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**User's Manual for EROSION/MOD1:
A Computer Program for Fluids-Solids Erosion**

Topical Report

**R.W. Lyczkowski
J.X. Bouillard
S.M. Folga
S.L. Chang**

September 1992

Work Performed Under Contract No.: DE-AC21-89MC24193

For
U.S. Department of Energy
Office of Fossil Energy
Morgantown Energy Technology Center
Morgantown, West Virginia

By
Argonne National Laboratory
Energy Systems Division
Argonne, Illinois

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Nomenclature

c	Compaction modulus
d_p	Particle diameter, m
dKE_s/dt	Energy dissipation rate as defined by Eq. 2.13 = e_{vs} , W/m ³
$\dot{E}$	Erosion rate, m/s
$\dot{E}_b, \dot{E}_d$	Brittle and ductile erosion rates, in Neilson-Gilchrist erosion model, Eq. 2.6, m/s
$\dot{E}_F$	Finnie erosion rate given by Eq. 2.1, m/s
$\dot{E}_{FM}$	Maximum erosion rate from Finnie's model given by Eq. 2.5, m/s
E_p, E_t	Young's modulus of particle and target, respectively, Pa
E_r	Reduced Young's modulus of elasticity given by Eq. 2.10, Pa
E_{sp}	Specific energy of eroding material (related to hardness), Pa
e	Coefficient of restitution of particle-target collision = V_2/V_1
e_v	Rate of kinetic energy dissipation per unit volume, W/m ³
G	Solids elastic modulus, Pa
k_p, k_t	Elastic constants defined by Eq. 2.10 = $(1 - \gamma_p^2)/(\pi E_p)$ and $(1 - \gamma_t^2)/(\pi E_t)$, respectively, Pa ⁻¹
g	Acceleration due to gravity, m/s ²
$\bar{I}$	Unity tensor
$\dot{m}_s$	Mass flux of solids = $\epsilon_s \rho_s \vec{v}_s $, kg/(m ² · s)
n	Empirical constant in Eq. 2.6
P	Pressure, Pa
P_t	Eroding flow stress or hardness of target, Pa
r	Radial coordinate, m
t	Time, s
U_{ED}	Kinetic energy dissipation per unit area, Eq. 2.14a, W/m ²
U_g, U_s	Gas and solid phase velocities in the x direction, respectively, m/s
U_{EDv}	Kinetic energy dissipation rate per unit area, Eq. 2.14b, W/m ²
U_{EDvCF}	Kinetic energy dissipation rate per unit area, Eq. 2.14c, W/m ²
U_{EDvREL}	Kinetic energy dissipation rate per unit area, Eq. 2.14d, W/m ²
V	Velocity, m/s
v_g, v_s	Gas and solid phase velocities in the y direction, respectively, m/s
V_{el}	Threshold velocity defined by Eq. 2.9 m/s
V_1, V_2	Approach and rebound particle velocities, respectively, m/s
$\vec{v}$	Velocity vector, m/s
$ \vec{v} $	Magnitude of velocity vector, m/s
x	Lateral coordinate, m
y	Axial coordinate, m
z	Axial coordinate, m

Greek Letters

α	Impingement angle, degrees
α_0	Transition impingement angle, degrees

$\bar{\bar{\beta}}_A, \bar{\bar{\beta}}_B,$	Fluid-particle friction tensors for hydrodynamic models A and B, respectively, ($= \beta_A \bar{\bar{I}}, \beta_B \bar{\bar{I}}$), kg/(m ³ ·s)
β_A, β_B	Fluid-particle interphase drag, kg/(m ³ ·s)
γ_p, γ_t	Poisson's ratio of particle and target, respectively
$\delta x, \Delta x, DX$	Cell size in Cartesian coordinates (x,y), m
$\delta y, \Delta y, DY$	Cell size in Cartesian coordinates (x,y), m
$\delta r, DR$	Cell size in cylindrical coordinates (r,z), m
$\delta z, DZ$	Cell size in cylindrical coordinates (r,z), m
ϵ	Fluid-phase volume fraction (porosity)
ϵ_s	Solids-phase volume fraction = (1 - ϵ)
ϵ^*	Compaction gas volume fraction
$\bar{\tau}_{kv}$	Viscous stress, Pa
μ_f	Fluid viscosity, Pa·s
ξ_b	Material-dependent brittle (deformation) wear factor, Pa
ρ	Density, kg/m ³
ρ_s, ρ_f	Solids- and fluid-phase microscopic densities, respectively, kg/m ³
ρ'_s, ρ'_f	Solids- and fluid-phase macroscopic (partial) densities, respectively, $\rho'_s = \rho_s (1 - \epsilon)$ and $\rho'_f = \rho_f \epsilon$, kg/m ³
ρ_p	Particle density, kg/m ³
σ_y	Elastic load limit (yield tensile strength), Pa
τ	Yield shear stress, Pa
$\bar{\tau}_{sv}$	Solids viscous stress, Pa
ϕ_c	Material-dependent ductile (cutting) wear factor, Pa
ϕ_s	Sphericity of solid particles (shape factor), $0 < \phi_s \leq 1$

Superscripts

$\cdot$	Denotes time rate of change
$\rightarrow$	Denotes a vector quantity
$=$	Denotes a tensor quantity
$n, n+1$	Time levels $n\Delta t$ and $(n+1)\Delta t$, respectively

Indexes and Subscripts

b	Brittle
d	Ductile
f	Fluid phase
i or I	Cell index in x or r direction
j or J	Cell index in y or z direction
p	Particle
s	Solids phase
t	Target
x	x-direction coordinate
y	y-direction coordinate
z	z-direction coordinate

Operators

$\frac{d}{dt^s}$ Total derivative following the solids $= \frac{\partial}{\partial t} + \vec{v}_s \cdot \nabla$

$\nabla \cdot$ Divergence

∇ Gradient

User's Manual for EROSION/MOD1: A Computer Program for Fluid-Solids Erosion

by

R.W. Lyczkowski, J.X. Bouillard, S.M. Folga, and S.L. Chang

Abstract

This report describes EROSION/MOD1, a computer program that was developed as a two-dimensional analytical tool for the general analysis of erosion in fluid-solids systems and the specific analysis of erosion in bubbling fluidized-bed combustors. Contained herein are implementations of Finnie's impaction erosion model, Neilson and Gilchrist's combined ductile and brittle erosion model, and several forms of the monolayer energy dissipation erosion model. These models and their implementations are described briefly. The global structure of EROSION/MOD1 that contains these models is also discussed. The input data for EROSION/MOD1 are given, and a sample problem for a fluidized bed is described. The hydrodynamic input data are assumed to come from the output of FLUFIX/MOD2.

1 Introduction

Excellent heat transfer from gas fluidized beds to the surfaces of immersed heat exchanger tubes is one of the most attractive features of bubbling atmospheric and pressurized fluidized-bed combustors (FBCs). Unfortunately, erosion (alternatively referred to as metal wastage) of these tube banks caused by vigorous solids motion, sometimes plagues their operation and must be mitigated to achieve reasonable lifetimes (~100,000 h) for commercial FBCs. Data recorded at the Grimethorpe cold model facility demonstrated that the tube metal loss rate could be reduced by more than one order of magnitude¹ to complete FBC experiments without catastrophic tube failures.

Earlier studies have indicated that one of the determining factors affecting the erosion of ductile and brittle materials is the velocity of the particles impinging upon the target. For flat surfaces impacted by single particles, the erosion rate has been shown^{2,3} to be proportional to the particle's velocity raised to the power 2 to 2.5.

More recently, simplified models for FBC erosion have been developed that attempt to relate the solids velocity to easily measurable quantities such as the bubble diameter, the bubble velocity, the bed porosity, and the fluidizing velocity.⁴⁻⁷ Gansley and O'Brien⁶ developed an analytical erosion expression by utilizing Davidson's classical bubble model, Finnie's single-particle impaction erosion model, and an empirical expression for the bubble diameter. Wood and Woodford⁵ treated erosion simply as the result of metal fatigue rather than metal cutting. Yates⁴ developed a simplified erosion model that, like Argonne National Laboratory's (ANL's) monolayer energy dissipation (MED) erosion model,⁷ is based on the premise that the erosion rate is proportional to the kinetic energy of the particles. Yates' erosion model predicts that this kinetic energy would be proportional to the fourth power of the bubble diameter. This functional dependency assumes that the solids move at the bubble rise velocity.

The computer program EROSION/MOD1 was developed as part of a coordinated effort by industry, government, and universities – known as the Cooperative R&D Venture, "Erosion of FBC Heat Transfer Tubes" – to better understand metal wastage in FBCs. This computer program utilizes detailed solids hydrodynamic as input calculations from the FLUFIX⁸ computer program. Three erosion models are included in the EROSION computer program: (1) Finnie's erosion model,² (2) Neilson and Gilchrist's,³ erosion model, and (3) ANL's MED erosion model⁷ developed during the course of the Cooperative R&D Venture.

Two reports that should be considered integral companions to this EROSION/MOD1 User's Manual are the FLUFIX/MOD1 User's Manual,⁸ which has been updated to become the FLUFIX/MOD2 User's Manual, and a review of erosion models that also served as a preliminary erosion computer program description.⁹ One is also referred to Bouillard's¹⁰ Chapter 6 and Appendix E. The availability of these resources makes it possible to keep the background and theory in this report to a minimum.

The methodologies employed in the EROSION/MOD1 computer program are not limited to erosion of FBC components, and may be used to predict erosion in any kind of solids processing device. These devices include circulating fluidized beds, fluid catalytic crackers (FCCs), and pneumatic and slurry transport pipelines.

2 Erosion Models

This section summarizes the erosion models contained in the EROSION/MOD1 computer program. An evaluation of the available literature is contained in Lyczkowski et al.⁹

2.1 Finnie's Ductile Erosion Model

Finnie's erosion model² is a single-particle impact cutting model. The particle approaching the surface at an angle α to its normal impingement angle removes the surface material in a manner similar to a machine cutting tool. The particle is assumed to be much harder than the surface, and so it does not break up. The surface material is assumed to deform plastically during the cutting process; hence, the surface material is considered ductile.

