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Matrix elements for the decays of S- and P-wave quarkonium: an exploratory study.

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Bodwin, Braaten and Lepage have shown that, in a non-relativistic expansion in powers of the heavy-quark velocity, the decay rates of S- and P-wave quarkonia can be written as sums of products of a hard parton matrix element, which can be calculated perturbatively, times a low-momentum expectation value of a parton operator in the quarkonium state. We report on attempts to calculate these low momentum matrix elements on the lattice in the non-relativistic formulation of the quark dynamics. The quenched approximation is used throughout.

The charmonium and $b\bar{b}$ systems are non-relativistic in the sense that $v^2 \ll 1$ where v is the heavy-quark velocity ($v^2 \sim 0.2 - 0.3$ for charmonium $v^2 \sim 0.1$ for Υ). The annihilation of the $Q\bar{Q}$ (Q is a b or c quark) pair is suppressed, and occurs, when it does, at distances $\sim 1/M_Q$. Bodwin, Braaten and Lepage [1] have shown that these facts allow one to write quarkonium decay rates as a sum of terms, each of which factors into a long-distance part times a short-distance part. The long-distance part involves the distribution of heavy quarks in quarkonium; the short-distance part can be calculated perturbatively. In particular, the S- and P-wave decay rates can be written at lowest non-trivial order in v^2 as

$$\begin{aligned} \Gamma(2^{s+1}S_J \rightarrow X) &= G_1(S)\hat{\Gamma}_1(Q\bar{Q}(2^{s+1}S_J) \rightarrow X) \\ \Gamma(2^{s+1}P_J \rightarrow X) &= H_1(P)\hat{\Gamma}_1(Q\bar{Q}(2^{s+1}P_J) \rightarrow X) \\ &\quad + H_3(P)\hat{\Gamma}_3(Q\bar{Q}(2^{s+1}S_J) \rightarrow X), \end{aligned} \quad (1)$$

where the X 's represent states of light partons. The $\hat{\Gamma}$'s are the short-distance ($p \sim M_Q$) parton-level decay rates. G_1 , H_1 and H_3 are the long-distance ($p \sim M_Q v$, $E \sim M_Q v^2$) matrix elements that we calculate on the lattice.

In our lattice calculations we have used 149 independent equilibrated quenched gauge configurations on a $16^3 \times 32$ lattice with $\beta = 6.0$. Heavy-quark, and hence quarkonium propagators were

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calculated using the non-relativistic formulation of Lepage and collaborators [2]. We used the lattice version of the quark action that is based on the euclidean lagrangian

$$\mathcal{L}_Q = \psi^\dagger(D_t - \frac{D^2}{2M_Q})\psi + \chi^\dagger(D_t - \frac{D^2}{2M_Q})\chi, \quad (2)$$

which is valid to the lowest non-trivial order in v^2 . We calculate the quark Green's function which obeys the evolution equation [2]

$$\begin{aligned} G(\mathbf{x}, t+1) &= (1 - H_0/2n)^n U_{\mathbf{x},t}^\dagger (1 - H_0/2n)^n \\ &\quad \times G(\mathbf{x}, t) + \delta_{\mathbf{x},0} \delta_{t+1,0}, \end{aligned} \quad (3)$$

with $G(\mathbf{x}, t) = 0$ for $t < 0$, and $H_0 = -\Delta^{(2)}/2M_0 - E_{sub}$. Here $\Delta^{(2)}$ is the gauge-covariant discrete laplacian and M_0 the bare quark mass. E_{sub} allows one to choose the zero-point energy.

For these preliminary calculations point sources and sinks were chosen for the quark Green's functions. This enabled us to use sources for the quark Green's functions at the site of the four-fermi operators that appear in G_1 , H_1 and H_3 , making it simple to calculate matrix elements in which the quarkonia are at zero momentum and on arbitrary time-slices, as well as to extract quarkonium wave functions (or their first derivatives) at the origin.

For charmonium we chose $M_0 = 0.5$, $n = 3$ and $E_{sub} = 1.11$, in lattice units. In converting to "physical units" we chose $1/a = 2 \text{ GeV}$. In

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order to improve statistics, we calculated propagators from 32 different sources per configuration. The 32 sources chosen were those with x , y and z coordinates either 1 or 9, and t coordinate 1, 9, 17 or 25. For comparison with other work in the field we give, in Table 1, our estimates for simple properties of S- and P-wave charmonium obtained from these propagators. Using mean field [2] estimates for the renormalization constants, we find that the renormalised quark mass is $M_R = 0.605$ (lattice units), and the mass of S-wave charmonium is $M_S = 3.36 \text{ GeV} + \mathcal{O}(v^4)$, calculated from the $p = 0$ state, and $M_S \text{ dyn} = 3.00 \text{ GeV} + \mathcal{O}(v^2)$, calculated from the coefficient of p^2 in the energy of the $p = 2\pi/16$ state.

