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On a Theory of Two-Beam Mechanisms of Charged Particle Acceleration in Electrodynamic Structures*

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Introduction

This work is devoted to the theoretical studies of two-beam mechanisms of charged particle acceleration in electrodynamic structures. The first section continues the outline of results of theoretical studies commenced in the intermediate report and considers the two-beam scheme of acceleration in the plasma waveguide. According to this scheme the strong current relativistic electron beam (REB) (driving beam) excites the intensive plasma waves accelerating the electrons of the second beam (driven beam). The driving beam is assumed to be density-modulated. The preliminary modulation of the driving REB is shown to enhance substantially the acceleration efficiency of relativistic electrons of the driven beam.

The second section deals with the two-beam acceleration in the vacuum corrugated waveguide. According to this scheme the excitation of electromagnetic waves and acceleration of driven beam electrons by them is accomplished under different Cherenkov resonances between the particles of beams and the corrugated waveguide field (two-resonance acceleration scheme) [1-3]. The electromagnetic field in the periodic structure is known to be the superposition of spatial harmonics. With the small depth of the periodic nonuniformity the amplitudes of these harmonics decrease fast with their number increasing [4]. Therefore, if the driving beam is in the Cherenkov resonance with the first spatial harmonic and the driven beam is in resonance with the zero space harmonic then the force accelerating the driven beam would be considerably bigger than the force decelerating the driving beam electrons. One may expect in this case that the energy of accelerated particles at the system exit would exceed their initial value considerably. Ref.[2] has suggested to apply the waveguide with not very deep corrugated lateral surface as the accelerating structure. The main advantage of corrugated waveguides against vacuum periodic structures of other types consists in their high electric strength as well as in the option to provide the favourable condition for the efficient interaction of strong current REB with the RF field of the structure. Corrugated waveguides have found a broad application in powerful relativistic

microwave backward oscillators (BWO) [5,6]. Application of corrugated waveguides also permits to advance to the short wavelength range of accelerating fields ($\lambda \leq 10\text{cm}$) and thereby to enhance their limiting values. This basically must provide high acceleration rate for charged particles $T \approx 100\text{ Mev/m}$.

1. Acceleration of charged particles by the modulated REB in the plasma waveguide

In the intermediate report the two-beam scheme of accelerating relativistic electrons in the plasma waveguide has been considered. The most efficient acceleration of electrons has been shown to occur at a distance corresponding, approximately, to the distance to the first maximum of the field amplitude. At the same time the delayed exponential growth of the field amplitude in the starting region of the interaction space has been shown to limit essentially the acceleration rate of electrons. In this region comprising around 30% of the total length of the accelerating gap the field amplitude is small (linear stage of the instability evolution) and therefore the process of energy increase is practically absent. To decrease the region of linear stage of the instability evolution (region to the first maximum of the field amplitude) and thereby to increase the acceleration rate of electrons it is of interest to consider the case when the driving REB is preliminarily density-modulated. In this case already formed bunches of electrons enter the interaction space. If the REB modulation frequency coincides with the frequency of the amplified wave the process of gathering electrons in bunches occurring at the linear stage of instability for the non-modulated beam would be absent. As a result, the distance at which the amplitude of the plasma wave achieves the first maximum must be considerably less than one for the non-modulated REB.

This section considers the acceleration of charged particles by the modulated REB in the plasma waveguide. At the waveguide entrance the density of the driving REB varies according to the harmonic law:

$$n_b(z=0) = N_b (1 + g \cos \omega_m t) \frac{\delta(r-r_1)}{2\pi r_1}, \quad (1.1)$$

where N_b is the beam particle density per unit length; g is the

modulation depth, ω_m is the modulation frequency. All the rest of variables and parameters in this section coincide with those given in the intermediate report. In what follows we assume the modulation frequency ω_m to be equal to the frequency ω of the amplified plasma wave and the beam to be in exact Cherenkov resonance with this wave ($\sigma=0$).