To apply Finnie's original erosion model to computer erosion rates, not for a single-particle impact, but rather for a continuum flux of particles hitting the surface as in a dense fluidized bed, the model was extended (Lyczkowski et al.⁹). They introduced the local solids, loading ϵ_s into the final expression of the erosion rate, $\dot{E}_F$

$$\dot{E}_F = \frac{C}{P_t} \dot{m}_s |\vec{V}_s|^2 f(\alpha), \quad (2.1)$$

where $\dot{m}_s$ is the flux of particles hitting the surface; that is,

$$\dot{m}_s = \rho_s \epsilon_s |\vec{V}_s| \quad (2.2)$$

Utilizing the parameters in Finnie's erosion model, the factor, C , in Eq. 2.1 = $1/8$. P_t is the target (Vicker's) hardness, $|\vec{V}_s|$ is the solids velocity magnitude, and $f(\alpha)$ is given by

$$f(\alpha) = \begin{cases} \sin(2\alpha) - 3\sin^2(\alpha) & \alpha \leq 18.43^\circ \\ \cos^2(\alpha)/3 & \alpha > 18.43^\circ \end{cases} \quad (2.3)$$

Below 18.43° , the surface is cut until the particle leaves the surface; above 18.43° , cutting ceases before the particle leaves the surface. The transition angle given in Eq. 2.3 is close to the angle of maximum erosion, $\alpha_{\max}$, given by

$$\alpha_{\max} = \frac{1}{2} \tan^{-1} (2/3) = 16.85^\circ \quad (2.4)$$

The disagreement with Finnie's model as used by Pourahmadi and Humphrey¹¹ was discussed by Lyczkowski et al.⁹

At 18.43° , Finnie's model yields an erosion rate for ductile target materials given by

$$\dot{E}_{FM} = 0.075 \frac{\dot{m}_s |\vec{V}_s|^2}{2P_t} \quad (2.5)$$

which is, in effect, the maximum value. Equation 2.5 predicts that only 7.5% of the particle's kinetic energy goes into erosion for a given target hardness, P_t .

Erosion models in general, and Finnie's erosion model in particular, cannot be used to calculate absolute erosion rates *a priori*. The reason for this is that assumptions need to be made concerning the most appropriate properties in the erosion models (e.g., yield strength, Young's modulus, Vickers hardness, etc.), as well as the submodels and their parameters. Finnie's erosion model can be used, together with estimates of target material flow stress, P_t (or Vicker's hardness), to back out a value for parameter C to match the data. On the other hand, if a value for C is assumed, then the value of the flow stress or hardness appropriate for erosion is backed out of the data.

The angular dependence of the erosion rate predicted by using Finnie's model is quite good up to 45° . Above 45° , Finnie's model underpredicts the erosion rate; at 90° , it predicts no erosion at all, whereas the analyzed data clearly indicate that this prediction is not correct.² The model described in the next section overcomes one of these shortcomings.

2.2 Neilson and Gilchrist's Combined Ductile and Brittle Erosion Model

The Neilson-Gilchrist (NG) erosion model³ is included in the EROSION/MOD1 computer program because it has the capability of computing erosion at a 90° angle of particle incidence relative to the eroding surface. It also has normal and parallel velocity threshold components below which erosion does not take place. However, in place of a single value for hardness, two material properties are needed to characterize cutting and deformation energy absorption.

The NG erosion model³ is a simplification of Bitter's earlier model.^{12,13} It consists of the sum of a ductile erosion rate, $\dot{E}_d$, and a brittle erosion rate, $\dot{E}_b$. The resulting wear expression, as extended by Lyczkowski et al.⁹, is given by

$$\dot{E}_{NG} = \dot{E}_d + \dot{E}_b = \begin{cases} \frac{\dot{m}_s |\vec{V}_s|^2 \cos^2(\alpha) \sin(\alpha)}{2\phi_c} + \frac{\dot{m}_s [|\vec{V}_s| \sin(\alpha) - V_{el}]^2}{2\xi_b} & \alpha \leq \alpha_0 \\ \frac{\dot{m}_s |\vec{V}_s|^2 \cos^2(\alpha)}{2\phi_c} + \frac{\dot{m}_s [|\vec{V}_s| \sin(\alpha) - V_{el}]^2}{2\xi_b} & \alpha > \alpha_0 \end{cases} \quad (2.6a)$$

with the provision that $\dot{E}_b = 0$ when $|\vec{V}_s| \sin(\alpha) < V_{el}$ and $\alpha_0 = \pi/2n$, where n is an empirical constant. This model was extended in the same way as Finnie's model (i.e., by introducing the mass flux of particles).

In addition, the ductile (cutting) and brittle (deformation) wear factors, ϕ_c and ξ_b , published by Neilson and Gilchrist³ for $210 \mu\text{m}$ aluminum oxide particles and aluminum plate target, were curve fit in dimensionless form, shown in Figure 1 (taken from Engel¹⁴) in the range $(100 \text{ m/s} \leq V \leq 200 \text{ m/s})$, as

$$\phi_c / (\tau/\rho_s) = 54.21 [1 + \exp(0.06793 (115.27 - V))] \quad (2.7)$$

and

$$\xi_b / (\sigma_y/\rho_s) = 82.52 [1 + \exp(0.07942 (110.13 - V))] \quad (2.8)$$

Figure 1 Neilson and Gilchrist Wear Factors for Aluminum Oxide Particles and Aluminum Plate Target (after Engel¹⁴).

where τ is the yield shear stress and σ_y is the yield tensile stress (plastic load limit) of the aluminum target ($\tau = 0.6 \sigma_y$ and $\sigma_y = 414 \text{ MPa}$).¹⁴ Since the ductile and brittle wear factors decrease with increasing particle impingement velocity, this may indicate some sort of dynamic softening of the target material.

Since data to determine the wear factors ϕ_c and ξ_b for conditions existing in FBCs are lacking, it is recommended that the values of these two factors evaluated at the lower bound of the experimental velocity data (107.8 m/s) be used as shown in Figure 1. If this is done, erosion rates are in rough agreement with the MED erosion model, as shown in Appendix A. N.B.: At present, only wear factors for aluminum target material are in the EROSION/MOD1 computer program.

With α_o chosen to be 18.43° , the same as in Finnie's erosion model described in Section 2.1, n becomes 4.89 from the relationship $\alpha_o = \pi/(2n)$. The wear factors calculated from the above empirical formula are in excellent agreement with the experimental data³ in a velocity range from 100 to 150 m/s.

Neilson and Gilchrist³ did not report the threshold velocity normal to the target, V_{el} , in their erosion analysis of aluminum plates because they stated it is usually small relative to the particle velocity. It can, however, be estimated from the theoretical expression from the Hertz contact theory given as¹⁴

$$V_{el} = 15.4 \sigma_y^{5/2} \rho_p^{-1/2} E_r^{-2} \quad (2.9)$$

where σ_y is the plastic load limit, ρ_p is the particle density, and E_r is the reduced Young's modulus of elasticity. The value of E_r is given by

$$E_r = \frac{1}{\left[\frac{(1 - \gamma_p^2)}{E_p} \right] + \left[\frac{(1 - \gamma_t^2)}{E_t} \right]} \equiv \frac{1}{(k_p + k_t)} \quad (2.10)$$

where γ_p and γ_t are the Poisson's ratios and E_p and E_t are the Young's moduli of elasticity for the particle and target, respectively.

Using the values¹⁵

$$\sigma_y = 0.414 \times 10^9 \text{ Pa},$$

$$\rho_p = 0.244 \times 10^4 \text{ kg/m}^3,$$

$$\gamma_p = 0.1 \text{ (estimated)},$$

$$E_p = 0.55 \times 10^{11} \text{ Pa (estimated)},$$

and

$$E_t = 0.20 \times 10^{12} \text{ Pa},$$

a reduced Young's modulus, $E_r = 0.14 \times 10^{12}$ Pa, and a threshold velocity, $V_{el} = 0.056$ m/s, are computed. The threshold velocity, V_{el} , can also be determined from particle rebound data by using the following relationship:¹²

$$V_2 = \sqrt{2V_1 V_{el} - V_{el}^2} \quad (2.11)$$

where V_1 and V_2 are the velocities before and after collision, respectively.

An assessment of the NG erosion model was carried out using solids velocities typical of those in fluidized beds. The results are summarized in Table 1. The results indicate that the erosion rate is strongly dependent on the solids velocity. The maximum erosion rate at a specified velocity occurs at an angle slightly below $\alpha_o = 18.43^\circ$. The angle at which maximum erosion occurs, α_{max} , is obtained by setting $d\dot{E}/d\alpha = 0$ from Eq. 2.6b. Table 1 shows that the erosion rate is roughly proportional to the velocity raised to the power 3.5. In other words, the particle velocity is a dominant parameter in determining the erosion rate for the NG model.

TABLE 1 Neilson-Gilchrist Erosion Rates for Various Impingement Velocities and Angles for Aluminum Target Eroded by Aluminum Oxide Particles

$\alpha = 90^\circ$		$V = 1$ m/s	
V (m/s)	Erosion Rate (mm/1000 h)	Impinging Angle, α (deg)	Erosion Rate (mm/1000 h)
1.00	1.107×10^{-2}	0.00	0.0
0.90	7.903×10^{-3}	10.00	6.451×10^{-2}
0.80	5.415×10^{-3}	20.00	1.090×10^{-1}
0.70	3.522×10^{-3}	30.00	9.419×10^{-2}
0.60	2.137×10^{-3}	40.00	7.606×10^{-2}
0.50	1.177×10^{-3}	50.00	5.682×10^{-2}
0.40	5.606×10^{-4}	60.00	3.874×10^{-2}
0.30	2.099×10^{-4}	70.00	2.401×10^{-2}
0.20	4.834×10^{-5}	80.00	1.441×10^{-2}
0.10	2.238×10^{-6}	90.00	1.107×10^{-2}
0.00	0.0		

2.3 The Monolayer Energy Dissipation Erosion Model

The MED erosion model is a significant extension of the so-called power dissipation erosion model for low-angle erosion caused by slurries, which was proposed by Ushimaru et al.¹⁶ Lyczkowski et al.⁹ discussed motivations of the power dissipation erosion model and extended it by deriving an early form of the MED erosion model.

A review of the most recently developed comprehensive MED erosion models was given by Bouillard et al.⁷ These models are based on the premise that the mechanical energy of the solids is irreversibly dissipated in the neighborhood of stationary surfaces by three competitive mechanisms:

1. Heat transfer between the gas and the solids particles, between the gas and the tube surfaces, and between the solids and the tube surfaces;

2. Erosion of obstacles; and
3. Attrition of solids particles.

Thus, the rate of energy dissipated in erosion represents only a fraction of the total power dissipation (which is related to the total entropy production).

The presently recommended form of the MED erosion model, which is essentially the one derived by Bouillard et al.⁷, as refined by Bouillard and Lyczkowski,¹⁷ is given by

$$\dot{E}_{MED} = \frac{(1-e^2)}{P_t} [(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s + (\vec{v}_s \cdot \bar{\beta}_B \cdot \vec{v}_s)/2] d_p \quad (2.12)$$

The factor $(1-e^2)$, where e is the target-particle coefficient of restitution, is given by $V_{s1}^2 - V_{s2}^2 = (1-e^2)V_{s1}^2$. The subscripts 1 and 2 denote solids velocities before and after collision with the target surface. Typically, $e^2 \sim 0.9$,¹⁸ and this is the default value used in the EROSION/MOD1 code.

