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Technical Progress Report

Studies of the Longitudinal Instability with an Electron Beam

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1. Introduction and Summary

This progress report covers the research on the "Longitudinal Instability with an Electron Beam" performed at the University of Maryland during the period from February 1 to August 31, 1993 of the two-year grant sponsored by the Department of Energy, under Grant No. DEFG02-92ER54178.

In the last six months our main research effort has continued the study of generation, propagation, and edge reflection/transmission of localized perturbations in space-charge dominated electron beams. In addition, we performed the measurement of the geometry factor g of a coasting beam in a periodic focusing channel. The experiments were performed with our short-pulse electron beam injector and the 5-m long periodic solenoid focusing channel. This study has produced very interesting, unexpected results which concern the fundamental beam physics. A brief report on the generation of space-charge waves in the form of localized perturbations, on the measurement of the geometry factor g , and on the beam edge effect on space-charge waves is given in Sections 2, 3, and 4 below.

We would also like to report a change of research personnel in our group. Dr. D. Kehne, who was assigned to study space-charge waves in the long channel in the first year of this research funding, left us in last May for CEBAF. His position in our research program is now vacant and a new research associate will be hired in the near future. A possible candidate is David Grove of LLNL whose expertise in computer simulation for HIF would be very valuable. D. X. Wang, who has performed the longitudinal compression experiment during the last four years, is finishing his Ph.D. program soon and will assume a new position also at CEBAF next month. Hyyong Suk, a Ph. D. student from the Physics Department, joined us last spring, participated in the study of space-charge waves, and will run the resistive-wall instability experiment.

2. Generation of space-charge waves

The generation of space-charge waves in the form of localized perturbations was reported in the last progress report. What we have discovered was the first experimental evidence that a single fast wave, or a single slow wave, or any combination in amplitudes and polarities of these two waves can be generated. A time-domain approach was developed for the theoretical analysis, and the dependence of space-charge waves on initial perturbation conditions (velocity, density and current) was examined. Further theoretical

refinement of the analysis shows the conditions for degeneracy of two waves, indicating that this is a phenomenon in space-charge dominated beams. A detailed description of this study can be found in reference [1], a copy of which is included in this report in the Appendix.

The experimental results and preliminary analysis of the generation of space-charge waves were presented at the Particle Accelerator Conference, Washington, D.C., last May and generated unexpected interest among the participants. A paper entitled "Generation of space-charge waves due to localized perturbations in a space-charge dominated beam" has been submitted to Physical Review Letters and received very positive comments and recommendation for publication from the referees. We expect it to be accepted for publication.

3. Measurement of geometry factor g

The geometry factor g relates the longitudinal electrical field in a beam with line charge density variation, and the beam and pipe sizes. It plays an important role in the longitudinal beam dynamics. The g factor can be expressed in the long-wavelength limit by [2]

$$g = 2 \ln \frac{b}{a} + \alpha, \quad (1)$$

where b is the pipe radius, a is the beam radius, and α is a constant for which different values (1, 0.5, and 0 etc.) can be found in the literature. When the longitudinal field on the beam axis is considered, the value $\alpha=1$ is usually used. On other hand, the value $\alpha=0$ corresponds to the longitudinal field on the beam surface, while $\alpha=0.5$ applies to the average longitudinal field across the beam cross section. Therefore, the right values of g and α are of fundamental interest for beam physics in longitudinal instability analysis.

As far as we know, there has been no direct measurement of the g factor, especially the constant α , in experiments. We proposed a novel method to measure independently the g factor and the beam radius " a " of a coasting electron beam in our periodic focusing channel, that leads to an experimental determination of the constant α .