The set of equations describing the excitation of plasma waves and acceleration of the driven REB's particles corresponds completely to the set of equations (20)-(24) at $\sigma=0$ given in the intermediate report. The only difference from the non-modulated REB consists in that ρ value entering Eqs. (20), (21), (23) assumes now the form: $\rho = \int_0^{2\pi} \Phi(\theta_0) \exp(i\theta) d\theta_0$, where $\Phi(\theta_0) = 1 + g \cos \theta_0$. The energy conservation law for the modulated REB following from the set of equations (20)-(22) has the form:

$$\begin{aligned} & (\nu_b/2\pi) \int_0^{2\pi} (1+q^2)^{1/2} \Phi(\theta_0) d\theta_0 + 0.5|A|^2 + 2(\nu_b/2\pi)^2 \gamma_0^4 \lambda_1^2 \Lambda_1 \times \\ & \times \{ [\int_0^{2\pi} \Phi(\theta_0) \cos \theta d\theta_0]^2 + [\int_0^{2\pi} \Phi(\theta_0) \sin \theta d\theta_0]^2 \} = \text{const.} \end{aligned} \quad (1.2)$$

Here the first term determines the energy contribution of the beam, and the second and third ones are the contributions of the amplified resonance wave and nonresonant harmonics into the energy balance of the system. From (1.2) we obtain the efficiency of exciting RF fields in the waveguide:

$$\begin{aligned} \eta = & \frac{1}{2\pi} \int_0^{2\pi} \frac{\gamma_0 - (1+q^2)^{1/2} \Phi(\theta_0)}{\gamma_0 - 1} d\theta_0 = \eta_1 + \eta_2 = \frac{|A|^2 - |A_0|^2}{2\nu_b(\gamma_0 - 1)} + \\ & + \frac{\nu_b \gamma_0^4 \lambda_1^2 \Lambda_1}{2\pi^2(\gamma_0 - 1)} \{ [\int_0^{2\pi} \Phi(\theta_0) \cos \theta d\theta_0]^2 + [\int_0^{2\pi} \Phi(\theta_0) \sin \theta d\theta_0]^2 - (\pi g)^2 \}, \end{aligned} \quad (1.3)$$

where η is the relative portion of total losses of REB kinetic energy, η_1, η_2 are the relative portions of beam energy losses due to the excitation of the resonant plasma waves and nonresonant harmonics of the plasma waveguide.

Let us consider briefly the linear stage of exciting plasma waves with the modulated REB. At the initial stage of the instability when the amplitude of the resonant plasma wave is small, the displacement

and deformation of injected bunches in collective electromagnetic fields don't play the essential role. Therefore the driving beam may be regarded as the given modulated current. In the approximation of a given current we neglect the perturbed quantities compared with their equilibrium ones (ϑ_0, q_0). Then, as it follows from excitation equation (20), the amplitude of the resonant wave grows linearly $|A| = \nu_b g \xi / 2$. This occurs because already formed bunches enter the waveguide. Linear growth of the field is a consequence of the resonance between the external force (modulated REB) and the plasma wave.

The processes of exciting RF fields by the modulated strong current REB and of accelerating the electrons of the driven beam by these fields are treated by simulation of Eqs.(20)-(24). Let us give the results of such modeling for the following parameters of the high-current REB: $\gamma_0 = 5$; $r_1 = 0.65 r_0$ ($K=2$); $g=0.5$; $\nu_b / 2\pi = 3.4 \cdot 10^{-4}$ ($I_b = 34 \text{ kA}$). The parameters of the driven REB were as follows: $\gamma_0^{\text{acc}} = 5$; $r_2 = 0.1 r_0$. The boundary condition for the amplified plasma wave was taken the same as for the non-modulated driving REB: $\text{Re} A_0 = 5 \cdot 10^{-3}$; $\text{Im} A_0 = 0$. The value of the driving REB current $I_b = 34 \text{ kA}$ for the system's parameters given is close to the limiting current I_b^* (see the intermediate report, section 4). Such a choice of I_b permits to reveal the strong current effects during beam-plasma interaction as well as to obtain the limiting admissible strengths of accelerating fields in the considered approximation of the linear plasma.

