BENCHMARK EIGENVALUE SPECTRUM CALCULATIONS WITH ANISOTROPIC EFFECTS

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Complete eigenvalue spectrum calculations are usually performed for benchmark analysis to compare the accuracy of various approximate methods. In one-speed criticality studies the most accurate methods either rely on analytical techniques (singular eigenfunction) or high order spatial expansion (Carlvik's method). The extension of these techniques to calculate higher order anisotropy is somewhat cumbersome and rarely done in practice.

In this paper, a low spatial order technique is examined for its capability to produce accurate benchmark results with higher order of anisotropy. In the past, the use of low order methods were generally discarded for this type of calculation due to the large computer memory and time requirements. With the recent development and availability of greatly improved computing power, these limitations are largely removed and low order spatial approximations may also produce accurate solutions.

The technique is similar to a finite-element technique (weak formulation) applied to the integral neutron transport equation. The neutron flux is expanded in volume basis functions

\[ \Phi(r) = \sum h_j(r) \Phi(r_j), \quad r_j = \text{internal node boundary points} \]

\[ h_j(r) = \text{spatial polynomials} \]

Inserting the above expansion into the integral transport equation and performing a volume weighing (with \( \delta(r_j) \) weights) leads to the eigenvalue formulation of the transport problem (in 1-D slab),

\[ \Phi = (c/2) A \Phi \quad \text{or} \quad A \Phi = \lambda \Phi, \]

where \( c \) is the multiplication factor and \( \Phi \) contains the neutron fluxes at the node volume boundaries, \( \Phi(r_j) \). The elements of \( A \) are the basis function weighted transport kernel, for example for a 1-D slab,

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\[ a_j = \int h_j(r) E_j(r_j - r) \, dr \]

The singularity at \( i = j \) is a weak singularity, thus its analytical integration over a limited interval remains well behaved.

The above formulation may easily be extended to higher order anisotropic scattering by generalizing the flux expansion in basis function polynomials to the higher order components of the angular flux expansion coupled with a spherical-harmonics expansion of the neutron flux and scattering cross section, i.e. (for 1-D slab geometry),

\[ \phi(x) = \frac{c}{2} \sum_k (2k+1) \int f_k(x') dx' \int P_l(\mu) P_k(\mu) e^{-r_{x,x'}} \frac{d\mu}{\mu} \]

The eigenvalue problem is formulated as a block matrix system of equations, \( A\Phi = \lambda \Phi \). In this case the matrix \( A \) contains submatrix blocks \( A_{jk} \) which couple the angular flux components \( \phi_j, \phi_k \) contained in \( \Phi \).

The size of the matrices \( A \) and \( \Phi \) is determined as \( M^2 \times N^2 \), where \( M \) is the order of angular expansion and \( N \) is the number of spatial meshes.

The formulation may also be generalized to include reflectors with vacuum boundary. The eigenvalue problem becomes a generalized matrix problem of the form

\[ B \Phi = (c/2) A\Phi, \]

where the elements of \( B \) are \( b_{ij} = \delta_{ij} c_2 \int h_j(r) E_i(r_j - r) \, dr \) (for a P\(_6\) expansion) with \( c_2 \) the reflector multiplicity. Again the inclusion of anisotropic scattering is straightforward without any major difficulty other than the increase in the sizes of the matrices \( A \) and \( B \).

A related problem of infinite fuel/reflectors array can also be handled in this formalism by considering reflecting boundary conditions at the outer boundary. The transport kernel effectively
becomes an infinite sum representing the contribution from an infinite number of regions to the point under consideration. The general form of the matrix elements becomes ($P_0$ expansion)

$$a_{ij} = \sum_{k=0}^{\infty} \int h_j(2k(a+d) \pm r) E_j(2k(a+d) \pm r_j \cdot r) \, dr$$

where $a$ and $d$ are the half-thicknesses of the fuel and reflector region respectively. The coefficients of $B$ or $b_j$'s are similarly modified from the $P_0$ form. In the actual calculations, the infinite sum is limited to a few elements with non-negligible contribution. The reflective boundary conditions are incorporated by summing the appropriate infinite sums forming the matrix elements and considering the angular expansion coefficient and the harmonic modes (even/odd) to be calculated ($\Phi_j = \Phi_{j \pm 1} N$, $J_j = -J_{j \pm 1} N$ for $P_1$ expansion).

The eigenvalue calculations were performed in double precision on an IBM-3090 computer and utilized an optimized ESSL vectorized routine (DGEEV/DGEGV) to determine the full spectrum of the eigenvalues of the transport matrix for each approximation.

Selected results for the different slab problems are listed in Table 1 for up to $P_1/P_2$ expansions. Comparison with Reference 3, 4 and 5 between the various even eigenvalues are also presented indicating the accuracy of the present technique. In all cases, the fundamental eigenvalue remains real, however, complex eigenvalues may appear at the higher modes with increasing anisotropy. Figure 1 depicts the first even (fundamental) and odd modes for different values of linear anisotropy for the reflected slab cases. Similar plots can also be obtained for the higher order even/odd modes indicating a similar pattern.

The calculations indicate that accurate benchmark calculations are in the realm of low order spatial expansion methods including the investigation of the effects of anisotropic scattering on the higher order eigenvalues.
REFERENCES:


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<table>
<thead>
<tr>
<th>EVEN MODES</th>
<th>HOMOGENEOUS SLAB CORE a=2. cm VACUUM BOUNDARY</th>
<th>REFLECTED FUEL SLAB CORE a=2. cm REFLECTOR d=1. cm</th>
<th>INFINITE FUEL LATTICE CORE a=2. cm REFLECTOR d=1. cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_0$ $\mu = 0.$</td>
<td>$P_1$ $\mu = .1$</td>
<td>$P_2$ $\mu_1 = .1$ $\mu_2 = .3$</td>
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<tr>
<td>1 REF. 3/4/5</td>
<td>1.277101821</td>
<td>1.2888639</td>
<td>1.3059805</td>
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</tbody>
</table>
Fig. 1 EVEN/ODD FLUX MODES
END

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