The g factor was measured by the propagation of localized space-charge waves on a coasting beam. The beam-frame velocity c_s of space-charge waves is determined by the longitudinal force or field, thus is related to the geometry factor g by [3]

$$c_s = \sqrt{\frac{eg\Lambda_0}{4\pi m\epsilon_0\gamma^5}}, \quad (2)$$

where Λ_0 is the line charge density of the beam. The time interval between two space-charge waves, which can be measured very accurately at different locations along the channel, is related to the traveling distance s by [4]

$$\Delta t = \frac{2c_s}{v_0^2 - c_s^2} \cdot s, \quad (3)$$

where v_0 is the beam velocity. Fig. 1 plots the time interval of the two waves as a function of the channel distance from the experiment, where the beam energy was 5.0 keV and the beam current was 62 mA. A least-square fitting of the experimental data yields $\Delta t = 2.66s$ ns where s is in the unit of meters. This results in the beam-frame velocity of $c_s \approx 2.33 \times 10^6$ m/s according to Eq. (3), which in turn yields the geometric factor $g = 2.45$ by Eq. (2).

Fig. 1. Time interval between two space-charge waves vs. drifting distance as measured by the current monitors. The solid line is a least-square fitting of the experimental data.

The beam radius "a" was measured by the phosphor screen plus CCD camera technique [5]. Due to slight mismatch in the periodic focusing channel, the beam performed envelope oscillation. The measured beam envelope in the last period of the channel for $\sigma_0=76^\circ$ is shown in Fig. 2. An average beam radius $a=0.56$ cm was obtained from this experiment. Using the measured beam radius "a" and the geometry factor g in Eq. (1) yields for the constant α the value $\alpha \approx 0$.

Fig. 2. Measured beam envelope in the last period of the channel.

A part of this experimental result was presented at the International Conference on Heavy Ion Inertial Fusion in Frascati, Italy, in last May. A paper entitled "Measurement of geometry factor g of a coasting beam in periodic focusing channel" is going to be submitted for publication.

4.. Beam edge effect on space-charge waves

The study of the beam edge effect on space-charge waves was a major goal in our first year's research proposal. The experiment was performed in the same setup as for the generation and propagation of space-charge waves. By placing the perturbation very close

to the beam edges, it was possible to determine what happens when the space-charge waves hit the beam edges. Due to the longitudinal expansion of the beam bunch, the beam edges in the experiment were not sharp, but had a rise and fall time of several nanoseconds which increased with distance along the channel. This made the experiment rather difficult. Nevertheless, we had enough experimental evidence to conclude that both transmission and reflection of space-charge waves on the beam edges would exist. Depending on the sharpness of the beam edge, the amount of the reflected wave and the transmitted wave would change accordingly. The sharper a beam edge was, the more of the perturbation was reflected.

Fig. 3. Current waveforms without and with perturbation (top two traces) and their difference (bottom trace), showing the original fast wave (F), the reflected slow wave (S_R), and the transmitted slow wave (S_T).

In Fig. 3 the top two traces are the current waveforms without and with the perturbation, measured by the fourth current monitor. The fast wave can be seen clearly on the perturbed current signal, but not the slow wave which hit the beam tail edge before it reached the fourth current monitor. The difference of the two current waveforms is shown

by the bottom trace. Besides the fast wave peak (F), two additional peaks appeared. One of them was the reflected slow wave (S_R) and the other was the transmitted slow wave (S_T). The transmitted slow wave continued to propagate on the beam falling edge, while the reflected slow wave turned back to follow the fast wave. In Fig. 4 the stars represent the time interval between the fast wave and the transmitted slow wave, while the circles are for the time interval between the fast wave and the reflected slow wave. It is clear that the reflected slow wave had almost the same speed as that of the fast wave, and that the transmitted slow wave had a rather high and constant speed throughout the falling edge. A similar experiment on the beam front edge was also performed and the same result was obtained.

Fig. 4. Time interval between the fast wave and the transmitted slow wave, and between the fast wave and the reflected slow wave. The solid line is a least-square fitting of the experimental data.

A paper entitled "behavior of space-charge waves on beam edges in a bunched electron beam" summarizes this research and will be submitted for publication.

5. Study of resistive-wall instability

Our theoretical study of the longitudinal instability in the past is summarized in a paper entitled "Longitudinal Instability of Space-Charge Dominated Beams in Transport Channels with Complex Wall Impedances", which has been published in Physics of Fluids B.

There was not much effort on the resistive-wall instability experiment in the last year. We plan to set up this experiment in September after we finish the experiments with the long periodic focusing channel.

