimensionless curvature at contact point B ($R = 1$).

Figure 5.15: Pressure drop correlation for high capillary numbers (high dimensionless curvature, $\kappa_D(0)$, at contact point **B**) ($R = 1$).

Figure B-1: Local coordinates, η, ξ .

5.2 COMPUTATION OF GAS-LIQUID DRAINAGE IN FRACTURED POROUS MEDIA

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INTRODUCTION

Gravity drainage in fractured porous media is related to, 1) matrix permeability, relative permeability, and capillary pressure, and 2) fracture two-phase gas-oil flow characteristics. Immiscible gas-oil flow for a homogeneous matrix porous media is a resolved issue. Fracture two-phase gas-oil flow is, however, complicated. The fracture permeability from the Poiseuille law ($k = 84.4 \times 10^5 (2l)^2$, k in md , l in cm) is for flow parallel to the fracture plane, and it may not apply to flow perpendicular to the fracture plane. Fracture relative permeability perpendicular to the fracture plane is also unknown. There is no reason to believe that straight line relative permeability would apply to gas-liquid flow across fractures. In the previous section, we have analyzed the liquid flow across a liquid bridge between two matrix blocks. An expression for the bridge liquid flow was derived ($k_T = 2.25 \times 10^5 l^2 R^2$, k_T in md , l in cm , see Nomenclature). As we will discuss later in this report, the proposed expression may provide fracture liquid transmissibility for the gas-oil flow across a fracture. It could replace the need for both the fracture absolute permeability and the relative permeability in the direction perpendicular to fracture plane. Our analysis in the previous section also demonstrated that there could be a substantial pressure drop across a liquid bridge in a fracture. Capillary pressure at the inflow end of a liquid bridge could be as much as 1 psi lower than capillary pressure at the outflow end. In other words, capillary pressure at the base of an upper matrix block could be 1 psi lower than capillary pressure at the top face of the matrix block underneath. Such a pressure drop provides the driving force across a fracture. Since the effect of gravity force across a fracture with an aperture of say 10 to 100 microns is negligible, the flow across both horizontal and vertical fractures requires the same pressure drop. A substantial capillary pressure variation along a thin fracture complicates the flow process (see Fig.

5.11 of Section 5.1). As a practical step towards the solution of the problem of gas-oil gravity drainage in fractured porous media, we examine the use of fracture transmissibility expression and fracture capillary pressure. Solution of the full problem may require integration of the flow equations for the fracture (see Section 5.1) and matrix to determine the position of the liquid bridge in the fractures and liquid bridge interface shape.

Both laboratory data and field studies have shown that capillary continuity across a fracture is a reality. However, fractures lower the rate of drainage. The experimental work of Refs. [6] and [5] has revealed that fracture aperture is the most important parameter to affect the flow rate across an idealized fracture. The larger the fracture aperture, the slower would be the rate of drainage. The contact area of the impermeable spacers and the blocks does not affect the rate of drainage [6].

In this report, the experiments of Refs. [6], [5], and [7] will be analyzed. One major goal of this study is to provide a framework for the modeling of gas-oil gravity drainage in fractured reservoirs. Another goal is to investigate how to enhance the rate of drainage.

ANALYSIS OF EXPERIMENTS

A number of experiments from Refs. [6] and [5] were selected to analyze. The experiments from Ref. [6] give both the cumulative production and the drainage rate data. In these experiments, three or four matrix blocks were stacked on the top of each other. Except direct contact experiments, the matrix blocks were separated by thin metallic spacers. Continuous desaturation of the matrix blocks in the stack confirms capillary continuity. Drainage performance of the single matrix blocks and the Tall block from which the three matrix blocks were cut are also available. Table 5.3 gives the dimensions of the matrix blocks LA, LB, LC, SA, SB, SC, SD, the Tall block, and the slabs used in the experiments. Note that in stacked block experiments, matrix blocks LC, LB, and LA were stacked such that LC was in the top, LB in the middle, and LA in the bottom. However, LA is cut from the top, LB from the middle, and LC from the bottom portion of the Tall block. In the second group of stacked block experiments, the blocks were stacked with SA, SB, SC, and SD from top to bottom. Three type of experiments were carried out with the

four-block stack: 1) spacers in the fracture space between the blocks, 2) direct contact, and 3) direct contact with a tilt angle.

Table 5.4 provides the permeability and porosity of the individual matrix blocks and the slabs. Normal decane was the liquid phase and atmospheric air the gas phase. Viscosity and density of normal decane at 76 °F are 0.866 cp and 0.724 g/cm³, respectively.

The capillary pressure of the Tall block is given by the model of Bentsen and Anli [1]

$$P_{cm} = P_{cm}^o - \sigma_m \ln \left(\frac{S_o - S_{orm}}{1 - S_{orm}} \right), \quad P_{cm} \geq P_{cm}^o \quad (5.97)$$

Parameters for the Tall block are $P_{cm}^o = 0.42$ psi, $S_{or} = 0.26$ $\sigma_m = 0.17$ psi [4]. With drainage data, liquid phase relative permeability can be inferred. The gas phase mobility is assumed infinity due to low air viscosity [2]. Therefore, the liquid phase relative permeability can be extracted from the history-matching of the drainage data using Corey-type relative permeability relationship [2],

$$k_{rom} = \left(\frac{S_o - S_{orm}}{1 - S_{orm}} \right)^{n_o} \quad (5.98)$$

$n_o = 3.5$, and $S_{orm} = 0.26$. The drainage performance of first group of individual matrix blocks, LA, LB and LC, are shown in Fig. 5.16 [4]. The difference in the drainage performance is mainly attributed to the difference in capillary pressure - especially the threshold capillary pressure. Using the liquid relative permeability of the Tall block, capillary pressure was adjusted to obtain a good match between measured drainage and rate data and simulation results for matrix blocks LA, LB, and LC. Fig. 5.17 shows the results for matrix block LA. The results for matrix blocks LB and LC are similar (also SA, SB, SC, and SD). Table 5.4 gives the parameters of the capillary pressure model. Note that in Fig. 5.16, the production during the early 5 minutes is mainly due to the film flow along the side walls which is not accounted for in the simulation model [3].

Using the parameters of Tables 5.4 and 5.5, both the cumulative production and drainage rate data of the Tall block were in good agreement with the simulation results (for the Tall block, the parameters of individual segments were used). Fig. 5.18 shows the calculated saturation profile for the Tall block at various times. This figure will be

used as a basis for the matrix block saturation in the stack to be discussed next. In all the simulation work, Eclipse numerical simulator was used [3].

Stacked Block Experiments of Refs. [6] and [5]

In the first group of stacked block experiments, three 60.5 cm blocks (blocks LC, LB, and LA of Table 5.3) were used. As mentioned before, the blocks were obtained by cutting the Tall block into three pieces. Therefore, when they were stacked, the total height was equivalent to the Tall block. In the second group of stacked block experiments, 45.5 cm blocks (blocks SA, SB, SC, and SD of Table 5.3) were used. Again the total height was equal to the height of the Tall block. The last experiment analyzed is the stacked slab experiment of Ref. [5]. Fracture aperture was provided using metal shims (spacers). The thickness and the number of spacers are given in Table 5.3.

In all the stacked block experiments, the permeability of the fracture is calculated based on the correlation developed in Section 5.1.

$$k_f = 2.25 \times 10^5 \mathcal{I} l^2 R^2 \quad (5.99)$$

where the film aspect ratio, R , is assumed to be unity and l is half fracture aperture. Eq. 5.99 is valid for the liquid bridge only. The k_f value based on Eq. 5.99 needs to be spreaded throughout the whole fracture surface to yield the appropriate pressure drop across the fracture aperture. Therefore, calculation of effective absolute permeability of the fracture is based on

$$k_{feff} = \frac{k_f A_{liq}}{L_x L_y} \quad (5.100)$$

The area occupied by the liquid film, A_{liq} , is assumed to be equal to total spacer area which is basically the area of contact. The fracture porosity, ϕ_f , is calculated based on the aperture area *not* occupied by the metal spacers. Therefore, the fracture porosity is

$$\phi_f = \frac{L_x L_y - \sum_{i=1}^{N_s} (A_s)_i}{L_x L_y} \quad (5.101)$$

where A_s is the spacer area and N_s is number of spacers. We propose the following expression for the fracture capillary pressure

$$P_{cf} = P_{cf}^o - \sigma_f \left| \ln \left(\frac{S_o - S_{orf}}{1 - S_{orf}} \right) \right|^{n_f}, \quad P_{cf} \geq P_{cf}^o \quad (5.102)$$

In Eq. 5.102, S_{orf} is the residual liquid saturation in the fracture space and P_{cf}^o is the threshold fracture capillary pressure. The residual liquid saturation in the fracture is assumed to be 1 percent. Initially, we assumed P_{cf}^o to be equal to $\frac{\sigma_f}{l}$, but simulation results revealed that due to dynamic effects, $P_{cf}^o < \frac{\sigma_f}{l}$. Therefore, in Eq. 5.102, P_{cf}^o and σ_f are adjustable parameters to be determined from history matching of data. However, parameter σ_f is taken to be directly proportional to the fracture aperture. In other words, the ratio of σ_f 's for two different fracture apertures is equal to the inverse of aperture ratio. Note that in Eq. 5.102, in order to increase sharply the capillary pressure slope around the residual liquid saturation, second power of the logarithmic term is considered ($n_f = 2$).

In the stacked block and stacked slab experiments to be analyzed next, we need to obtain P_{cf}^o and σ_f for only one of the experiments by matching data and simulation results ($n_f = 2$). The P_{cf}^o and σ_f for the other two experiments need not to be matched.

Case R1, Four 2 cm $\times$ 2 cm $\times$ 50 μ m Spacers - In this test, four 50 μ m thick spacers were used. Measurement of the fracture aperture by a feeler gauge showed uniform thickness of 40-50 μ m [6]. The comparison of experiments with the simulation results are shown in Fig. 5.19. Both rate and the cumulative production data show good agreement with the simulation results. The two bumps in the experimental data (Fig. 5.19a) are due to interruptions to measure average block saturation. Saturations are measurement by interrupting the test and weighing individual blocks. Final calculated average block saturations and the final measured average block saturations are listed in Table 5.6. Similar to single block tests, initial high rate which lasts about 6-9 minutes, is due to film flow along the side faces of the blocks. The fluid in the aperture between the matrix blocks is also mainly produced in the early period. Fig. 5.20 presents the progress of the calculated saturation profiles along the stack. As the final average block saturation measurements and the calculated saturation profiles show, the bottom block (LA) drains the least while the top two blocks (LC and LB) drain much more than the bottom block.

Bottom block drains to its threshold capillary pressure of 41 cm liquid height. Due to capillary continuity, the top two blocks drain substantially over their threshold heights of 35.3 cm and 34.1 cm, respectively.

Comparison of saturation profiles of the top and middle matrix blocks of Fig. 5.20 with the saturation of the corresponding regions of the rock matrix of the Tall block (shown in Fig. 5.18) shows that capillary continuity does not mean rapid drainage. As we will see later, fracture capillary pressure is the cause of the matrix drainage slow down. At very long times ($t \rightarrow \infty$), the saturation profile of the stacked blocks will become the same as the Tall block.

Fig. 5.21 shows the plot of fracture capillary pressure for 50 μm spacers. Parameters P_{cf}^o and σ_f were the adjustable parameters of our simulation (see Table 5.5). It should be noted that the calculated P_{cf}^o is about half of the value of $\frac{\sigma}{r}$ which implies the significance of dynamic capillary pressure in the fractures.

Case R2, Sixteen 2 cm $\times$ 2 cm $\times$ 50 μm Spacers - The only difference between Case R1 and Case R2 is the number of spacers. In Case R2, the number of spacers is increased to 16 to examine the effect of contact area. Since P_{cf} was available from Case R1, the simulation results are purely prediction. Fig. 5.22 shows that the simulation results are in excellent agreement with the data. Also, measured final average block saturations agree with the computed values (see Table 5.6). The drainage performance of this case (R2) and the previous case with 4 spacers (R1) is almost the same. The small discrepancy between the two can be attributed to interruptions of the former experiment due to saturation measurements. The rate versus time plot for the case with 16 spacers (R2) does not have those interruptions. The saturation profiles (not shown) are the same as the case with 4 spacers. The comparison of Cases R1 and R2 indicates that the area of contact (so as the permeability of the fracture) within a given range does not have a pronounced affect on the recovery performance of the stacked blocks.

Case R3, Four 2 cm $\times$ 2 cm $\times$ 100/50 μ m Spacers - This is a multi-stage test. The first part was conducted by using 100 μ m spacers (R3a). At time=29863 min, 100 μ m spacers were replaced by 50 μ m spacers (R3b). In the last stage at time=70270 min, spacers were removed and blocks were put in direct contact. In this study, we have only simulated the response of the first two stages. Fig. 5.23 presents the experimental data and the simulation results. The first part of the experiment with 100 μ m spacers was interrupted twice (at 3243 min and 18349 min) for saturation measurements. During the first part of the test with 100 μ m spacers at time=18349 min, the measured rate decreases sharply (second saturation measurement). the measured rate decreases sharply (Fig. 5.23a). Rupture or evaporation of the liquid film on the spacers at the time of saturation measurement is the likely cause of drainage rate change [6]. This sharp break in the rate was avoided in the second test period by wetting the spacers with few droplets of the liquid at the time of remounting [6]. Yet, overall agreement between the data and simulation results are very good. There was no adjustment of capillary pressure for case R3a. All the results are predictions. Fracture capillary pressure which was used in the calculations is shown in Fig. 5.21. The P_{cf}^o and σ_f values are half of the same parameters of the fracture with 50 μ m spacers. The calculated and measured average block saturations are also compared in Table 5.6. The saturation profiles before changing the spacers are shown in Fig. 5.24. The bottom block (LA) is drained by the same amount as 50 μ m cases (R1 and R2) regardless of the fracture aperture. The overall recovery performance of this test with 100 μ m spacers (Fig. 5.23) is lower than the cases with 50 μ m spacers (Figs. 5.19 and 5.22).

At time 29850 min, 100 μ m spacers were replaced with 50 μ m spacers. As a result, the rate increased rapidly to 6 cc/hr from 0.2 cc/hr. The main reason for the rate increase is due to the difference in capillary pressure characteristics of the 50 μ m spacers and 100 μ m spacers. The liquid from the matrix blocks starts flowing to reach dynamic capillary conditions in the fracture space created by 50 μ m spacers. The same response is also visible from the cumulative production plot (Fig. 5.25b).

Use of fracture capillary pressure corresponding to 50 μm aperture underpredicted the recovery. During various tests it has been observed that the thickness of the spacers and the fracture aperture between the matrix blocks were not equal [6]. Feeler gauge measurements revealed fracture aperture less than 50 microns. Measurements showed fracture aperture varied from 40 to 50 microns [6]. We varied fracture aperture and consequently fracture capillary pressure to match the simulated results and data for the second period of Case R3 (i.e., R3b). An aperture of 37.5 μm yielded good agreement between data and simulation results (see Fig. 5.25).

Measured and calculated average block saturations before the start of the direct contact portion of the same run is reported in Table 5.6. The saturation profiles after the change of 100 μm spacers are also shown in Fig. 5.26. Note that the liquid phase saturation in the middle block increases first and then decreases. We did not account for this saturation direction change for the matrix capillary pressure and relative permeability. Recently, Vidal [8] has observed a new drainage behavior once saturation direction changes. Incorporation of the mechanism may alter the results.

