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Integrated Seismic Study of Naturally Fractured Tight Gas Reservoirs

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This was the seventh quarter of the contract. During this quarter we (1) continued the large task of processing the seismic data, (2) collected additional geological information to aid in the interpretation, (3) tied the well log data to the seismic via generation of synthetic seismograms, (4) began integrating regional structural information and fracture trends with our observations of structure in the study area, (5) began constructing a velocity model for time-to-depth conversion and subsequent AVO and raytrace modeling experiments, and (6) completed formulation of some theoretical tools for relating fracture density to observed elastic anisotropy.

Review of Study Area; Available Seismic and Well Information

The study area is located at the southern end of the Powder River Basin in Converse County in east-central Wyoming. It is a low permeability fractured site, with both gas and oil present. Reservoirs are highly compartmentalized due to the low permeabilities, and fractures provide the only practical drainage paths for production. The two formations of interest are:

- The Niobrara: a fractured shale and limey shale to chalk, which is a reservoir rock, but also its own source rock.
- The Frontier: a tight sandstone lying directly below the Niobrara, brought into contact with it by an unconformity.

Figure 1 shows a basemap with the seismic lines being analyzed for this project (labeled RMC 21, 22, 23 and 24), plus locations of 13 wells that we are using to supplement the analysis. The arrows point to two wells for which we have constructed synthetic seismograms, as discussed below.

Well Log Synthetics

We have generated normal incidence synthetic seismograms for two wells: the Apache #36-1 State, located along the southeast part of line 2, and the Energetics #43-28 Sims, located along line 3 (see Figure 1). The first stage in the process involved blocking the digitized sonic and density logs for input to the synthetic generation program. This was accomplished with the Sierra Geophysics **LOGPREP**^{TM} program, which allows for various combinations of layers and smoothing of spikes. Through trial and error, the final blocked log configurations shown in Figure 2 were achieved. These plots show the sonic,

density and velocity (1/sonic) logs for both wells; each consists of 30 blocked layers over several thousand feet, with large amounts of smoothing. Increasing the number of layers to 50 or 100 seemed to weaken the correlation of lithologic boundaries with events on the synthetic, as did further reducing the number of layers. Note that the area of interest (the Niobrara and Frontier formations, Figure 2), contains the most variability, which translates into numerous finely spaced layers.

The models for the two wells were subsequently input into Sierra's **VESPA**^{TM} (**ViscoElastic Seismic Profile Algorithm**) program, which simulates seismic shot records and generates 3-D frequency-dependent synthetics for flat layered models. The program solves, in this case, for the vertical displacement by a Hankel transform-type integral over wavenumber for particular source and receiver depths. For this experiment, no surface waves, PS conversions or interbed multiples were considered, and the solution was for the zero-offset case. The resulting impulse response was then convolved with a 20 Hz Ricker wavelet and scaled to match the real data as closely as possible.

Figure 3 shows the comparison between synthetic and real data for the Apache #36-1 State and Energetics #43-28 Sims wells, respectively, with the layer boundaries tied in from log tops also shown. Qualitatively, ignoring the large spike at the top of the model, the synthetics appear to match the real data quite well in the zone of interest. The Frontier reflection, the strong peak at 2.53 s at the Apache well along line 2 (Figure 3), shows the most consistent correlation, and we are now able to identify this event and others on the real data with confidence. The Niobrara is a less consistent trough, but it shows interesting P-wave amplitude variability which we are attempting to correlate with fracture occurrence and orientation.

Structural Trends and Fractures

Preliminary structure maps for the Niobrara and Frontier, based only on tops for the 13 wells for which we have information, show a prominent northeast-plunging anticline which ramps up steeply to the southwest near the flexure. This crest of this anticline appears to lie just to the southeast of the intersection of lines 2 and 3 at the Niobrara level, and slightly farther west at the Frontier level. The isopach between the Niobrara and Frontier thickens dramatically both to the southwest and southeast of this structural high.

The structural trend is subparallel with the fracture orientation of N55°E - N60°E determined for the Frontier formation in the Apache #36-1 State well, as described in last quarter's report. The other fracture orientation, for a group of fractures within the Niobrara at this well location, was perpendicular to this trend, approximately N30°W to N40°W. As stated in a previous report, northwest-trending fractures were found by Lewis *et al.* (1991, Geophysics) for the Niobrara at Silo Field in southeastern Wyoming. Laubach (1992, GRI Tight Gas Sands Report), in a study of the Green River Basin and Moxa Arch area of southwestern Wyoming, noted that Cretaceous and younger deformation has produced north-, northeast- and east-trending fractures in the foreland basins. There is also a possible change in orientation of the regional flexure trend in the eastern part of the study area from northwest to northeast (Amoco Corp., personal comm.). This change may be associated with the transition to the bordering Hartville Uplift (Figure 4a). In addition, Mitchell and Rogers (1993, *The Mountain Geologist*, RMAG) have correlated trends in isopach maps (Figure 4b) and resistivity maps with predominant fracture orientations and regional tectonic trends. In summary, there does appear to be a change from a northwest orientation in predominant structural and fracture trends just north of the study area to a northeast orientation within the study area, and we will attempt to integrate this information into our interpretation.

Data Processing

Last quarter we reported some preliminary results on P-wave and S-wave processing of the various lines. This quarter we have additionally processed the P-wave data for line 3 and some of the S-wave components for line 3. We have returned to the preliminary AVO analysis that we reported for line 2: Amoco gave us some gain corrections that they discovered. We have incorporated these and continuing with the analysis.

Theoretical Formulation Relating Fracture Occurrence to Seismic Anisotropy

During the quarter we finished a theoretical study relating fracture density and orientation to observable seismic anisotropy. Previous studies, most notably those by Hudson, have established relations between dry fractures and seismic properties, but suffer from several deficiencies: (1) they are based on idealized fracture shapes, which may not always be adequate to describe natural fractures, (2) they are limited to low fracture densities, and (3) they do not adequately treat the differences between dry and fluid-

saturated fractures and the resulting frequency dependence (which makes laboratory results different from field results). We took an approach which is fairly general with regard to fracture geometry and density and focused on the problem of relating dry (gas-saturated) and fluid saturated conditions. We plan to publish the work in Geophysics. A copy of the manuscript is attached.

Plans for Next Quarter; Depth Conversion Strategies

We have begun the process of depth converting portions of lines 2 and 3 near the ties to the Apache and Energetics wells. An accurate and reasonable velocity model is necessary for obtaining a structure map which ties the well tops and for future raytrace modeling. We will continue developing a velocity model which honors the well log data and produces a reasonable earth model.

Figure 2: Blocked sonic and density logs for Apache #36-1 State well (top) and Energetics #43-28 Sims well (bottom). Five formation boundaries are shown, including top Niobrara and top First Frontier sand; section is divided into 30 layers in each case.