The expression for the solids kinetic energy dissipation rate per unit volume was derived by Lyczkowski et al.⁹ For reference, it is quoted as follows:

$$\begin{aligned} e_{vs} = - \frac{dKE_s}{dt} \\ = (\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s = - \left\{ \begin{array}{c} \epsilon_s \rho_s \vec{v}_s \cdot \frac{\partial \vec{v}_s}{\partial t} + \epsilon_s \rho_s \vec{v}_s \cdot (\vec{v}_s \cdot \nabla \vec{v}_s) \\ 1 \qquad \qquad \qquad 2 \\ + (\epsilon_s \vec{v}_s) \cdot \nabla P + \vec{v}_s \cdot \bar{\beta}_A \cdot (\vec{v}_s - \vec{v}_f) - G(\epsilon_s) \vec{v}_s \cdot \bar{I} \cdot \nabla \epsilon \\ 3 \qquad \qquad \qquad 4 \qquad \qquad \qquad 5 \\ - \epsilon_s \rho_s \vec{v}_s \cdot \vec{g} - \nabla \cdot [(\epsilon_s \bar{\tau}_{sv}) \cdot \vec{v}_s] \end{array} \right\} \quad (2.13) \\ 6 \qquad \qquad \qquad 7 \end{aligned}$$

The six terms (1-6) in braces are contained in the prototype EROSION code described by Lyczkowski et al.⁹ The seventh term $\nabla \cdot [(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s]$ contained in Eq. 2.13, as well as other forms of the kinetic energy dissipation per unit volume, as discussed by Bouillard et al.,⁷ are used in the EROSION code. Hence, there are four forms of the kinetic energy dissipation rate per unit volume in EROSION/MOD1. They are given by

$$U_{ED} = (\text{Terms 1-7 in Eq. 2.13}) d_p \quad (2.14a)$$

$$U_{EDv} = [(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s] d_p \quad (2.14b)$$

$$U_{EDvCF} = [(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s + \bar{\beta}_B \vec{v}_s^2/2] d_p \quad (2.14c)$$

$$U_{EDvREL} = \left[(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s + \epsilon_s \bar{\beta}_B (\vec{v}_f - \vec{v}_s) \right] d_p \quad (2.14d)$$

To obtain the corresponding erosion rate, Eqs. 2-14a through 2.14d are used successfully in the brackets in the MED erosion model given by Eq. 2.12 to obtain

$$\dot{E}_{ED} = (1-e^2) U_{ED} d_p / P_t \quad (2.15a)$$

$$\dot{E}_{EDv} = (1-e^2) U_{EDv} d_p / P_t \quad (2.15b)$$

$$\dot{E}_{EDvCF} = (1-e^2) U_{EDvREL} d_p / P_t \quad (2.15c)$$

and

$$\dot{E}_{EDvREL} = (1-e^2) U_{EDvREL} d_p / P_t \quad (2.15d)$$

Bouillard and Lyczkowski¹⁷ provides a discussion of a simplified MED erosion model, based upon Eq. 2.12, that uses simplified solids rheology and hydrodynamics.

3 Erosion Model Implementation and Finite-Difference Equations

The application of algebraic and differential erosion models to fluid-solids systems is described in this section, as well as the finite difference equations for the monolayer energy dissipation models.

3.1 Coupling of Hydrodynamic and Erosion Models

A typical FLUFIX⁸ computational cell next to a tube surface is shown in Figure 2. If the tube is round, it must be approximated with a "staircase" approximation, as indicated because Cartesian coordinates are used. The normal velocities of the gas and solids phases are set equal to zero on all the approximate tube surfaces. The gas tangential velocities are set to zero and solids tangential velocities are either set to zero or partial slip.¹⁹ If the obstacle is rectangular and aligned along the coordinate direction, no approximation is necessary.

For the NG and Finnie algebraic erosion models described in Sections 2.1 and 2.2, the solid phase velocities in each direction are resolved at the cell center and the magnitude of the velocity is obtained. The angle of the resolved velocity vector at the cell center with respect to each horizontal and vertical approximate tube surface is then obtained. A further discussion of the angle determination is found in Section 6. The porosity is known at the cell center, so the mass flux of the solid is computed from the magnitude of the velocity resolved at the cell center and the porosity times the density (which is constant). Given the tube material hardness, E_{sp} , or flow pressure, P_t , and some estimate for the restitution coefficient, e , the erosion rate is obtained. This is done by using as inputs the transient FLUFIX outputs saved from the solution of the hydrodynamic equations.

The MED erosion models require derivatives of the solids phase velocities to compute the energy dissipation rates at the cell centers around the obstacle. In addition, the gas and solids phase velocities are required to compute the energy dissipation rates caused by drag, and gradients of pressure are needed to evaluate the pressure work terms. A detailed description of the finite-differencing for the prototypic MED erosion model was given in Reference 9. The additional finite difference equations in the EROSION/MOD1 code are described in Section 3.2.

From Eq. 2.13, it is clear that the terms representing the irreversible rate of internal energy conversion $(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s$ are equivalent to all seven of the terms in the braces. Boundary condition ambiguities arise in evaluating these terms, as discussed by Lyczkowski et al. on pages 52-55. It has been our experience that the evaluation of Eq. 2.14a is subject to more discretization error than the direct evaluation of Eq. 2.14b, which constitutes a portion of the models given by Eqs. 2.14c and 2.14d.

Therefore, the discussion of the finite difference equations in this section is limited to the two ways to calculate the rate of irreversible conversion to internal energy in subroutines CHECK (default) and HAPPY.

FIGURE 2 Coupling of Hydrodynamic and Erosion Models

3.2 Finite Difference Approximations of the Irreversible Rate of Conversion to Internal Energy

The total rate of irreversible conversion to internal energy $\epsilon_s(\bar{\tau}_s : \nabla \vec{v}_s)$ is comprised of four terms for a two-dimensional coordinate system. In (x,z) Cartesian coordinates, they are given by

$$\epsilon_s(\bar{\tau}_{sv} : \nabla \vec{v}_s) = \epsilon_s \left[\tau_{xxs} \frac{\partial U_s}{\partial x} + \tau_{xzs} \frac{\partial U_s}{\partial z} + \tau_{zxs} \frac{\partial V_s}{\partial x} + \tau_{zzs} \frac{\partial V_s}{\partial z} \right] \quad (3.1)$$

or

$$\epsilon_s(\bar{\tau}_{sv} : \nabla \vec{v}_s) = \epsilon_s \left\{ \tau_{xxs} \frac{\partial U_s}{\partial x} + \tau_{zzs} \frac{\partial V_s}{\partial z} + \tau_{zxs} \left(\frac{\partial U_s}{\partial z} + \frac{\partial V_s}{\partial x} \right) \right\} \quad (3.2)$$

since $\tau_{zxs} = \tau_{xzs}$. The finite-differencing method used for Eq. 3.2 is the central differencing method, with the evaluation of the various velocity gradients at the cell center. Two subroutines were written, using the same basic equations for the derivatives. One accounts explicitly for the presence of the obstacle on these derivatives by forcing no-slip boundary conditions (subroutine CHECK) and is a special case of the general expression with no boundary condition prescriptions (subroutine HAPPY). The latter allows arbitrary boundary condition prescription (i.e., partial solids slip).²⁰

The basic finite difference equations in subroutine HAPPY are, as in the K-FIX program:²⁰

$$\begin{aligned} [\epsilon_s(\bar{\tau}_{sv} : \nabla \vec{v}_s)](IJ) = & [1 - \epsilon(IJ)] \left\{ \tau_{xxs}(IJ) \left[\frac{UL(IJ) - UL(IMJ)}{DX} \right] + \tau_{xzs}(IJ) \left[\frac{VL(IJ) - VL(IJM)}{DZ} \right] \right. \\ & + \tau_{zxs}(IJ) \left[\frac{(VL(IPJ) + VL(IPJM)) - (VL(IMJ) + (VL(IMJM)))}{(4DX)} \right] \\ & \left. + \tau_{zzs}(IJ) \left[\frac{(UL(IJP) + UL(IMJP)) - (UL(IJM) + (UL(IMJM)))}{(4DZ)} \right] \right\} \end{aligned} \quad (3.3)$$

The indices used (IJ, IMJ, etc.) are shown in Figure 3.

The finite difference expressions for the three shear terms τ_{xxs} , τ_{zxs} , and τ_{zzs} are given by

$$\tau_{xxs}(IJ) = \mu_s \left\{ \frac{4}{3} \left[\frac{UL(IJ) - UL(IMJ)}{DX} \right] - \frac{2}{3} \left[\frac{VL(IJ) - VL(IJM)}{DZ} \right] \right\} \quad (3.4a)$$

$$\tau_{zxs}(IJ) = \mu_s \left\{ \frac{4}{3} \left[\frac{UL(IJ) - UL(IMJ)}{DX} \right] - \frac{2}{3} \left[\frac{VL(IJ) - VL(IJM)}{DZ} \right] \right\} \quad (3.4b)$$

$$\tau_{zzs}(IJ) = \mu_s \left\{ \left[\frac{(VL(IPJ) + VL(IPJM)) - (VL(IMJ) + VL(IMJM))}{(4DX)} \right] + \right.$$

FIGURE 3 Single-Subscript Notation for Cell and Cell-Edge Quantities About Cell (i,j)

$$\left[\frac{(\text{UL}(\text{IJP}) + \text{UL}(\text{IMJP})) - (\text{UL}(\text{IJM}) + \text{UL}(\text{IMJM}))}{(4 \text{ DZ})} \right] \quad (3.4c)$$

The normal stresses τ_{xxs} and τ_{zzs} and velocity derivatives are differenced over one cell length, while two cell lengths are used for the off-diagonal shear stress τ_{xzs} . These equations are general, in that no boundary conditions have been applied explicitly, and they hold for no-, free-, and partial-solids slip. Of course, if free slip is used, erosion is not computed for the cell.

The formulation given by Eqs. 3.3 and 3.4 (in subroutine HAPPY) was modified to account for no-slip solid boundary conditions by using the four flags discussed in Section 3.1. The results of this modification are contained in subroutine CHECK and are summarized below.