Table 1

Properties of S- and P-wave charmonium from our simulations. $R(r)$ are the radial wave functions. The lattice quantities include mean field renormalizations

	LATTICE	EXPERIMENT
$M_P - M_S$	0.40–0.48 GeV	0.43 GeV
$ R_{1S}(0) ^2$	0.599(4) GeV ³	0.48(4) GeV ³
$ R'_{1P}(0) ^2$	0.032–0.053 GeV ⁵	0.054(13) GeV ⁵

The matrix elements we wish to calculate are defined as

$$G_1 = \langle {}^1S | \psi^\dagger \chi \chi^\dagger \psi | {}^1S \rangle / M_Q^2 \quad (4)$$

$$H_1 = \langle {}^1P | \psi^\dagger(i/2) \vec{D} \chi \chi^\dagger(i/2) \vec{D} \psi | {}^1P \rangle / M_Q^4 \quad (5)$$

$$H_3 = \langle {}^1P | \psi^\dagger T^a \chi \chi^\dagger T^a \psi | {}^1P \rangle / M_Q^2, \quad (6)$$

each of which has the generic form

$$W = \langle \Phi | O(\mathbf{x}, t) | \Phi \rangle \quad (7)$$

where $|\Phi\rangle$ is the quarkonium state and O the 4-fermi operator. On the lattice, we measure W^* defined by

$$W^*(T, T') = \frac{\langle 0 | \phi(-T) O(\mathbf{x}, t) \phi^\dagger(T') | 0 \rangle}{\langle 0 | \phi(-T) \phi^\dagger(0) | 0 \rangle \langle 0 | \phi(0) \phi^\dagger(T) | 0 \rangle} \quad (8)$$

where $\phi(t) = \sum_{\mathbf{x}} \chi^\dagger(\mathbf{x}, t) \psi(\mathbf{x}, t)$ for the S-wave matrix element and $\phi(t) = \sum_{\mathbf{x}} \chi^\dagger(\mathbf{x}, t) \vec{D} \psi(\mathbf{x}, t)$

for the P-wave matrix elements. Then as $T, T' \rightarrow \infty$

$$G_1^*(T, T') \rightarrow G_1 \frac{2\pi M_Q^2}{3|R_{1S}(0)|^2} = 1 + \mathcal{O}(v^4) \quad (9)$$

$$H_1^*(T, T') \rightarrow H_1 \frac{2\pi M_Q^4}{9|R'_{1P}(0)|^2} = 1 + \mathcal{O}(v^2) \quad (10)$$

$$H_3^*(T, T') \rightarrow H_3 \frac{2\pi M_Q^2}{9|R'_{1P}(0)|^2} \quad (11)$$

In Fig. 1a we have plotted G_1^* as a function of T , with separate points for each value of T' , with the restriction $T+T' < 32$. We notice that G_1^* is very close to 1, as expected. In Fig. 1b we have made a similar plot for H_1^* , distinguishing those points with $T < 8$ from those with $T \geq 8$. Here, again the values cluster around 1 as expected. Since the statistical errors in these points are comparable with their departure from unity, it is unclear whether the much larger variation for H_1^* than for G_1^* is due to the expected $\sim v^2$ departure from unity, the increased errors, or both.

Finally, in Fig. 1c we have plotted H_3^* against T . The scatter of points is considerable. The best we can say is that H_3^* appears to lie in the range 0.1–0.2, but there is really no good evidence that we have seen a plateau. Experimental data suggest a value [1] around 0.5 (with an error of about 50%). It is easy to see why our measurement would be expected to underestimate this quantity. Noise dominates the signal for $T, T' \gtrsim 8$. However, we know that even at $T = 8$ the coefficient of the exponential in each of the propagators in the denominator of (10) has not reached its asymptotic value, and that this value is approached from above. The numerator, on the other hand, is zero at zero separation, and hence the coefficient of the exponential in the numerator approaches its asymptotic value from below. Thus, the overall effect of not being able to attain large enough separations is to cause us to underestimate H_3^* .

Since this conference, we have extended this analysis to the $b\bar{b}$ system. Here one might hope that, since the radius of the Υ is less than half that of charmonium, the use of a point source would be less of a handicap. Indeed, for G_1^* and H_1^* this appears to be the case. Unfortunately,

Figure 1. Matrix elements as a function of T . a) G_1^* . b) H_1^* . c) H_8^* . Separate points are plotted for each T' . Points with $T' < T$ are labelled o; those with $T' \geq T$ are labelled x in (b) and (c).

for H_8^* this is not true. $H_8^* \sim 1/M_Q^2$, and so decreases by roughly an order of magnitude in going from the charmonium to the $b\bar{b}$ system. The signal/noise ratio is, if anything, worse for χ_8 .

In summary we have found that the colour singlet matrix elements involved in the decay of S- and P-wave quarkonia (G_1 and H_1) appear to be related to the quark wave-functions at the origin in the expected manner. To make reliable estimates of the colour-octet matrix element (H_8) contributing to P-wave decay would require much higher statistics and, possibly, a longer lattice than we have used. This operator is of considerable interest, since it involves an explicit gluon exchange and cannot, therefore, be estimated in a potential model. At least for the charmonium case, it involves the exchange of gluons which are too soft for one to believe the perturbative estimates.

In the above we have used mean-field estimates of the renormalization constants. For a more thorough analysis, a complete perturbative calculation will be needed. We are now investigating the possibility of improving our signal/noise ratio by using extended sources in a way that is similar to their use in B parameter calculations [3]. Extensions of our calculations to include contributions which are higher order in v^2 are under consideration.

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