6. Related research not sponsored by this contract

The longitudinal compression experiments are closely related to the Heavy Ion Fusion program, but funded by the High Energy Physics Office of the Department of Energy. We have performed the longitudinal expansion and compression for both parabolic and rectangular bunches. Some of the important findings from this research are the following:

a. We have experimentally verified the reversibility of the fluid equations which govern the motion of longitudinal expansion and compression of beams. This results in the reformation principle of rectangular beams, which could possibly provide a new acceleration mechanism for high current bunched beams. A paper to report this result will be submitted to Physical Review Letters for publication.

b. We have for the first time experimentally verified the linear force in a parabolic beam, verified the longitudinal envelope equation for the free expansion and drifting compression of parabolic bunches. Two papers on this topic have been published in Applied Physics Letters.

c. We have measured the longitudinal beam energy spread due to coupling from the transverse motion and due to nonlinearity of transverse focusing. The preliminary result was presented at the Particle Accelerator Conference, Washington, D.C., in last May, and was very well received. The longitudinal energy spread is a fundamental beam physics problem. It has significant importance to high energy physics accelerators, high current accelerators for heavy ion fusion, and other devices such as high power microwaves and free electron lasers, etc. We plan to pursue this study in our future research programs.

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12. D. X. Wang, J. G. Wang, M. Reiser, and I. Haber, "Longitudinal Dynamics of Space-Charge Dominated Parabolic Bunches", to be submitted for publication to PR-E.
13. J. G. Wang, D. X. Wang, M. Reiser, and H. Suk, "Behavior of Space-Charge Waves on Beam Edges in a Bunched Electron Beam", to be submitted for publication to PRL.
14. D. X. Wang, J. G. Wang, M. Reiser, and I. Haber, "Reformation of Space-Charge Dominated Rectangular Bunches", to be submitted for publication to PRL.

* Included in the Appendix.

Appendix

Copies of relevant papers

Generation of space-charge waves due to localized perturbations in a space-charge dominated beam*

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Abstract

It has been found for the first time in experiments with an electron beam that space-charge waves can be generated not only in pairs. A single slow wave, a single fast wave, or combinations of the two waves with different amplitude and polarity relations have been produced experimentally in a space-charge dominated beam. Analytical solutions derived from one-dimensional fluid equations show the amplitude and polarity of each space-charge wave as a function of the initial perturbation parameters. The theory provides the correct interpretation of the experimental observations.

PACS #: 29.27.Bd, 52.35.-g, 29.17.+w, 29.20.-c.

The longitudinal instability of charged particle beams is a very important issue in particle accelerators and other applications such as microwave devices. It is especially critical to high-current beam acceleration and transport where the beam physics is dominated by space charge rather than temperature, as is the case, for instance, in induction linac drivers for heavy ion inertial fusion [1]. This instability is caused by the interaction of slow space-charge waves with a resistive wall or a microwave circuit. We designed an experiment to investigate the growth of small slow-wave perturbations in space-charge dominated electron beams as a low-cost, practical choice to obtain an understanding of the underlying physics of the instability and to check code predictions [2].

The investigation of the behavior of space-charge waves has a long history in the fields of microwave generation and particle acceleration [3,4], where the analysis is usually carried out with sinusoidal signals and space-charge waves are always generated in pairs. There has so far, to the best of our knowledge, been no report of experimental evidence that only one single wave was produced. In order to study the physics of the longitudinal instability and beam-wave interaction, it is desirable to generate space-charge waves in the form of localized perturbations instead of sinusoidal signals. Bisognano et. al. studied localized perturbations by computer simulation [5]. They pointed out that "a localized perturbation on a beam will generally consists of a linear combination of forward and backward traveling eigenfunctions", implying other possible solutions. Still, there is very little information about experiments and detailed analysis of this topic in the literature.