IFLAG = 1 (no-slip solid surface above)

The shear stress τ_{xzs} becomes

$$\tau_{xzs}(\text{IJ}) = \mu_s \left[\left[\frac{\text{VL}(\text{IPJM}) - \text{VL}(\text{IMJM})}{4 \text{ DX}} \right] + \left[\frac{-\text{UL}(\text{IJ})}{(\text{DZ}/2)} \right] \right] \quad (3.5)$$

and for the tangential velocities

$$\frac{\partial U_s}{\partial z} = - \frac{\text{UL}(\text{IJ})}{(\text{DZ}/2)} \quad (3.6)$$

$$\frac{\partial V_s}{\partial x} = \frac{\text{VL}(\text{IPJM}) - \text{VL}(\text{IMJM})}{(4 \text{ DX})} \quad (3.7)$$

with the term $\frac{\partial U_s}{\partial x}$ differenced over the interval $\Delta Z/2$ next to the obstacle and the velocity at the surface of the obstacle set equal to zero.

Only the three terms above need to be modified, and the same logic is used for each change. Because of this, only the final result will be given for the other three flags.

IFLAG = 2 (no-slip solid surface to the left)

$$\tau_{xz} = \mu_s \left[\left[\frac{\text{VL}(\text{IJ})}{(\text{DX}/2)} \right] + \left[\frac{\text{UL}(\text{IJP}) - \text{UL}(\text{IJM})}{(4 \text{ DZ})} \right] \right] \quad (3.8)$$

$$\frac{\partial U_s}{\partial z} = \frac{\text{UL}(\text{IJP}) - \text{UL}(\text{IJM})}{(4 \text{ DZ})} \quad (3.9)$$

$$\frac{\partial V_s}{\partial x} = \frac{\text{VL}(\text{IJ})}{(\text{DX}/2)} \quad (3.10)$$

IFLAG = 3 (no-slip solid surface below)

$$\tau_{xz} = \mu_s \left[\left[\frac{\text{VL}(\text{IPJ}) - \text{VL}(\text{IMJ})}{(4 \text{ DX})} \right] + \left[\frac{\text{UL}(\text{IJ})}{(\text{DZ}/2)} \right] \right] \quad (3.11)$$

$$\frac{\partial U_s}{\partial z} = \frac{UL(IJ)}{(DX/2)} \quad (3.12)$$

$$\frac{\partial V_s}{\partial x} = \frac{VL(IPJ) - VL(IMJ)}{(4 DX)} \quad (3.13)$$

IFLAG = 4 (no-slip solid surface to the right)

$$\tau_{xz} = \mu_s \left\{ \left[\frac{-VL(IJ)}{(DX/2)} \right] + \left[\frac{UL(IMJP) - UL(IMJM)}{(4 DZ)} \right] \right\} \quad (3.14)$$

$$\frac{\partial U_s}{\partial z} = \frac{UL(IMJP) - UL(IMJM)}{(4 DZ)} \quad (3.15)$$

$$\frac{\partial V_s}{\partial x} = \frac{-VL(IJ)}{(DX/2)} \quad (3.16)$$

The reversible rate of work done by the solids viscous stress can be evaluated by

1. Using velocities averaged at cell centers to compute cell-centered gradients as originally developed⁹ (subroutine VISWRK), and
2. Using cell-edge velocities similar to that done in the FLUFIX computer programs^{8,19} (subroutine VSCWRK) (default).

The rate of irreversible conversion to internal energy can be evaluated by:

1. Using the same formulation for this term as done in the K-FIX computer program,²⁰ which uses cell-edge solids velocities (subroutine HAPPY); and
2. using a modification of the K-FIX formulation to better account for the presence of the obstacles by using the same cell-centered solids velocities (subroutine CHECK) (default).

The energy dissipation computations contained in this report used the second of each of the above two methods. The sensitivity and implications of these various methods have been investigated on a limited basis.

Cell-centered solids velocities are used to compute algebraic erosion models such as the Finnie² and NG³ models described in this manual.

5 Global Structure of the EROSION/MOD1 Computer Program

The global structure of the erosion computer program is described in this section. First, however, it is necessary to briefly describe the EROSION/MOD1 subroutines.

5.1 EROSION/MOD1 Subroutines and Their Functions

An alphabetical listing of the EROSION/MOD1 subroutines, together with brief descriptions, follows:

CHECK	Calculates the rate of irreversible conversion to internal energy setting surface velocities to zero explicitly (default).
FINNIE	Calculates the erosion for the Finnie erosion model.
FLIC	Sets the cell flags on the basis of input data.
HAPPY	Calculates the rate of irreversible conversion to internal energy.
INDEX, INDEXA	Calculates the indexes for array quantities and reflects cell quantities in the rigid cells.
KDRAGG	Calculates the Richardson-Zaki equation for fluid-phase volume fraction $\epsilon \geq 0.8$. ^{8,19}
KDRAGL	Calculates the Ergun equation for fluid-phase volume fraction $\epsilon \geq 0.8$. ^{8,19}
KDRAGS	Calculates the values of the interphase friction function; calls KDRAGG and KDRAGL.
NEILSN	Calculates the erosion for NG erosion model.
SETC	Sets the C array, which is used to store equations of state or transport properties data, coefficients of functions, and related constants.
VISWRK	Calculates the reversible rate of work done by solids viscous stress using cell centers.
VSCWRK	Calculates the reversible rate of work done by solids viscous stress using cell edges (default).

5.2 Erosion Program Structure Synopsis

Figure 4 shows the data flow in and out of EROSION/MOD1. The input and control data are read from UNIT 5. Up to three sequential sets of post-processor files, which consist of IFR2 time-dependent porosity, pressure, and gas and solid velocities data sets called "dumps"^{8,19} are read from UNITS 8-12, 30-33, and 40-43. Figure 5 shows the flow structure and subroutine calling sequence. The cells are swept and the calculation procedure continues until $IFR > IFR2$. The time-averaging is then performed for the various energy dissipation and erosion rates and

FIGURE 4 Data Flows in and out of EROSION/MOD1

FIGURE 5 EROSION/MOD1 Program Structure and Subroutine Calling Sequences

written to UNIT 6. These dissipation rates are then written to UNITS 51 through 56 for further analysis or plotting.

6 Description of EROSION /MOD1 Input Data

In this section, the EROSION/MOD1 input data are described. These input data are a subset of the FLUFX input data and must agree with those data.

Line No. 1

IFR0,IFR1,IFR2,IFR3 (Format 4I5)

- IFR0 = Dump number at which to start time-averaging (usually = 1).
- IFR1 = Number of dumps in first file.
- IFR2 = Total number of dumps on all three files.
- IFR3 = Number of dumps on first two files.

Line No. 2

IB2,JB2,IPRES,IDRAG (Format 4I5)

- IB2 = Total number of cells in x or r direction.
- JB2 = Total number of cells in y or z direction.
- IPRES = 0 or 1
 IPRES = 0, selects Hydrodynamic Model A.
 IPRES = 1, selects Hydrodynamic Model B.
- IDRAG = 0 or 1
 IDRAG = 0, vector form of interphase drag is used.
 IDRAG = 1, scalar form of interphase drag is used.

Line No. 3

NO, DX, DZ,BHITE (Format I12, 3F12.4)

- NO = Number of Interior Obstacle Cells.
- DR = δr or δx , the cell dimension in the r or x direction, cm.
- DZ = δz or δy , the cell dimension of the z or y direction, cm.
- BHITE = Height of fluidized bed where solids are present, cm.

Line No. 4

KFLIN(M), M = 1,4 (Format 6I12)

KFLIN(M) = Cell flag types for the inflow boundaries, BOTTOM-L and BOTTOM-R openings (M= 1,2) and LEFT=B and LEFT-T openings (M = 3,4). Refer to Figure 9 of References 8 or 19 for lines 4 through 9.

Line No. 5

FLO(M),M = 1,4 (Format 4F12.4)

FLO(1) and FLO(2) are the x or r coordinates of the left inflow opening on the BOTTOM-L boundary, cm; FLO(3) and FLO(4) are the x or r coordinates of the right inflow opening on the BOTTOM-R boundary, cm.

Line No. 6

FLO(M),M = 5,8 (Format 4F12.4)

FLO(5) and FLO(6) are the y or z coordinates of the bottom inflow opening on the LEFT-B boundary, cm; FLO(7) and FLO(8) are the y or z coordinates of the top inflow opening on the LEFT-T boundary, cm.

Line No. 7

KFLOUT(M), M = 1,4 (Format 6I12)

KFLOUT(M) = Cell flag types for the outflow boundaries, TOP-L and TOP-R openings (M = 1,2) and RIGHT-B and RIGHT-T openings (M = 3,4).

Line No. 8

FLO(M),M = 9,12 (Format 4F12.4)

FLO(9) and FLO(10) are the x or r coordinates of the left outflow opening on the TOP-L boundary, cm; FLO(11) and FLO(12) are the x or r coordinates of the right outflow opening on the TOP-R boundary, cm.

Line No. 9

FLO(M), M = 13,16 (Format 4F12.4)

FLO(13) and FLO(14) are the y or z coordinates of the bottom outflow opening on the RIGHT-B boundary, cm; FLO(15) and FLO(16) are the y or z coordinates of the top outflow opening on the RIGHT-T boundary, cm.

Line No. 10

NSL(M), M = 1,4 (Format 4I12)

NSL(M) indicates free-slip or no-slip boundary conditions for rigid walls around the computing mesh perimeter. A value of 0 for free-slip or 1 for no-slip is assigned for the bottom (M = 1), left (M = 2), top (M = 3), and right (M = 4) boundaries, in that order. The assigned values are ignored across inflow or outflow openings. This flag overrides KFLIN or KFLOUT = 2 or 3.

Line No. 10 + N*

NSO(N), (OB(M,N), M = 1,4) (Format I12, 4F12.4)

NSO(N) = 0 for free-slip boundary conditions for obstacle N.
 = 1 for no-slip boundary conditions for obstacle N.

OB(1,N) = x or r coordinate of the left side of obstacle N, cm.

OB(2,N) = x or r coordinate of the right side of obstacle N, cm.

OB(3,N) = y or z coordinate of the bottom of obstacle N, cm.

OB(4,N) = y or z coordinate of the top of obstacle N, cm.

Line No. 11+N

SLOPE, THZERO, RLO (Format 3F10.6)

SLOPE, THZERO, parameters for the elastic modulus $G = \exp[-\text{SLOPE}(e - \text{THZERO})]$,
 dynes/cm².

RLO = Density, g/cm³ (default value is 2.611 g/cm³).**Line No. 12 + N**

VPRIM, THPRIM, MUL (Format 3F10.6)

VPRIM, THPRIM not used.

MUL = solids viscosity, poise. The default value is 1.0 poise.

Line No. 13+N

DP, HARD, PHI, MMUG (Format 3F10.5, F12.8)

DP = Particle diameter, cm.

HARD = Eroding surface hardness, kgf/mm².PHI = Particle sphericity, ϕ_s ($0 < \phi_s \leq 1$).

MMUG = Fluid viscosity, poise, used in the interphase drag. The default value is 1.82×10^{-4} poise.