In the second group of stacked block experiments, shorter set of blocks were used (blocks SA, SB, SC, and SD in Tables 5.3 and 5.4). Again, the properties of the blocks SA, SB, SC, and SD are very close to the ones used in the Tall block. The total height of the new blocks were equal to the Tall block height. Therefore, the results were also compared to the Tall block performance. other tests as shown in Table 5.4.

Case R4, Blocks in Direct Contact - In this test, blocks are stacked on top of each other with no spacers in the fracture space between them. The hydraulic fracture aperture parallel to the fracture planes for this case is 150 μm [6]. However, the aperture to be used in the model for constant σ_f of the fracture capillary pressure should be less than 35 μm (see the results of Case R3). Fracture capillary pressure is the most important parameter to be used in the fine grid simulation of the drainage performance experiments. With few trials, an aperture of 23 μm and the corresponding σ_f constant of the fracture capillary

pressure model proved helpful to match the measured rate and cumulative production data. Fracture permeability mainly affects the initial rate. A fracture permeability of 650 md gave an excellent match of the initial rate data. Fig. 5.21 shows the fracture capillary pressure used in the fine grid simulation. The parameters of the P_{cf} model are given in Table 5.5.

Fig. 5.27 compares the simulation results with the experimental data. There is an excellent match of the rate and the drainage data. For the purpose of comparison, the data of the Tall block are also shown.

Case R5, Tilted Blocks in Direct Contact - This test is similar to Case R4 except the stack was tilted 45° from the vertical axis along its long axis [5]. In other words, the tilting is through the plane which passes through the diagonal of the top and bottom faces.

An aperture of $26 \mu m$ yields good agreement between the data and the simulation results (see Fig. 5.28). This value is only $3 \mu m$ higher than the aperture obtained from the vertical case (Case R4, $23 \mu m$). Comparison of Figs. 5.27 and 5.28 shows that tilted stack produces less than the vertical stack, as expected.

Case R6, Stacked Slab Experiments of Ref. [5] - As a final case, the drainage performance of the 12 slab arrangement of Ref. [5] was simulated. Both horizontal and vertical fractures are present in this arrangement. In the test to be simulated here, the 12 slabs are arranged in 4 rows on top of each other, where every row consists of 3 slabs. Average permeability and porosity of each slab used in the numerical simulation are given in Table 5.4.

Both horizontal and vertical fractures are assigned the same equivalent fracture properties. The match of simulation results with the experimental data is shown in Fig. 5.29. From the history match, the average fracture aperture is found to be $30 \mu m$ for both horizontal and vertical fractures. This value is very close to other direct contact

experiments.

The crossplot of the measured versus calculated average block saturations of the stacked block experiments reported in Tables 5.6 and 5.7 are shown in Figs. 5.30 and 5.31, respectively. Some of the points are scattered around the ideal line. All cases exhibit small scattering around the ideal line (see Figs. 5.30 and 5.31). Yet, the level of scattering is very reasonable considering the variation of the fracture aperture in the experiments. In essence, our simulation study matched very well with the rate and cumulative production data as well as the measured average block and slab saturations.

Stacked Block Experiments of Labastie

Labastie [7] has performed drainage experiments on a stack of two identical matrix blocks. The permeability and porosity of each block were 4.6 darcy, and 44 percent, respectively. The height of each block was 35 cm, and the threshold height was also 35 cm for the given rock/fluid system. Matrix capillary pressure and relative permeability were also available. Rock and fluid data for Labastie's experiment are given in Table 5.8.

The aperture in the fracture space between the two blocks was varied by inserting stainless steel spacers (discs) of various thickness. An experiment with 3 spacers of 1 cm diameter and 300 μm thickness produced substantial amount of oil. Labastie [7] reports that fine grid numerical simulation results did not agree with drainage performance data. He used the Young-Laplace equation of capillarity to calculate a constant fracture capillary pressure. The calculated fracture capillary pressure is equal to 6 percent of the plateau capillary pressure of the matrix. Labastie used fracture permeability calculated from aperture. But, he reasoned that since the liquid flow in the fracture takes place around the contacts in the form of film flow, the fracture permeability for flow parallel to fracture planes cannot be used. In Refs. [4] and [5], we have emphasized that due to substantial pressure drop in the fracture space between two matrix blocks, capillary pressure varies substantially across the fracture aperture under flow conditions. Fracture capillary pressure is lower at one end and higher at the other end. Flow takes place from

the low to the high fracture capillary pressure. Therefore, under flow conditions (dynamic conditions), one may not use the Young-Laplace equation. Our theoretical analysis of film flow across a fracture also demonstrates that fracture transmissibility (i.e., permeability) across a contact is much less than the fracture permeability parallel to the fracture planes (in line with Labastie's reasoning). The guidelines provided from our study may help to perform fine grid simulation of his experiment.

The fracture capillary pressure based on experiments analyzed earlier can be used for the numerical simulation of Labastie's experiment with some minor adjustments. Direct extension of the fracture capillary pressure function parameters, σ_f and P_{cf}^o , along with the area weighted effective fracture permeability, $k_{f,eff}$, did not give satisfactory results.

The fracture capillary pressure for Labastie's experiment, is found to be steeper around the fracture residual liquid saturation in comparison to the experiments analyzed previously (see Fig. 5.21). Therefore, we increased the exponent n_f to 5.25. In a sequence of trials, we found that the early part of the recovery curve is always predicted steeper (i.e., high initial rate), although the late part of the recovery curve is matched. There are two parameters that control the initial rate in this problem: 1) fracture threshold capillary pressure, P_{cf}^o , along with the multiplier of the logarithmic term, σ_f , 2) fracture permeability, $k_{f,eff}$. We selected the latter to be consistent with our other runs so that the parameters, P_{cf}^o and σ_f remains the same as extended from the analysis of the previous cases. A drastic reduction in fracture permeability to match the early data caused the exponent n_f in Eq. 5.102 to be significantly higher than the other cases presented. The parameters of the fracture capillary pressure model used in the simulation to obtain the match in Fig. 5.32 are reported in Table 5.9.

Here, we would like stress that the fracture permeability alone is not the main controlling parameter. For $n_f = 2$, if k_f is varied from 0.6 md to 42000 md, the calculated recovery changes from 2 to 4 percent at the end of the experiment, whereas the measured recovery is about 26 percent [7].

SENSITIVITY ANALYSIS

Comparison of the recovery of the individual blocks in a stacked block arrangement and single block recoveries demonstrates that due to capillary continuity, a good portion of the oil in the block is produced. On the other hand, the top blocks always produce less oil than the corresponding segments of the Tall block before gravity capillary equilibrium is established. A practical important question is, then, how to enhance the recovery performance of fractured porous media especially where fracture aperture is high. In the following, we will study the effect of surface tension reduction and hence the reduction of the capillary pressure in both the matrix and the fractures. In addition, the effect of fracture permeability, transmissibility, and contact area will be investigated.

Effect of Capillary Pressure

The capillary pressure will be reduced by half in the following sensitivity studies. In terms of miscibility considerations, such reduction is not significant. In other words, the system is not close to the range of miscibility.

The effect of capillary pressure reduction is first studied on the Tall block to provide a base case. Figs. 5.33 and 5.34 present the production performance and the saturation history of the Tall block with reduced capillary pressure (reduced surface tension), respectively. As Fig. 5.33 shows, the total production increases about 20 % as compared to experimental data. The increase in recovery is mainly due to a reduction of the threshold height.

Stacked Blocks with 100 μm Aperture

To study the effect of capillary pressure reduction, the case with 100 μm spacers is considered. First, both matrix and fracture capillary pressures are reduced by half. Examination of the production data and the simulation results of the low capillary pressure case (see Fig. 5.35), indicates that production rate and cumulative production increases significantly. At the termination (at 497.5 hrs.), the total recovery of the reduced capillary

pressure case is almost twice of the experiment. In this case, even the initial rate is significantly higher than the rate value extrapolated from the experimental data.

Comparison of the calculated saturation profiles of Fig. 5.24 and Fig. 5.36 shows that the drainage of the top two blocks increases significantly when the capillary pressure is reduced. The bottom block also produces significantly more mainly due to reduction in threshold capillary pressure. However, a large amount of oil still remains in the top two blocks as compared to the equivalent regions of the Tall block (see Fig. 5.34). This means that regional heterogeneities have a detrimental effect on gravity drainage process as was demonstrated by Correa and Firoozabadi [2].

Next, different combinations of reduced capillary pressure are studied. First, we assume that only fracture capillary pressure is reduced by half keeping everything else the same as in case R3. This is basically equivalent to a system with increased fracture aperture. As expected, both production rate and the total recovery of this system is lower at all times as compared to the experiment where P_{cf} is higher (Fig. 5.37). The reduced P_{cf} basically reduces the driving force in the fracture.

Finally, Case R3a is simulated by only reducing the capillary pressure of the matrix by half. As seen in Fig. 5.38, this is the most favorable case. The matrix produces more due to reduced capillary pressure while the fracture driving force remains about the same. An alternative explanation is that capillary pressure contrast between the matrix and the fracture is reduced. The least amount of production occurs when there is no capillary continuity between the blocks (zero fracture capillary pressure) where the capillary contrast is maximum. Therefore, stacked block experiments are somewhere between the Tall block case and the capillary discontinuous case. The closer the capillary pressures of the two media the higher the production is.

Effect of Fracture Aperture

After investigating the sensitivity to reduction of capillary pressure (or surface tension), a superficial case with high fracture aperture is studied. The rest of the parameters were the same. The production response of the system with doubled aperture width remained

the same. The main parameter which can be isolated from this run is the gravity effect in the fracture. The conclusion of this particular case is that the effect of gravity is not significant *within* the fracture with respect to capillary pressure difference since fracture aperture is very small.

Effect of Fracture Permeability

In order to examine the effect of fracture permeability, several cases have been investigated. The results were similar, and we will report only one case where the fracture permeability is increased 1000 $\times$. Increasing the fracture absolute permeability did not have any effect for the range of the fracture permeability considered. However, the effect of increasing fracture permeability becomes noticeable when very low fracture permeabilities are considered. This effect can easily be seen from the average single phase absolute permeability expression,

$$\bar{k} = \frac{\sum_{i=1}^{n_b} L_{zi} + \sum_{i=1}^{n_b-1} t_i}{\sum_{i=1}^{n_b} \frac{L_{zi}}{k_{mi}} + \sum_{i=1}^{n_b-1} \frac{t_i}{k_{fi}}} \quad (5.103)$$

Eq. 5.103 indicates that the average permeability decreases when k_{fi} is of the same order as t_i . The initial drainage response is directly related to $\bar{k}$ as Eq. 5.104 demonstrates.

$$q_{oi} = \frac{\bar{k} L_x L_y}{\mu_o L_z} [\Delta \rho g L_z - P_c^o] \quad (5.104)$$

In other words, when $\frac{t_i}{k_{fi}}$ is significant, initial rate will decrease. Therefore, increasing k_f when it is of the order of t_i has positive effect on the initial performance and thus the early production. However, the limit is reached around $k_f = k_m$.

CONCLUSIONS

1. Drainage performance across a stack of matrix blocks is mainly influenced by 1) matrix capillary pressure, and 2) fracture capillary pressure.

2. The fracture capillary pressure is the most critical parameter to influence fracture flow characteristics of gas-oil gravity drainage process.
3. Fracture permeability and fracture liquid transmissibility have a negligible effect on gas-oil gravity drainage in fractured porous media.
4. Reduction of matrix capillary pressure may have a significant influence on gas-oil gravity drainage, especially when the fracture aperture is large.

Nomenclature

A_{liq}	=	area covered by liquid, cm^2
A_s	=	area covered by a single spacer, cm^2
h_m^o	=	matrix threshold height, cm
$\mathcal{I}$	=	liquid film shape factor defined in Section 5.1
k_m	=	absolute permeability of matrix
k_f	=	absolute permeability of fracture
$k_{f_{eff}}$	=	effective absolute permeability of fracture
k_{ro}	=	relative permeability of liquid (oil) phase
k_{ro}^o	=	end point relative permeability of liquid (oil) phase
l	=	half fracture aperture, $\frac{t}{2}$, μm
L_x	=	thickness of the block in x -direction, cm
L_y	=	thickness of the block in y -direction, cm
L_z	=	height of the block (z -direction), cm
n_o	=	relative permeability exponent
n_f	=	exponent in Eq. 5.102
P_{cm}	=	matrix capillary pressure, psi
P_{cf}	=	fracture capillary pressure, psi
P_{cm}^o	=	matrix threshold capillary pressure, psi
P_{cf}^o	=	fracture threshold capillary pressure, psi

R	=	liquid film aspect ratio defined in Section 5.1
S_o	=	liquid (oil) saturation
S_{orm}	=	residual liquid (oil) saturation in matrix
S_{orf}	=	residual liquid (oil) saturation in fracture
t	=	fracture aperture, μm

Greek Symbols

σ_m	=	logarithmic slope of the matrix capillary pressure, <i>psi</i>
σ_f	=	logarithmic slope of the fracture capillary pressure, <i>psi</i>
σ	=	interfacial tension, $\frac{\text{dynes}}{\text{cm}}$
ϕ_f	=	fracture porosity
ϕ_m	=	matrix porosity
θ	=	tilt angle (<i>Degrees</i>)
μ_o	=	liquid (oil) viscosity, <i>cp</i>
$\Delta\rho$	=	density difference of liquid (oil) and gas (air) at room conditions, $\frac{\text{gm}}{\text{cm}^3}$

References

- [1] Bentsen, R.G. and Anli, J.: "A New Displacement Capillary Pressure Model," *Journal of Canadian Petroleum Technology* (September-October 1991) 75-79.
- [2] Correa, A.C. and Firoozabadi, A.: "Concept of Gravity Drainage in Layered Porous Media," Chapter 6.
- [3] ECLIPSE: "ECLIPSE 100 Reference Manual," No. 92A, ECL (1992).

- [4] Firoozabadi, A. and Markeset, T.: "Laboratory Study of Capillary Crossflow in Fractured Porous Media," Chapter 4.
- [5] Firoozabadi, A. and Markeset, T.: "An Experimental Study of Capillary and Gravity Crossflow in Fractured Porous Media," paper SPE 24918 presented at the 1992 67th Annual Technical Conference and Exhibition, Washington, DC, October, also Chapter 4.
- [6] Firoozabadi, A. and Markeset, T.: "An Experimental Study of the Gas-Liquid Transmissibility," paper SPE 24919 presented at the 1992 67th Annual Technical Conference and Exhibition, Washington, DC, October, SPE Reservoir Engineering Journal (to appear), also Chapter 4.
- [7] Labastie, A.: "Capillary Continuity Between Blocks of a Fractured Reservoir," paper SPE 20515 presented at the 1990 SPE Annual Technical Conference and Exhibition, New Orleans, LA, September 23-26.
- [8] Vidal, J.: "Gas/Oil Gravity Drainage: Laboratory Study of Oil Reimbibition and Streaming Between Stacked Matrix Blocks," paper SPE 25074 presented at the 1992 SPE European Petroleum Conference, Cannes, France, November 16-18.