Figure 3: Tie between synthetic seismogram to left and real data to right for Apache #36-1 State (top) and Energetics #43-28 Sims (bottom) wells (see Figure 1 for locations). Wavelet on synthetics is a 20 Hz Ricker. Other major formation boundaries are also labeled. Blocked velocity logs are shown to left. Large amplitude spike near the top of the synthetic should be ignored.

Figure 4(a): Map of structural features in east half of Wyoming, from Mitchell and Rogers (1993) showing Powder River Basin (shaded) and adjacent Hartville Uplift to the southeast. Field site for DOE investigation is marked by dot.

Figure 4(b): Isopach map of Sage Breaks Member (Frontier equivalent) showing change in trend of thickness variations from NW-SE in northern two-thirds of area to NE-SW in southern third, implying possible change in fracture orientation in this area (from Mitchell and Rogers, 1993). DOE field site for this study is outlined in southwest corner.

PORE FLUID EFFECTS ON SEISMIC VELOCITY IN ANISOTROPIC ROCKS

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ABSTRACT

A simple new technique predicts the high and low frequency saturated velocities in anisotropic rocks entirely in terms of measurable dry rock properties, without the need for idealized crack geometries. Measurements of dry velocity vs. pressure and porosity vs. pressure contain all of the necessary information for predicting the frequency-dependent effects of fluid saturation. Furthermore, these measurements automatically incorporate all pore interaction, so there is no limitation to low crack density. The velocities are found to depend on five key interrelated variables: frequency, the distribution of compliant crack-like porosity, the intrinsic or non-crack anisotropy, fluid viscosity and compressibility, and effective pressure. The sensitivity of velocities to saturation is generally greater at high frequencies than at low frequencies. The magnitude of the differences, from dry to saturated and from low frequency to high frequency, is determined by the compliant or crack-like porosity. Predictions of saturated velocities based on dry data for sandstone and granite show that compressional velocities generally increase with saturation and with frequency. However, the degree of compressional wave anisotropy may either increase or decrease upon saturation depending on the crack distribution, the effective pressure, and the frequency at which the measurements are made. Shear wave velocities can either increase or decrease with saturation, and the degree of anisotropy depends on the microstructure, pressure, and frequency. Consequently great care must be taken when interpreting observed velocity anisotropy; measurements at low frequencies, typical

of insitu observations, will generally be different than at high frequencies, typical of the laboratory.

INTRODUCTION

Most crustal rocks are found experimentally to be seismically anisotropic. This is usually understood as a combination of microstructural fabric (Jones, 1983), fine scale layering (Backus, 1962), and aligned microcracks (Nur and Simmons, 1969). Although each of these may be equally important, crack effects have been emphasized most in hydrologic, exploration, and tectonic studies, because of their sensitivity to stress (Nur, 1971; Lockner et al., 1977; Crampin, 1978) and their impact on permeability (Walls, 1983). Nur (1971) showed experimentally that nonhydrostatic stress can induce both P- and S-wave anisotropy in an otherwise isotropic rock, by preferentially closing cracks that are perpendicular or nearly perpendicular to the axis of greatest compression. Lockner et al. (1977) showed that increasing the differential stress to near failure can further increase anisotropy by creating new cracks. Numerous field studies have confirmed fracture-related anisotropy in situ (eg., Alford, 1986; Crampin et al., 1985; Willis et al., 1986; Winterstein and Meadows, 1991; Queen and Rizer, 1990)

Despite this emphasis, surprisingly few experimental (Jones, 1983; Lucet and Tarif, 1988; Tarif, 1986; Zamora and Poirier, 1990) and theoretical (Brown and Koringa, 1975; Hudson, 1980, 1981, 1991) studies exist on the critical anisotropic effects of *pore fluids and pore pressure*, which are most pronounced in cracked rocks.

The sensitivity of seismic velocity to pore fluids, in general, was demonstrated years ago in the laboratory for *isotropic* rocks (Nur and Simmons, 1969), and is the basis for modern direct seismic hydrocarbon detection and EOR monitoring (Nur, 1989). High reflectivity contacts between fluid-saturated and gas-saturated rocks are often detectable as "bright spots" on conventional P-wave stacked sections, and as amplitude vs. offset (AVO) anomalies on prestack data (Ostrander, 1982). High pore pressure (Spencer and Nur, 1976; Jones, 1983), perhaps coupled with anisotropy

(Lueschen, et al., 1990), may be the key to puzzling low velocity zones in the crust, and low-strength, low velocity fault zones (Raleigh and Evernden, 1982).

Every one of these fluid effects is enhanced by compliant, crack-like porosity. Therefore, it can clearly be misleading to interpret in situ seismic anisotropy without a complete understanding of pore fluid effects in anisotropic cracked media. This is particularly true when cracks are superimposed on an intrinsically anisotropic mineral matrix.

Most previous analysis techniques (eg., Hudson, 1980, 1981; Crampin, 1984; Thomsen, 1986) for cracked anisotropic rocks share one or more features which can limit their usefulness, particularly when comparing laboratory and field data: (1) they are based on idealized (and unrealistic) crack geometries, such as ellipsoidal cracks, (2) they are therefore intrinsically limited to low crack densities, (3) they often ignore the anisotropic mineral matrix, and (4) Their treatment of frequency-dependent saturation effects is incomplete.

We present here a simple alternative technique for predicting the relation between dry and saturated velocities in anisotropic rocks entirely in terms of *measurable dry rock properties*, without the need for idealized crack geometries. Measurements of dry velocity vs. pressure and porosity vs. pressure contain all of the necessary information for predicting the frequency-dependent effects of fluid saturation. Furthermore, these measurements automatically incorporate all pore interaction, so there is no limitation to low crack density.

We find that the velocities depend on five key interrelated variables:

- Frequency
- The distribution of compliant, crack-like porosity
- The intrinsic or non-crack anisotropy
- Fluid viscosity and compressibility
- Effective pressure

The sensitivity of velocities to saturation is generally greater at high frequencies than at low frequencies. The magnitude of the differences, from dry to saturated and from low frequency to high frequency, is determined by the compliant or crack-like porosity.

At low frequencies our results reduce to those of Brown and Korrington (1975), which are an anisotropic extension of Gassmann's (1951) relations. For high frequencies, we present a simple new method for predicting the amount of local flow or "s squirt" dispersion (Mavko and Nur, 1975; O'Connell and Budianski, 1977), which is an extension of our previous results for the isotropic case (Mavko and Jizba, 1991).

Using this formulation, we show that velocities and velocity anisotropy in saturated rocks can be quite different at low (in situ) frequencies than at high (laboratory) frequencies. Consequently great care must be used when applying laboratory observations to the field, or when using laboratory observations to validate a model.