* N = 1, N0, N0 > 0

Default Values:

DP, 0.0503 cm; HARD, 30 kgf/mm² (294 MPa); PHI, 0.86; MMUL, 1.0 poise; MMUG, 1.82 x 10⁻⁴ poise; RLO, 2.611 g/cm³; MUL, 1.0 poise.

7 Sample Problem for a Fluidized Bed

The geometry of the sample problem given here for a fluidized bed is shown in Figure 6. This sample problem is the same as the one described in the FLUFIX/MOD2 Users Manual.¹⁹ The computational region is 19.685 cm wide and 58.44 cm high. The cell dimensions are $\Delta x = \delta x = 0.635$ cm and $\Delta y = \delta y = 4.87$ cm, so that the number of computational cells is 31 in the x direction and 12 in the y direction, for a total of 372. In Figure 6, the numbers in parentheses refer to key cell numbers (I,J). Symmetry about the central jet is assumed; hence, the actual bed width is 39.37 cm. The jet half-width is 0.635 cm (one cell width). The jet velocity is 578 cm/s, and the secondary air velocity of 26.0 cm/s maintains the bed without a jet at minimum fluidization. The particle diameter is 503 μm , and the density is 2.44 g/cm³. The obstacle is placed two nodes above the jet and is two nodes (1.27 cm) wide by two nodes (9.74 cm) high. Because the initial bed height is 29.22 cm (six cells high), the obstacle lies completely within the bed. Although this configuration is not typical of FBC geometries, it was selected because (1) it is similar to the model without the obstacle, so that prior experience is relevant, and (2) it serves to further validate the hydrodynamic model.

The boundary conditions are described here. At the inlet ($J = 2$), the axial gas velocity of 26.0 cm/s is set equal to the minimum fluidization superficial velocity as computed from the Ergun equation (with solids velocity set to zero) by using an assumed gas volume fraction at minimum fluidization, ϵ_{mf} , of 0.42. Because no solids are entering, the inlet porosity is set to 1.0. The pressures in the dummy cells at the top ($J = 14$) are set equal to atmospheric pressure (1.013×10^5 Pa), and $V_s = 0$ at the exit ($J = 13$); that is, wire mesh is simulated to prevent solids carry-over. The pressure in the bottom row of dummy cells ($J = 1$) is set equal to atmospheric pressure plus 1.2 times the total bed weight (1.0549×10^5 Pa). On all solid surfaces except the inlet, outlet, and line of symmetry, no-slip boundary conditions are used (i.e., normal and tangential velocities for each phase are set equal to zero). Free-slip boundary conditions are used along the line of symmetry and at the inlet and outlet. Initially, the radial gas velocity is zero, the axial gas velocity is equal to the interstitial gas velocity at minimum fluidization, and the solids radial and axial velocities are zero. The bed porosity is uniform at 0.42. The initial pressure distribution corresponds to the hydrostatic bed height. At time, t , greater than zero ($0+$), the gas flow through the jet is increased to 578 cm/s. A fixed-time stop of 0.1 ms is used. Typical running time on an IBM 3033 computer is approximately one hour for each second of transient time. The running time is approximately four times faster on a CRAY-XMP.

The FLUFIX/MOD2 code was run to 0.4 s with post-processing files produced every 100 time steps (0.005 s) for a total of 80 "dumps." The input data and sample output can be found in the users manual.¹⁹ The EROSION/MOD1 sample problem input is given in Table 2. The EROSION input data file can easily be generated from the FLUFIX input data file by using a text editor. All of the input values must agree. There are no checks for inconsistencies.

Table 3 presents the sample problem output. The output beginning at line 46 (TIME = 0.0050) and ending at line 63 summarizes all the cells IJ for which erosion has been computed. These surfaces are numbered for each IJ and are denoted by $K = 1$ through $K = 16$ in column 3, lines 47 through 63. The quantities RES(K), TAU(V(K), and VISC(K) are not of interest. These quantities represent partial results for one of the MED erosion models evaluated by using values at the first dump.

The table of flags, FL, starting at line 31 and ending at line 44, defines the entire computational domain. Erosion is only computed for surfaces when the flag FL = 1 is next to a

FIGURE 6 EROSION Sample Problem Geometry

TABLE 2 EROSION/MOD1 Sample Problem Input Data File

Line No.

1.	001	80	80	80				
2.	33	14	1	01				
3.			1	0.6350	4.8700	29.22		
4.			5		5	2	2	
5.	0.0		0.6350		0.6350	19.685		
6.	0.0		0.0		0.0	0.0		
7.			7		7	3	3	
8.	0.0		19.685		19.685	19.685		
9.	0.0		0.0		0.0	0.0		
10.			0		0	0	1	
11.			1	0.0	1.27	9.74	19.48	
12.	600.00		0.420	2.44				
13.	15.0		0.40	1.00				
14.	0.0503		30.0	1.0	0.000182			

TABLE 3 EROSION/MOD1 Sample Problem Output

[illegible]

TABLE 3 (cont'd)

60.	32	9	13	0.000E+00	0.000E+00	0.000E+00	4
61.	32	10	14	0.000E+00	0.000E+00	0.000E+00	4
62.	32	11	15	0.000E+00	0.000E+00	0.000E+00	4
63.	32	12	16	0.000E+00	0.000E+00	0.000E+00	4
64.							
65.	TIME=0.010000			DT=0.005000 SEC			
66.	TIME=0.015000			DT=0.005000 SEC			
67.	TIME=0.020000			DT=0.005000 SEC			
68.	TIME=0.025000			DT=0.005000 SEC			
69.	TIME=0.030000			DT=0.005000 SEC			
70.	TIME=0.035000			DT=0.005000 SEC			
71.	TIME=0.040000			DT=0.005000 SEC			
72.	TIME=0.045000			DT=0.005000 SEC			
73.	TIME=0.050000			DT=0.005000 SEC			
74.	TIME=0.055000			DT=0.005000 SEC			
75.	TIME=0.060000			DT=0.005000 SEC			
76.	TIME=0.065000			DT=0.005000 SEC			
77.	TIME=0.070000			DT=0.005000 SEC			
78.	TIME=0.075000			DT=0.005000 SEC			
79.	TIME=0.080000			DT=0.005000 SEC			
80.	TIME=0.085000			DT=0.005000 SEC			
81.	TIME=0.090000			DT=0.005000 SEC			
82.	TIME=0.095000			DT=0.005000 SEC			
83.	TIME=0.100000			DT=0.005000 SEC			
84.	TIME=0.105000			DT=0.005000 SEC			
85.	TIME=0.110000			DT=0.005000 SEC			
86.	TIME=0.115000			DT=0.005000 SEC			
87.	TIME=0.120000			DT=0.005000 SEC			
88.	TIME=0.125000			DT=0.005000 SEC			
89.	TIME=0.130000			DT=0.005000 SEC			
90.	TIME=0.135000			DT=0.005000 SEC			
91.	TIME=0.140000			DT=0.005000 SEC			
92.	TIME=0.145000			DT=0.005000 SEC			
93.	TIME=0.150000			DT=0.005000 SEC			
94.	TIME=0.155000			DT=0.005000 SEC			
95.	TIME=0.160000			DT=0.005000 SEC			
96.	TIME=0.165000			DT=0.005000 SEC			
97.	TIME=0.170000			DT=0.005000 SEC			
98.	TIME=0.175000			DT=0.005000 SEC			
99.	TIME=0.180000			DT=0.005000 SEC			
100.	TIME=0.185000			DT=0.005000 SEC			
101.	TIME=0.190000			DT=0.005000 SEC			
102.	TIME=0.195000			DT=0.005000 SEC			
103.	TIME=0.200000			DT=0.005000 SEC			
104.	TIME=0.205000			DT=0.005000 SEC			
105.	TIME=0.210000			DT=0.005000 SEC			
106.	TIME=0.215000			DT=0.005000 SEC			
107.	TIME=0.220000			DT=0.005000 SEC			
108.	TIME=0.225000			DT=0.005000 SEC			
109.	TIME=0.230000			DT=0.005000 SEC			
110.	TIME=0.235000			DT=0.005000 SEC			
111.	TIME=0.240000			DT=0.005000 SEC			
112.	TIME=0.245000			DT=0.005000 SEC			
113.	TIME=0.250000			DT=0.005000 SEC			
114.	TIME=0.255000			DT=0.005000 SEC			
115.	TIME=0.260000			DT=0.005000 SEC			
116.	TIME=0.265000			DT=0.005000 SEC			
117.	TIME=0.270000			DT=0.005000 SEC			
118.	TIME=0.275000			DT=0.005000 SEC			
119.	TIME=0.280000			DT=0.005000 SEC			

TABLE 3 (cont'd)