Table 5.3: Dimensions of the blocks, $L_x \times L_y \times L_z$, spacer thickness, t , and number of spacers, N_s , used in the experiments (Firoozabadi and Markeset, 1992).

CASE	L_x <i>cm</i>	L_y <i>cm</i>	L_z <i>cm</i>	N_s 2×2 <i>cm</i>	t μm	Liquid Level <i>cm</i>	Tilt Angle Degrees
LC	14.71	14.71	1×60.48	–	–	1.1	0
LB	14.71	14.71	1×60.44	–	–	1.1	0
LA	14.71	14.71	1×60.48	–	–	1.0	0
TALL	15.14	15.16	1×182.9	–	–	1.45	0
SA	14.71	14.71	1×45.49	–	–	0.25	0
SB	14.71	14.71	1×45.49	–	–	0.5	0
SC	14.71	14.71	1×45.49	–	–	0.3	0
SD	14.71	14.71	1×45.49	–	–	0.8	0
R1	14.71	14.71	3×60.47	4	50	1.1	0
R2	14.71	14.71	3×60.47	16	50	0.9	0
R3	14.71	14.71	3×60.47	4	100/50	1.15	0
R4	14.71	14.71	4×45.49	–	–	0.9	0
R5	14.71	14.71	4×45.49	–	–	1.0	45
R6	4.89	14.95	4×45.53	–	–	0.9	0

Table 5.4: Rock and fluid properties.

VARIABLE	UNITS	LC	LB	LA	SA	SB	SC	SD	SLABS
k_m	(md)	660	700	670	890	725	725	750	715
σ_m	(psi)	0.18	0.18	0.17	0.138	0.180	0.180	0.149	0.138
P_{cm}^o	(psi)	0.364	0.351	0.412	0.364	0.355	0.355	0.380	0.364
h_m^o	(cm)	35.34	34.08	40.04	35.34	34.47	34.47	36.89	35.34
ϕ_m	(fraction)	0.219	0.218	0.214	0.223	0.216	0.222	0.219	0.217

Table 5.5: Fracture permeability and fracture capillary pressure parameters for the stacked block experiments.

	θ	k_f	S_{orf}	σ_f	P_{cf}^o	n_f
CASE	(Deg.)	(mD)	(frac.)	(psi)	(psi)	
R1 (50 μm)	0	167	0.01	-2.35×10^{-3}	6.25×10^{-2}	2
R2 (50 μm)	0	666	0.01	-2.35×10^{-3}	6.25×10^{-2}	2
R3a (100 μm)	0	666	0.01	-1.175×10^{-3}	3.125×10^{-2}	2
R3b (37.5 μm)	0	94	0.01	-3.133×10^{-3}	8.333×10^{-2}	2
R4 (23 μm)	0	650	0.01	-5.11×10^{-3}	13.59×10^{-2}	2
R5 (26 μm)	45	650	0.01	-4.52×10^{-3}	11.98×10^{-2}	2
R6 (30 μm)	0	450	0.01	-3.92×10^{-3}	10.42×10^{-2}	2

Table 5.6: Measured and calculated final saturations for the short blocks, LC, LB and LA, in stacked block experiments.

CASE	Time <i>min</i>	Calculated $\bar{S}_o$			Measured $\bar{S}_o$		
		LC	LB	LA	LC	LB	LA
R1	47527	0.5240	0.5741	0.9031	0.477	0.598	0.904
R2	47850	0.5228	0.5731	0.9017	0.529	0.534	0.905
R3a	29863	0.6846	0.7036	0.9035	0.687	0.661	-
R3b	70270	0.4487	0.5108	0.9039	0.370	0.521	0.906

Table 5.7: Measured and calculated final saturations for the short blocks, SA, SB, SC, and SD, in stacked block experiments.

CASE	Time <i>min</i>	Calculated $\bar{S}_o$				Measured $\bar{S}_o$			
		SA	SB	SC	SD	SA	SB	SC	SD
R4	21368	0.3303	0.4082	0.5110	0.9733	0.347	0.401	0.448	0.981
R5	48861	0.3342	0.4283	0.6203	0.9982	0.366	0.378	0.548	1.0

Table 5.8: Rock and fluid data used in Labastie (1990).

VARIABLE	UNITS	
μ_o	(cp)	2.03
$\Delta\rho$	(gm/cc)	0.781
S_{orm}	(fraction)	0.02
k_m	(md)	4600
σ	(mN/m)	24.1
P_{cm}^o	(psi)	0.3836
h_m^o	(cm)	35
ϕ_m	(fraction)	0.44

Table 5.9: Fracture permeability and fracture capillary pressure parameters for Labastie's experiment (1990).

VARIABLE	UNITS	300 μm
k_f	(<i>md</i>)	0.60
n_f		5.25
S_{orf}	(fraction)	0.01
σ_f	(<i>psi</i>)	-4.00×10^{-4}
P_{cf}^o	(<i>psi</i>)	1.04×10^{-2}

Figure 5.16: Comparison of rate and cumulative production history of short blocks.

Figure 5.17: Comparison of rate and cumulative production data of matrix block LA with simulation results.

Figure 5.18: Calculated saturation profiles for the Tall block at different times.

Figure 5.19: Comparison of rate and cumulative production data of Case R1 with simulation results.

Figure 5.20: Calculated saturation profiles for Case R1 at different times.

Figure 5.21: Fracture capillary pressure for the experiments.

Figure 5.22: Comparison of rate and cumulative production data of Case R2 with simulation results.

Figure 5.23: Comparison of rate and cumulative production data of Case R3a with simulation results.

Figure 5.24: Calculated saturation profiles for Case R3a at different times before changing the 100 μm spacers.

Figure 5.25: Comparison of rate and cumulative production data of Case R3b with simulation results until time=70270 min.

Figure 5.26: Calculated saturation profiles for Case R3b at different times after replacing 100 μm spacers by 50 μm spacers.

Figure 5.27: Comparison of rate and cumulative production data of Case R4 with simulation results.

Figure 5.28: Comparison of rate and cumulative production data of Case R5 (45° tilted) with simulation results.

Figure 5.29: Comparison of rate and cumulative production data of Case R6 with simulation results.

Figure 5.30: Comparison of measured versus calculated average block saturations for the stacked block experiments.

Figure 5.31: Comparison of measured versus calculated average block saturations for the stacked block and stacked slab experiments.

Figure 5.32: Comparison of cumulative production data of Labastie (1990) with simulation results.

Figure 5.33: Comparison of cumulative production data of the Tall block experiment with simulation results for the reduced capillary pressure case.

Figure 5.34: Calculated saturation profiles of the Tall block with reduced capillary pressure at different times.

Figure 5.35: Comparison of rate and cumulative production data of Case R3a experiment with simulation results for the reduced matrix and fracture capillary pressure case.

Figure 5.36: Calculated saturation profiles for Case R3a with reduced matrix and fracture capillary pressure at different times.

Figure 5.37: Comparison of cumulative production data of Case R3a experiment with simulation results for the reduced fracture capillary pressure case.

Figure 5.38: Comparison of cumulative production data of Case R3a experiment with simulation results for the reduced matrix capillary pressure case.

5.3 ANALYSIS OF REINFILTRATION IN FRACTURED POROUS MEDIA

Tor Barkve, and Abbas Firoozabadi

INTRODUCTION

Fractured petroleum reservoirs under gas-oil gravity drainage are commonly described as a large collection of individual matrix blocks feeding oil to the fracture network. The oil, then, is assumed to flow towards the producing wells. Several publications¹⁻⁷ have addressed the topic of reinfiltration, where drained oil from an upper matrix block may reinfiltrate into a matrix block underneath. This process could result in a reduced drainage rate. In two recent publications^{8,9}, theory of reinfiltration in fractured porous media was presented, and it was shown that both gravity and capillary forces are driving forces of the reinfiltration process. The high reinfiltration rates found in the example problems of Refs.8 and 9 could indicate that the oil flow path primarily is in the matrix system.

The solutions derived for the one- and two-dimensional models in Refs.8 and 9 are in terms of infinite sums. These solutions cannot be directly used to study the influence of parameters representing capillarity, gravity and block dimensions. The models and the solutions were also limited to an isotropic matrix block. The purpose of this report is to study the effect of, 1) capillary and gravity forces, 2) matrix block dimensions, and 3) anisotropy in permeabilities, on the drainage- and reinfiltration process.

In the following, flow equation for a two-dimensional anisotropic porous media and the proposed scaling are presented. Solutions of the model equation are then used to study effects of various parameters on the reinfiltration and the drainage rates; first for a one-dimensional matrix block, then for a non-tilted and a tilted two-dimensional block. Finally, based on a numerical model, the effect of hysteresis on reinfiltration- and drainage rates is discussed.

FLOW EQUATION FOR A 2D ANISOTROPIC MATRIX BLOCK

The saturation distribution in a two-dimensional anisotropic porous media can be

described by the following equation:

$$\phi \frac{\partial S_g}{\partial t} = \frac{k_x k'_o \Delta \rho g \sin \theta}{\mu_o} \frac{\partial S_g}{\partial x} + \frac{k_z k'_o \Delta \rho g \cos \theta}{\mu_o} \frac{\partial S_g}{\partial z} + \frac{k_x k'_o P_{co}}{\mu_o} \frac{\partial^2 S_g}{\partial x^2} + \frac{k_z k'_o P_{co}}{\mu_o} \frac{\partial^2 S_g}{\partial z^2} \quad (5.105)$$

Symbols are defined in the Nomenclature. Fig. 5.39 shows the geometry of the problem. The assumptions that are made in the derivation of Eq.5.105 include: 1) linear oil relative permeability, 2) logarithmic dependence of capillary pressure on saturation, 3) infinite gas mobility, and 4) incompressible fluids. Pavone, Bruzzi and Verre¹⁰ made these assumptions to analytically solve the one-dimensional flow problem. Firoozabadi, Ishimoto and Dindoruk⁹ combined the isotropic form of the above equation with various boundary- and initial conditions to derive expressions for the saturation, drainage- and reinfiltration rates for an isotropic matrix block. The scaling of the flow equations by Firoozabadi et al., does not directly reveal the sensitivity of the reinfiltration process to parameters representing gravity, capillarity and block dimensions. In the following, an alternative scaling approach based on the maximum gravity rate in the z -direction, q_G , will be introduced. (Later on, the topic of maximum drainage rate will be discussed.)

$$q_G = \frac{k_z k'_o}{\mu_o} \Delta \rho g \quad (5.106)$$

In the following, q_G will be called the gravity rate. Dimensionless time is defined by:

$$t_D = \frac{q_G}{\phi l_z} t \quad (5.107)$$

Dimensionless lengths and dimensionless gas saturation are defined in the same manner as in Refs.8, 9 and 10, also shown in the Nomenclature. The normalized flow equation can be written as:

$$\frac{\partial S}{\partial t_D} = \alpha \sin \theta \frac{\partial S}{\partial x_D} + \cos \theta \frac{\partial S}{\partial z_D} + \frac{\alpha^2}{\epsilon} \frac{\partial^2 S}{\partial x_D^2} + \frac{1}{\epsilon} \frac{\partial^2 S}{\partial z_D^2} \quad (5.108)$$

In Eq.5.108, beside the tilt angle θ , two dimensionless parameters govern the reinfiltration process:

$$\epsilon = \frac{\Delta \rho g l_z}{P_{co}} \quad (5.109)$$

$$\alpha = \frac{k_x l_z}{k_z l_x} \quad (5.110)$$

Parameter ϵ is a measure of gravity to capillary forces and will be called the Bond number. A large capillary pressure, i.e. large P_{co} , gives a small Bond number, whereas a dominating gravity force results in a large Bond number. The parameter P_{co} in Eq.5.109 can also be regarded as a measure of capillary-pressure gradients. Parameter α will be called the aspect ratio and includes the ratio of 1) matrix block height to its width l_z/l_x , and 2) horizontal to vertical permeability.

Flow rates can be scaled to the gravity rate given by Eq.5.106. The dimensionless flow rates in the x - and z -direction respectively are given by,

$$q_{Dx} = -\kappa(1-S)\sin\theta + \frac{\alpha}{\epsilon} \frac{\partial S}{\partial x_D} \quad (5.111)$$

$$q_{Dz} = -(1-S)\cos\theta + \frac{1}{\epsilon} \frac{\partial S}{\partial z_D} \quad (5.112)$$

In Eq.5.111 and Eq.5.112, the first term on the right hand face represents the gravity contribution, and the second term represents the capillary pressure contribution to the flow rate. Positive flow is assumed to be in the direction of positive coordinate axes. The parameter κ is the anisotropy ratio:

$$\kappa = \frac{k_x}{k_z} \quad (5.113)$$

This parameter enters only in the expression for the rates, and the solution for saturation is independent of κ . For a tilted block, the gravity component of the flow in the x -direction is, however, proportional to κ . Since gas saturation is assumed to be zero at $x_D = 0$ and $x_D = 1$, Eq.5.111 shows that the gravity component of the flow rate is equal at these two faces. An increase in κ , holding everything else constant, leads to a proportional increase in gravity flow through the block, but creates no change in the net flow from the matrix block. In the following, κ will be assumed equal to 1, although influence of the aspect ratio α will be considered.

The average dimensionless flow rates along the x_D - and z_D -faces of the matrix block are given by

$$\bar{q}_{Dx} = \int_0^1 q_{Dx} dz_D \quad (5.114)$$

$$\bar{q}_{Dz} = \int_0^1 q_{Dz} dx_D \quad (5.115)$$

Only average rates will be considered in the present discussion, and the term "average" will generally be omitted in the following. At steady-state condition, the reinfiltration balances the drainage, and the material balance equation can be expressed as

$$\alpha \sum_x \bar{q}_{Dx}^\infty + \sum_z \bar{q}_{Dz}^\infty = 0 \quad (5.116)$$

In the above equation, $\bar{q}^\infty$ denotes the steady-state rate, and the summation takes place over each face in the x_D - and z_D -directions, respectively.

Next, the influence of the Bond number, the aspect ratio, and the tilt angle on the drainage- and reinfiltration rates will be discussed.

ONE-DIMENSIONAL SOLUTION IN TERMS OF BOND NUMBER

The one-dimensional model is mainly a basis for the two-dimensional study. The matrix block is assumed to be initially saturated with oil. The upper face has a no-oil-flow boundary condition, and the bottom face is held at constant zero gas saturation^{8,10}.

From the derivation of Pavone et al¹⁰, the solution for the dimensionless gas saturation in terms of the new variables can be expressed as

$$S_{1D} = 1 - e^{\epsilon \cos \theta z_D} - \epsilon \cos \theta e^{\frac{1}{2}\epsilon(1-z_D)} \sum_{n=0}^{\infty} A_n e^{-\omega_n^2 t_D} \sin(a_n z_D) \quad (5.117)$$

The saturation variable is here given the index 1D to distinguish it from two-dimensional cases. A non-zero tilt angle is also included in the solution. Appendix A provides the coefficients of Eq.5.117, in terms of the new dimensionless variables.