Fluid Mechanisms

If a set of loads applied to a dry rock cause a decrease in pore volume, then saturating the rock with fluid will usually increase the rock's stiffness under the same set of loads. Under undrained conditions pore compression induces pore fluid pressure, which, in turn, resists the deformation. Gassmann (1951) quantified this for isotropic rocks with a simple expression relating the saturated rock bulk modulus to the dry rock bulk modulus:

$$\frac{1}{K_{dry}} - \frac{1}{K_{sat}} = \frac{\left(\frac{1}{K_{dry}} - \frac{1}{K_0}\right)^2}{\left(\frac{1}{K_{dry}} - \frac{1}{K_0}\right) + \phi\left(\frac{1}{K_f} - \frac{1}{K_0}\right)}, \quad (1)$$

where K_0 , K_{dry} , K_{sat} , and K_f are the bulk moduli of the mineral grains, the dry composite, the saturated composite, and the pore fluid, and ϕ is the porosity. This result assumes isotropy, but is otherwise completely independent of pore geometry, and elastic interaction between pores is completely described. The result is limited to low frequencies so that inertial and scattering effects can be ignored and so that the pore pressure induced by the externally applied compression has time to equilibrate

throughout the pore space.

Brown and Korringa (1975) extended Gassmann's relation to anisotropic media:

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(sat)} = \frac{(S_{ij\alpha\alpha}^{(dry)} - S_{ij\alpha\alpha}^0)(S_{kl\alpha\alpha}^{(dry)} - S_{kl\alpha\alpha}^0)}{(\beta_{dry} - \beta_0) + (\beta_f - \beta_0)\phi}, \quad (2)$$

$$\beta_{dry} = S_{\alpha\alpha\beta\beta}^{(dry)} = 1/K_{dry},$$

$$\beta_0 = S_{\alpha\alpha\beta\beta}^0 = 1/K_0,$$

$$\beta_f = 1/K_f,$$

where $S_{ijkl}^{(sat)}$ is the predicted saturated rock compliance, while $S_{ijkl}^{(dry)}$ and S_{ijkl}^0 are the compliances of the dry rock and mineral material. This form again assumes that the frequencies are low, so that the induced pore pressures are everywhere equalized.

Biot's (1956a, b) formulation relaxed some of the requirements of Gassmann's low frequency theory, by including the dynamic coupling of fluid and solid stresses and the effects of contrasting inertial forces in the fluid and solid. The low frequency limit of Biot's formulation for bulk modulus is identical to Gassmann's, and a modest dispersion is predicted at higher frequencies. Biot (1962a, b) later extended this work to anisotropic media.

Measured seismic velocities in saturated rocks at ultrasonic frequencies (Winkler, 1985, 1986) are often much faster than predicted by either the Gassmann or Biot formulations. Both experiments (Murphy et al., 1984; Wang and Nur, 1988, 1990) and models (Mavko and Nur, 1979; O'Connell and Budiansky, 1974, 1977; Mavko and Jizba, 1991; Thomsen, 1985, 1986, 1991) suggest that the cause is related to the grain-scale microscopic flow field, referred to as "local flow" or "squirt" (Mavko and Nur, 1975). For most crustal rocks, particularly those with crack porosity, the Biot dispersion is small compared with squirt dispersion, and can often be ignored.

To understand the squirt mechanism, consider a saturated rock sample with the pore space vented to zero pore pressure. When an increment of stress is applied

to the rock at very low frequencies, pore fluid can easily flow in and out of every pore; the rock behaves as though it is drained, and its moduli are essentially the same as dry rock moduli. At higher frequencies, viscous and inertial effects cause the thinnest pores to be isolated with respect to flow from the larger drained pores, and the external stress induces increments of pore pressure. Stiff portions of the pore space tend to have relatively low induced pore pressure; the compliant pores have relatively high induced pore pressure. The unequilibrated pore pressures make the rock stiffer than at low frequencies.

Our formulation considers the saturated compliant, crack-like fraction of the pore space (which we sometimes also call the soft porosity) to be part of the viscoelastic rock frame (Biot, 1962a,b), separate from the larger, stiffer fraction of the pore space where "global" flow might occur. This is in contrast to the Biot-Gassmann-Brown-Korrington approach, which assumes that pore pressures are uniform throughout the pore space. We call the high frequency limit of this frame the "unrelaxed frame" and label its compliances $S_{ijkl}^{(wet)}$ which are different from $S_{ijkl}^{(sat)}$.

We proceed now to derive expressions for the *unrelaxed frame compliances*, $S_{ijkl}^{(wet)}$, in terms of the dry or relaxed compliance, $S_{ijkl}^{(dry)}$, at the same pressure, and the dry compliance at very high confining pressure, $S_{ijkl}^{(highP)}$. These unrelaxed frame compliances can then be substituted into the Brown and Korrington or Biot formulations in place of the dry compliances to predict $S_{ijkl}^{(sat)}$ and model the total dispersive behavior of the saturated rock.

DERIVATION OF UNRELAXED FRAME MODULI

We estimate the effect of the uneven pore pressure distribution on the unrelaxed frame compliance, $S_{ijkl}^{(wet)}$, using the Betti-Rayleigh reciprocity theorem (Walsh, 1965; Jaeger and Cook, 1969). Consider the two sets of tractions applied to a rock with total volume V , as shown in Figure 1.

The rock on the left is loaded by externally applied tractions $\Delta T_i = \Delta \sigma_{ij} n_j$,

where $\hat{n}$ is the outward unit normal vector at each point on the surface, and $\Delta\sigma_{ij}$ is an arbitrary constant stress tensor corresponding to the incremental loading of a passing wave. Internally applied normal tractions, $\Delta P(x, y, z)$, (which are equal to the as yet unknown spatially variable incremental pore pressures induced by the wave) are applied to the unrelaxed compliant fraction of the pore space. Zero internal tractions are assumed on the drained fraction of the pore space. Therefore, it has the appearance of a partially saturated rock with fluid in the thin, compliant pores only, and it deforms with the unrelaxed frame compliance. The rock on the right has the same tractions ΔT_i applied to the external surfaces and zero tractions applied to the internal surfaces, so it deforms with the effective dry rock compliances, $S_{ijkl}^{(dry)}$. Applying the reciprocity theorem, we can write:

$$\Delta\sigma_{ij}\Delta\sigma_{kl}S_{ijkl}^{(dry)}V - \sum_n \Delta P_n \Delta v_n = \Delta\sigma_{ij}\Delta\sigma_{kl}S_{ijkl}^{(wet)}V. \quad (3)$$

The summations result from separating the integrals of work done on the pore walls into the sum of integrals over subsets of the pore space having approximately equal induced pore pressure, ΔP_n . Correspondingly, v_n is the volume of that subset of pore space and Δv_n is the dry rock stress-induced change of pore volume.