Let us dwell, first, on the excitation of the plasma waveguide by the high-current REB. Fig.1 shows the amplitude $|A(\xi)|$ and the phase $\text{Arg } A(\xi)$ of the wave field in resonance with the beam versus the longitudinal coordinate ξ . The same Figure shows also the similar dependences obtained in the absence of the RF space charge of the high-current REB. In the initial region of the interaction space the wave amplitude attains the maximum value at a distance considerably less than one for the unmodulated REB $\xi \approx 90$ (compare with the results given in the intermediate report). The linear growth of the amplitude is followed by trapping oscillations. Comparison of dependences depicted in the Figure shows that the account of RF space charge field changes the plasma-beam instability saturation considerably: the magnitude of amplitude oscillations decreases and at $\xi > 700$ the amplitude of the plasma wave assumes a constant value.

This is the result of partial destruction of bunches of charged particles and of their phase mixing. The maximum of total energy losses of the high-current REB is achieved at the point $\xi \approx 90$ (the region of the first maximum of the amplitude) and it is equal to $\eta \approx 27\%$.

Fig.2 shows the dependences of the field amplitude $|\tilde{A}(\xi)|$ and phase $\text{Arg } \tilde{A}(\xi)$ in the region of the driven REB ($r=r_2$). At $\xi=0$ the field amplitude equals $|\tilde{A}|=0.072$. It exceeds considerably the amplitude of the amplified signal applied across the entrance in the region of the driven beam ($|A_0|K=0.01$). This difference is due to the contribution into the resulting strength of the accelerating field at the entrance to the plasma waveguide made by the second term in (23) that is nonvanishing for the modulated REB at $\xi=0$. Thus in contrast to the non-modulated REB case the accelerated particles get into the field with a sufficiently large amplitude already at $\xi=0$.

On increasing ξ the amplitude $|\tilde{A}|$ growth is changed by irregular space oscillations. The maximum value of the field amplitude in the region of the driven REB $|\tilde{A}_{\text{max}}| \approx 0.44$ is achieved at the point $\xi \approx 90$. This corresponds to the distance to the first maximum of the amplitude of the resonant plasma wave (see Fig.1a). At $\xi > 90$ the amplitude falls and its value does not exceed $|\tilde{A}|=0.26$. Comparison of dependences shown in Figs.1a and 2a shows that the field of the RF space charge of the driving REB enhances considerably the accelerating field amplitude over all length of the plasma waveguide. Thus, e.g., the value of $|\tilde{A}_{\text{max}}|$ is approximately 4 times bigger than the maximum amplitude of the resonance wave in the region of the driven REB $|A_{\text{max}}|K=0.12$.

One can draw the following conclusions from the results given.

a) The preliminary modulation of the high-current REB reduces substantially the distance to the first maximum of the accelerating field amplitude.

b) RF space charge field of the high-current modulated REB increases the field amplitude in the region of the driven beam over all length of the accelerating gap. Already at $\xi=0$ the electrons of the driven beam get into the accelerating field exceeding considerably the field of the resonant plasma wave.

Therefore the preliminary modulation of the high-current REB may

play an important role in enhancing the efficiency of acceleration of driven beam electrons.

Consider now the acceleration process of the driven beam electrons. The optimum length of the waveguide l at which the most efficient increase of electron energy occurs has been determined according to the technique described in the intermediate report. This length is $l=96$ for the parameters considered. Fig.3 depicts the electron momenta of the driven beam vs their initial phases ϕ_0^{acc} at various points of the accelerating gap. Already at the beginning of the accelerating gap the bunches of accelerated and decelerated particles are formed. At the exit the maximum energy of accelerated particles $W_{\text{max}}=mc^2(\gamma_{\text{max}}^{\text{acc}}-1)$, $\gamma_{\text{max}}^{\text{acc}}=[(q_{\text{max}}^{\text{acc}})^2+1]^{1/2}$ exceeds 4 times their initial energy. The number of accelerated particles in the bunch at the exit of the waveguide amounts to ca. 17% of the total number of particles (Fig.3d). The energy increase of the bulk electrons belonging to the accelerated bunch possesses the markedly monotonous pattern. Fig.4 gives the good illustration of this picturing the dependence of the accelerated particle momentum with the initial phase $\phi_0^{\text{acc}}=\pi$ versus the longitudinal coordinate ξ .