120.	TIME=0.285000	DT=0.005000 SEC			
121.	TIME=0.290000	DT=0.005000 SEC			
122.	TIME=0.295000	DT=0.005000 SEC			
123.	TIME=0.300000	DT=0.005000 SEC			
124.	TIME=0.305000	DT=0.005000 SEC			
125.	TIME=0.310000	DT=0.005000 SEC			
126.	TIME=0.315000	DT=0.005000 SEC			
127.	TIME=0.320000	DT=0.005000 SEC			
128.	TIME=0.325000	DT=0.005000 SEC			
129.	TIME=0.330000	DT=0.005000 SEC			
130.	TIME=0.335000	DT=0.005000 SEC			
131.	TIME=0.340000	DT=0.005000 SEC			
132.	TIME=0.345000	DT=0.005000 SEC			
133.	TIME=0.350000	DT=0.005000 SEC			
134.	TIME=0.355000	DT=0.005000 SEC			
135.	TIME=0.360000	DT=0.005000 SEC			
136.	TIME=0.365000	DT=0.005000 SEC			
137.	TIME=0.370000	DT=0.005000 SEC			
138.	TIME=0.375000	DT=0.005000 SEC			
139.	TIME=0.380000	DT=0.005000 SEC			
140.	TIME=0.385000	DT=0.005000 SEC			
141.	TIME=0.390000	DT=0.005000 SEC			
142.	TIME=0.395000	DT=0.005000 SEC			
143.	TIME=0.400000	DT=0.005000 SEC			
144.	INODE= 80				
145.	CELL NUMBER= ①				
146.	TIME-AVERAGED ENERGY DISSIP=	9.020E-05	WATT/CM**2	1.105E+00	MM/1000 H
147.	TIME-AVERAGED TOTAL KINETIC ENERGY=	2.278E-03	WATT/CM**2	2.790E+02	MM/1000 H
148.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.739E-06	WATT/CM**2	2.159E-01	MM/1000 H
149.	TIME-AVERAGED MEAN DIRECT RESIDUE=	9.864E-07	WATT/CM**2	1.208E-02	MM/1000 H
150.	TIME-AVERAGED CLOSED SOLUTION=	1.840E-05	WATT/CM**2	2.353E-01	MM/1000 H
151.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	1.277E-04	WATT/CM**2	1.564E+00	MM/1000 H
152.	TIME-AVERAGED NEILSON-GILCHRIST =	1.875E-03	WATT/CM**2	6.107E-01	MM/1000 H
153.	TIME AVERAGED ANGLE = 4.704E+01				
154.					
155.					
156.	CELL NUMBER= ②				
157.	TIME-AVERAGED ENERGY DISSIP=	1.112E-05	WATT/CM**2	1.361E-01	MM/1000 H
158.	TIME-AVERAGED TOTAL KINETIC ENERGY=	1.110E-03	WATT/CM**2	1.359E+02	MM/1000 H
159.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.313E-05	WATT/CM**2	1.618E+00	MM/1000 H
160.	TIME-AVERAGED MEAN DIRECT RESIDUE=	7.517E-07	WATT/CM**2	9.204E-03	MM/1000 H
161.	TIME-AVERAGED CLOSED SOLUTION=	1.088E-05	WATT/CM**2	1.362E-01	MM/1000 H
162.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	8.610E-05	WATT/CM**2	1.054E+00	MM/1000 H
163.	TIME-AVERAGED NEILSON-GILCHRIST =	1.236E-03	WATT/CM**2	4.027E-01	MM/1000 H
164.	TIME AVERAGED ANGLE = 4.010E+01				
165.					
166.					
167.	CELL NUMBER= 3				
168.	TIME-AVERAGED ENERGY DISSIP=	1.208E-05	WATT/CM**2	1.480E-01	MM/1000 H
169.	TIME-AVERAGED TOTAL KINETIC ENERGY=	1.876E-04	WATT/CM**2	2.297E+01	MM/1000 H
170.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	3.205E-07	WATT/CM**2	3.925E-02	MM/1000 H
171.	TIME-AVERAGED MEAN DIRECT RESIDUE=	4.525E-06	WATT/CM**2	5.541E-02	MM/1000 H
172.	TIME-AVERAGED CLOSED SOLUTION=	1.106E-05	WATT/CM**2	1.354E-01	MM/1000 H
173.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	1.994E-04	WATT/CM**2	2.442E+00	MM/1000 H
174.	TIME-AVERAGED NEILSON-GILCHRIST =	7.705E-06	WATT/CM**2	2.509E-03	MM/1000 H
175.	TIME AVERAGED ANGLE = 6.407E+00				
176.					
177.					
178.	CELL NUMBER= ④				
179.	TIME-AVERAGED ENERGY DISSIP=	-7.045E-07	WATT/CM**2	-8.627E-03	MM/1000 H

TABLE 3 (cont'd)

180.	TIME-AVERAGED TOTAL KINETIC ENERGY=	2.236E-03	WATT/CM**2	2.738E+02	MM/1000 H
181.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	4.852E-07	WATT/CM**2	5.941E-02	MM/1000 H
182.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.382E-05	WATT/CM**2	1.692E-01	MM/1000 H
183.	TIME-AVERAGED CLOSED SOLUTION=	2.377E-05	WATT/CM**2	2.960E-01	MM/1000 H
184.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	2.792E-04	WATT/CM**2	3.419E+00	MM/1000 H
185.	TIME-AVERAGED NEILSON-GILCHRIST =	1.450E-05	WATT/CM**2	4.722E-03	MM/1000 H
186.	TIME AVERAGED ANGLE = -5.796E-01				
187.					
188.					
189.	CELL NUMBER= 5				
190.	TIME-AVERAGED ENERGY DISSIP=	1.576E-05	WATT/CM**2	1.930E-01	MM/1000 H
191.	TIME-AVERAGED TOTAL KINETIC ENERGY=	5.428E-04	WATT/CM**2	6.646E+01	MM/1000 H
192.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.381E-06	WATT/CM**2	1.691E-01	MM/1000 H
193.	TIME-AVERAGED MEAN DIRECT RESIDUE=	9.486E-06	WATT/CM**2	1.162E-01	MM/1000 H
194.	TIME-AVERAGED CLOSED SOLUTION=	2.430E-05	WATT/CM**2	2.975E-01	MM/1000 H
195.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	2.115E-04	WATT/CM**2	2.590E+00	MM/1000 H
196.	TIME-AVERAGED NEILSON-GILCHRIST =	3.003E-05	WATT/CM**2	9.779E-03	MM/1000 H
197.	TIME AVERAGED ANGLE = 9.119E-01				
198.					
199.					
200.	CELL NUMBER= 6				
201.	TIME-AVERAGED ENERGY DISSIP=	9.813E-06	WATT/CM**2	1.202E-01	MM/1000 H
202.	TIME-AVERAGED TOTAL KINETIC ENERGY=	1.223E-03	WATT/CM**2	1.498E+02	MM/1000 H
203.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	8.731E-08	WATT/CM**2	1.069E-02	MM/1000 H
204.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.177E-05	WATT/CM**2	1.441E-01	MM/1000 H
205.	TIME-AVERAGED CLOSED SOLUTION=	2.423E-05	WATT/CM**2	2.967E-01	MM/1000 H
206.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	2.555E-04	WATT/CM**2	3.129E+00	MM/1000 H
207.	TIME-AVERAGED NEILSON-GILCHRIST =	2.127E-06	WATT/CM**2	6.926E-04	MM/1000 H
208.	TIME AVERAGED ANGLE = -1.372E-01				
209.					
210.					
211.	CELL NUMBER= 7				
212.	TIME-AVERAGED ENERGY DISSIP=	1.230E-05	WATT/CM**2	1.506E-01	MM/1000 H
213.	TIME-AVERAGED TOTAL KINETIC ENERGY=	8.653E-04	WATT/CM**2	1.060E+02	MM/1000 H
214.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	0.000E+00	WATT/CM**2	0.000E+00	MM/1000 H
215.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.294E-05	WATT/CM**2	1.584E-01	MM/1000 H
216.	TIME-AVERAGED CLOSED SOLUTION=	3.189E-05	WATT/CM**2	3.905E-01	MM/1000 H
217.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	2.157E-04	WATT/CM**2	2.641E+00	MM/1000 H
218.	TIME-AVERAGED NEILSON-GILCHRIST =	0.000E+00	WATT/CM**2	0.000E+00	MM/1000 H
219.	TIME AVERAGED ANGLE = 0.000E+00				
220.					
221.					
222.	CELL NUMBER= 8				
223.	TIME-AVERAGED ENERGY DISSIP=	-1.045E-05	WATT/CM**2	-1.279E-01	MM/1000 H
224.	TIME-AVERAGED TOTAL KINETIC ENERGY=	2.686E-04	WATT/CM**2	3.288E+01	MM/1000 H
225.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	8.228E-08	WATT/CM**2	1.008E-02	MM/1000 H
226.	TIME-AVERAGED MEAN DIRECT RESIDUE=	3.645E-07	WATT/CM**2	4.464E-03	MM/1000 H
227.	TIME-AVERAGED CLOSED SOLUTION=	1.325E-05	WATT/CM**2	1.662E-01	MM/1000 H
228.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	4.010E-05	WATT/CM**2	4.910E-01	MM/1000 H
229.	TIME-AVERAGED NEILSON-GILCHRIST =	6.789E-05	WATT/CM**2	2.211E-02	MM/1000 H
230.	TIME AVERAGED ANGLE = -1.799E+01				
231.					
232.					
233.	CELL NUMBER= 9				
234.	TIME-AVERAGED ENERGY DISSIP=	-1.767E-05	WATT/CM**2	-2.164E-01	MM/1000 H
235.	TIME-AVERAGED TOTAL KINETIC ENERGY=	2.824E-04	WATT/CM**2	3.458E+01	MM/1000 H
236.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	4.467E-07	WATT/CM**2	5.440E-02	MM/1000 H
237.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.188E-06	WATT/CM**2	1.454E-02	MM/1000 H
238.	TIME-AVERAGED CLOSED SOLUTION=	1.360E-05	WATT/CM**2	1.675E-01	MM/1000 H
239.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	5.012E-05	WATT/CM**2	6.137E-01	MM/1000 H

TABLE 3 (cont'd)

240.	TIME-AVERAGED NEILSON-GILCHRIST =	4.720E-05	WATT/CM**2	1.537E-02	MM/1000 H
241.	TIME AVERAGED ANGLE = -8.705E+00				
242.					
243.					
244.	CELL NUMBER= 10				
245.	TIME-AVERAGED ENERGY DISSIP=	-5.314E-06	WATT/CM**2	-6.507E-02	MM/1000 H
246.	TIME-AVERAGED TOTAL KINETIC ENERGY=	9.945E-04	WATT/CM**2	1.218E+02	MM/1000 H
247.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	5.405E-06	WATT/CM**2	6.618E-01	MM/1000 H
248.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.379E-05	WATT/CM**2	1.689E-01	MM/1000 H
249.	TIME-AVERAGED CLOSED SOLUTION=	3.493E-05	WATT/CM**2	4.277E-01	MM/1000 H
250.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	2.194E-04	WATT/CM**2	2.687E+00	MM/1000 H
251.	TIME-AVERAGED NEILSON-GILCHRIST =	1.272E-04	WATT/CM**2	4.142E-02	MM/1000 H
252.	TIME AVERAGED ANGLE = 1.176E+00				
253.					
254.					
255.	CELL NUMBER= 11				
256.	TIME-AVERAGED ENERGY DISSIP=	2.908E-05	WATT/CM**2	3.561E-01	MM/1000 H
257.	TIME-AVERAGED TOTAL KINETIC ENERGY=	1.266E-03	WATT/CM**2	1.550E+02	MM/1000 H
258.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.706E-06	WATT/CM**2	2.089E-01	MM/1000 H
259.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.766E-05	WATT/CM**2	2.162E-01	MM/1000 H
260.	TIME-AVERAGED CLOSED SOLUTION=	5.439E-05	WATT/CM**2	6.660E-01	MM/1000 H
261.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	2.397E-04	WATT/CM**2	2.935E+00	MM/1000 H
262.	TIME-AVERAGED NEILSON-GILCHRIST =	3.715E-05	WATT/CM**2	1.210E-02	MM/1000 H
263.	TIME AVERAGED ANGLE = -4.313E-02				
264.					
265.					
266.	CELL NUMBER= 12				
267.	TIME-AVERAGED ENERGY DISSIP=	-8.912E-05	WATT/CM**2	-1.091E+00	MM/1000 H
268.	TIME-AVERAGED TOTAL KINETIC ENERGY=	2.120E-03	WATT/CM**2	2.596E+02	MM/1000 H
269.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.989E-05	WATT/CM**2	2.435E+00	MM/1000 H
270.	TIME-AVERAGED MEAN DIRECT RESIDUE=	1.612E-05	WATT/CM**2	1.974E-01	MM/1000 H
271.	TIME-AVERAGED CLOSED SOLUTION=	3.468E-05	WATT/CM**2	4.246E-01	MM/1000 H
272.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	5.620E-05	WATT/CM**2	6.882E-01	MM/1000 H
273.	TIME-AVERAGED NEILSON-GILCHRIST =	4.484E-04	WATT/CM**2	1.460E-01	MM/1000 H
274.	TIME AVERAGED ANGLE = 9.556E-01				
275.					
276.					
277.	CELL NUMBER= 13				
278.	TIME-AVERAGED ENERGY DISSIP=	-8.344E-05	WATT/CM**2	-1.022E+00	MM/1000 H
279.	TIME-AVERAGED TOTAL KINETIC ENERGY=	3.576E-03	WATT/CM**2	4.379E+02	MM/1000 H
280.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	4.525E-05	WATT/CM**2	5.541E+00	MM/1000 H
281.	TIME-AVERAGED MEAN DIRECT RESIDUE=	2.281E-05	WATT/CM**2	2.794E-01	MM/1000 H
282.	TIME-AVERAGED CLOSED SOLUTION=	4.622E-05	WATT/CM**2	5.660E-01	MM/1000 H
283.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	4.101E-05	WATT/CM**2	5.022E-01	MM/1000 H
284.	TIME-AVERAGED NEILSON-GILCHRIST =	1.041E-03	WATT/CM**2	3.389E-01	MM/1000 H
285.	TIME AVERAGED ANGLE = 9.998E-01				
286.					
287.					
288.	CELL NUMBER= 14				
289.	TIME-AVERAGED ENERGY DISSIP=	-2.130E-05	WATT/CM**2	-2.608E-01	MM/1000 H
290.	TIME-AVERAGED TOTAL KINETIC ENERGY=	2.949E-03	WATT/CM**2	3.611E+02	MM/1000 H
291.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.592E-08	WATT/CM**2	1.950E-03	MM/1000 H
292.	TIME-AVERAGED MEAN DIRECT RESIDUE=	6.756E-06	WATT/CM**2	8.273E-02	MM/1000 H
293.	TIME-AVERAGED CLOSED SOLUTION=	8.186E-06	WATT/CM**2	1.002E-01	MM/1000 H
294.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	1.045E-05	WATT/CM**2	1.279E-01	MM/1000 H
295.	TIME-AVERAGED NEILSON-GILCHRIST =	3.581E-07	WATT/CM**2	1.166E-04	MM/1000 H
296.	TIME AVERAGED ANGLE = 7.724E-02				
297.					
298.					
299.	CELL NUMBER= 15				