The volume change Δv_n can be written in terms of an effective dry pore compliance, $W_{ijkl}^{(n)}$ that relates the average strain of the n th piece of pore space to the applied stress:

$$\Delta v_n = W_{\alpha\alpha kl}^{(n)} \Delta\sigma_{kl} v_n. \quad (4)$$

The induced pressure in the n th unrelaxed pore can be written as (see appendix):

$$\Delta P_n = \frac{W_{\alpha\alpha ij}^{(n)} \Delta\sigma_{ij}}{W_{\alpha\alpha\beta\beta}^{(n)} + (\beta_f - \beta_0)}, \quad (5)$$

where β_f and β_0 are the compressibilities of the pore fluid and of the rock without the soft pores. Combining equations (3), (4), and (5) and taking partial derivatives with respect to $\Delta\sigma_{ij}$ and $\Delta\sigma_{kl}$ leads to:

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} = \sum_n \frac{\phi^{(n)} W_{\alpha\alpha ij}^{(n)} W_{\alpha\alpha kl}^{(n)}}{W_{\alpha\alpha\beta\beta}^{(n)} + (\beta_f - \beta_0)}, \quad (6)$$

where $\phi^{(n)}$ is the porosity of the n th pore. Equation (6) expresses the unrelaxed wet frame compliances entirely in terms of the dry rock properties and the pore fluid modulus. Since the low frequency relaxed frame is the same as the dry frame, the difference $S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)}$ is also equal to the frame dispersion.

Following the work of Mavko and Jizba (1991), we now imagine that each piece of soft porosity is locally thin and "crack-like", but has otherwise quite general geometry. If we define local crack coordinate systems with the x_3 axis normal to the crack faces and transform the compliance tensor, $W_{ijkl}^{(n)}$, to that coordinate system, then equation (6) can be written as

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} = \sum_m \frac{\phi^{(m)} \bar{W}_{\alpha\alpha rs}^{(m)} \bar{W}_{\alpha\alpha tu}^{(m)}}{\bar{W}_{\alpha\alpha\beta\beta}^{(m)} + (\beta_f - \beta_0)} \beta_{ir}^{(m)} \beta_{js}^{(m)} \beta_{kt}^{(m)} \beta_{lu}^{(m)}, \quad (7)$$

where the $\beta_{ij}^{(m)}$ are the direction cosines between the crack coordinate axes and the global coordinate axes (not to be confused with β_f and β_0 , the fluid and mineral compressibilities). We have grouped together into sets all pores having approximately the same orientation, so that $\bar{\phi}^{(m)}$ and $\bar{W}_{rstu}^{(m)}$ now refer to the porosity and average pore compliance of the m th set.

We now take a closer look at the dry pore deformation, $\phi^{(n)} W_{ijkl}^{(n)}$, again using the reciprocity theorem (Figure 2). The rock on the left has the same tractions ΔT_i applied to all exterior surfaces, as before. The inside surfaces are traction free, so that the rock deforms with the dry compliances. The rock on the right has the same tractions applied externally and to the inside surfaces of the soft pores, so that

it deforms approximately as though the soft pores were not there – as though filled with mineral material, or compressed closed by high confining pressure – with effective high pressure dry compliances, $S_{ijkl}^{(highP)}$. Then we can write:

$$\Delta\sigma_{ij}\Delta\sigma_{kl}S_{ijkl}^{(dry)}V - \sum_n \Delta\sigma_{ij}\Delta\sigma_{kl}W_{ijkl}^{(n)}v_n = \Delta\sigma_{ij}\Delta\sigma_{kl}S_{ijkl}^{(highP)}V, \quad (8)$$

which leads to

$$\Delta S_{ijkl}^{(dry)} = S_{ijkl}^{(dry)} - S_{ijkl}^{(highP)} = \sum_n \phi^{(n)} W_{ijkl}^{(n)}, \quad (9)$$

where $\Delta S_{ijkl}^{(dry)}$ is the change in dry compliance between the pressure of interest and very high confining pressure.

In terms of the crack-local coordinate systems, equation (9) can be written as

$$\Delta S_{ijkl}^{(dry)} = \sum_m \bar{\phi}^{(m)} \bar{W}_{rstu}^{(m)} \beta_{ir}^{(m)} \beta_{js}^{(m)} \beta_{kt}^{(m)} \beta_{lu}^{(m)}. \quad (10)$$

From equations (9) and (10) we see that, in principle, measurements of the change in dry rock compliances with pressure, $\Delta S_{ijkl}^{(dry)}$, contain information about the dry pore compliances $\phi^{(m)} W_{ijkl}^{(m)}$. When substituted into equations (6) and (7), they allow us to predict the high frequency, unrelaxed, wet compliances, in terms of the measured dry rock behavior. We illustrate this with examples of isotropic, transversely isotropic, and orthorhombic symmetries.

Isotropic

Approximating the summation in equation (10) as an integral over all crack orientations, and noting that for an isotropic rock all pore sets act similarly, equation (10) can be written as:

$$\Delta S_{ijkl}^{(dry)} = \phi_{soft} \bar{W}_{rstu} \frac{1}{4\pi} \int \beta_{ir} \beta_{js} \beta_{kt} \beta_{lu} d\Omega, \quad (11)$$

where $\bar{W}_{rstu}$ is the representative average compliance of one crack set, averaged over all crack shapes, including all elastic interactions with the other cracks, and ϕ_{soft} is the total soft porosity – the porosity that closes under high confining pressure. Using this, the change in dry compressibility with pressure can be written as:

$$\Delta S_{\alpha\alpha\beta\beta}^{(dry)} = \phi_{soft} \bar{W}_{\alpha\alpha\beta\beta}. \quad (12)$$

To estimate the wet compliances, we similarly approximate the sum in equation (7) with an integral over all crack orientations, and use isotropy to bring the porosity and compliances outside of the integral:

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} = \frac{\phi_{soft} \bar{W}_{\alpha\alpha\beta\beta} \bar{W}_{\alpha\alpha\beta\beta}}{\bar{W}_{\alpha\alpha\beta\beta} + (\beta_f - \beta_0)} \frac{1}{4\pi} \int \beta_{ir} \beta_{js} \beta_{kt} \beta_{lu} d\Omega. \quad (13)$$

For crack-like features we expect the $\bar{W}_{ijkl}$ to be sparse (see appendix), so that equation (13) can be simplified to

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} \approx \frac{\phi_{soft} \bar{W}_{\alpha\alpha\beta\beta}}{1 + (\beta_f - \beta_0)/\bar{W}_{\alpha\alpha\beta\beta}} \frac{1}{4\pi} \int n_i n_j n_k n_l d\Omega, \quad (14)$$

where $\hat{n}$ is the unit normal to the crack faces. Values of the tensor $G_{ijkl} = (1/4\pi) \int n_i n_j n_k n_l d\Omega$ are given in Table 1. Combining with equation (12) gives

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} \approx \frac{\Delta S_{\alpha\alpha\beta\beta}^{(dry)}}{1 + \phi_{soft}(\beta_f - \beta_0)/\Delta S_{\alpha\alpha\beta\beta}^{(dry)}} G_{ijkl}, \quad (15)$$

which shows that the difference between high frequency (unrelaxed) and low frequency (relaxed) frame compliances is a simple function of the change of dry compressibility

with pressure. This agrees with the previous results of Mavko and Jizba (1991). Often the soft porosity correction term in the denominator is quite small and may be neglected, so that for practical purposes the above equation may be written as:

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} \approx \Delta S_{\alpha\alpha\beta\beta}^{(dry)} G_{ijkl}. \quad (16)$$

Transversely Isotropic

A critical point to remember is that the measured *dry* compliances $\Delta S_{ijkl}^{(dry)}$ are affected by both the *volumetric* and *shear* components of crack deformation; the amount of the shear vs. volumetric effect depends on the orientation of each crack set relative to the applied field. In contrast, the change in *wet* compliance $\Delta S_{ijkl}^{(wet)}$ is affected only by the decrease in *volumetric* compliance when pore fluid is added. The compressional effect can be seen in equation (3) as the crack-by-crack products of pressure increment times volume change, and also in equation (6) as the product of volumetric pore strains $W_{\alpha\alpha ij} W_{\alpha\alpha kl}$. To extract only the volumetric part of the dry compliances, it is necessary to carefully treat the crack orientations. For the isotropic case this was very simple, because we assume that all orientations exist. But for anisotropic rocks, we must take more care.