In this letter we report new experimental results on the generation and propagation of space-charge waves due to localized perturbations in space-charge dominated electron beams. The experimental setup consists of a short-pulse electron beam injector [6] and a 5-m long periodic focusing channel [7]. The electron beam is generated by a gridded electron gun, and is matched into the transport channel where the beam is focused by 36 short solenoid lenses with a period of 13.6 cm. The beam pipe has a radius of 1.9 cm and the beam radius is less than 1 cm depending on the beam current, energy and focusing

conditions. The diagnostics includes five fast wall-current monitors and three time-resolved beam energy analyzers along the channel at different locations. Typical beam parameters in the experiment are: 50 ns in pulse duration, 40 mA in current and 2.5 keV in energy, resulting in a generalized perveance of $K=4.8 \times 10^{-3}$ to be defined in connection with Eq. (7).

The initial perturbation is introduced to the beam by modulating the cathode-grid pulse with a small bump as shown in Fig. 1. This corresponds to a positive (to increase) velocity perturbation on the beam particles, which in turn produces the initial density, or current perturbations. The space-charge waves in the form of localized perturbations are then generated and propagate along the beam. The relative strength of the initial current, or density perturbation over the given velocity perturbation can vary over a wide range, depending on the operating conditions of the gun. For instance, an initial velocity perturbation will cause a small, initial current change if the gun is operated in the space-charge limited regime; by contrast, a rather large, initial beam current change can be induced by the same initial velocity perturbation if the gun is operated in the temperature-limited regime [8]. Thus, the space-charge waves can be generated in various ways.

Figure 2 shows the beam current waveforms measured at two different locations along the transport channel. The slow and fast waves appearing in the beam current waveforms, are generated in almost equal amplitudes and opposite polarities. The evolution of the waveforms along the channel shows that the two space-charge waves propagate apart from each other. This effect is what is usually discussed in the literature about generation and propagation of space-charge waves.

Figure 3 shows two beam current waveforms taken at the same location as that in Fig. 2, but with different initial perturbation conditions. Only one fast wave with a positive polarity (current increase) has been generated on the electron beam current, which propagates toward the beam front. By contrast, Fig. 4 shows the beam current waveforms with only one slow wave, which has a negative polarity (current decrease) and propagates

toward the beam tail. These new phenomena happen in a space-charge dominated beam and require specific, initial perturbation conditions on the beam parameters.

Figure 5 shows the space-charge wave signals from a beam energy analyzer, where the beam particles with an energy above the average beam energy pass through the energy analyzer and form the bumps on the oscilloscope traces. All the eight scope traces are from the same energy analyzer and the same location, but with different initial perturbation conditions. On the top trace, only one fast velocity wave appears, corresponding to the case in Fig. 3, while on the bottom trace only one slow wave appears, corresponding to the case in Fig. 4. On the fourth trace, two velocity waves with almost equal amplitude and the same polarity appear, similar to the case in Fig. 2. Other traces show that one velocity wave dominates over another. In obtaining this set of data, the only difference is a change of the ratio of the initial current perturbation strength over the initial velocity perturbation strength by adjusting the gun operating conditions.

In order to interpret the experimental observations, the cold fluid model has been employed. The solution for generating space-charge waves is solved in the time domain for a localized perturbation instead of in the frequency domain for a sinusoidal wave. The specific solutions under the given initial and boundary conditions are obtained instead of eigenmode solutions from dispersion equations. All possible initial conditions of velocity, density, and current perturbations occurring in the experiment are taken into consideration.

The one-dimensional cold-fluid model consists of the linearized continuity equation and momentum transfer equation [1] :

$$\begin{cases} \frac{\partial \lambda_1}{\partial t} + v_0 \frac{\partial \lambda_1}{\partial z} + \lambda_0 \frac{\partial v_1}{\partial z} = 0 \\ \frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial z} = \frac{e}{m\gamma^3} E_z = \frac{-eg}{4\pi\epsilon_0 m\gamma^5} \frac{\partial \lambda_1}{\partial z} \end{cases} \quad (1)$$

where $\lambda(z,t)$ and $v(z,t)$ are the line charge density and the particle velocity, the subscripts 0 and 1 representing the unperturbed and perturbed quantities, respectively, $e/(m\gamma^3)$ denotes the ratio of the charge and the "longitudinal mass" of the charged particles with γ being the relativistic factor, $E_z(z,t)$ is the longitudinal electrical field induced by the a. c. component of the beam current, which is $(-g/4\pi\epsilon_0\gamma^2)(\partial\lambda_1/\partial z)$ in the long wavelength limit, i.e. the perturbation wavelength is much larger than the pipe radius, ϵ_0 is the permittivity of free space, and g is a geometric factor of order unity.