Now we present the estimates of the maximum strength of the longitudinal component of the electric field in the driven beam region and of the electron acceleration rate. Plasma density is taken to be $n_p=10^{14}\text{cm}^{-1}$. For the driving REB parameters given above the maximum strength of the accelerating field is equal to (see Fig.2a): $|E_{\text{max}}|=\tilde{A}_{\text{max}}|mc^2\omega/\text{ev}_0|=0.44mc^2\omega/\text{ev}_0$. As was shown in the intermediate report, the resonance frequency ω is close to the plasma frequency ω_p . Then the maximum amplitude of the accelerating field is equal to $|E_{\text{max}}|=420\text{Mev/m}$. The length of the plasma waveguide equals $L=96v_0/\omega_p\approx 5\text{cm}$, correspondingly. At this length the maximum increase of energy, as it follows from Figs.3,4 amounts to $\Delta W=mc^2(\gamma_{\text{max}}^{\text{acc}}-\gamma_0^{\text{acc}})\approx 5.8\text{Mev}$. Thus the acceleration rate is $T=116\text{Mev/m}$. The estimates given show that the modulation of the driving REB involves a remarkable increase of the acceleration rate (compare with the similar estimates given in the intermediate report). The increase of the acceleration rate is due first of all, to changes in the field amplitude distribution over the accelerating gap, as a result of which accelerated electrons are in the

field of a sufficient amplitude already at $\xi=0$. Besides, in contrast to the non-modulated REB, the delayed exponential growth of the field amplitude to the first maximum is absent. These factors help to enhance considerably the acceleration rate of electrons without increasing the maximum strength of the accelerating field.

In conclusion, it should be noted the following. The above given values of the highest accelerating field intensity and the acceleration rate of driven beam electrons were calculated for the case of a homogeneous-density plasma. However, it is known that plasma inhomogeneity, which does exist in real plasma systems, may cause violation of synchronism between the beam and the plasma wave due to the change in the phase velocity of the wave. This may be one of the essential factors that impede the effective plasma-wave excitation and hence, the electron acceleration. By way of example we shall give the requirements to the inhomogeneity scale for the linear plasma density profile

$$n_p(z) = n_{p0}(1 + z/L_p), \quad (1.4)$$

L_p is the characteristic length of inhomogeneity; $n_{p0} = n_p(z=0)$. For this purpose we shall derive the expression to describe the variation of the longitudinal wave number h_1 , belonging to the resonant plasma wave, as a function of the coordinate z :

$$E(z) \equiv 1 - h_1(z)/h_1(z=0), \quad h_1(z=0) = \omega/v_0.$$

Using the expression for the dispersion of the cylindrical plasma waveguide of radius r_0 , located in a strong longitudinal magnetic field

$$(\lambda_1/r_0)^2 = [(\omega/c)^2 - h_1^2](1 - \omega_p^2/\omega^2), \quad (1.5)$$

$$\lambda_1 = 2.405, \quad \omega_p = (4\pi n_p e^2/m)^{1/2},$$

and taking into account (1.4), we obtain the expression for $E(z)$:

$$E(z) = 0.5 \frac{\omega_p^2(z=0) r_0^2 z}{\gamma_0^4 \lambda_1^2 \beta_0^2 c^2 L_p}. \quad (1.6)$$

It is evident that at a fixed frequency of the enhanced plasma wave ω , expression (1.6) describes the variation of the phase velocity of the plasma wave relative to the beam velocity: $E(z) \equiv 1 - h_1(z)/h_1(z=0) = (v_0 - v_{ph})/v_0$, $v_{ph} = \omega/h_1$. For a premodulated beam the breakdown of

plasma-beam instability occurs at such a plasma inhomogeneity, when the phase difference between the beam and the wave gained over the length of interaction is equal to π . Hence we come to the inequality defining the admissible values of plasma inhomogeneity length, at which no breakdown of plasma wave excitation take place:

$$L_p > \frac{\omega_p^3(z=0) r_0^2 L^2}{4\pi \gamma_0^4 v_0^3 \lambda_1^2}. \quad (1.7)$$

For the above given parameters of the system and $r_0=2\text{cm}$, it follows from (1.7) that $L_p > 14\text{cm}$. So, the distance over which the plasma density is doubled at the expense of inhomogeneity must be essentially greater than the length of the accelerating gap.