For example, the simplest (though not necessarily correct) model for transversely isotropic rocks assumes that all of the soft, crack-like porosity is parallel. We leave general the crack geometries, crack density, crack interaction, etc. With only this single crack set, equation (10) simplifies to

$$\Delta S_{ijkl}^{(dry)} = \phi_{soft} \bar{W}_{ijkl}. \quad (17)$$

Equation (7) also simplifies for a single crack set and when combined with equation (17) gives

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} = \frac{\Delta S_{\alpha\alpha ij}^{(dry)} \Delta S_{\alpha\alpha kl}^{(dry)}}{\Delta S_{\alpha\alpha\beta\beta}^{(dry)} + (\beta_f - \beta_0)\phi_{soft}}. \quad (18)$$

We now show how the measured dry compliances $\Delta S_{ijkl}^{(dry)}$ can be uniquely decomposed into their volumetric and shear components for any general transversely isotropic crack distribution. This includes not only a single crack set or a superposition of a single set over an isotropic distribution but also any other rotationally symmetric distribution of cracks. The extracted volumetric component is then used to calculate the unrelaxed high frequency saturated compliances. We assume sparseness of the crack compliance tensor $\bar{W}_{ijkl}^m$ (see appendix) and also assume the ratio of the normal to shear crack compliance to be unknown, but the same for the m different crack sets. The measured change in the dry compliances with pressure, equation (10), can be resolved into a sum of as yet unknown volumetric and shear components G_{ijkl} and H_{ijkl} respectively as :

$$\Delta \tilde{S}_{ijkl}^{(dry)} = \frac{\Delta S_{ijkl}^{(dry)}}{\Delta S_{\alpha\alpha\beta\beta}^{(dry)}} \approx G_{ijkl} + \alpha H_{ijkl}, \quad (19)$$

where

$$G_{ijkl} = \int f(\Omega) \beta_{i3} \beta_{j3} \beta_{k3} \beta_{l3} d\Omega, \quad (20)$$

$$H_{ijkl} = \int f(\Omega) [(\beta_{i2} \beta_{k2} + \beta_{i1} \beta_{k1}) \beta_{j3} \beta_{l3} + (\beta_{j2} \beta_{k2} + \beta_{j1} \beta_{k1}) \beta_{i3} \beta_{l3} + (\beta_{i2} \beta_{l2} + \beta_{i1} \beta_{l1}) \beta_{j3} \beta_{k3} + (\beta_{j2} \beta_{l2} + \beta_{j1} \beta_{l1}) \beta_{i3} \beta_{k3}] d\Omega, \quad (21)$$

and

$$\alpha = (\Delta \tilde{S}_{\alpha\alpha\beta\beta}^{(dry)} - 1)/4, \quad (22)$$

where $f(\Omega)$ is the normalized crack orientation distribution function such that its integral over all angles equals unity. α , which is computed from the dry data, is the

ratio of the representative shear to normal compliance of one crack set, including all elastic interactions with the other cracks. In the above expressions the tilde is used to denote the normalized change in dry compliances, and the β_{ij} are the components of the Euler rotation matrix that relates the crack coordinates to the laboratory coordinate frame. The tensor G_{ijkl} represents the fraction of total compliance that is due to volumetric deformation of cracks with different orientations, for a given externally applied load σ_{ij} .

Now making use of the orthonormality properties of the rotation matrix *viz.* $\beta_{i1}\beta_{j1} + \beta_{i2}\beta_{j2} + \beta_{i3}\beta_{j3} = \delta_{ij}$ and $\beta_{\alpha i}\beta_{\alpha j} = \delta_{ij}$ one can uniquely solve for the independent components of G_{ijkl} :

$$G_{1111} = \Delta\tilde{S}_{1111}^{(dry)} - \frac{4\alpha}{1-4\alpha}(\Delta\tilde{S}_{1122}^{(dry)} + \Delta\tilde{S}_{1133}^{(dry)}), \quad (23)$$

$$G_{1122} = \Delta\tilde{S}_{1122}^{(dry)} / (1 - 4\alpha), \quad (24)$$

$$G_{1133} = \Delta\tilde{S}_{1133}^{(dry)} / (1 - 4\alpha), \quad (25)$$

$$G_{3333} = \Delta\tilde{S}_{3333}^{(dry)} - \frac{8\alpha\Delta\tilde{S}_{1133}^{(dry)}}{1-4\alpha}, \quad (26)$$

$$G_{2323} = \frac{\Delta\tilde{S}_{2323}^{(dry)}}{(1-4\alpha)} - \frac{\Delta\tilde{S}_{1111}^{(dry)} + \Delta\tilde{S}_{3333}^{(dry)}}{4(1-4\alpha)} + \frac{G_{1111} + G_{3333}}{4}. \quad (27)$$

Having thus extracted the volumetric part we can calculate the high frequency wet compliances from equation (7) as:

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} \approx \frac{\Delta S_{\alpha\alpha\beta\beta}^{(dry)}}{1 + \phi_{soft}(\beta_f - \beta_0) / \Delta S_{\alpha\alpha\beta\beta}^{(dry)}} G_{ijkl}, \quad (28)$$

where ϕ_{soft} is the total soft porosity – the porosity that closes under high confining pressure. Note that if the $\Delta S_{ijkl}^{(dry)}$ are isotropic, then G_{ijkl}^{TI} reduces to the isotropic form in Table 1. If there is a single crack set, then equation (28) reduces to a simplified form of equation (18). (For a single crack set, equation (18) allows a completely general crack compliance tensor W_{ijkl} , while equation (28) assumes a sparse W_{ijkl} as described in the appendix.)