By applying the double Laplace transformations both for space z and time t , Eq. (1) then can be converted to algebraic equations for $\tilde{v}_1$, $\tilde{\lambda}_1$, and the perturbed beam current $\tilde{i}_1$ in the k - s domain.

Further, we consider the following initial and boundary conditions:

- a. There is no perturbation anywhere along the z -axis when $t < 0$;
- b. At $z=0$ for $t > 0^+$ a localized velocity perturbation is introduced to the beam in the form

$$v_1(0, t) = \delta v_0 h(t), \quad (2a)$$

where δ is a small, positive quantity to specify the strength of the perturbation, and $h(t)$ is any smooth function with an amplitude of unity. The function $h(t)$ is supposed to vanish when t is equal to or smaller than zero.

- c. Assume that the initial current perturbation has the form

$$i_1(0, t) = \eta i_0 h(t), \quad (2b)$$

with η as a small quantity to specify the perturbation strength. The value of η could be negative if a velocity increase causes a current decrease. Thus the initial density perturbation is determined by

$$\lambda_1(0, t) = (\eta - \delta) \lambda_0 h(t). \quad (2c)$$

Therefore, the perturbed beam parameters are

$$\tilde{\lambda}_1(k, s) = \frac{\tilde{h}(s)}{D} [s\eta i_0 + k(\eta - \delta)\lambda_0(v_0^2 - c_s^2)] \quad (3a)$$

$$\tilde{v}_1(k, s) = \frac{\tilde{h}(s)}{D} \{s[\delta v_0^2 + (\eta - \delta)c_s^2] + k\delta v_0(v_0^2 - c_s^2)\} \quad (3b)$$

and

$$\tilde{i}_1(k, s) = v_0 \tilde{\lambda}_1(k, s) + \lambda_0 \tilde{v}_1(k, s) \quad (3c)$$

Here $c_s = (eg\lambda_0/4\pi\epsilon_0 m\gamma^3)^{1/2}$ is the velocity of space-charge waves in the beam frame, and $D = (s + kv_0)^2 - (kc_s)^2$ is the dispersion function.

Inverse Laplace transforms yield the perturbed beam density, velocity and current in the real time-space domain as

$$\begin{aligned} \lambda_1(z, t) = & -\frac{\lambda_0}{2} \left[\delta \frac{v_0}{c_s} - (\eta - \delta) \right] h\left(t - \frac{z}{v_0 - c_s}\right) + \\ & + \frac{\lambda_0}{2} \left[\delta \frac{v_0}{c_s} + (\eta - \delta) \right] h\left(t - \frac{z}{v_0 + c_s}\right) \end{aligned} \quad (4a)$$

$$\begin{aligned} v_1(z, t) = & +\frac{v_0}{2} \left[\delta - (\eta - \delta) \frac{c_s}{v_0} \right] h\left(t - \frac{z}{v_0 - c_s}\right) + \\ & + \frac{v_0}{2} \left[\delta + (\eta - \delta) \frac{c_s}{v_0} \right] h\left(t - \frac{z}{v_0 + c_s}\right) \end{aligned} \quad (4b)$$

$$\begin{aligned} i_1(z, t) = & -\frac{i_0}{2} \left[\delta \frac{v_0}{c_s} - \eta + (\eta - \delta) \frac{c_s}{v_0} \right] h\left(t - \frac{z}{v_0 - c_s}\right) + \\ & + \frac{i_0}{2} \left[\delta \frac{v_0}{c_s} + \eta + (\eta - \delta) \frac{c_s}{v_0} \right] h\left(t - \frac{z}{v_0 + c_s}\right) \end{aligned} \quad (4c)$$