2. Two-beam acceleration of charged particles in a corrugated waveguide

2.1. Statement of the problem. Input equations.

Let us consider a segment of length L of a corrugated waveguide with an ideally conducting lateral surface. The radius of the waveguide walls varies in accordance with the harmonic law:

$$r(z) = r_0 (1 + \alpha \cos k_0 z), \quad (2.1)$$

where $k_0 = 2\pi/D$, D being the waveguide spatial period, r_0 is the average radius of the waveguide, $\alpha = \Delta r / r_0 \ll 1$ is the periodical inhomogeneity parameter of the electrodynamical structure, Δr is the corrugation depth. The waveguide is completely consistent on both sides (reflected waves are absent). Along the z -axis of the waveguide two tubular monoenergetic REB propagate (Fig.5). The radii of the driving beam and the driven one are respectively r_1 and r_2 ($r_1 < r_2 \approx r_0$).

In the acceleration scheme under consideration, the driving high-current REB of a relatively low initial energy $W_0 = mc^2(\gamma_0 - 1)$ ($\gamma_0 > 1$ is the relativistic factor of the driving beam, m is the electron mass, c is the speed of light), which excites the electromagnetic field, is in the Cherenkov resonance with the fundamental radial mode ($s=1$) of the first space harmonic ($n=1$) of the backward wave: $\omega_0 = [k_0 - h_1(\omega_0)]v_0$, where v_0 is the initial driving REB velocity, ω_0

is the resonance frequency of the wave, h_1 is the longitudinal wave number (the subscript "1" denotes the radial mode number). The driven beam is injected from the exit side of the waveguide ($z=L$) with the initial energy $W_0^{acc} = mc^2(\gamma_0^{acc} - 1)$, $\gamma_0^{acc} > \gamma_0$ (γ_0^{acc} is the relativistic factor of the driven beam) and is in the Cherenkov resonance with the zero space harmonic of the backward wave ($n=0$): $\omega_0 = h_1 v_0^{acc}$, v_0^{acc} being the initial velocity of the driven REB (Fig.6). The zero and the first space harmonics are parametrically interrelated by a spatial period of the structure, D .

The physical model adopted here to describe the processes of RF field excitation by a high-current REB and the acceleration of driven beam electrons by these fields involves the following statements:

1. The electrodynamic structure under study is in a strong longitudinal magnetic field, therefore there is no transverse displacement of beam electrons (the motion is one dimensional).

2. The beams are infinitely thin, therefore their stratification is neglected.

3. Assuming the driven beam current to be small in comparison with the driving beam current, we shall neglect its influence on the RF field generation in the corrugated waveguide. So, the accelerated electrons move in the given electromagnetic field.

4. TE and TM-waves propagate independently in the electrodynamic structure under study. Since only TM waves perturb the longitudinal component of the beam velocity as well as the beam density, we shall consider below the excitation of this type of the waves.

The original set of equations describing the RF field excitation by a high-current in a corrugated waveguide comprises the Maxwell's equations written in the cylindrical coordinate system:

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{1}{c} \frac{\partial H_\phi}{\partial t}, \quad (2.2)$$

$$\frac{\partial H_\phi}{\partial z} = -\frac{1}{c} \frac{\partial E_r}{\partial t}, \quad (2.3)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) = \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_b, \quad (2.4)$$

where E_z , E_r , H_ϕ are the components of the TM-wave electric and mag-

netic fields, j_b is the diving REB current density. To solve equations (2.2)-(2.4), we make use of the Fourier-series expansion of the variables which enter into these equations.

$$\begin{aligned}\vec{E} &= \text{Re} \int_0^\infty \vec{E}_\omega \exp(-i\omega t) d\omega, \quad \vec{H} = \text{Re} \int_0^\infty \vec{H}_\omega \exp(-i\omega t) d\omega, \\ j_b &= \text{Re} \int_0^\infty j_\omega \exp(-i\omega t) d\omega\end{aligned}\quad (2.5)$$

Then, for the complex-valued variables $E_{r\omega}$, $E_{z\omega}$, $H_{\varphi\omega}$, j_ω we obtain the following equations:

$$\frac{\partial E_{r\omega}}{\partial z} - \frac{\partial E_{z\omega}}{\partial r} = i \frac{\omega}{c} H_{\varphi\omega}, \quad (2.6)$$

$$\frac{\partial H_{\varphi\omega}}{\partial z} = i \frac{\omega}{c} E_{r\omega}, \quad (2.7)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_{\varphi\omega}) = -i \frac{\omega}{c} E_{z\omega} + \frac{4\pi}{c} j_\omega. \quad (2.8)$$