Orthorhombic

For three mutually perpendicular crack sets superimposed on a general orthorhombic background equation (10) becomes:

$$\Delta S_{ijkl}^{(dry)} = \sum_{m=1}^3 \bar{\phi}^{(m)} \bar{W}_{rstu}^{(m)} \beta_{ir} \beta_{js} \beta_{kt} \beta_{lu}. \quad (29)$$

Assuming crack-like behavior of each set (sparse W_{ijkl}) we get:

$$\begin{aligned} \Delta S_{ijkl}^{(dry)} &= \bar{\phi}^{(1)} \bar{W}_{\alpha\alpha\beta\beta}^{(1)} \delta_{i1} \delta_{j1} \delta_{k1} \delta_{l1} + \bar{\phi}^{(2)} \bar{W}_{\alpha\alpha\beta\beta}^{(2)} \delta_{i2} \delta_{j2} \delta_{k2} \delta_{l2} + \bar{\phi}^{(3)} \bar{W}_{\alpha\alpha\beta\beta}^{(3)} \delta_{i3} \delta_{j3} \delta_{k3} \delta_{l3} \\ &= \Delta S_{1111}^{(dry)} \delta_{i1} \delta_{j1} \delta_{k1} \delta_{l1} + \Delta S_{2222}^{(dry)} \delta_{i2} \delta_{j2} \delta_{k2} \delta_{l2} + \Delta S_{3333}^{(dry)} \delta_{i3} \delta_{j3} \delta_{k3} \delta_{l3}. \end{aligned} \quad (30)$$

Similarly the unrelaxed wet moduli, equation (7), become:

$$\begin{aligned} S_{ijkl}^{(dry)} - S_{ijkl}^{(wet)} &\approx \frac{\Delta S_{1111}^{(dry)}}{1 + \phi_{soft}^{(1)}(\beta_f - \beta_0)/\Delta S_{1111}^{(dry)}} \delta_{i1} \delta_{j1} \delta_{k1} \delta_{l1} \\ &+ \frac{\Delta S_{2222}^{(dry)}}{1 + \phi_{soft}^{(2)}(\beta_f - \beta_0)/\Delta S_{2222}^{(dry)}} \delta_{i2} \delta_{j2} \delta_{k2} \delta_{l2} \\ &+ \frac{\Delta S_{3333}^{(dry)}}{1 + \phi_{soft}^{(3)}(\beta_f - \beta_0)/\Delta S_{3333}^{(dry)}} \delta_{i3} \delta_{j3} \delta_{k3} \delta_{l3}. \end{aligned} \quad (31)$$

For more general orthorhombic crack distributions the nine independent elements of the G matrix are given by:

$$G_{1111} = \Delta \tilde{S}_{1111}^{(dry)} - \frac{4\alpha}{1-4\alpha} (\Delta \tilde{S}_{1122}^{(dry)} + \Delta \tilde{S}_{1133}^{(dry)}), \quad (32)$$

$$G_{2222} = \Delta \tilde{S}_{2222}^{(dry)} - \frac{4\alpha}{1-4\alpha} (\Delta \tilde{S}_{1122}^{(dry)} + \Delta \tilde{S}_{2233}^{(dry)}), \quad (33)$$

$$G_{3333} = \Delta \tilde{S}_{3333}^{(dry)} - \frac{4\alpha}{1-4\alpha} (\Delta \tilde{S}_{1133}^{(dry)} + \Delta \tilde{S}_{2233}^{(dry)}), \quad (34)$$

$$G_{1122} = \Delta \tilde{S}_{1122}^{(dry)} / (1-4\alpha), \quad (35)$$

$$G_{1133} = \Delta \tilde{S}_{1133}^{(dry)} / (1-4\alpha), \quad (36)$$

$$G_{2233} = \Delta \tilde{S}_{2233}^{(dry)} / (1-4\alpha), \quad (37)$$

$$G_{2323} = \frac{\Delta \tilde{S}_{2323}^{(dry)}}{(1-4\alpha)} - \frac{\Delta \tilde{S}_{2222}^{(dry)} + \Delta \tilde{S}_{3333}^{(dry)}}{4(1-4\alpha)} + \frac{G_{2222} + G_{3333}}{4}, \quad (38)$$

$$G_{1313} = \frac{\Delta \tilde{S}_{1313}^{(dry)}}{(1-4\alpha)} - \frac{\Delta \tilde{S}_{1111}^{(dry)} + \Delta \tilde{S}_{3333}^{(dry)}}{4(1-4\alpha)} + \frac{G_{1111} + G_{3333}}{4}, \quad (39)$$

$$G_{1212} = \frac{\Delta \tilde{S}_{1212}^{(dry)}}{(1-4\alpha)} - \frac{\Delta \tilde{S}_{1111}^{(dry)} + \Delta \tilde{S}_{2222}^{(dry)}}{4(1-4\alpha)} + \frac{G_{1111} + G_{2222}}{4}, \quad (40)$$

and the unrelaxed wet frame compliances are calculated from equation (28). To predict the total dispersion these unrelaxed wet frame compliances computed from the squirt formalism are then taken as the effective dry compliances in the Brown and Korrington or Biot equations.

DISCUSSION

We have presented a technique for predicting the saturated velocities at both high and low frequencies in anisotropic rocks in terms of measureable dry rock velocities. At low frequencies, representative of in situ measurements of water saturated rocks, the saturated elastic compliances can be derived from the dry compliances and total porosity, using equation (2). At very high frequencies, as with ultrasonic laboratory measurements, the saturated compliances depend on the distributions of crack orientations and compliances, and can be related to the change of dry rock compliance with pressure, plus a correction proportional to the difference between the compressibilities of the liquid and the mineral. As pointed out by Mavko and Jizba (1991) this has the simple interpretation that filling a compliant pore or crack with unrelaxed fluid resists compression, similar to filling it with mineral, or equivalently, compressing it closed. The second order term compensates for the finite compressibility of the fluid.

The isotropic result, equations (15) and (16), is consistent with the results of Mavko and Jizba (1991). The tensor G_{ijkl} (Table 1) represents the fraction of total compliance due to volumetric deformation of cracks with different orientations, for a given externally applied load σ_{ij} . This form highlights the result found before that for an isotropic rock, the dispersions in bulk and shear are expected to be proportional, (Mavko and Jizba, 1990).

The first transversely isotropic result, equation (18), is appropriate when all of the soft, crack-like, porosity is aligned normal to the x_3 direction. This would be indicated by little or no change of the $\Delta S_{1111}^{(dry)}$ and $\Delta S_{2222}^{(dry)}$ dry compliances with stress. The second transversely isotropic result, equation (28), would be appropriate when the dry measurements indicate transversely isotropic behavior, but with stress-induced changes in more than the 3333 component. The general analysis of crack distributions, using equations (23)-(27) will indicate that a single set of cracks predominates when $G_{3333} \rightarrow 1$ and $G_{1111}, G_{2222}, G_{1122}, G_{1133}, G_{2233} \ll G_{3333}$.

The orthorhombic result, equation (31), is appropriate when the crack-like porosity lies in 1, 2, or 3 perpendicular sets. This would be recognized when the change in dry compliances $\Delta S_{1122}^{(dry)}, \Delta S_{1133}^{(dry)}, \Delta S_{2233}^{(dry)}$ are much smaller than $\Delta S_{1111}^{(dry)}, \Delta S_{2222}^{(dry)}, \Delta S_{3333}^{(dry)}$.