Based on Eq.5.117 and a relationship between saturation and rate analogous to Eq.5.112, Ref.8 gives an expression for the drainage rate of the matrix block. Ref.8 also provides an expression for the recovery, R , using time integration of q_D . An alternative expression for R can be obtained from the global material balance:

$$\begin{aligned} R &= \int_0^1 S_{1D} dz_D \\ &= \left(1 + \frac{1}{\epsilon \cos \theta} e^{-\epsilon \cos \theta} - \frac{1}{\epsilon \cos \theta}\right) - \cos \theta e^{\frac{1}{2}\epsilon \cos \theta} \sum_{n=0}^{\infty} \frac{A_n}{\omega_n^2} e^{-\omega_n^2 t_D} \end{aligned} \quad (5.118)$$

The ultimate recovery is given as the steady-state part of this equation:

$$R^{\infty} = 1 + \frac{1}{\epsilon \cos \theta} e^{-\epsilon \cos \theta} - \frac{1}{\epsilon \cos \theta} \quad (5.119)$$

For a zero tilt angle and a Bond number equal to 1, the total recovery is approximately 37 percent. For a large Bond number and a small tilt angle, the recovery is approximately $(1 - \frac{1}{\epsilon \cos \theta})$.

The initial dimensionless rate predicted by the model is 1, and it is independent of the Bond number. This result is easily seen from Eq.5.112 using an initial zero saturation gradient. The result can also be derived from the expression for the drainage rate given in Ref.8. In terms of dimensional variables, the initial rate equals the maximum drainage rate given by Eq.5.106.

Maximum Drainage Rate

Pavone et al¹⁰ included a threshold capillary pressure in their one-dimensional model equations, but later neglected it when solving the equations. From their choice of boundary conditions, it is not clear how a non-zero threshold capillary pressure can be included in the model. In a more complete formulation, coupling between pressure and saturation is made through the flux term, consisting of the fractional-flow function multiplied by the total velocity. This term was neglected when deriving Eq.5.105, since it is assumed that the gas mobility is infinity. The assumption of infinite gas mobility is not a good assumption for a small region in the matrix block where the gas saturation and gas fractional flow are close to zero. The initial small amount of gas that enters into the matrix block has a finite mobility, and the rate will be smaller than predicted by the assumption of infinite gas mobility. For a data set with a small Bond number, only a small amount of gas enters the matrix block, and the error caused by assuming infinite gas mobility may be large. For a large Bond number, the error will only be significant in a short initial period. The initial error will not change main conclusions concerning the reinfiltration effect to be presented in the following sections.

Note that the initial rate found from the one-dimensional model equals the gravity rate. The effect of a threshold capillary pressure on the initial rate can not be predicted

by this model.

No rigorous derivation of the initial rate from a matrix block can be found in the literature. Van Golf-Racht¹¹ gives an expression for the initial drainage rate for a simplified model based on gravity segregation and infinite gas-mobility assumptions. An expression for the initial rate including the effect of threshold capillary pressure can be found by integrating Darcy's law, assuming one-dimensional flow:

$$\int_B^T \frac{q_g}{\lambda_g} dz = \rho_g g l_z - [p_g^T - p_g^B] \quad (5.120)$$

$$\int_B^T \frac{q_o}{\lambda_o} dz = \rho_o g l_z - [p_o^T - p_o^B] \quad (5.121)$$

The superscripts T and B are used to denote the top and the bottom faces of the matrix block, respectively. λ_g is the gas-phase mobility and λ_o is the liquid-phase mobility. No assumption is made here concerning gravity segregation or fluid distribution in the matrix block. The capillary pressure is introduced by subtracting the above two equations:

$$\int_B^T \frac{q_o}{\lambda_o} dz - \int_B^T \frac{q_g}{\lambda_g} dz = \Delta \rho g l_z - P_c^{TH} \quad (5.122)$$

The capillary pressure is defined to be zero at the bottom face corresponding to the gas/oil contact. At the top face, the capillary pressure equals the threshold value P_c^{TH} . The matrix block is initially saturated with oil, and the second integral on the left side of Eq. 5.122 is zero, although the integrand may locally have a non-zero value at the top face. For the same reason, the mobility in the integral for the oil phase may be replaced by the end-point value λ_o' . Neglecting compressibility effects, this leaves the following expression for the initial rate:

$$q_o = \lambda_o' \left(\Delta \rho g - \frac{P_c^{TH}}{l_z} \right) \quad (5.123)$$

Deviation from this expression can only be caused by induced pressure gradients, multi-dimensional effects or compressibility effects.

TWO-DIMENSIONAL NON-TILTED BLOCK

Appendix B provides the solution for Eq. 5.108 for the two-dimensional non-tilted matrix block for which the side- and bottom faces are kept at a liquid saturation of unity

and the top face has a no-oil-flow boundary condition. For this case, reinfiltration may take place along the two vertical faces. In addition to the introduction of new variables, the solution by Firoozabadi et al⁹ has been further simplified. It should be noted that due to the symmetry condition, the length l_x used in Ref.9 represents half the width of the matrix block. In our studies, the total block width l_x will be used, for instance in Eq.5.110, to be able to compare the results of various cases.

The saturation- and rate equations of Appendix B show dependency on the Bond number and the aspect ratio. The solution can incorporate the state of saturation at the start of reinfiltration, as in the original formulation⁹. For this purpose, the matrix block is assumed initially to have a gas saturation distribution given by $S(t_D = 0) = S_{1D}(t_{D0})$ where S_{1D} is the solution of the one-dimensional problem given by Eq.5.117 at time t_{D0} .

The solution for the drainage rate at $z_D = 0$ and the reinfiltration rate at $x_D = 1$ both consist of transient and steady-state terms. The steady-state term as given by Eq.B-9 represents the equilibrium rate where the drainage rate equals the reinfiltration rate. The equilibrium drainage rate $\bar{q}_{Dz}^\infty$ is seen to be independent of the start-up time t_{D0} and depends only on the Bond number and the aspect ratio. For the transient part, two extreme cases will be studied; 1) no initial drainage prior to the reinfiltration ($t_{D0} = 0$), and 2) complete drainage before the start of reinfiltration ($t_{D0} = \infty$).

Steady State Behavior

At steady state, the drainage rate of the matrix block, i.e the equilibrium rate, is supported by reinfiltration only, and therefore, it is a direct measure of the magnitude of the reinfiltration. Fig. 5.40 shows the equilibrium rate as a function of the aspect ratio for various Bond numbers. This figure shows that the aspect ratio has a significant influence on the steady-state rate. The rate increases as the aspect ratio increases. For a cubic block with Bond number equal to 1, the dimensionless equilibrium drainage rate is approximately equal to 0.9. This implies that the reinfiltration rate is only some 10 percent less than the maximum drainage rate. As the aspect ratio is increased, the equilibrium rate becomes close to the maximum drainage rate, see Fig. 5.40. At steady state, the

following relationship provides an approximate definition of the parameter region where the equilibrium rate equals the maximum drainage rate:

$$\alpha \geq 2e^{0.03\epsilon} \quad (5.124)$$

The above relationship was obtained by plotting level curves for the equilibrium rate in a (α, ϵ) -coordinate system.

Fig. 5.41 shows the variation of the equilibrium rate versus the Bond number for several values of the aspect ratio. For given block dimensions, i.e constant aspect ratio, an increase in the Bond number results in a decrease in the equilibrium rate. A large capillary pressure leads to a reduced Bond number and an increase in equilibrium rate. Fig. 5.41 shows that even for highly gravity-dominated flow, i.e for large Bond number, the equilibrium rate is comparable to the maximum drainage rate for moderately large aspect ratios.

Transient Behavior

For a fully saturated matrix block, the initial reinfiltration rate is zero. However, as the matrix block desaturates, the reinfiltration process becomes active. Transient characteristics of the reinfiltration process can be investigated using the following reinfiltration/drainage ratio R_{ID} :

$$R_{ID} = \frac{2\alpha \bar{q}_{Dx}}{\bar{q}_{Dz}} \quad (5.125)$$

The aspect ratio α is included to account for the scaling of the areas, and the factor 2 represents the number of vertical faces open to reinfiltration. This ratio will always vary between 0 and 1. Figs. 5.42 and 5.43 show variation in the reinfiltration/drainage ratio for various values of the Bond number and the aspect ratio. Fig. 5.42 shows that the Bond number strongly influences the transient behavior of the reinfiltration process. For Bond number equal to 1, the equilibrium is reached at dimensionless time of about 1, and as the Bond number increases, the equilibrium process is significantly delayed.

Fig. 5.43 indicates that for $\epsilon = 1$, aspect ratios in the region 0.5–1 result in a slow process toward steady state. Similar results were found using a broad range of Bond

numbers, although results are not shown here. The large transient period for α in the range 0.5–1 is a result of a balance between the surface area open to reinfiltration and the travel distance of the reinfiltrated oil. For a large aspect ratio, the area of reinfiltration along the side faces compensates almost immediately for the distance, and equilibrium is established in a very short period. For small aspect ratios, the matrix block is draining mainly through a small fraction of the horizontal face close to the vertical face, and, therefore, the travel distance for reinfiltrated oil is short. A maximum in the duration of the transient period is realized when the surface area of reinfiltration is relatively small, and the reinfiltrated oil travels a relatively long distance. This could happen when the aspect ratio is in the range of 0.5 to 1.

The slow establishment of equilibrium in a near-square block can be observed when ultimate net recoveries are considered. Although the drainage rate for the parameter region given by Eq.5.124 is independent of time, the reinfiltration rate will have a transient period where the rate increases from zero to its equilibrium value. Therefore, a net recovery from the matrix block will always exist. The transient period will be longer if there were no reinfiltration. Splitting all rates in a transient component $\bar{q}_D^T$ and a steady-state component $\bar{q}_D^\infty$, the ultimate net recovery from the matrix block can be written in terms of the integral,

$$R^\infty = \int_0^\infty [-\bar{q}_{Dz}^T + 2\alpha\bar{q}_{Dx}^T] dt_D \quad (5.126)$$

The steady-state component has been eliminated by using the material balance equation, i.e Eq.5.116. The expression for the recovery R^∞ is given by Eq.B-11. The variation in ultimate net recovery as function of the aspect ratio is plotted in Fig. 5.44. This figure shows that a maximum net recovery is realized for an aspect ratio in the range 0.5 to 1. This observation is in accord with the slow establishment of equilibrium in a near-square matrix block.

Fig. 5.45 shows variation in the ultimate net recovery as function of the Bond number. The ultimate recovery from the one-dimensional model given by Eq.5.119 is also included

in this figure. For an aspect ratio and a Bond number of 1, the net recovery is approximately 14 percent. The corresponding recovery from the 1D model with no reinfiltration is 37 percent. Consistent with the results given in Fig. 5.42, net recovery increases with increasing Bond number. Despite reinfiltration, a significant amount of oil in the matrix block is recovered for large values of the Bond number.

If the block is fully drained before the start of reinfiltration, the initial reinfiltration rate will be very large, as was shown in Ref.9. Due to the large magnitude of this initial rate, we define the drainage/infiltration rate ratio, R_{DI} , as the inverse of the right-hand side in Eq.5.125. This ratio, then, expresses how quickly the drainage responds to the reinfiltration. As for the ratio R_{ID} , R_{DI} will also vary between 0 and 1. Figs. 5.46 and 5.47 show plot of R_{DI} versus time for various values of Bond number and aspect ratio, respectively. The data in these figures are calculated using $t_{D0} = \infty$. Fig. 5.46 shows that high Bond numbers tend to slow down the approach to steady state, while Fig. 5.47 implies that the aspect ratio has a limited influence on the transient period.

TWO-DIMENSIONAL TILTED BLOCK

In this section, we investigate the effect of an inclined fracture system on the reinfiltration process. The matrix block has a tilt angle θ , as shown in Fig. 5.39. Three faces of the matrix block are kept at a liquid saturation of unity, and the face at $z_D = 1$ has a no-oil-flow boundary condition. The solution of Eq.5.108 for the tilted block and the corresponding rates in terms of the new variables are provided in Appendix C. This Appendix also describes numerical techniques used to achieve accurate calculation of the infinite series.

Figs. 5.48, 5.49 and 5.50 all show the steady-state rates as a function of the tilt angle. The angle varies from 0° to 90° . Fig. 5.48 shows the equilibrium rate of the three active faces of the matrix block. This figure also shows the total equilibrium drainage from the two faces $z_D = 0$ and $x_D = 0$. The total drainage rate is defined as:

$$\bar{q}_D^\infty = \bar{q}_{Dz}^\infty + \max[\alpha \bar{q}_{Dx}^\infty, 0] \quad (5.127)$$

In Fig. 5.48, reinfiltration rates are shown negative, whereas drainage rates are positive. Along the face $x_D = 0$, a net reinfiltration takes place for angles up to approximately 30° , but for larger tilt angles, the gravity component of flow dominates over the capillary component. Note that the total drainage rate has a minimum for the tilt angle where the rate from the face $x_D = 0$ is zero.

Using the expressions for the dimensionless rates given by Eq.5.111 and Eq.5.112 the initial total drainage rate from a two-dimensional block can be calculated,

$$\bar{q}_D(t_D = 0) = \cos \theta + \alpha \sin \theta \quad (5.128)$$

For an aspect ratio of 1, a tilt angle of 45° results in a maximum initial drainage rate. The steady state total drainage rate has a maximum for a tilt angle larger than 45° , due to the unsymmetric boundary conditions.

The total drainage rate has a minimum at a critical angle where gravity balances the reinfiltration along the side face $x_D = 0$. Fig. 5.49 shows a small variation in this critical angle as function of the Bond number. By increasing the aspect ratio, the gravity will dominate over the capillarity at progressively smaller tilt angles, see Fig. 5.50.

Steady-state gas-saturation contours for a tilted block of tilt angle $\theta = 45^\circ$ are displayed in Figs. 5.51 and 5.52. The aspect ratio for both cases is equal to 1, whereas the Bond number for Figs. 5.51 and 5.52 are 3 and 10, respectively. Comparison of these two figures clearly indicates the influence of the gravity on the saturation profile. When the Bond number is increased from 3 to 10, the dominance of the gravity force compared to capillarity increases.

INFLUENCE OF UNSCALED PARAMETERS

The sensitivity of the reinfiltration rate to unscaled parameters such as density difference and matrix permeability can only be investigated indirectly by using the parameters and scaling constants presented in the preceding sections. According to the Eq.5.106 and Eq.5.107, a change in matrix permeability will stretch the time scale and will change the magnitude of the flow rates. For an isotropic block, the absolute permeability does not

enter into the definition of any of the model parameters, and does not influence variation in the dimensionless solutions. A reduced matrix permeability will reduce the drainage rate and the reinfiltration rate in the same manner. Horizontal permeability is usually larger than the vertical permeability, and thus anisotropy is in favor of the reinfiltration process.

Increasing the block height l_z leads to an increase both in the Bond number and the aspect ratio. Consequently, the magnitude of gravity forces compared to capillary forces increases. At the same time, a larger fraction of the matrix-block surface is open for reinfiltration. The aspect ratio has a strong influence on the ratio of drainage to reinfiltration rates, and for tall matrix blocks, the reinfiltration is close to the maximum drainage rate.

It should be noted that for a three-dimensional matrix block, the ratio of the reinfiltrating and draining surfaces will be larger than in a two-dimensional model. Therefore, the reinfiltration will be more pronounced than predicted by the two-dimensional model.