According to the general theory of waveguide excitation [7], the expressions for the fields entering into (2.6)-(2.8) can be written as

$$\vec{E}_\omega = \sum_{s=1}^\infty (C_{s\omega} \vec{E}_s + C_{-s\omega} \vec{E}_{-s}) + \frac{4\pi}{i\omega} j_\omega \vec{e}_z, \quad (2.9)$$

$$\vec{H}_\omega = \sum_{s=1}^\infty (C_{s\omega} \vec{H}_s + C_{-s\omega} \vec{H}_{-s}), \quad (2.10)$$

where $\vec{E}_{\pm s}$, $\vec{H}_{\pm s}$ are the eigenwave fields of the electrodynamic structure without a beam ($\vec{E}_{-s} = \vec{E}_s^*$, $\vec{H}_{-s} = -\vec{H}_s^*$), $C_{\pm s\omega}$ are their amplitudes. The summation in (2.9), (2.10) is taken over all radial modes of the waveguide. The indices s and $-s$ refer to direct and backward waves, respectively.

The amplitudes $C_{\pm s\omega}$ satisfy the following equations of excitation:

$$\frac{dC_{\pm s\omega}}{dz} = \pm \frac{1}{N_s} \int_{S_1} j_\omega \vec{E}_{\pm s} \cdot \vec{e}_z ds_1, \quad (2.11)$$

where $N_s = \frac{c}{4\pi} \int_{S_1} \{(\vec{E}_s \times \vec{H}_{-s}) - (\vec{E}_{-s} \times \vec{H}_s)\} \cdot \vec{e}_z ds_1$ is the norm of the s -th radial mode, S_1 is the waveguide cross section.

Assuming that we have the excitation of a narrow wavepacket with

the frequency interval $2\Delta\omega$ and the average frequency ω_0 ($\omega = \omega_0 \pm \Delta\omega$, $\Delta\omega/\omega_0 \ll 1$), and employing the inverse Fourier transformation for (2.11) we obtain the equations which describe the space-time variation of the fields of direct and counter backward waves in the electrodynamic structure:

$$\frac{1}{v_{gs}} \frac{\partial C_{\pm s}}{\partial t} \pm \frac{\partial C_{\pm s}}{\partial z} = \frac{2}{N_s T_0} \int_{s_1} ds_1 \int_{t-T_0/2}^{t+T_0/2} j_b E_{\mp sz} \exp(i\omega_0 t') dt', \quad (2.12)$$

where $C_{\pm s}(z, t) = \int_{-\Delta\omega}^{\Delta\omega} C_{\pm s\omega} \exp[\pm i(\Delta\omega'/v_{gs})z - i\Delta\omega' t] d\Delta\omega'$, $T_0 = 2\pi/\omega_0$, $v_{gs} = 1/(dh_s/d\omega)$ is the group velocity of waves; the $v_{gs}, N_s, E_{\mp sz}$

values are taken at the frequency ω_0 . Note, that by separating the narrow frequency interval $\Delta\omega/\omega_0 \ll 1$, we impose thereby the requirement on the slow variation of the functions $C_{\pm s}(z, t): |(\partial C_{\pm s}/\partial t)T_0/C_{\pm s}| \ll 1$.

To obtain the self-consistent set of equations describing the processes of RF field generation by a diving REB one must use the kinetic Vlasov equation for the electron distribution function:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} - eE_z \frac{\partial f}{\partial p} = 0 \quad (2.13)$$

with a boundary condition at the interaction space input

$$f(t, p, r, z=0) = f_0(t, p, r), \quad (2.14)$$

where e is the charge of the electron; v, p are, respectively, the velocity and momentum of electrons, f is their distribution function. The beam current density is given by

$$j_b = -e \int f v dp. \quad (2.15)$$

The problem of integrating the kinetic equation (2.13) is equivalent to the integration of the corresponding characteristic set of equations

$$\frac{dt}{dz} = \frac{1}{v}, \quad (2.16)$$

$$\frac{dp}{dz} = -\frac{e}{m} E_z, \quad (2.17)$$

the solution of which can be represented as

$$t=t_l(t_0, p_0, z), \quad p=p_l(t_0, p_0, z), \quad (2.18)$$

where t_0 is the time of electron entrance into the waveguide, p_0 is the initial electron momentum value. Note, that expression (2.18) describe the motion of beam particles along the Lagrange trajectories.