Equations (4a), (4b), and (4c) show that each perturbed beam parameter consists of two terms: the one with $h[t - z/(v_0 - c_s)]$ is the slow wave, while the one with $h[t - z/(v_0 + c_s)]$ is the fast wave. The amplitude and polarity of each wave is determined by the coefficients before the function $h(t)$. It is easily seen that the condition for generating only one fast wave is

$$\frac{\eta}{\delta} = 1 + \frac{v_0}{c_s} \quad (5)$$

while the condition for producing only one slow wave is

$$\frac{\eta}{\delta} = 1 - \frac{v_0}{c_s} \quad (6)$$

The beam velocity v_0 is usually much larger than the wave velocity c_s . These two conditions require a rather larger, relative initial current change for a given velocity perturbation. Note that the relation

$$\frac{v_0}{c_s} = \gamma \left(\frac{1}{2} g K \right)^{-1/2}, \quad (7)$$

where $K = (2i_0/I_0\beta^3\gamma^3)$ is the generalized beam perveance, with $\beta = (1 - 1/\gamma^2)^{1/2}$, i_0 being the beam current and $I_0 = 1.7 \times 10^4$ A for electrons. The generalized perveance K is a measure for the effects of space charge in a beam. A beam with a large perveance K yields a small value of v_0/c_s , thereby reducing the required value of $\ln \eta/\delta$ in a practical device. Thus Eqs. (5) and (6) can be more easily satisfied in a space-charge dominated beam.

Fig. 6 shows the relative amplitude of the velocity waves, i.e. the ratio of the velocity wave amplitude over the initial perturbation amplitude δv_0 , versus the quantity η/δ for $c_s/v_0 = 0.1$. It is only at the point A where $\eta/\delta = 1$ indicating the initial density perturbation is zero, that the two velocity waves have the same polarity and the same relative amplitude of $1/2$. Elsewhere, the two velocity waves have different amplitudes, even different polarity when $\eta/\delta < 1 - v_0/c_s$ or $\eta/\delta > 1 + v_0/c_s$. The sum of the two amplitudes is always 1, i.e. equal to the initial perturbation amplitude, as expected. The slow wave vanishes completely at $\eta/\delta = 1 + v_0/c_s = 1.1$, while the fast wave vanishes at $\eta/\delta = 1 - v_0/c_s = 0.9$. This can be compared with the scope traces in Fig. 5. Note that each scope trace in Fig. 5 is not an energy spectrum of the space-charge waves, i.e. the peaks of the bumps in each trace do not directly provide the information about the amplitudes of the perturbed beam velocity. Further, a quantitative measurement of the value δ was not available in the experimental setup. Therefore, there was no attempt made to more quantitatively compare the experimental data in Fig. 5 with the theoretical plot in Fig. 6.

According to Eq. (4c), the two current waves have the same amplitudes with the opposite polarity only when $\eta=0$, i.e. without an initial current perturbation. This is the case in Fig. 2. When $\eta/\delta=(1+v_0/c_s)$, only one fast wave is generated as in the case of Fig. 3. When $\eta/\delta=(1-v_0/c_s)$, only one slow wave exists as shown in Fig. 4. The relative amplitude of the current waves, i.e. the ratio of the current wave amplitude over the initial perturbation amplitude η_0 , versus the quantity η/δ is plotted in Fig. 7, where $c_s/v_0=0.1$ is also assumed. A similar plot can be made for the density waves according to Eq. (4a), as shown in Fig. 8.

In summary, the generation of space-charge waves due to localized perturbations to the beam parameters has been studied experimentally and analytically. It is found for the first time in experiments that a single wave, i.e. either a fast (forward) wave, or a slow (backward) wave, can be generated in a space-charge dominated beam. The analytical solutions provide all the information about the wave amplitude, polarity and the condition for degeneracy. The ability to generate single localized space-charge perturbations is especially useful for the experimental investigation of wave reflection from bunch ends and of wave interaction with a resistive wall (longitudinal instability). It may also find applications in other fields such as microwave generation.

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Figure captions:

Fig. 1. Grid-cathode voltage waveform with a perturbation bump in the gun (10 ns/div. horizontally, relative scale vertically).

