DOE/ER/13927--4

DE92 007534

EFFECTS OF CAPILLARITY ON MICROSCOPIC FLOW IN POROUS MEDIA

Progress Report
for Period June 1, 1991 - May 31, 1992

Michael J. Miksis
Department of Engineering Sciences
and Applied Mathematics
Northwestern University
Evanston, Illinois 60208

January 1992

Prepared for
THE U.S. DEPARTMENT OF ENERGY
AGREEMENT NO. DE-FG02-88ER13927

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED *ok*

Abstract

The central theme of this proposal is to study the effects of capillarity on the motion of a fluid interface and to apply these results to flow in porous media. Here we report on several problems considered this year. In particular we have investigated a new similarity solution of a moving boundary problem driven only by surface tension, we have started an investigation on the effect of roughness on the motion of a contact line and we have started both a numerical and analytical investigation of the motion of fluid interfaces in a pore. In addition we report on a new method to derive macroscopic effective equation of motion of two-phase flows at low volume fraction.

Several projects have been under investigation during the first year of this grant renewal. In the following I will be specific about the work.

The central theme in this proposal is to study the effects of capillarity on the motion of a fluid interface and to apply these results to flow in porous media. During this year we have studied several problems in this direction. For example in collaboration with my graduate student, J. Tsai, we have studied the surface tension driven motion of a wedge of fluid (see the enclosed report, "The effect of surface tension on the motion of a viscous wedge of fluid", submitted to Phys. Fluids A). Here we report on a similarity solution of the Stokes equations for the motion of a viscous wedge of fluid. The similarity solution holds for a free wedge or for a wedge of fluid bounded by a solid plane boundary. The latter case has applications to the motion of a contact line. The motion is completely driven by surface tension. This problem can be thought of as the local free surface problem that occurs when a free interface is formed, e.g., when a bubble of air becomes unstable and divides to form a foam in a porous media, or it can be related to the local dynamics of an interface near a contact line as the interface relaxes into an equilibrium solution. The enclosed report derives the similarity solution, presents solutions and describes a numerical method to solve for the free surface shapes. Extensions of this work to other viscosity ratios is currently under way and expected to be completed soon.

The major project which we have started, and for which we expect to concentrate our efforts on for the completion of this year and during the next, is the motion of a fluid interface on the microscopic scale in a porous media, e.g., in a pore. For example, we are currently studying how a gas bubble moves inside of a pore with non-parallel walls. The purpose of this investigation is to understand how a foam is formed. This foam formation process is called "snap off" in the literature and is not completely understood. We are studying this process using a lubrication model and computationally. The asymptotic model will be valid for small Capillary numbers and in the lubrication limit. Here we are able to derive a first order time and forth order space evolution equation for the bubble interface in the pore. Solutions of this evolution equation will be studied both analytically and numerically. The numerical solution of the complete free boundary problem consists of solving the Stokes equations for the liquid within the pore by using a boundary integral method. This code is partially written and we expect to have results by this summer. Once completed we will be able to generalize these techniques to study the motion of a fluid interface within a pore with contact line motion and in a variety of geometries.

Most of the models of contact line motion assume that the surface along which the contact line is moving is smooth. This idealization is fine for many applications but for the motion of an fluid interface within a porous material this may be too much of an idealization. One could argue that the slip coefficient and prescribed contact angle-slip velocity relation accounts for the observed roughness of the surfaces. But in many cases the roughness is on such a large scale that the slip and contact angle relations are not effected by it, but the scale of the roughness is smaller than the scale over which measurements are made. Hence it is interesting to consider the dynamics of an interface with a contact line as it moves over a rough surface. We have initiated a project on this topic. In

particular we have developed a model for the motion of a droplet over a rough surface. This was done by using lubrication theory. In particular we have found that if the advancing contact angle is sufficiently small, then an infinite number of steady states are possible for the droplet, while if the contact angle is sufficiently large, only a unique steady state exists. We are now trying to generalize this result to a finer scale of roughness.

Many interesting problems in two-phase flow occur for situations where the volume fraction of one of the phases is very small and is in the form of discrete inclusions, e.g., bubbly liquids, dusty gasses, fogs, etc. In collaboration with L. Ting of the Courant Institute of NYU (see the enclosed report "Wave propagation in a bubbly liquid at small volume fraction", submitted to Chem. eng. Comm.), we have developed a theory using the method of matched asymptotic expansions, to systematically develop effective equations of motion for these two-phase flows. We apply this technique to a bubbly liquid. The result of this analysis is a system of effective equations which generalize previous theories. In particular, in the recent review article by Miksis and Ting 1991 (Advances in Applied Mechanics, vol. 28) a set of uniform equations were postulated for wave propagation in a bubbly liquid. Here these equations were derived.

Also recently published is the article, Bounds for Mobility of Foams Flowing through Porous Media (Phys. Fluids A vol. 3) by the principle investigator. This work develops bounds on the mobility of a foam (a two-phase flow) flowing through a porous media by studying the dissipation of the two-phase flow. The results generalize the variational bounds used to estimate the fluid permeability of a one-phase flow in porous media and the results are related to the one-phase permeabilities.

During this period I have taken one summer month in salary plus 5% academic year. I plan to do the same during 92-93.

- "The Effect of Surface Tension on the Motion of a Wedge of Viscous Fluid", (with J. Tsai), Applied Mathematics Technical Report 9109, Northwestern University, 1991, Phys. Fluids A, Submitted.
- "Wave Propagation in a Bubbly Liquid at Small Volume Fraction", (with L. Ting), Applied Mathematics Technical Report 9106, Northwestern University, 1991, Chem. Eng. Comm., to appear.
- "Bounds for Mobility of Foams Flowing Through Porous Media", Phys. Fluids A, 3, 2541-2545, 1991.
- "Effective Equations for Multiphase Flows - Waves in a Bubbly Liquid", (with L. Ting), Advances in Applied Mechanics, 28, eds. J. W. Hutchinson and T. Y. Wu, 141-260, 1991.

END

**DATE
FILMED**

3 / 25 / 92