EFFECTS OF HYSTERESIS

The analytical models that were developed to study the reinfiltration process do not take into account the effect of hysteresis. For a partially saturated block, once a boundary surface is kept at a liquid saturation of say unity, the hysteresis in capillary pressure and relative permeability influences the rate of reinfiltration. It was pointed out in Ref.8 that the use of direction-dependent capillary pressure slows down the reinfiltration rate. By using a numerical simulator, the sensitivity of the reinfiltration rate to the direction-dependent P_c is investigated in the following.

Consider a cylindrical block with a diameter of 9.96 *cm* and a height of 60 *cm*. The porosity of the block is 20 percent and its permeability is 444 *mD*. The viscosity of the liquid saturating the block is 0.95 *cp* and its density is 0.724 *g/cm*³. Gas-phase viscosity is assumed to be 0.01 *cp* and its density 0.0012 *g/cm*³. Fig. 5.53 shows the drainage- and imbibition capillary pressure of the matrix block. Table 5.10 provides the liquid-

and gas-phase relative permeabilities. It is assumed that the hysteresis effect is absent in relative-permeability data.

The block is initially fully saturated and is surrounded by gas. It is allowed to reach gravity-capillary equilibrium. The final liquid saturation is given by the drainage capillary pressure of Fig. 5.53. The top face of the block is then assigned a liquid saturation of unity. Fig. 5.54 shows the computed rate of reinfiltration across the face. The results are obtained using the Eclipse simulator¹² and a 4×28 radial grid. The maximum drainage rate of the block is approximately $1.55 \text{ cm}^3/\text{min}$. The rates of reinfiltration at time $t = 0.005 \text{ hr}$ are 30 and $50 \text{ cm}^3/\text{min}$ with and without hysteresis effect, respectively. As the saturation gradient at the top face of the block decreases, the computed reinfiltration rates (with and without hysteresis) also decrease. At time 0.5 hr and later, the computed rate of reinfiltration and the computed rate of drainage become the same. Fig. 5.54 reveals that the P_c -hysteresis effect decreases the rate of reinfiltration, as expected. However, even with hysteresis effect, the initial rate of reinfiltration is very high,— twenty times the maximum rate of drainage.

Fig. 5.55 shows the rate of drainage for two cases; 1) with hysteresis effect, and 2) no hysteresis effect. At first, the block is at capillary-gravity equilibrium and the block face is then contacted with the liquid face at the top face. After the saturation in the block builds up, the drainage starts. The drainage rate is higher for the zero hysteresis case than the case for which hysteresis is present. After about 3 hours, the drainage rate for both cases become equal. At this time, and later on, the drainage rate remains at its maximum value; about $1.55 \text{ cm}^3/\text{min}$.

The methodology of Killough¹³ was used to compute the scanning capillary-pressure curve in the above calculations. The results in Figs. 5.54 and 5.55 are based on a value of 0.7 for the scanning P_c -curve parameter. If the value of this parameter is reduced to 0.07, the reinfiltration at time 0.005 hr reduces to $20 \text{ cm}^3/\text{min}$, which is 12 times more than the maximum drainage rate. We could conclude that the reinfiltration rate decreases due to the history-dependency of the P_c -curve. The rate is, however, higher than the maximum

drainage rate whenever large saturation gradients exist at the boundary.

DISCUSSION AND CONCLUSION

In this report, it is clearly shown that the reinfiltration process is governed both by capillary and gravity forces. Therefore, it is suggested that the word "reinfiltration" be used in favor of "reimbibition", as the latter could imply the capillarity effect only.

The main conclusions of this report are:

1. The combination of gravity and capillary forces generally provides conditions for a pronounced reinfiltration process in a fractured porous media. For a wide range of capillary- and gravity forces, the reinfiltration rate is of the same order of magnitude as the maximum drainage rate.
2. The aspect ratio, i.e $k_x l_z / k_z l_x$, has an important effect on the rate of reinfiltration at steady state conditions. The Bond number, i.e the ratio of gravity to capillarity, plays an important role for the reinfiltration in the transient period.

Nomenclature

g	Gravitational acceleration
k	Matrix permeability
k'_o	End-point oil relative permeability. Equivalent to C_o in Ref.8.
l_x, l_z	Matrix block dimensions
p	pressure
P_c	Gas/oil capillary pressure
P_c^{TH}	Threshold capillary pressure
P_{co}	Characteristic gas/oil capillary pressure
q	Flow rate
$\bar{q}$	Average flow rate
q_G	Gravity rate defined by Eq.5.106

q^T	Transient component of flow rate
q^∞	Steady-state component of flow rate
R	Recovery = Net dimensionless cumulative flow rate
R^∞	Ultimate recovery
S_g	Gas saturation
S	Normalized gas saturation
t	Time
$x_D = x/l_x$	Dimensionless x -coordinate
$z_D = z/l_z$	Dimensionless z -coordinate

Greek Symbols

α	Aspect ratio defined by Eq.5.110
$\Delta\rho = \rho_o - \rho_g$	Gas-oil density difference
ϵ	Bond number defined by Eq.5.109
$\kappa = \frac{k_x}{k_z}$	Anisotropy ratio
λ	Mobility
μ	Viscosity
ρ	Density
θ	Tilt angle

Subscripts

D	Dimensionless
G	Gravity
g	Gas phase
o	Oil phase
x	x -direction
z	z -direction

References

- [1] Lefebvre du Prey, E.J.L. : "Drainage en cascade de blocs d'un reservoir fissure," *Revue Institute Franscais du Petrole*, (Jan-Fev, 1976) 173-178.
- [2] Saidi, A.M., Tehrani, D.H. and Wit, K. : "Mathematical simulation of fractured reservoir performance, based on physical model experiments," *Proc. 10th World Petroleum Conference*, Bucharest (1979).
- [3] Festøy, S. and van Golf Racht, T.D. : "Gas gravity drainage in fractured reservoirs through new dual continuum approach," *SPE Reservoir Engineering* (Aug, 1989) 271-278.
- [4] Por, G.J., Boerrigter, P., Maas, J.G. and de Vries, A. : "A fractured reservoir simulator capable of modeling block-block interaction," Paper 19807, *Proc. SPE 64th Annual Technical Conference and Exhibition*, San Antonio (1979) 373-331.
- [5] Fung, L. : "Simulation of block-to-block processes in naturally fractured reservoirs," Paper SPE 20019, *SPE 60th California Regional Meeting*, Ventura (1990).
- [6] da Silva, F.V. and Meyer, B. : "Analysis of gravity-capillary equilibrium in chalk fractured reservoirs," *Third North Sea Chalk Symposium*, Copenhagen (1990).
- [7] Coats, K. : "Implicit compositional simulation of single-porosity and double-porosity reservoirs," Paper SPE 18427, *SPE Reservoir Simulation Symposium*, Houston (1989).
- [8] Firoozabadi, A. and Ishimoto, K. : "Theory of reinfiltration in fractured porous media. Part I: One dimensional model," Paper SPE 21796, *SPE Western Regional Meeting*, Long Beach (1991), to appear in SPE Advanced Technology Series.

- [9] Firoozabadi, A., Ishimoto, K. and Dindoruk, B. : "Theory of reinfiltration in fractured porous media. Part II: Two dimensional model," Paper SPE 21798, *SPE Western Regional Meeting*, Long Beach (1991).
- [10] Pavone, D., Bruzzi, P. and Verre, R. : "Gravity drainage at low interfacial tension," *Proc. 5th Hung. Hydrocarbon Inst. Impr. Oil Recovery European Symposium* (1989) 165-174.
- [11] van Golf-Racht, T.D. : "Fundamentals of fractured reservoir engineering," Elsevier Scientific Publishing Co, *Developments in Petroleum Science* (1982).
- [12] Eclipse Reference Manual, Version 90A, Exploration Consultants Ltd (1990).
- [13] Killough, J.E. : "Reservoir simulation with history-dependent saturation functions," *Trans. AIME* vol. 261 (1976) 37-48.
- [14] Gradshteyn, I.S. and Ryshik, I.M. : "Tables of integrals, series and products," Academic Press, Inc., New York (1966).

Appendix A:

Coefficients of the 1D Solution

The solution for the one-dimensional model is given in Eq. 5.117. For the eigenvalues a_n , the same equation can be applied for both 1D and 2D cases:

$$\epsilon \cos \theta \sin a_n + 2a_n \cos a_n = 0 \quad (\text{A} - 1)$$

The Fourier frequencies are given by

$$\omega_n^2 = \frac{1}{4\epsilon} (4a_n^2 + \epsilon^2) \quad (\text{A} - 2)$$

The form of the coefficients A_n presented here deviates from the originally form^{8,10}, but is mathematically equivalent:

$$A_n = \frac{4a_n \sin a_n}{\epsilon \omega_n^2 [2a_n - \sin(2a_n)]} \quad (\text{A} - 3)$$

Appendix B:

Solution for the 2D Non-tilted Matrix Block

From Ref.9, the solution can be written in terms of the new variables as:

$$S = \epsilon e^{\frac{\epsilon}{2}(1-z_D)} \left\{ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [A_{nm} + B_{nm}] \cos(b_m x_D) \sin(a_n z_D) e^{-\sigma_{nm}^2 t_D} - \sum_{m=1}^{\infty} C_m \cos(b_m x_D) \sinh(c_m z_D) \right\} \quad (B-1)$$

Similar to the one-dimensional case, the eigenvalues a_n are defined by Eq.A-1, using a zero tilt angle. The following notation has been introduced to simplify the equations:

$$b_m = \left(m - \frac{1}{2}\right)\pi \quad (B-2)$$

$$c_m^2 = \frac{1}{4}\epsilon^2 + 4\alpha^2 b_m^2 \quad (B-3)$$

$$\sigma_{nm}^2 = \frac{1}{2\alpha} (a_n^2 + c_m^2) \quad (B-4)$$

The coefficients given in Ref.9 can be further simplified using this notation together together with the solution of the one-dimensional problem:

$$A_{nm} = \frac{2(-1)^m}{b_m} \left[e^{-\omega_n^2 t_{D0}} - 1 \right] A_n \quad (B-5)$$

$$B_{nm} = \frac{(-1)^m}{2b_m} \frac{\epsilon \omega_n^2}{\alpha \sigma_{nm}^2} A_n \quad (B-6)$$

$$C_m = \frac{4(-1)^m}{b_m} (2c_m \cosh c_m + \epsilon \sinh c_m)^{-1} \quad (B-7)$$

A zero tilt angle should be used for the coefficients ω_n and A_n , defined by Eq.A-2 and Eq.A-3 respectively.

The average drainage rate from the bottom face $z_D = 0$ can be expressed as:

$$\bar{q}_{Dz} = \bar{q}_{Dz}^{\infty} - e^{\frac{\epsilon}{2}} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^m \frac{a_n}{b_m} (A_{nm} + B_{nm}) e^{-\sigma_{nm}^2 t_D} \quad (B-8)$$

$$\bar{q}_{Dz}^{\infty} = -1 + e^{\frac{\epsilon}{2}} \sum_{m=1}^{\infty} (-1)^m \frac{c_m}{b_m} C_m \quad (B-9)$$

$\bar{q}_{Dz}^\infty$ is the steady-state component, i.e the average equilibrium rate. The average reinfiltration rate at the side face $x_D = 1$ is given similarly by

$$\bar{q}_{Dx} = 2\alpha\bar{q}_{Dz}^\infty + 2\alpha e^{\frac{\epsilon}{2}} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^m \frac{a_n b_m}{\epsilon \omega_n^2} (A_{nm} + B_{nm}) e^{-\sigma_{nm}^2 t_D} \quad (B-10)$$

The material balance relationship, Eq.5.116, has been used for the steady-state component. Finally, the ultimate net recovery from the matrix block is given by integrating the net average rate over time:

$$\begin{aligned} R &= \int_0^\infty (-\bar{q}_{Dz} + 2\alpha\bar{q}_{Dx}) dt_D \\ &= e^{\frac{\epsilon}{2}} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^m \frac{a_n}{\sigma_{nm}^2 b_m} [A_{nm} + B_{nm}] \\ &\quad - 4\alpha^2 e^{\frac{\epsilon}{2}} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^m \frac{a_n b_m}{\epsilon \omega_n^2 \sigma_{nm}^2} [A_{nm} + B_{nm}] \end{aligned} \quad (B-11)$$

The saturation S is contained in the interval $[0, 1]$, but the solutions all contain an exponential factor with argument $\epsilon/2$. Hence, the decay rate of the sum of the infinite series is approximately $e^{-\epsilon/2}$ as function of the Bond number ϵ . The first terms in the sums are generally of order 1, and for large Bond numbers, it is difficult to calculate the infinite sums numerically with sufficient accuracy. This is especially noticeable for the two-dimensional sums. In the present study, the material balance condition was used to test the numerical results wherever possible, and the techniques described in Appendix C are used to accelerate convergency of summation calculations.

Appendix C:

Solution for the 2D Tilted Matrix Block

From Ref.9, the solution for the gas saturation in a tilted block can be written in terms of new variables as

$$\begin{aligned} S &= \epsilon \cos \theta e^{\frac{\epsilon}{2} \cos \theta (1-z_D) - \frac{\epsilon}{2\alpha} \sin \theta x_D} \left\{ \right. \\ &\quad \sum_{m=1}^{\infty} D_m \sin(d_m x_D) \sinh(e_m z_D) \\ &\quad \left. - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E_{nm} \sin(d_m x_D) \sin(a_n z_D) e^{-\nu_{nm}^2 t_D} \right\} \end{aligned} \quad (C-1)$$

The eigenvalues a_n are defined in Eq.A-1 as for the 1D model and the 2D non-tilted model. The definition of e_m and ν_{nm} has the same form as the definition of c_m and σ_{nm} used in Appendix B:

$$d_m = m\pi \quad (C-2)$$

$$e_m^2 = \frac{1}{4}\epsilon^2 + \alpha^2 d_m^2 \quad (C-3)$$

$$\nu_{nm}^2 = \frac{1}{\alpha} (a_n^2 + e_m^2) \quad (C-4)$$

To simplify further, the following functions are defined:

$$F_m^+ = \frac{\alpha^2 d_m}{\alpha \nu_{mn}^2 - \epsilon \omega_n^2} [1 - (-1)^m e^{\frac{\epsilon}{2\alpha} \sin \theta}] \quad (C-5)$$

$$F_m^- = \frac{\alpha^2 d_m}{\alpha \nu_{mn}^2 - \epsilon \omega_n^2} [1 - (-1)^m e^{-\frac{\epsilon}{2\alpha} \sin \theta}] \quad (C-6)$$

$$G_m = \frac{e^{(e_m - \frac{\epsilon}{2} \cos \theta)} - 1}{2e_m - \epsilon \cos \theta} + \frac{e^{-(e_m + \frac{\epsilon}{2} \cos \theta)} - 1}{2e_m + \epsilon \cos \theta} \quad (C-7)$$

The Fourier coefficients can then be written as:

$$D_m = 4(2e_m \cosh e_m + \epsilon \cos \theta \sinh e_m)^{-1} F_m^+ \quad (C-8)$$

$$E_{nm} = \frac{2\epsilon \omega_n^2}{\alpha \nu_{mn}} A_n F_m^+ \quad (C-9)$$

The coefficients A_n and ω_n are again taken from Eq.A-3 and Eq.A-2 respectively, for a tilt angle θ . The average rate along the face $z_D = 0$ is given by:

$$\bar{q}_{Dz} = -\cos \theta + \cos \theta e^{\frac{\epsilon}{2} \cos \theta} \left[\sum_{m=1}^{\infty} e_m D_m F_m^- - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_n E_{nm} F_m^- \right] \quad (C-10)$$

The average rate along the face $x_D = 0$ is given by:

$$\begin{aligned} \bar{q}_{Dx}(x_D = 0) = & -\sin \theta \\ & + \cos \theta e^{\frac{\epsilon}{2} \cos \theta} \left[\sum_{m=1}^{\infty} d_m D_m G_m \right. \\ & \left. - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_n d_m}{\epsilon \omega_n^2} E_{nm} e^{-\nu_{nm}^2 t_D} \right] \end{aligned} \quad (C-11)$$

The average rate along the face $x_D = 1$ is:

$$\begin{aligned} \bar{q}_{Dx}(x_D = 1) = & -\sin \theta \\ & + \cos \theta e^{\frac{\epsilon}{2} \cos \theta} \left[\sum_{m=1}^{\infty} (-1)^m d_m D_m G_m \right. \\ & \left. - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^m \frac{a_n d_m}{\epsilon \omega_n^2} E_{nm} e^{-\nu_{nm}^2 t_D} \right] \end{aligned} \quad (C-12)$$

A method of comparison was used to accelerate convergency in the infinite-sum calculations. In general, let the original sum be denoted $s(\theta) = \sum s_m(\theta)$. Close inspections shows that for a zero tilt angle, the terms $s_m(0)$ decay approximately as $1/m^2$, and the original form of the sum can then be replaced by the following calculation:

$$s(0) = s_{\infty} + \sum_{m=0}^{\infty} \left[s_m(0) - \sigma_0 \frac{(2m-1)}{m^2} \right] \quad (C-13)$$

The constant σ_0 is found as the ratio $s_m(0)/m^2$ for a sufficiently large value of m . The sum in Eq.C-13 converges much more rapidly than the original form, and the sum of the comparing series s_{∞} can be found from standard tables, e.g Ref.14.