Fig. 2. Beam current waveforms with perturbations measured at the channel distances of $z=0.624$ m and $z=3.48$ m, respectively ($z=0$ is the cathode position), where F is for the fast wave and S is for the slow wave (Ordinate is the current amplitude in relative scale).

Fig. 3. Beam current waveforms with only one fast wave, taken at the same locations as that in Fig. 2.

Fig. 4. Beam current waveforms with only one slow wave as measured at the same locations as that in Fig. 2.

Fig. 5. Velocity waves measured at $z=3.75$ m by a beam energy analyzer. The eight traces corresponds to eight different initial perturbation conditions in the gun. From top to bottom the fast wave amplitude decreases while the slow wave amplitude increases.

Fig. 6. Relative velocity wave amplitude, i.e. the ratio of the velocity wave amplitude over the initial perturbation amplitude δv_0 , vs. initial perturbation parameters η/δ for $c_s/v_0=0.1$, where the solid line is for slow wave while the dashed line is for fast wave.

Fig. 7. Relative current wave amplitude, i.e. the ratio of the current wave amplitude over the initial perturbation amplitude ηi_0 , vs. initial perturbation parameters η/δ for $c_s/v_0=0.1$.

Fig. 8. Relative density wave amplitude, i.e. the ratio of the density wave amplitude over the initial perturbation amplitude $(\eta-\delta)\lambda_0$, vs. initial perturbation parameters η/δ for $c_s/v_0=0.1$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

LONGITUDINAL EXPANSION AND COMPRESSION OF ELECTRON BUNCHES WITH RECTANGULAR LINE CHARGE PROFILES *

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Abstract

The longitudinal expansion and compression of a space-charge dominated rectangular electron bunch have been investigated at the University of Maryland Electron Beam Transport Facility. In this paper we report the experimental results and comparison with theoretical prediction and 1-D PIC simulations.

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1. Introduction

The longitudinal dynamics of space-charge dominated beams has been studied experimentally, theoretically, and numerically at the University of Maryland Electron Beam Transport Experiment. One of the objectives is to study beams having a rectangular line charge profile such as those found in induction linacs for Heavy Ion Inertial Fusion drivers. In previously experimental study [1] of the end erosion of a rectangular pulse the bunch lifetime has not been sufficiently long to explore the beam dynamics as the two waves launched at each end reach the bunch center. Understanding the details of this regime can be important to achieving longitudinal bunch compression [2]. Recently we have conducted the longitudinal expansion and compression experiments of a space-charge dominated rectangular electron beam to verify theoretical prediction and to check simulation codes.

2. 1-D cold fluid model

For a bunch which is long compared with the pipe diameter, the longitudinal space-charge field can be approximately calculated as [3, 4]

$$E_z(z, s) = - \frac{g}{4\pi\epsilon_0\gamma^2} \frac{\partial\Lambda(z, s)}{\partial z} \quad (1)$$

where z is the distance from the center of the bunch, $\Lambda(z, s)$ is the line charge density, s is the distance traveled by the beam bunch center, and the geometry factor g is a function of the ratio of the pipe radius b to the beam radius a [4].

In the experiment the longitudinal temperature is low enough so that the longitudinal beam dynamics can be described by 1-D cold fluid equations, i.e. the continuity equation and momentum equation:

$$\begin{cases} \frac{\partial\Lambda}{\partial t} + v \frac{\partial\Lambda}{\partial z} + \Lambda \frac{\partial v}{\partial z} = 0 \\ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = \frac{e}{m\gamma^3} E_z = - \frac{eg}{4\pi\epsilon_0 m\gamma^5} \frac{\partial\Lambda}{\partial z} \end{cases} \quad (2)$$

which can be solved by the method of characteristics. In general there are velocity and density waves along the bunch propagating in the beam frame towards both directions. If the waves overlap, the problem is nonlinear and in general must be solved numerically. However, if there is no wave overlap, i.e. in the "simple-wave" region, then an analytical solution exists. An approximate formula is found in the region not too far away from the "simple wave" region. The lengthy formula and detailed derivations can be found in ref. [2].