TABLE 3 (cont'd)

300.	TIME-AVERAGED ENERGY DISSIP=	-1.990E-05	WATT/CM**2	-2.437E-01	MM/1000 H
301.	TIME-AVERAGED TOTAL KINETIC ENERGY=	5.071E-03	WATT/CM**2	6.209E+02	MM/1000 H
302.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	1.906E-06	WATT/CM**2	2.334E-01	MM/1000 H
303.	TIME-AVERAGED MEAN DIRECT RESIDUE=	8.229E-06	WATT/CM**2	1.008E-01	MM/1000 H
304.	TIME-AVERAGED CLOSED SOLUTION=	9.896E-06	WATT/CM**2	1.212E-01	MM/1000 H
305.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	8.246E-06	WATT/CM**2	1.010E-01	MM/1000 H
306.	TIME-AVERAGED NEILSON-GILCHRIST =	4.013E-05	WATT/CM**2	1.307E-02	MM/1000 H
307.	TIME AVERAGED ANGLE = -4.107E-02				
308.					
309.					
310.	CELL NUMBER= 16				
311.	TIME-AVERAGED ENERGY DISSIP=	-8.051E-05	WATT/CM**2	-9.858E-01	MM/1000 H
312.	TIME-AVERAGED TOTAL KINETIC ENERGY=	8.218E-03	WATT/CM**2	1.006E+03	MM/1000 H
313.	TIME AVERAGED DISSIPATED POWER FROM FINNIE=	5.067E-06	WATT/CM**2	6.204E-01	MM/1000 H
314.	TIME-AVERAGED MEAN DIRECT RESIDUE=	9.321E-06	WATT/CM**2	1.141E-01	MM/1000 H
315.	TIME-AVERAGED CLOSED SOLUTION=	1.166E-05	WATT/CM**2	1.428E-01	MM/1000 H
316.	TIME-AVERAGED RELATIVE VELOCITY MODEL=	9.337E-06	WATT/CM**2	1.143E-01	MM/1000 H
317.	TIME-AVERAGED NEILSON-GILCHRIST =	1.066E-04	WATT/CM**2	3.471E-02	MM/1000 H
318.	TIME AVERAGED ANGLE = -5.583E-02				
319.					
320.					
321.	MESSAGE SUMMARY: MESSAGE NUMBER - COUNT				
322.	208 511 OR OVER				

cell having a flag $FL = 3$ (no slip or partial slip). The obstacle is clearly seen by the 2×2 array of flags $FL = 3$ at lines 40 and 41. The row of flags $FL = 2$ represents the dummy cells along the line of symmetry. The indices of the flags go from $I = 1$ to $I = 33$ and $J = 1$ to $J = 14$ (not numbered). Thus, the cells counterclockwise around the obstacle (circled for clarity) in Table 3 are (2,3), (3,3), (4,4), (4,5), (3,6), and (2,6) and the corresponding surfaces are $K = 1, 2, 4, 6, 8$, and 9 (also circled for clarity in Table 3). The row of cells $I = 32, J = 2$ to 13 are the cells next to the solid (right) wall. The remaining surface numbers, $K = 3, 5$, etc., constitute the surface of the fluidized-bed side wall corresponding to the vertical row of flags $FL = 3$, for which erosion is also calculated. The CELL NUMBERS 1 through 16 correspond to $K = 1$ through 16 in lines 47 through 63. For example, CELL NUMBER 1 corresponds to $I = 2, J = 3$, the lower surface of the obstacle next to the line of symmetry.

Table 4 shows the key for the time-averaged printed output for the first column of results representing the kinetic energy dissipated per unit volume W/cm^2 for each quantity indicated. The time dependent energy dissipation results are written to UNITS 51 through 56, as shown in Figure 4 in Section 5.2.

The corresponding time-averaged erosion results are given in the last column, starting at line 146, in mm/1,000 h. These rates are for aluminum tube material having a hardness of 294 MN/m² (30 kgf/mm²). The erosion rates for the MED ($\dot{E}_{EDVCF}$), NG, and Finnie's erosion models are summarized in Figure 7 along with the impingement angles. The numbers at the bottom of each cell are the cell numbers from the printout given in Table 4.

A note of precaution is in order. It has been determined that if the FLUFX standard problem¹⁹ is run with the default convergence parameter, $EPSG = 5 \times 10^{-4}$, slightly different hydrodynamic results are produced when using different computing platforms or compilers. This difference results in slightly different erosion rates and impingement angles. However, no such variable is encountered with the EROSION/MOD1 code run on different computer platforms. The computer platforms used were a B&W SUN workstation, an ANL CRAY X-MP/18, and an IBM 3033 with Fortran 66 and Fortran 77 compilers. These results are available from the first author upon request.

The convention used to determine the sign of the angle, α , is that the angle is negative if $\vec{v}_s \cdot \vec{n} > 0$ and positive if $\vec{v}_s \cdot \vec{n} < 0$. If the angle is negative, no erosion is computed in the NG, and Finnie erosion models (the solids flow away from the surface). This convention can be used to determine the direction of the velocity normal to a given surface from the printout of the time-averaged impingement angles.

		<table><tr><td>0.166</td><td>0.168</td></tr><tr><td>0.022</td><td>0.015</td></tr><tr><td>0.010</td><td>0.054</td></tr><tr><td>$\alpha=-18.0^\circ$</td><td>$\alpha=-8.71^\circ$</td></tr><tr><td>8</td><td>9</td></tr></table>		0.166	0.168	0.022	0.015	0.010	0.054	$\alpha=-18.0^\circ$	$\alpha=-8.71^\circ$	8	9	
0.166	0.168													
0.022	0.015													
0.010	0.054													
$\alpha=-18.0^\circ$	$\alpha=-8.71^\circ$													
8	9													
<table><tr><td>Erosion Rates</td></tr><tr><td>Energy Dissipation Model Neilson-Gilchrist Model Finnie Model</td></tr><tr><td>Units are: mm/1000 h</td></tr></table>	Erosion Rates	Energy Dissipation Model Neilson-Gilchrist Model Finnie Model	Units are: mm/1000 h	Obstacle	<table><tr><td>0.296</td></tr><tr><td>0.0007</td></tr><tr><td>0.011</td></tr><tr><td>$\alpha=-0.14^\circ$</td></tr><tr><td>6</td></tr></table>		0.296	0.0007	0.011	$\alpha=-0.14^\circ$	6			
	Erosion Rates													
	Energy Dissipation Model Neilson-Gilchrist Model Finnie Model													
Units are: mm/1000 h														
0.296														
0.0007														
0.011														
$\alpha=-0.14^\circ$														
6														
<table><tr><td>0.296</td></tr><tr><td>0.004</td></tr><tr><td>0.059</td></tr><tr><td>$\alpha=-0.58^\circ$</td></tr><tr><td>4</td></tr></table>		0.296	0.004	0.059	$\alpha=-0.58^\circ$	4								
0.296														
0.004														
0.059														
$\alpha=-0.58^\circ$														
4														
		<table><tr><td>0.235</td><td>0.136</td></tr><tr><td>0.611</td><td>0.403</td></tr><tr><td>0.216</td><td>1.618</td></tr><tr><td>$\alpha=47.0^\circ$</td><td>$\alpha=40.1^\circ$</td></tr><tr><td>1</td><td>2</td></tr></table>		0.235	0.136	0.611	0.403	0.216	1.618	$\alpha=47.0^\circ$	$\alpha=40.1^\circ$	1	2	
0.235	0.136													
0.611	0.403													
0.216	1.618													
$\alpha=47.0^\circ$	$\alpha=40.1^\circ$													
1	2													

FIGURE 7 Summary of Computed Erosion Rates for the MED ($\dot{E}_{EDvCF}$), NELSON-GILCHRIST, and Finnie Erosion Models