For a non-zero tilt angle, the original form is replaced by the following calculation:

$$s(\theta) = s(0) + s_c + \sum_{m=0}^{\infty} [s_m(\theta) - s_m(0) - \sigma \sigma_m] \quad (C-14)$$

where $\sigma_m = 1/m^2$ or $\sigma_m = (-1)^m/m^2$ depending on whether or not the original series is oscillating. The sum $s(0)$ is obtained from Eq.C-13. The constant σ is computed in the same manner as for σ_0 . The sum s_c can be found from standard tables, e.g Ref.14.

Table 5.10: Liquid and gas relative permeabilities.

Figure 5.39: Two-dimensional matrix block.

Figure 5.40: Equilibrium rate versus aspect ratio (non-tilted block).

Figure 5.41: Equilibrium rate versus bond number (non-tilted block).

Figure 5.42: Reinfiltration to drainage ratio versus time (non-tilted block).

Figure 5.43: Reinfiltration to drainage ratio versus time (non-tilted block).

Figure 5.44: Net recovery versus aspect ratio (non-tilted block).

Figure 5.45: Net recovery versus bond number (non-tilted block).

Figure 5.46: Drainage to reinfiltration ratio versus time (non-tilted block).

Figure 5.47: Drainage to reinfiltration ratio versus time (non-tilted block).

Figure 5.48: Equilibrium rates versus tilt angle.

Figure 5.49: Total equilibrium drainage rate versus tilt angle.

Figure 5.50: Total equilibrium drainage rate versus tilt angle.

Figure 5.51: Equilibrium gas-saturation contours of a tilted block.

Figure 5.52: Equilibrium gas-saturation contours of a tilted block.

Figure 5.53: Drainage and imbibition capillary pressure of the example problem.

Figure 5.54: Computed reinfiltration rate with and without hysteresis effect.

Figure 5.55: Computed reinfiltration rate with and without hysteresis effect.

Table 1: Liquid- and gas relative-permeabilities for the hysteresis example.

S_o	k_{ro}	k_{rg}
0.20	0.000	0.700
0.40	0.005	0.300
0.48	0.010	0.250
0.56	0.040	0.170
0.70	0.160	0.080
0.80	0.370	0.040
0.84	0.460	0.025
0.88	0.600	0.014
0.92	0.720	0.005
1.00	1.000	0.000

Figure 1: Two-D Matrix Block.

FIG 2: EQUILIBRIUM RATE VS. ASPECT RATIO (NON-TILTED BLOCK)

FIG 3: EQUILIBRIUM RATE VERSUS BOND NUMBER (NON-TILTED BLOCK)

FIG 4: REINFILTRATION TO DRAINAGE RATE RATIO VS TIME (NON-TILTED BLOCK)

FIG 5: REINFILTRATION TO DRAINAGE RATE RATIO VS TIME (NON-TILTED BLOCK)

FIG 6: NET RECOVERY VS. ASPECT RATIO (NON-TILTED BLOCK)

FIG 7: NET RECOVERY VS. BOND NUMBER (NON-TILTED BLOCK)

FIG 8: DRAINAGE TO REINFILTRATION RATE RATIO VS. TIME (NON-TILTED BLOCK)

RATE RATIO

FIG 9: DRAINAGE TO REINFILTRATION RATE RATIO VS. TIME (NON-TILTED BLOCK)

FIG 10: EQUILIBRIUM RATES VERSUS TILT ANGLE

EQUILIBRIUM RATE

FIG 11: TOTAL EQUILIBRIUM DRAINAGE RATE VERSUS TILT ANGLE

EQUILIBRIUM RATE

FIG 12: TOTAL EQUILIBRIUM DRAINAGE RATE VERSUS TILT ANGLE

CONTOUR INTERVAL 0.0500

FIG 13: EQUILIBRIUM GAS-SATURATION CONTOURS OF A TILTED BLOCK.

CONTOUR INTERVAL 0.05

FIG 14 : EQUILIBRIUM GAS-SATURATION CONTOURS OF A TILTED BLOCK.

Fig. 15- Drainage and imbibition capillary pressure of the example problem.

Fig. 16- Computed reinfiltration rate with and without hysteresis effect.

Fig. 17- Computed reinfiltration rate with and without hysteresis effect.

CHAPTER 6

CONCEPT OF GAS-OIL GRAVITY DRAINAGE IN LAYERED POROUS MEDIA

Antonio Cláudio F. Corrêa, and Abbas Firoozabadi

OVERVIEW

Gas-oil gravity drainage in layered porous media may have similarities with gas-oil drainage in fractured porous media. In this chapter through the study of gas-oil drainage in layered media, the effect of capillary pressure contrast will be studied. In addition, the stability of gas-oil drainage in fractured porous media will also be examined.

INTRODUCTION

Oil drainage in gas-oil gravity drainage process in homogeneous porous media has two important characteristics: 1) the gas-oil front is stable, and 2) the recovery performance is nearly independent of capillary pressure over a wide range (where residual oil to gas, S_{org} , is not affected). Due to both low values of S_{or} to gas and the above two characteristics, gravity drainage is recognized as an efficient process for homogeneous reservoirs.

To our knowledge, there is no published work on gas-oil immiscible drainage in layered heterogeneous porous media. Our research work in gas-oil flow in fractured porous media has led us to question the use of averaging permeability in the study of gas-oil gravity drainage flow in layered porous media. An important practical question is the difference in characteristics of gravity drainage in homogeneous and layered reservoirs.

In this chapter, we will first formulate the gas-oil gravity drainage process in a homogeneous porous media by means of a non-linear form of the diffusivity equation. Then the concept of gas-oil gravity drainage in layered porous media will be presented. Features of gravity drainage in layered media will be shown in three numerical examples. The chapter concludes with a discussion.

MODELLING OF GRAVITY DRAINAGE IN HOMOGENEOUS MEDIA

Modelling of the gravity drainage process may be accomplished by combining the continuity equation and Darcy's law. For isothermal immiscible flow with constant porosity and incompressible oil phase, one may derive:

$$\frac{\partial}{\partial z} \left[\frac{k k_{ro}}{\mu_o} \left(\frac{\partial p_o}{\partial z} + \rho_o g \right) \right] = \phi \frac{\partial S_o}{\partial t} . \quad (6.1)$$

In this work we will assume that gas can, without resistance, instantaneously occupy any void space left by oil draining out of the rock (i.e., infinite gas mobility or inviscid gas flow).

Pressure and saturation of the oil phase are related by the capillary pressure curve. Assuming that the gas phase density is constant, such that $p_g(z) = p_g(z = 0) + \rho_g g z$,

and assigning a zero value to $p_g(z = 0)$, then the capillary pressure is related to the oil pressure by:

$$P_c(S_o) = \rho_g g z - p_o. \quad (6.2)$$

(Note that $p_g(z = 0) = 0$ is a matter of convenience and that constant values will not change the derivations.)

Drainage capillary pressure may be represented by¹:

$$P_c(S_o) = P_c^0 - \sigma \ln \left(\frac{S_o - S_{or}}{1 - S_{or}} \right), \quad P_c \geq P_c^0, \quad (6.3)$$

where P_c^0 is the threshold capillary pressure. Other symbols are defined in the nomenclature.

Oil relative permeability is assumed to be described by the power-law model:

$$k_{ro}(S_o) = k_{ro}^0 \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^\lambda. \quad (6.4)$$

Equations 6.1 to 6.4 may be used to model the gravity drainage process in a one-dimensional porous media. For the purpose of this study, we seek a formulation in terms of pressure. The reasoning for pressure formulation will be discussed later. Writing Eq. 6.3 in terms of S_o , one can obtain:

$$\begin{aligned} S_o(P_c) &= S_{or} + (1 - S_{or}) e^{-(P_c - P_c^0)/\sigma}; \quad P_c \geq P_c^0, \\ &= 1; \quad P_c < P_c^0. \end{aligned} \quad (6.5)$$

From Eqs. 6.4 and 6.5 it follows that the oil relative permeability may be described as a function of the capillary pressure, $k_{ro} = k_{ro}(P_c)$. For constant absolute permeability and constant viscosity, the relationship given by Eq. 6.2 is used to transform Eq. 6.1 to:

$$\frac{\partial}{\partial z} \left[\frac{k_{ro}}{k_{ro}^0} \left(\Delta \rho g - \frac{\partial P_c}{\partial z} \right) \right] = \frac{\phi \mu_o}{k k_{ro}^0} \frac{dS_o}{dP_c} \frac{\partial P_c}{\partial t} \quad (6.6)$$

in which $\Delta \rho = \rho_o - \rho_g$. Note that dS_o/dP_c in Eq. 6.6 has units of pressure reciprocal. It may be regarded as an apparent compressibility, defined as:

$$c_o = - \frac{dS_o}{dP_c} \quad (6.7)$$

Combining Eqs. 6.5 and 6.7, the following expression for the apparent compressibility is derived:

$$\begin{aligned} c_o &= \frac{1-S_{or}}{\sigma} e^{-(P_c - P_c^0)/\sigma} ; \quad P_c \geq P_c^0 , \\ &= 0 ; \quad P_c < P_c^0 . \end{aligned} \quad (6.8)$$

Defining the capillary potential as:

$$\Phi(z, t) = \Delta \rho g z - P_c(z, t) , \quad (6.9)$$

and noting that $\partial \Phi / \partial t = -\partial P_c / \partial t$ then Eq. 6.6 transforms to a non-linear form of the diffusivity equation given by:

$$\frac{\partial}{\partial z} \left(\frac{k_{ro}}{k_{ro}^0} \frac{\partial \Phi}{\partial z} \right) = \frac{\phi \mu_o c_o}{k k_{ro}^0} \frac{\partial \Phi}{\partial t} \quad (6.10)$$

Using the dimensionless variables $z_D = z/h$, $t_D = k k_{ro}^0 t / \phi \mu_o c_o^0 h^2$ and $\Phi_D(z_D, t_D) = \Phi(z, t) / \Delta \rho g h$, Eq. 6.10 becomes a dimensionless gravity drainage equation:

$$\frac{\partial}{\partial z_D} \left(\tau \frac{\partial \Phi_D}{\partial z_D} \right) = c \frac{\partial \Phi_D}{\partial t_D} , \quad (6.11)$$

in which $\tau = k_{ro}/k_{ro}^0$ and $c = c_o/c_o^0$ are the normalized oil relative permeability and apparent compressibility, respectively. The parameter c_o^0 is the apparent compressibility evaluated at P_c^0 , which is given by $c_o^0 = (1 - S_{or})/\sigma$.

Equation 6.11 describes the gas-oil flow assuming an incompressible oil-phase. When only the oil phase is flowing, as it happens below the gas front, the right-hand side of Eq. 6.11 vanishes and an incompressible single-phase flow equation is obtained. Another interesting aspect of Eq. 6.11 is that when both linear oil relative permeability and zero threshold pressure are considered, it becomes equivalent to the saturation equation of Ref. 2.

The oil flowrate at a given point of the medium may be computed from:

$$q_o(z, t) = \frac{k k_{ro} A}{\mu_o} \left(\frac{\partial p_o}{\partial z} + \rho_o g \right), \quad (6.12)$$

in which flow is positive downwards. Defining a dimensionless oil flow rate as:

$$q_D(z_D, t_D) = \frac{q_o(z, t) \mu_o}{k k_{ro} A \Delta \rho g}, \quad (6.13)$$

then it follows that:

$$q_D(z_D, t_D) = \tau \frac{\partial \Phi_D}{\partial z_D}. \quad (6.14)$$

Equation 6.11 is not restricted to the capillary pressure and relative permeability models of Eqs. 6.3 and 6.4, respectively. It may be applied to other capillary pressure and relative permeabilities models, as long there is a one-to-one functional relationship between capillary pressure and oil saturation.

VERIFICATION OF THE MODEL

A numerical model based on the equations described in the previous section has been developed. Fully-implicit finite difference and non-uniform gridding techniques have been employed in the numerical simulator. The model has been used to interpret the results of a gravity drainage experiment performed in a long homogeneous block of Berea sandstone.³ Normal decane and air were used as displaced and displacing fluids, respectively.

The long core as well as the space between its vertical faces and the glass walls have

been fully saturated with normal decane before the experiment began. Because the top face of the core was open to air at atmospheric pressure, the oil pressure at the top face at the start of drainage was equal to the negative of the threshold capillary pressure of the rock.

Flow started by opening a valve located at the bottom of the apparatus, allowing liquid to be produced at atmospheric conditions. Initially a very fast transient develops in the system due to the instantaneous dropping of the liquid pressure at the bottom of the apparatus from $p_o = \Delta\rho_g g h - P_c^0$ at $t = 0$ to $p_o = 0$ at $t = 0^+$. Because of low rock and liquid compressibilities and limited dimensions of the porous medium, such transient effects last a few seconds only. Hence, liquid can be considered incompressible at experimental conditions, and a dynamic pressure gradient as being instantaneously established.