Making use of the law of phase volume conservation on the Lagrange trajectories of beam particles we represent the desired distribution function f in the following form:

$$f(t, p, r, z) = \int dt_0 \int dp_0 f_0(t_0, p_0, r) \delta[t - t_l(t_0, p_0, z)] \cdot \delta[p - p_l(t_0, p_0, z)] v_0 / v \quad (2.19)$$

Here $f_0(t_0, p_0, r) = N_b \delta(r - r_1) \delta(p_0 - P_0) \Phi(t_0) / 2\pi r_1$ is the initial distribution function of electrons; N_b is the particle density per unit length; $\delta(x)$ is the Dirac function, $\Phi(t_0)$ is the distribution function of electrons in initial times of their entrance into the waveguide. Further on, we assume that a non-modulated high-current REB is injected into the waveguide $\Phi(t_0) = 1$. Substituting (2.15) into (2.12) and taking into account expression (2.19) we obtain the following equations for $C_{\pm s}(z, t)$:

$$\frac{1}{v_{gs}} \frac{\partial C_{\pm s}}{\partial t} \pm \frac{\partial C_{\pm s}}{\partial z} = - \frac{2I_b}{N_s T_0} \int r dr \int \delta(r - r_1) r_1^{-1} E_{\mp s z} \exp(i\omega_0 t_l) dt_0, \quad (2.20)$$

where $I_b = eN_b v_0$ is the beam current. Limits of integration of $X(t, z)$ in t_0 follow from the solution of the functional equation:

$$t_l(t_0, p_0, z) = t \pm T_0/2. \quad (2.21)$$

In the approximation of a small change in the beam particle velocity during interaction with resonance wave (this approximation is valid for Cherenkov oscillators with an inertial mechanism of beam particle grouping), the limits of integration in t_0 are substantially simplified and can be represented explicitly. To this approximation the t_l value can be defined as: $t_l = t_0 + z/v_0 + \Delta t_l$ ($\Delta t_l \ll z/v_0$). Neglecting Δt_l and replacing t by $t - z/v_0$, we obtain the following relationship for the limits of integration:

$$t - T_0/2 \leq t_0 \leq t + T_0/2. \quad (2.22)$$

In view of substitution made, Eqs.(2.20) take on the form:

$$\left(\frac{1}{v_{gs}} + \frac{1}{v_o}\right) \frac{\partial C_{\pm s}}{\partial t} \pm \frac{\partial C_{\pm s}}{\partial z} = - \frac{2I_b}{N_s T_o} \int_{r_1}^{t-T_o/2} r dr \int \delta(r-r_1) r_1^{-1} E_{\pm sz} \exp(i\omega_o t_l) dt_o. \quad (2.23)$$

Eqs.(2.16),(2.17),(2.23) are a self-consistent set of equations describing the dynamics of RF field excitation by the driving REB in the electrodynamic structure. For further analysis of the equations derived, it is necessary to define explicitly the function E_z . For this purpose, according to Floquet's theorem, we represent the eigenfields of the periodic structure under study as space harmonic series:

$$E_{\pm sz} = \sum_{n=-\infty}^{\infty} a_n F_{sn}(r) \exp[i(\pm h_s + k_o n)z], \quad a_o = 1, \quad (2.24)$$

where a_n is the n -th space harmonic amplitude, F_{sn} is the function describing the radial field distribution in the waveguide.