TABLE 4 Key for Energy Dissipation Printed Outputs

Variable	Definition	
Energy Dissipation	= UED	
Total Kinetic Energy	= $ \vec{v}_s ^2/2$	
Dissipated Power from Finnie	= KEF	= $\dot{m}_s \vec{v}_s ^2 f(\alpha)/8$
Mean Direct Residue	= UEDv	= $\left[(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s \right] d_p$ Eq. 2.14b
Closed Solution	= UEDvCF	= $\left[(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s + \bar{\beta}_B v_s^2/2 \right] d_p$ Eq. 2.14c
Relative Velocity Model	= UEDvREL	= $\left[(\epsilon_s \bar{\tau}_{sv}) : \nabla \vec{v}_s + \epsilon_s \bar{\beta}_B (\vec{v}_g - \vec{v}_s)^2 \right] d_p$ Eq. 2.14d
Neilson-Gilchrist Angle	= KENG	= (Eq. 2.6) $\times \xi_b$
	= Impingement Angle, α , Degrees	

8 Limitations of Current Erosion Models and Recommendations for Improvements

It would not serve the reader well if the impression were left that the erosion models contained in the code EROSION/MOD1 had no limitations and could not be improved. In fact, quite a few uncertainties still exist, some of which have been alluded to at various points in this manual. The major limitations, assumptions, and recommendations for future improvements will now be discussed.³

1. The extension of the Finnie and NG erosion models that uses the mean mass flux may over predict erosion rates. Another approach recently developed that uses the kinetic theory of granular flow may be a more rational way to extend these models.²²
2. The factor C in Finnie's erosion model may be inappropriate for fluidized beds. The value is taken from the original model to be $1/8$ and might be much lower for the nearly horizontal slow flow of particles near surfaces in fluidized beds.
3. The factor P_t in Finnie's model is taken to be the Vickers hardness. It could be the yield strength, tensile strength, or any one of other hardnesses reported in the literature. This could easily produce a spread of one-half order of magnitude in predicted erosion rates.
4. As already discussed, the wear factors for the NG model are only for aluminum and are used at the lowest range of the experimental velocity data. Wear factors for other materials, and extension of them to low particle flows, are needed.
5. The particles in all the erosion models are assumed to not break up. This implies they are much harder than the eroding surface. It is known that the erosion rate decreases dramatically when the particles become less hard than the eroding surface.¹⁴ Clearly, the part of the kinetic energy dissipation going into attrition increases and predominates over the portion that causes erosion.
6. The MED erosion models depend upon an empirical restitution coefficient that is presently taken to be a reasonable value of $e^2 = 0.9$, which could vary with impingement angle and velocity surface roughness and particle and eroding surface hardness. Raising e^2 from 0.9 to 0.99 would lower the predicted MED erosion rates an order of magnitude.
7. The exact expression for the solids kinetic energy dissipation rate is not yet clear. We presently recommend Eq. 2.14b on the basis of the work of Bouillard and Lyczkowski.¹⁷ We feel that the form given by Eq. 2.14d is probably not correct. In short, the fraction of the energy dissipation caused by interphase drag that contributes to erosion is not a certainty.
8. There are two ways to evaluate $\epsilon_s : \bar{\tau}_{sv} : \nabla \cdot \vec{v}_s$ one way uses cell-edge velocities and the other uses cell-centered velocities. These two means of evaluations render different results, with a variance of 20%.
9. Only a single average particle size is used in the erosion models. The methodology should be extended to include at least two particle sizes, and a methodology should be developed to calculate the erosion due to each mass fraction of the particle size distribution.

In spite of the limitations enumerated above, reasonable predictions of erosion rates and patterns have, in our opinion, been produced that are on the correct order of magnitude if the

solids hydrodynamics is correctly computed. Generally speaking, erosion rates are over-predicted at present and are hence conservative estimates. Items 1 through 9 should be reexamined in light of the present discussion. This is a major undertaking.

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Appendix A:

Assessment of Neilson-Gilchrist Erosion Model

A comparison of computed erosion rates in a fluidized bed for the Neilson-Gilchrist, erosion model was made with previously calculated erosion rates for Finnie's and the prototypical⁹ monolayer energy dissipation (MED) erosion models. The results are tabulated in Table A.1. The input values for all three models used were taken from the time-averaged hydrodynamic calculations previously performed.

Because of uncertainties in the extrapolated values from the empirical formulae for the ductile and brittle wear factors given by Eqs. 2.7 and 2.8, to very much lower velocities typical of FBCs (~1 m/s), the erosion rates for the Neilson-Gilchrist erosion model were computed for three cases for which the velocity, V , were set to: (1) the magnitude of the time-averaged computed particle velocity, $|\bar{v}_s|$ (~1 m/s); (2) 107.8 m/s (the lower bound of the experimental particle velocity data); and (3) 200 m/s (the upper bound of the experimental particle velocity data).

As can be seen in Table A.1, the Neilson-Gilchrist erosion rates with a velocity of 107.8 m/s or 200 m/s are about 1,000 times larger than those based on solids velocities calculated from the hydrodynamic model and are generally about the same as the early monolayer energy dissipation erosion model rates (except when they are zero). This is the reason for the recommendation to use the wear factors evaluated at 107.8 m/s, the lower bound of the experimental velocity data (Section 2.2). It is instructive to compare results in Table A.1 with those shown in Figure 7.

TABLE A.1 Comparison of Erosion Models

Erosion Range (mm/1,000 hrs) for Aluminum ^a					
Location ^b	Neilson and Glichrist			Finnie	Monolayer Energy Dissipation dp=500 μ m
	$V = \vec{V}_a $ (~ 1 m/s)	V = 107.8 m/s	V = 200 m/s		
1	6.69×10^{-4}	0.172	3.82	0.216	0.235
2	4.58×10^{-4}	0.61	1.5	1.62	0.136
4	0.0	0.0	0.0	0.06	0.296
6	0.0	0.0	0.0	0.01	0.296
8	1.18×10^{-4}	0.30	0.68	0.01	0.166
9	3.12×10^{-3}	0.035	0.09	0.05	0.168

^a A hardness of 30 kgf/mm (294MN/m²) was used for the Finnie and Monolayer Energy Dissipation models.

^b Refer to Figure 7

Appendix B:

EROSION/MOD1 Principal Fortran Symbols and Definitions

In the following list of Fortran symbols, the array quantities are indicated by an index. They are all part of a common block. The array dimensions of most of them need to be changed when the input data file is changed. The indexes shown in the following table and the respective array dimensions to be used are: $I \rightarrow IB2$ and $IJ \rightarrow IB2 \cdot JB2$. The rest are left unchanged unless otherwise specified.

Fortran Symbol	Definition
BHITE	Input date line No. 3
C(100)	Storage for constants initialized in SETC
DP	d_p , particle diameter, cm, input data line No. 13 + N
ERNG(K)	Neilson-Gilchrist model energy dissipation, W/m^2
EROEF(K)	Finnie model energy dissipation, W/m^2
EROEF1(K)	Total solids kinetic energy dissipation, W/m^2
DX	δx , read in line No. 3
DZ	δz , read in line No. 3
EPS	The cutting wear factor, J/kg
EP1	Young's modulus of the solids, Pa
ERNG	Energy dissipation rate, erg/cm^2
ET1	Young's modulus of the target, Pa
FL(IJ)	Cell flags set up in FLIC
FLO(M)	Input data line Nos. 5, 6, 7, and 9
HARD	Eroding surface hardness, kgf/mm^2
I	i, computing mesh column index
IB	Number of cells in the radial direction, excluding the two fictitious columns; $IB = IB2 - 2$
IB1	$IB1 = IB2 - 1$

IB2	Total number of cells in the radial direction, read in line No. 2
IDRAG	Read in line No. 2
IFLAG	Flag of erosion node (1:under, 2:right, 3:top, 4:left)
IFR0, IFR1 IFR2, IFR3	Dump control numbers read in line No. 1
IJ	Index of quantities for cell (i,j); $IJ = I + (J-1) \cdot IB2$
IJB	Index of cell-centered quantities associated with cell i, j-1
IJBR	Index of cell-centered quantities associated with cell (i+1, j-1)
IJL	Index of cell-centered quantities associated with cell (i-1,j)
IJM	Index of cell (i,j-1)
IJP	Index of cell (I,j+1)
IJR	Index of cell-centered quantities associated with cell (i+1,j)
IJRR	Index of cell-centered quantities associated with cell (i+2,j)
IJT	Index of cell-centered quantities associated with cell, (i,j+1)
IJTL	Index of cell-centered quantities associated with cell (i-1,j+1)
IJTR	Index of cell-centered quantities associated with cell (i+1,j+1)
IJTT	Index of cell-centered quantities associated with cell (i,j+2)
IMJ	Index of cell (i-1, j)
IMJM	Index of cell (i-1,j-1)
IPJ	Index of cell (i+1,j)
IPJM	Index of cell (i+1,j-1)
IPJP	Index of cell (i+1,j+1)
IPRES	Read in line No. 2
J	j, computing mesh row index (z direction)
JB	Number of cells in the axial direction, excluding the two fictitious rows $JB = JB2-2$
JB1	$JB1 = JB2 - 1$

JB2	Total number of cells in the axial direction, read in line No. 2
KDRAGX(IJ)	r component of gas-solids friction
KDRAGY(IJ)	z component of gas-solids friction
KFLIN(K)	Read in line No. 4
KFLOUT	Read in line No. 7
MMUG	Viscosity of gas used in the interphase drag, input data line No. 13 + N
MUL	Viscosity of solids used in viscous stress, input data line No. 12 + N
NO	Input data line No. 3
NSL(M)	Input data line No. 10
NSO(M)	Input data line No. 10 + N
OB(M,N)	Input data line No. 10 + N
P(IJ)	Pressure in cell (i,j)
PHI	ϕ_s , the shape factor, input data line No. 13 + N
PHIN	The deformation wear factor (J/Kg), energy dissipation, W/m^2
RES(K)	Prototype energy dissipation model
RLO	ρ_s , solids density, read in line No. 11 + N
SIGMY	The elastic load limit of target, Pa
SLOPE	Input data line No. 11 + N
SP	Poisson's ratio of the impinging solids
ST	Poisson's ratio of the target
TAUV(K)	The irreversible conversion to internal energy, W/m^2
TAUVCF(K)	The closed form solution energy dissipation, W/m^2
TAUVRL(K)	The relative velocity energy dissipation, W/m^2
TH(IJ)	$(\epsilon_g)_{i,j}^{n+1}$
THO	Input data line No. 11 + N
THZERO	Input data line No. 11+N

UL(IJ)	$(U_s)_{i+\frac{1}{2},j}^n$
VG(IJ)	$(V_g)_{i,j+\frac{1}{2}}^n$
VL(IJ)	$(V_s)_{i,j+\frac{1}{2}}^n$
ZN1	Unit normal vector on the obstacle (radial component)
ZN2	Unit normal vector on the obstacle (axial component)
ZT1	Unit tangential vector on the obstacle (radial component)
ZT2	Unit tangential vector on the obstacle (axial component)

Notice

The EROSION/MOD1 computer program may be obtained from:

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P. O. Box 1020
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