The initial condition to be used in conjunction with Eq. 6.11 may be obtained from the equation for the dynamic liquid pressure profile, a straight line defined by $p_o(z = 0, t = 0^+) = 0$ and $p_o(z = h, t = 0^+) = \rho_g g h - P_c^0$, yielding:

$$\Phi_D(z_D, 0^+) = \left(1 - \frac{P_c^0}{\Delta\rho_g g h}\right) z_D. \quad (6.15)$$

Substituting this expression into Eq. 6.14 and noting that initially $\tau = 1$, a formula for the initial rate of drainage is then obtained:

$$q_D(z_D, 0^+) = 1 - \frac{h^0}{h}, \quad 0 \leq z_D < 1, \quad (6.16)$$

in which the relation $P_c^0 = \Delta\rho_g g h^0$ has been used, where h^0 is the threshold height of the medium. A similar expression was derived by Barkve and Firoozabadi⁴.

During the experiment, the bottom face of the tall block was kept immersed in oil at atmospheric pressure, allowing the following lower boundary condition to be used:

$$\Phi_D(0, t_D) = 0. \quad t_D > 0. \quad (6.17)$$

The fact that the top face of the tall block was opened to air, and that no oil was supplied through it during the experiment, gives the following no-flow upper boundary condition:

$$\left(\frac{\partial \Phi_D}{\partial z_D} \right)_{z_D = 1} = 0, \quad t_D > 0. \quad (6.18)$$

Initial and boundary conditions described by Eqs. 6.15, 6.17 and 6.18 have been incorporated in a numerical simulator for interpreting the tall block experiment. The measured rock, fluid and rock/fluid data are $k = 750 \text{ mD}$, $\phi = 0.22$, $h = 181.45 \text{ cm}$, $A = 229.5 \text{ cm}^2$, $\mu_o = 0.866 \text{ cp}$, $\Delta\rho = 0.724 \text{ g/cm}^3$, $\sigma = 0.17 \text{ psi}$, $P_c^o = 0.42 \text{ psi}$ and $S_{or} = 0.26$. Air density was assumed to be negligible. These values have been used in the numerical model to match the experimental data. The only experimentally unknown parameter was the exponent of the relative permeability curve.

Figure 6.1 displays the history matching of the experimental data. The relative permeability exponent used in the history matching was $\lambda = 4$. A good match has been obtained for both oil rate and cumulative production, except for the early period. At very early time, the disagreement between experimental data and simulated results is due to the film flow along the vertical faces of the tall block, which has not been accounted for in the one-dimensional numerical model. However, the estimated volume of oil production from film flow (70 cm^3) has been added to the simulated cumulative production of Figure 6.1.

The computed saturation history and pressure profiles for the experiment are displayed in Figure 6.2. Both the initial and equilibrium pressure profiles, at $t = 0^+$ and $t = \infty$, respectively, are also displayed. As the medium desaturates, the slope of the pressure profile in the single phase region, below the gas front, decreases, and consequently oil drainage rate also decreases. At capillary-gravity equilibrium, $dp_o/dz = -\rho_o g$, and therefore $q_o = 0$.

MULTILAYER GRAVITY DRAINAGE MODEL

We next consider the case of a heterogeneous media composed of several horizontal ho-

homogeneous layers in perfect capillary contact.

Assuming both an inviscid gas phase and an incompressible oil phase, Eq. 6.11 may be used to describe the vertical flow through each layer, yielding the following system of partial differential equations for an N-layer arrangement:

$$\frac{\partial}{\partial z_D} \left(\kappa_j \tau_j \frac{\partial \Phi_{Dj}}{\partial z_D} \right) = \omega_j c_j \frac{\partial \Phi_{Dj}}{\partial t_D};$$

$$z_{D(j-1)} < z_D < z_{Dj}, \quad j = 1, N, \quad (6.19)$$

in which $z_{D0} = 0$ and $z_{DN} = 1$. The dimensionless variables of Eq. 6.19 have been defined with respect to the average properties of the layers and the total thickness, h , that is: $z_D = z/h$, $t_D = \overline{k k_{ro}^0} t / \mu_o \overline{\phi c_o^0} h^2$ and $\Phi_D(z_D, t_D) = z/h - P_c(z, t) / \Delta \rho g h$. The average permeability and storativity are given respectively by $\overline{k k_{ro}^0} = \left[\sum_1^N (h_j/h) / (k_j k_{roj}^0) \right]^{-1}$ and $\overline{\phi c_o^0} = \sum_1^N \phi_j c_{oj}^0 (h_j/h)$. The normalized relative permeability and apparent compressibility of each layer are $\tau_j = k_{roj} / k_{roj}^0$ and $c_j = c_{oj} / c_{oj}^0$, respectively.

The following parameters, describing respectively the permeability and the storativity ratios with respect to the average values, have also been introduced in Eq. 6.19:

$$\kappa_j = \frac{k_j k_{roj}^0}{\overline{k k_{ro}^0}}, \quad (6.20)$$

$$\omega_j = \frac{\phi_j c_{oj}^0}{\overline{\phi c_o^0}}. \quad (6.21)$$

The dimensionless oil flow rate of layer j , defined as:

$$q_{Dj}(z_D, t_D) = \frac{q_{oj}(z, t) \mu_o}{\overline{k k_{ro}^0} A \Delta \rho g}, \quad z_{j-1} < z < z_j, \quad (6.22)$$

may be computed at any point of the corresponding layer by:

$$q_{Dj}(z_D, t_D) = \kappa_j \tau_j \frac{\partial \Phi_{Dj}}{\partial z_D}, \quad z_{D(j-1)} < z_D < z_{Dj}. \quad (6.23)$$

In order to solve the general gravity drainage problem described by the non-linear system of Eq. 6.19, equations for adjacent layers are coupled assuming both pressure and flow continuity at the interface, that is:

$$\Phi_{D(j-1)}(z_{Dj}, t_D) = \Phi_{Dj}(z_{Dj}, t_D), \quad (6.24)$$

and:

$$\kappa_{j-1} \tau_{j-1} \left[\frac{\partial \Phi_{D(j-1)}}{\partial z_D} \right]_{z_{Dj}} = \kappa_j \tau_j \left[\frac{\partial \Phi_{Dj}}{\partial z_D} \right]_{z_{Dj}} \quad (6.25)$$

Equations 6.19, 6.24 and 6.25 may be used to model the gravity drainage process in layered media provided certain criteria are met. For the purpose of this study, the following boundary conditions will be employed:

$$\Phi_{D1}(0, t_D) = 0, \quad t_D > 0, \quad (6.26)$$

and:

$$\left[\frac{\partial \Phi_{DN}}{\partial z_D} \right]_{z_D=1} = 0, \quad t_D > 0. \quad (6.27)$$

The understanding of the gravity drainage process can be strongly improved by analyzing the case of a two-layer media.

GRAVITY DRAINAGE CONCEPT

Two cases may be devised - the more permeable layer either on top or in the bottom of a two-layer media. Because these cases do present very different features, they will be studied independently. However, in order to appreciate the concept of gas-oil gravity drainage in layered media, we will first present the pressure profiles of single-phase free drainage in layered systems. In either case, k_1 denotes the permeability of the bottom medium and k_2 the permeability of the top layer. (The properties used to compute the pressure profiles of Figs. 6.3 to 6.6 are given in the numerical examples section).

Single-Phase Flow in Layered Media

For a homogeneous medium with zero pressure at both end faces, $dp_o/dz = 0$, and therefore, $q_o = k A \rho_o g / \mu_o$, which will be referred as the free drainage rate of the medium. That is, the free drainage rate is associated to a vertical oil pressure profile along the porous medium.

In a two-layer media the single-phase flowrate in layers 1 and 2 are given by:

$$q_{o1} = \frac{k_1 A}{\mu_o} \left(\frac{dp_{o1}}{dz} + \rho_o g \right), \quad (6.28)$$

and:

$$q_{o2} = \frac{k_2 A}{\mu_o} \left(\frac{dp_{o2}}{dz} + \rho_o g \right), \quad (6.29)$$

where flow is positive downwards.

We will assume the pressure at the top face of layer 2 and at the bottom face of layer 1 to be zero. Let's denote the pressure of the neighboring faces of the layers by p_i . Equating the flow rates in the two layers, the following expression for p_i is derived:

$$p_i = \frac{\rho_o g (k_2 - k_1)}{\frac{k_1}{h_1} + \frac{k_2}{h_2}}. \quad (6.30)$$

The pressure profiles across the two-layer system for the cases $k_2 > k_1$ and $k_2 < k_1$ are shown in Fig. 6.3. Note that the slopes of the pressure profiles in layers 1 and 2 change sign, depending on the sign of $k_2 - k_1$. The flow rate delivered by the system is given by:

$$q_o = \frac{\bar{k} A \rho_o g}{\mu_o}, \quad (6.31)$$

where $\bar{k}$ is the harmonic average permeability given by:

$$\bar{k} = h \cdot \left(\frac{h_1}{k_1} + \frac{h_2}{k_2} \right)^{-1}, \quad (6.32)$$

in which $h = h_1 + h_2$.

According to Eq. 6.31, single-phase flow rate is not dependent on the order of the layers. Equation 6.32 indicates that $\bar{k}$ lies in the k_1 to k_2 range. The flow process for two-phase gas-oil gravity drainage is of a different nature, as we will witness next.

More Permeable Layer on Top

As flow starts, a dynamic pressure profile instantaneously develops in the system. Such initial pressure profile may be computed by neglecting the time derivative of Eq. 6.19 and solving the resulting ordinary differential equation (with $\tau = 1$), together with the coupling conditions and the lower boundary given by Eqs. 6.24 to 6.26. The upper boundary condition is obtained by setting $\Phi_{D2}(1, 0) = 1 - h_2^0/h$, in which $h_2^0 = P_{c2}^0/\Delta\rho g$ is the threshold height of the top layer (more permeable layer in this case). The oil pressure at the interface, at the start of drainage, can be computed by equating the rates of both layers and applying the boundary conditions, yielding:

$$p_i = \frac{\rho_o g k_2}{\frac{k_1}{h_1} + \frac{k_2}{h_2}} \cdot \left[\left(1 - \frac{k_1}{k_2}\right) - \frac{h_2^0}{h_2} - \frac{\rho_g}{\rho_o} \frac{h - h_2^0}{h_2} \right] \quad (6.33)$$

From the above equation p_i can be either positive or negative. In this case, because $k_1/k_2 < 1$, then we have $p_i > -\Delta\rho g h_2^0(h_1/h) - \rho_g g h_1$. (This limiting condition can be obtained by setting $k_1 = k_2$ in Eq. 6.33). Note that the term $-\rho_g g h_1$ corresponds to the gas pressure at the layers' interface. Therefore p_i will be always greater than $-P_{c2}^0 - \rho_g g h_1$, that is, the oil pressure at the layers' interface will be greater than the gas pressure at the same location. Referring to a fluid gradient as the rate of increase in fluid pressure with depth ($-dp/dz$), we could say that in this case the dynamic oil gradient along the top layer will be always greater than the gas gradient ($-dp_o/dz > \rho_g g$). As we will see, this condition leads to a stable gravity drainage process.

The equation for the initial rate of drainage is:

$$q_{oi} = \frac{\bar{k} A \Delta\rho g}{\mu_o} \cdot \left(1 - \frac{h_2^0}{h}\right), \quad (6.34)$$

in which $\bar{k}$ is the harmonic average permeability of the system. Equation 6.34 indicates that the initial rate delivered by a layered medium with the more permeable layer on top is less than the single-phase flow rate of the system.

Figure 6.4 displays the initial and equilibrium pressure profiles for the layered system. Observe that the oil pressure profile in the bottom layer (less permeable) has been shifted clockwise with respect to the vertical axis, indicating that the initial rate of drainage through that layer is greater than its free gravity drainage rate. The opposite effect appears in the top layer (more permeable), which produces initially at a rate less than its free gravity drainage rate. The system acts as if there is a positive head being applied to the bottom layer. In the limit of infinite permeability of the top layer, the system will be equivalent to a single layer subject to an initial hydrostatic head with magnitude equal to the height of the top layer.

Another interesting aspect is that initially the oil pressure in the bottom layer and in part of the top layer is positive. This may appear inconsistent, because it implies a negative capillary pressure, which cannot be defined for a drainage process. However, we must keep in mind that initially only oil is flowing through the system and, because no gas is present, the concept of capillary pressure becomes meaningless.

More Permeable Layer in the Bottom

In this case $k_1 > k_2$, which implies, from Eq. 6.33, that the oil pressure at the interface is negative ($p_i < 0$). The limiting condition is again obtained by making $k_1 = k_2$ in Eq. 6.33, which results in $p_i < -P_{c2}^0 \cdot (h_1/h) - \rho_g g h_1$. Depending on permeability and thickness ratios, p_i may assume values: 1) in the interval $-P_{c2}^0 - \rho_g g h_1 < p_i < -P_{c2}^0 \cdot (h_1/h) - \rho_g g h_1$, and 2) less than $-P_{c2}^0 - \rho_g g h_1$. For the first case, the initial dynamic oil gradient along the top layer is greater than the gas gradient ($-dp_o/dz > \rho_g g$), and the system behaves as the case of more permeable layer on top.

A more complex situation occurs when $p_i < -P_{c2}^0 - \rho_g g h_1$, which defines a dynamic oil gradient along the top layer less than the gas gradient ($-dp_o/dz < \rho_g g$). Recall that the

assumption of infinite gas mobility implies in identical static and dynamic gas gradients. The initial pressure profile for this case is displayed in Fig. 6.5. Imposing the same flow rate in both layers causes the rotation of the pressure profile either clockwise or counterclockwise with respect to the vertical axis, according to the departure of the flow rate of each layer from its free gravity drainage rate.

Figure 6.5 implies that the gas pressure decreases along the top layer. This contradicts the assumption of infinite gas mobility. Therefore, $-dp_o/dz < \rho_g g$ may not realize. This type of behavior leads to the identification of an unstable flow displacement.

In such displacement, gas can finger through the top layer to contact the bottom layer instantaneously, to allow gradual desaturation of the bottom layer to take place. Although pressure continuity at the interface exists, both media shall no longer produce at the same rate. Hence, the initial pressure profile must be revised. To the best of our knowledge, no experimental or theoretical evidences of such unstable gravity drainage displacement have been reported in the literature.

A stability criterion can be derived by noticing that the gas front can not instantaneously invade the less permeable layer when $p_i > -P_{c2}^0 - \rho_g g h_1$. This assures that Eq. 6.34 remains valid. Thus, no gas fingering will exist if the dynamic oil gradient along the top layer exceeds the gas gradient ($-dp_o/dz > \rho_g g$), which defines the following criterion for a stable gravity drainage process:

$$q_o < \frac{k_2 A \Delta \rho g}{\mu_o} . \quad (6.35)$$

This equation also holds in case of restricted gravity drainage, where the system produces initially below its maximum drainage rate, which happens when the outlet oil pressure, p_{out} is larger than the gas pressure. Equating the flow rate for both layers and substituting the result into Eq. 6.35, the stability criterion becomes:

$$p_{out} > p_g(z=0) + \Delta \rho g h_1 \left(1 - \frac{h_2^0}{h_1} - \frac{k_2}{k_1} \right) . \quad (6.36)$$

If the system is draining at its maximum gravity drainage rate, such that $p_{out} = p_g(z = 0)$, then flow will be stable if:

$$\frac{k_2}{k_1} > 1 - \frac{h_2^0}{h_1} \quad (6.37)$$

The above condition could be hardly satisfied in layered reservoirs, and therefore one may expect unstable drainage behavior for many layered oil reservoirs.

The problem is now how to find both the initial rate and the initial pressure profile in the case of unstable flow.