The components of the G matrix for any of the symmetries are calculated directly

from the dry data without assuming any specific crack orientation distribution function $f(\Omega)$. In computing these components, care must be taken to ensure that they always correspond to real cracks and not to anti-cracks ($f(\Omega) < 0$). We found that for real data (which are rarely perfect) better results are obtained by slightly damping the denominator in equations (23) to (27) and keeping the soft porosity correction term in equation (28).

Finally, we emphasize that equations (15), (16), (18), (28), and (31) are for the unrelaxed wet compliances, $S_{ijkl}^{(wet)}$ which describe the stiffening effect of pore fluid in the most compliant parts of the pore space. These must then be substituted into equation (2) in place of $S_{ijkl}^{(dry)}$ for the completely saturated behavior $S_{ijkl}^{(sat)}$.

For cracked media saturated with fluid, it is very important to differentiate the two limits or regimes of behaviour depending on the degree of isolation of the cracks with respect to fluid flow : a) low frequency or isobaric behaviour when the fluid may flow and equilibrate the pressure gradients induced by the passing wave, and b) high frequency or adiabatic behaviour when regions of unequilibrated pore pressures are effectively isolated with respect to fluid flow. The unequilibrated pore pressures always makes the rock stiffer in both compression and shear than the isobaric case.

Most crack models of rocks, that are based on idealized ellipsoidal inclusions (Eshelby, 1957; Walsh, 1965; O'Connell and Budiansky, 1974; Nishizawa, 1982) or scatterers (Kuster and Toksöz, 1974; Hudson, 1980, 1981), are high frequency theories in terms of *fluid effects*. This should not be confused with the fact that these models are often termed low frequency models as the wavelengths are assumed to be much larger than crack dimensions. For example, Hudson's (1980, 1981) model of anisotropic cracked rocks treats fluid filled cavities as isolated with respect to fluid flow. Therefore, Hudson's formulation for saturated rocks is best suited for high frequency laboratory conditions but is inappropriate for interpretation of field data. A better approach for low frequency (field) data is to use Hudson's dry rock formulation and then use equation (2) to predict the low frequency saturated velocities.

To illustrate this we make a comparison of the results obtained from Hudson's

theory and the high frequency squirt model for saturated cracked rocks. Hudson's formulation is limited to small concentrations of highly idealized penny-shaped cracks, while our expressions are valid for *all* crack geometries and crack densities *including* idealized penny-shaped cracks. Therefore, Hudson's formulation can be considered a particular case of the more general formulation. As an example, we consider a rock with a single set of cracks with their normals aligned along the 3-axis. The dry stiffnesses were generated using Hudson's equations, and taken as the input for the squirt model to calculate the saturated stiffnesses which were then compared with those predicted from Hudson's theory. Figure (3) shows, for example, C_{33} as a function of crack density for both dry and saturated conditions. Figure (4) shows the quasi-P and two quasi-S velocity variation as a function of angle from the symmetry axis (3-axis), again for the dry and saturated cases. The saturated velocities (and stiffnesses) calculated from squirt and Hudson's theory agree very closely, as we expect them to do, since both are high frequency theories.

Figure (5) shows angular P and S velocity variation for a conical distribution of cracks i.e. randomly oriented cracks with all their normals at 30 degrees to the 3-axis. As in the previous example, dry stiffnesses were calculated using Hudson's model which were then used as input for the squirt calculations, equation (28). The dashed lines represent dry P and S velocities. The saturated velocities from Hudson's theory and the high frequency squirt calculations are shown by crosses and open circles respectively. The low frequency saturated velocities, calculated from equation (2) are represented by squares. Once again we observe that the Hudson and squirt saturated velocities (crosses and open circles) show similar behaviour but, depending on direction of propagation, may be quite different from the low frequency saturated velocities (squares). This difference is more pronounced for the two S waves. The low and high frequency saturated velocities are different because various subsets of the cracks experience unequal induced pore pressures depending on the direction of wave propagation. At high frequencies, these unequilibrated pore pressures make the rock stiffer.

These examples emphasize the difference between high and low frequency satu-

rated moduli (and velocities). They also show that idealized inclusion models (such as Hudson's) are high frequency models with respect to fluid effects, and can be considered to be a particular case of our more general formulation.

Next, we present results of modelling laboratory data using the anisotropic squirt formalism. Figure (6) shows V_{p1} and V_{p3} , the fast and slow P velocities in transversely isotropic Arizona sandstone under both dry and saturated conditions, measured at ultrasonic frequencies as a function of confining pressure, (Podio-Lucioni, 1968). The circles are the dry data, and triangles represent saturated measurements. As expected, the low frequency Brown and Korringa predictions are lower than the saturated high frequency measurements. The Biot velocities explain much of the dispersion at high pressure when most of the crack-like porosity is closed, but does a poor job at low pressures. The high frequency squirt predictions capture both the total dispersion and the pressure dependence of the high frequency saturated velocities.

Figure (7) compares low and high frequency saturated P and S velocities for transversely isotropic granite calculated using dry pressure-dependent data from Lo, et al. (1986). This data set did not have the measured saturated velocities, so we present the predicted saturated velocities for both low and high frequency regimes of fluid behaviour. The measured dry data are plotted as points, and the predicted values are plotted as curves. For the compressional velocities, the triangles are the velocities in the slow x_3 direction, and the circles are the velocities in the fast x_1 direction. For the shear velocities, the triangles are the velocities for one of the equivalent polarizations propagating in the slow x_3 direction, and the circles are the velocities propagating in the x_1 direction with the fast x_2 polarization. Note that the dry data show anisotropy, even at high pressure, when presumably most of the cracks are closed. This intrinsic anisotropy accounts for much of the total anisotropy at the lower pressures. Therefore, interpretation of observed anisotropy entirely in terms of cracks can be misleading in rocks such as these.

The dashed curves show the low frequency prediction of saturated velocity, using equation (2), and the solid curves the high frequency prediction of saturated velocity,

using equation (28). We found that for these rocks a single crack set, equation (17), was a poor representation of the crack-related anisotropy. The more complete analysis of the G_{ijkl} indicated a distributed range of crack orientations, and ignoring the distribution gave unrealistic results. As expected, the high frequency velocities are almost always faster than the corresponding low frequency values.

The low frequency theory predicts no change in the shear compliances ΔS_{2323} and ΔS_{1313} and hence a small drop in S velocities in the principal directions due to the density effect. For the low porosity granite (0.9%) the dry and low frequency saturated S velocities are barely distinguishable. Although not plotted, the low frequency shear compliances in the non-principal directions are predicted to stiffen with saturation. The high frequency theory predicts an increase in the saturated shear compliances relative to the low frequency saturated values and therefore an increase in the S velocities. For the low porosity granite the high frequency velocity is always higher than the dry or low frequency saturated values. An interesting observation is the cross-over of the two high frequency saturated S velocity curves at low pressures. This suggests that at high frequencies, the observed velocity anisotropy can be smaller or even in the opposite sense of the anisotropy observed in dry rocks or in saturated rocks at low frequencies. Therefore, finding relatively low velocity anisotropy in the laboratory (at high frequencies) does not preclude finding higher anisotropy in the field.