3. Experimental setup

The experimental configuration is illustrated schematically in Fig. 1. The electron bunch is produced and then matched into a 5-m long periodic focusing channel. Typical initial beam parameters are an energy 2.5 keV, a current 36 mA, and pulse duration 10-50 ns. Beam current profiles are measured by five fast, 2 GHz bandwidth current monitors located along the channel. Two retarding field energy analyzers and one deflecting field energy analyzer along the channel are used to measure the longitudinal beam velocity distribution.

4. Experimental results

4.1 Longitudinal expansion

Without a time-varying acceleration an initially rectangular bunch, with a constant energy will expand from both edges due to the very strong space-charge forces. The rarefaction waves propagate at speed of c_s inwards and at $2c_s$ outwards, where

$$c_s = \sqrt{\frac{egI}{4\pi\epsilon_0 v m_e}}, \quad (3)$$

and $I = \Lambda\beta_0 c$ is the beam current. Both the current profiles and velocity distributions of such a drift-expanding rectangular bunch were measured as shown in Fig. 2 and Fig. 3a, 3b, 3c, respectively. The experimental data is compared in the figures with theory and simulation using the 1-D Particle In Cell (PIC) code. The top curves in Fig. 2 and Fig. 3a are typical current and velocity profiles, respectively, in the simple wave region showing the edge erosion while the center region remains unperturbed. When the waves meet at the beam center, a change in behavior occurs where the beam velocity relative to the beam center varies approximately linearly and density or current profile has a cusp shape where the peak current has not yet changed. In this experiment we deliberately adjusted the pulse duration so that the waves meet at the second energy analyzer. As shown in Fig. 3b the velocity distributions from both measurement and simulation are close to linear. The bottom curves in Fig. 2 and Fig. 3c show the current and velocity profiles in the non-simple wave region. It is noted that there is an asymmetrical droop at the center region of the current profile. That is caused by the velocity tilt since the velocity is no longer flat in the center region. The theoretical curves obtained from the approximation formula are in reasonably good agreement with experiment and simulation. The waves overlap in the center region and the peak current significantly reduces. During the whole process the bunch length expands at a speed of $4c_s$.

4.2 Longitudinal compression

By using the time-varying acceleration voltage of the induction gap, a rebunching velocity gradient was imparted to the beam. In this experiment the voltage slope is about 50 eV/ns. The measured current profiles of the compressing bunch are shown in Fig. 4. The peak current increases to nearly 5 times the initial one as the bunch length shortens slightly while the leading and falling edges spread out due to the space-charge force. Simulation work is currently underway.

It was observed in the experiment that changing focusing strength of the matching lenses over wide range had little effect on the longitudinal behavior. Simulation work done by the WARP code for a parabolic beam also indicates that the longitudinal dynamics is insensitive to the transverse mismatch [5].

5. Conclusion

Experiment on the longitudinal expansion and compression of a space-charge dominated rectangular electron bunch has been carried out. Good agreement is found

between theory, experiment and simulation for drift expansion. The linear velocity distribution predicted by theory was experimentally demonstrated at the point where the rarefaction waves meet at the beam center. The approximate solution in the nonsimple-wave region is found to be in good agreement with experiment and simulation. The initial rectangular bunch has been longitudinally compressed, achieving nearly five times peak current amplification. In the compression experiment we found that the current profiles were not significantly changed when the transverse focusing was changed. Further simulation work is underway.

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Figure Captions:

- Fig. 1.** Experimental configuration.
- Fig. 2.** Current profiles of expanding rectangular bunch at different locations, where the dash curve represents the initial current profile, the circles experiment, the triangles simulation, and the solid curves theory.
- Fig. 3.** Velocity distributions of expanding rectangular bunch at different locations, where the circles represent experiment, the triangles simulation, and the solid curves theory.
- Fig. 4.** Current profiles of compressing bunch at five different locations.

Injector

Periodic Solenoid Focusing Channel

Gridded
Electron
Gun

Induction
Module

Diagnostic
Chamber

Solenoid
Focusing
Lens

Fast Beam
Current
Monitor

Retarding
Energy
Analyzer

Deflecting
Energy
Analyzer

DATE

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END