If gas fingers through the top zone, it may spread along the top face part of the bottom layer. From pressure continuity concept, the capillary pressure at the interface must be equal to the threshold pressure of the bottom medium. Because gas fingering is a localized phenomenon, the top zone shall produce under an independent gravity drainage mechanism. Desaturation of the bottom zone takes place from the very beginning, defining a new initial flowing pressure profile, which is presented in Figure 6.6. Thus, the initial rate of drainage from the bottom layer can be computed from:

$$q_{oi} = \frac{k_1 A \Delta \rho g}{\mu_o} \cdot \left(1 - \frac{h_1^0}{h_1} \right) \quad (6.38)$$

We will next describe some numerical examples of gravity drainage process in layered media.

NUMERICAL EXAMPLES

In the numerical examples that follows, the properties of the less permeable layer are the same as those of the tall block. The more permeable layer has $k = 7,500 mD$, $\sigma = 0.085 psi$ and $P_c^0 = 0.21 psi$. Other properties are identical to those of the tall block. Each layer is half of the tall block's height.

Figure 6.7 displays both rate and cumulative production for the two cases to be described in this section, as well as results for two homogeneous media with properties of the individual layers. Ultimate recovery for each case can be computed by integrating

Eq. 6.5 along each layer at conditions of capillary-gravity equilibrium. In order to have a general picture of the saturation behavior in layered media, saturation histories for both homogeneous and layered systems are presented in Figure 6.8. We will discuss each case separately.

More Permeable Layer on Top

As we have seen, when the outlet oil pressure and the gas pressure are equal, the layered system of this case undergoes a stable gravity drainage process. Pressure and saturation profiles at several times are presented in Figure 6.9.

Initially only the top layer desaturates. The position of the gas front is defined by a sharp change in the pressure gradient of the top layer. In the two-phase flow region pressure is nearly constant. However, because the capillary pressure curve of the top layer is very flat, large saturation changes are observed for small variations in the oil pressure. The saturation profile in the top layer is characterized by a shock front with a decreasing oil saturation tail, which is a distinctive feature of immiscible displacement processes at low capillary pressure.

Below the gas front, the pressure profile is linear. In the top layer the flowing pressure profile is shifted counterclockwise, indicating that the system is producing at a rate less than the top layer's free drainage rate. (See the pressure profiles at $t = 100$ and $t = 0$ minutes). The opposite effect appears in the bottom layer, which yields more than its free gravity drainage rate.

As the gas front reaches the interface, there is a large saturation change in the bottom face of the top layer in a short time span. Due to capillary continuity and to the large difference in the capillary pressure curve of the two layers, the bottom layer starts to desaturate only when very low saturation values are attained in the bottom face of the top layer. In fact, this happens when the capillary pressure at the interface becomes equal to the bottom layer's threshold pressure. Such a constraint causes a large change in the pressure profile of the bottom layer in a very short time, dropping substantially the rate

delivered by the system, as shown in Fig. 6.7 ($t \approx 250 \text{ min}$).

As time progresses, oil desaturation continues to take place in both layers, and the oil pressure profile progressively shifts to the left and rotates counterclockwise, indicating a decrease in the oil rate. However, when gas starts invading the bottom layer, most of the oil in the top layer has already been produced, and desaturation in the bottom layer proceeds as in a standard gravity drainage mechanism. As it can be seen in Fig. 6.7, this is a very efficient process in terms of recovery, because the flow rate is nearly constant for a long time. Desaturation of the bottom layer may not even happen when its threshold height is greater than its thickness ($P_{c1}^0 > \Delta \rho g h_1$).

More Permeable Layer in the Bottom

The evolution of both pressure and saturation profiles, for the case of the more permeable layer in the bottom, is presented in Fig. 6.10. For the layer properties used in this example, the displacement process is unstable. We must emphasize that, although this is an unstable process, capillary continuity exists at the interface, due to the localized nature of the gas fingering phenomenon.

At early times, the saturation profile in the top layer (less permeable) is diffused, indicating that capillarity plays a role in the mechanism. Due to the inherent flow instability, desaturation of the bottom layer starts at the beginning of the displacement. In both layers, sharp changes in the pressure profiles characterize the corresponding gas fronts.

Because the bottom layer is free to drain, the initial rate of drainage of the two-layer system is close to that of an equivalent homogeneous medium with properties of the more permeable layer. Also, when the top layer's gas front is far from the media interface, the drainage mechanism in the top layer acts independently.

The initial rate delivered by the top layer depends on the interface oil pressure, which, in this case, is equal to minus the bottom layer's threshold pressure. The more permeable the bottom layer is (less capillary threshold), the less is the initial rate delivered by the

upper layer.

An important conclusion can be drawn by observing the production history of Fig. 6.7. Although the oil rate is high at the start of drainage, gas infiltrating underneath the top layer slows down the production of the less permeable medium. This is mainly associated to the slopes of the capillary pressure curves of both media and to relative permeability effects.

At low oil saturations, large change in capillary pressure causes only a small variation in the oil saturation. Also, at these levels of oil saturations, there is a significant reduction in the effective oil permeability. These constraints cause the capillary pressure in the uppermost part of the bottom layer (more permeable) to be maintained at relatively low levels, as can be observed in Fig. 6.10. Hence, due to the capillary continuity, oil saturation at the bottom of the top layer (less permeable) is high, reducing the rate of drainage of such medium. This is likely to be amplified in a multilayer media.

Four-layer System

We now consider a four layer system composed of the two distinct porous media whose properties have been previously described, alternately placed on top of each other, with the more permeable layer in the bottom. Each layer has one fourth of the tall block's height, and they are labeled from bottom to top as layers 1 to 4. Figure 6.11 displays both rate and cumulative production histories of the system, and Fig. 6.12 displays oil pressure and oil saturation profiles at several times.

Initially flow is unstable in the top layer. The initial pressure profile has been computed considering such unstability, letting the third layer to undergo gravity drainage from the beginning of the displacement. This situation leads to a stable gravity drainage mechanism throughout the lower three layers. The initial rate of drainage is then given by a system composed of two high permeability layers with a low permeability layer in between.

At short time, the displacement is characterized by frontal gas displacements in the two uppermost layers. At time around 75 minutes the bottom face of layer 3 (more

permeable) starts to desaturate. Because layer 2 (less permeable) has a higher capillary threshold than layer 3, oil saturation at the bottom face of layer 3 has to drop significantly before gas enters layer 2, what takes place at around 100 minutes. This is associated to a reduction of the oil rate delivered by the system.

As layer 2 desaturates, undergoing a stable displacement process, the oil pressure at the interface of layers 1 and 2, as well as the oil rate, stay practically constant. This period last from 100 to 120 minutes, when gas reaches the bottom face of layer 2.

Because the capillary pressure at the interface of layers 1 and 2 at the moment that gas reaches this position is equal to layer's 2 threshold pressure (less permeable), in order to honor the capillary continuity among layers either the top part of layer 1 has to desaturate instantaneously or there should be a reversal in the flow direction. The latter option does not reflect mass conservation and the former alternative actually happens. The flow reversal may be observed in the oil pressure profile at 120 minutes. Note that the oil pressure in the bottom part of layer 2 has increased such that the capillary pressure becomes less than the layer's threshold pressure (0.42 *psi*), indicating resaturation. Such flow reversal causes the oil gradient in the lower part of layer 2 to become negative, yielding an unstable situation. This allow gas to finger towards the bottom layer, which becomes free to produce at a rate close to its free drainage rate, yielding a spike in the rate history.

In actual reservoirs, sharp changes in rate may be damped by compressibility or other local heterogeneity effects, but the overall tendency presented here could explain the unexpected healing of some oil wells. These effects become more evident in a ten-layer case.

Ten-layer System

We consider a ten-layer system composed of alternate more and less permeable homogeneous layers, with a more permeable layer in the bottom. The layers properties are the same as given for the four-layer case described previously. The height of each individual

layer is 181.45 *cm* (same height of the tall block), so that the system is 18.15 *m* thick.

Figure 6.13 displays rate and cumulative recovery for this case, labeled as normal IFT (interfacial tension). The first three peaks in rate are due to unstable flow, that is gas front reaching the interfaces of layers 8-9, 6-7 and 4-5, respectively. The fourth spike in rate corresponds to the moment at which the gas front reaches the interface of layers 1-2, allowing the bottom layer to drain.

Figure 6.14 presents both pressure and saturation profiles at different times. From the saturation profile we observe that the recovery at 10^7 *min* (19.6 years) is almost complete, with a small amount of oil left (above residual oil) in the bottom portion of the low permeability layers, and also in the lower portion of the bottom layer (due to capillary threshold).

To show the effect of capillary pressure on gravity drainage in layered media, we have computed a case in which the parameter σ of the capillary pressure model was increased four fold, for both more and less permeable layers. The residual saturation was kept the same. Rate and production results are given in Fig. 6.13, labeled as augmented IFT. As we can see, there is a considerable reduction in recovery after 10^7 *min* of production. Figure 6.15 displays pressure and saturation profiles for this case, and it shows that there is a large amount of oil left in the less permeable layers even at time 10^7 *min*. Although the amount of oil left in the bottom layer due to capillary holdup is greater than in the previous case (normal IFT), the difference is not significant.

To verify this, we have run a case of a homogeneous medium with permeability 2214 *md* (geometrical average of more and less permeable layers), and other properties as given for the ten-layer normal-IFT case. The results are given in Fig. 6.16 labeled as low IFT. The coefficient of the capillary pressure model (σ) was then multiplied by four and a new run was performed. Results are also presented in Fig. 6.16 as the high IFT curves. As we may observe, the difference of the two recovery curves of Fig. 6.16 is much less than that of Fig. 6.13, and it is mainly due to the difference between the capillary threshold pressures of the high and low IFT cases. This emphasizes the point that gravity drainage

in homogeneous and layered systems have different features.

DISCUSSION AND CONCLUSIONS

In this report we have derived a non-linear form of the diffusivity equation to study gas-oil gravity drainage in both homogeneous and layered porous media. The non-linear diffusivity equation in terms of oil pressure offers the advantage of a continuous pressure function in a layered media. Equations in terms of oil saturation have a discontinuity in the boundary which arises from capillary continuity concept.

Gravity drainage has been thought as a stable process. In this report we have shown that drainage in a layered system may be unstable. A criterion for stability in layered systems has been derived. Based on our criteria one suspects that gravity drainage in many field situations could be an unstable process. Our work is based on the assumption of infinite gas mobility. This assumption could be removed. In exchange the problem would require an additional equation for the gas phase pressure. However, the concepts discussed in this report are general enough to be applied to real field problems.

The main conclusions of this report are:

1. A non-linear form of the diffusivity equation can be used to study two-phase gas-oil gravity-drainage flow in both homogeneous and layered heterogeneous porous media,
2. The order of layer placement affects the gravity drainage performance of layered systems,
3. Gravity drainage in layered systems can be, under certain conditions, an unstable flow process,
4. Early arrival of the gas phase underneath the less permeable layer reduces the rate of drainage of this layer, making the gravity drainage process less effective.

Nomenclature

A	=	Cross-sectional area, m^2
c	=	Normalized apparent compressibility, c_o/c_o^0
c_o	=	Apparent compressibility, Pa^{-1}
d	=	Total derivative
g	=	Gravity acceleration, m/s^2
h	=	Height, m
k	=	Absolute permeability, m^2
k_r	=	Relative permeability
p	=	Phase pressure, Pa
P_c	=	Capillary pressure, Pa
q	=	Flow rate, m^3/s
S	=	Saturation
t	=	Time, s
z	=	Elevation from datum, m
$\Delta\rho$	=	Oil-gas density difference, kg/m^3
∂	=	Partial derivative
κ	=	permeability ratio, $k k_{ro}^0/k_1 k_{ro1}^0$
λ	=	Exponent of the relative permeability model
μ	=	Viscosity, $Pa \cdot s$
ρ	=	Density, kg/m^3
σ	=	Parameter of the capillary pressure model, Pa
τ	=	Normalized relative permeability, k_{ro}/k_{ro}^0
ϕ	=	Porosity
Φ	=	Oil potential, m
ω	=	Storativity ratio, $\phi c_o^0/\phi_1 c_{o1}^0$

Subscripts

c	=	Capillary
D	=	Dimensionless
g	=	Gas
i	=	Interface or initial
j	=	Layer index
N	=	Number of layers
o	=	Oil
out	=	Outlet
r	=	Residual
l	=	Bottom layer index

Superscripts

0	=	Threshold or end-point
$-$	=	Average

References

1. Bentsen, R. G., and Anli, J.: "A New Displacement Capillary Pressure Model", *J. Can. Pet. Tech.*, (Sept.-Oct., 1991) 75-79.
2. Firoozabadi, A. and Ishimoto, K.: "Theory of Reinfiltration in Fractured Porous Media: Part 1 - One-Dimensional Model", paper SPE 21796 presented at the SPE Western Regional Meeting, Long Beach, Ca, March 20-22, 1991, to appear in SPE Advanced Technology Series.
3. Firoozabadi, A., and Markeset, T.: "Laboratory Study of Capillary Crossflow in Fractured Porous Media", Chapter 4.

4. Barkve, T., and Firoozabadi, A.: "An Analysis of Reinfiltration in Fractured Porous Media", SPE 24900, paper presented at the 1992 SPE Annual Technical Conference and Exhibition, Washington D.C., Oct. 4-7, 1992, also Chapter 5.

Figure 6.1: Tall block production history.

Figure 6.2: Saturation and pressure profiles, Tall block drainage.

Figure 6.3: Pressure profiles for single-phase flow.

Figure 6.4: Initial and equilibrium pressure profiles, more permeable layer on top.

Figure 6.5: Initial and equilibrium pressure profiles, more permeable layer at the bottom.

Figure 6.6: Initial pressure profile, unstable flow, more permeable layer in the bottom.

Figure 6.7: Production histories in layered systems.

H - More permeable
L - Less permeable

—●— $t = 1 \text{ min}$
—■— $t = 10 \text{ min}$
—▲— $t = 100 \text{ min}$

—+— $t = 1,00 \text{ min}$
—◆— $t = 10,000 \text{ min}$
—○— $t = 100,000 \text{ min}$

Figure 6.8: Saturation profiles, homogeneous and layered media.

$t = 100 \text{ min}$
 $t = 200 \text{ min}$
 $t = 260 \text{ min}$

$t = 280 \text{ min}$
 $t = 10,000 \text{ min}$
 $t = 100,000 \text{ min}$

Figure 6.9: - Saturation and pressure profiles, more permeable layer on top.

—○— $t = 10 \text{ min}$
 —■— $t = 100 \text{ min}$
 —▲— $t = 500 \text{ min}$

—+— $t = 1,00 \text{ min}$
 —◆— $t = 10,000 \text{ min}$
 —○— $t = 100,000 \text{ min}$

Figure 6.10: Saturation and pressure profiles, more permeable layer at the bottom.

Figure 6.11: Rate and cumulative production for 4-layer system.

Figure 6.12: Saturation and pressure profiles for 4-layer system.

Figure 6.13: Rate and cumulative production for 10-layer system.

Figure 6.14: Saturation and pressure profiles for the 10-layer system.

Figure 6.15: Saturation and pressure profiles for the 10-layer system, augmented IFT.

Figure 6.16: Production history for homogeneous systems.