The compressional velocities always increase from dry to saturated rocks and increase from low frequency to high frequency. At some pressures the saturated velocities have equivalent or slightly smaller anisotropy compared with the dry rocks; but we also find that the high frequency saturated anisotropy can be larger than the dry anisotropy. Therefore, it is not correct to say that saturation always decreases anisotropy.

We summarize some key points illustrated by these examples:

1. Velocities and velocity anisotropy in dry rocks are generally expected to be different than in saturated rocks.

2. Velocities and velocity anisotropy are generally expected to be different at low frequencies, typical of field measurements, and high frequencies typical of laboratory measurements. Therefore, care must be taken when extrapolating from the laboratory to field situations. Our formulation gives a simple method for doing so in a geometry-independent way.
3. A rotationally symmetric distribution of cracks may often be a better model of crack induced transversely isotropic rocks than just a single set of aligned cracks.
4. Assigning all of the observed anisotropy to the presence of cracks may not always be correct as a significant fraction of the anisotropy could be due to the background matrix.

Finally, an often overlooked point is that inclusion models for velocity anisotropy, such as Hudson's (1980, 1981), are usually *high frequency* theories, in terms of fluid effects. Hudson treats fluid-filled cavities as isolated with respect to fluid flow. As we have discussed, at high frequencies compliant cracks are isolated from the stiff portions of the pore space and from compliant cracks at other orientations; the resulting unequilibrated pore pressures make the rock stiffer. At low frequencies, pore fluid has sufficient time to flow and equilibrate the induced pore pressures throughout the pore space and hence cracks and pores are effectively connected with respect to fluid flow. Consequently Hudson's work is best suited for describing ultrasonic laboratory measurements. Interpretations of low frequency field observations using Hudson's formulation of saturated cracks is inappropriate. A better approach would be to use a dry rock model and to predict low frequency velocities using our equation (2).

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APPENDIX

Stress Induced Pore Pressure

We derive the stress induced pore pressure in the n th pore using linear superposition, as shown in Figure 8. The change in volume, Δv_n , can be written as the sum:

$$\Delta v_n = v W_{ijkk}^{(n)} (\Delta \sigma_{ij} - \Delta P_n \delta_{ij}) + \Delta P_n \beta_0 v_n \quad (41)$$

Equating this with the volume change of the fluid within the pore, $\Delta v_n = \Delta P_n \beta_f v_n$, gives equation (5).

Sparse Crack Compliance Tensor

For crack like features we expect the intrinsic compliance tensor in the crack coordinate system, $\bar{W}_{ijkl}$, to be sparse. The largest components of the tensor are the normal compliance $\bar{W}_{3333}$ and the shear compliances $\bar{W}_{2323}$ and $\bar{W}_{1313}$. The other components are approximately zero. This is a general property of planar crack formulations and reflects an approximate decoupling of normal and shear deformation of the crack and decoupling of the in plane and out of plane compressive deformation. This allows us to write $\bar{W}_{\alpha\alpha\beta\beta} \approx \bar{W}_{3333}$ and $\bar{W}_{ij\alpha\alpha} \approx \bar{W}_{3333} \delta_{i3} \delta_{j3}$.

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FIGURE CAPTIONS

Fig. 1 Two sets of tractions applied to the same rock sample. The rock on the left deforms with the unrelaxed frame compliance. The rock on the right deforms with the effective dry rock compliances.

Fig. 2 Two sets of tractions applied to the same rock sample. The sample on the left deforms with the effective dry rock compliances. The sample on the right deforms with the effective high pressure dry compliances as though the soft pores were compressed closed by high confining pressure.

Fig. 3 Comparison of high frequency predicted saturated C_{33} for transversely isotropic cracked rock for various crack densities. The dry stiffnesses (dashed line) are calculated from Hudson's theory. These are then used in the squirt theory to predict the high frequency saturated stiffnesses. The squirt predictions (circles) match very well with saturated stiffnesses calculated from Hudson's model (triangles).

Fig. 4 Comparison of high frequency saturated P (qP) and S (qS_1 , qS_2) velocity variation as a function of angle from the symmetry axis (3-axis) for transversely isotropic cracked rock containing a single crack set with normals aligned along the 3-axis. The dry velocities (dashed line) are calculated from Hudson's theory. These are then used in the anisotropic squirt formalism to predict the high frequency saturated velocities. Saturated squirt predictions (circles); saturated Hudson's model (crosses).

Fig. 5 Angular P (qP) and S (qS_1 , qS_2) velocity variation for conical crack distribution. Cracks randomly oriented with their normals all at 30° to the 3-axis. Dashed line: Hudson dry; squares: saturated low frequency; circles: squirt, high frequency saturated; crosses: Hudson saturated.

Fig. 6 Comparison of predicted and measured saturated anisotropic velocities for transversely isotropic sandstone using ultrasonic measurements from Podio-Lucioni, 1968. Circles are the dry V_{p1} (fast) and V_{p3} (slow) measured velocities which are used to predict the measured saturated velocities (triangles). The Biot and Brown and Korringa predictions of saturated V_{p1} and V_{p3} are shown on the left while the anisotropic squirt predictions are represented by the curves on the right. Calculations based on the squirt theory does a much better job of explaining the high frequency saturated velocities.

Fig. 7 Comparison of low and high frequency predicted saturated velocities for transversely isotropic granite. a) P velocities and b) S velocities. The measured dry data from Lo, et al. (1986) are plotted as points and predicted values as curves. For P velocities, the triangles are the dry velocities in the slow x_3 direction and the circles are the dry velocities in the fast x_1 direction. For the shear velocities, the triangles are the dry velocities for one of the equivalent polarizations propagating in the slow x_3 direction, and the circles are the dry velocities propagating in the x_1 direction with the fast x_2 polarization. The dashed curves denote the low frequency predictions of saturated fast and slow velocities using equation (2) while the solid curves are the high frequency ones using equation (28).

Fig. 8 Decomposition of stresses applied to the saturated rock, in order to estimate the pore pressure induced in the n th pore.

TABLE CAPTION

Table 1. Values of the tensor G_{ijkl} for the case of isotropic symmetry.

Table 1. Values of G_{ijkl}^* .							
	ij	11	22	33	23	13	12
kl							
11		$\frac{1}{5}$	$\frac{1}{15}$	$\frac{1}{15}$	0	0	0
22		$\frac{1}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	0	0	0
33		$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{5}$	0	0	0
23		0	0	0	$\frac{1}{15}$	0	0
13		0	0	0	0	$\frac{1}{15}$	0
12		0	0	0	0	0	$\frac{1}{15}$
* Elements with any permutation of a given set, $ijkl$, are equal.							

fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

fig. 6

fig. 7

fig-8

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