It will be recalled that the driving REB is in the Cherenkov resonance with the first space harmonic ($n=1$) of the fundamental radial mode ($s=1$) of the backward wave. As to the nonresonant terms, they contribute, together with the beam term in (2.9), to the RF space charge field. Since the space harmonic amplitudes a_n decrease as α^n with their growing number [4], we shall restrict ourselves to considering only the nonresonant terms of the zero order in α . Hence, the resultant expression for the longitudinal electric field component in the region of the driving REB can be written as

$$E_z(r_1, z, t) = \text{Re}\{[C_{-1}(t, z) I_o(\alpha r_1) a_1 + \frac{2i\omega_o I_b G}{\pi v_o^2 \gamma_o^2} \int_{t-T_o/2}^{t+T_o/2} \exp[i\omega_o t_l - i(-h_1 + k_o)z] \cdot \exp[-i\omega_o t_l + i(k_o - h_1)z] dt_l\} \quad (2.25)$$

here $\alpha = (-h_1 + k_o)/\gamma_o$; $G = [K_o(\alpha r_1) I_o(\alpha r_o) - K_o(\alpha r_o) I_o(\alpha r_1)] I_o(\alpha r_1) / I_o(\alpha r_o)$; I_o is the zero-order modified Bessel function, K_o is the zero-order Modonald function. The second term in (2.25) describes the RF space charge field of a high-current REB, $C_{-1}(t, z)$ is a slowly varying complex amplitude of the resonating backward wave which satisfies Eq.(2.23). As a result, the self-consistent set of equations describing the RF field generation can be represented by:

$$\left(\frac{1}{v_{g1}} + \frac{1}{v_o}\right) \frac{\partial C_{-1}}{\partial t} - \frac{\partial C_{-1}}{\partial z} = - \frac{I_b a_1 I_o(\alpha r_1) \omega_o}{\pi N_1} \frac{t+T_o/2}{t-T_o/2} \int \exp[i\omega_o t_l - 1(k_o - h_1)z] dt_o, \quad (2.26)$$

$$\frac{dt_l}{dz} = (p_l^2 + m^2 c^2)^{1/2} (p_l c)^{-1}, \quad (2.27)$$

$$\frac{dp_l}{dz} = -e(p_l^2 + m^2 c^2)^{1/2} (p_l c)^{-1} E_z(r_1, z, t). \quad (2.28)$$

For further analysis, it appears more convenient to write this set of equations in a dimensionless form:

$$\frac{\partial A_-}{\partial \tau} - \beta_- \frac{\partial A_-}{\partial \xi} = \nu_b \int_{\tau-\pi}^{\tau+\pi} \exp(i\theta) d\theta_o, \quad (2.29)$$

$$\frac{d\theta}{d\xi} = \beta_o (1 + q^{-2})^{1/2} - 1, \quad (2.30)$$

$$\frac{dq}{d\xi} = -\text{Re}\{[A_- + i\sigma \int_{\tau-\pi}^{\tau+\pi} \exp(i\theta) d\theta_o] \exp(-i\theta)\} (1+q^{-2})^{1/2}, \quad (2.31)$$

where $A_- = C_{-1} e a_1 I_o(\alpha r_1) [mc^2 (k_o - h_1)]^{-1}$, $\tau = \omega_o (t - z/v_o)$, $\xi = (k_o - h_1)z$, $\beta_o = v_o/c$, $\beta_- = \beta_{g1}/(1+\beta_{g1})$, $\beta_{g1} = v_{g1}/v_o$, $\theta = \omega_o t_l - (k_o - h_1)z$, $\theta_o = \theta(\xi=0)$, $q = p_l/mc$ is the dimensionless momentum of the electrons,

$$\sigma = \frac{2I_b G}{\pi I_A \beta_o \gamma_o^2}, \quad I_A = mc^3/e \approx 17 \text{ kA}, \quad \nu_b = \frac{2I_b [\lambda_1 a_1 I_o(\alpha r_1)]}{\pi I_A \beta_o^3 \gamma_o^4 (\alpha r_o)^4 J_1^2(\lambda_1) (1+\beta_{g1})},$$

J_1 is the first-order Bessel function, $\lambda_1 = 2.405$ is the first root of the zero-order Bessel function. Note that for writing N_1 we made use of the expression for the smooth waveguide. This is valid for a small value of the periodic inhomogeneity $\alpha \ll 1$, so we have:

$$N_1 = -v_{g1} \omega_o^2 r_o^4 J_1^2(\lambda_1) / (2\lambda_1^2 c^2).$$

For the given set of equations in the case of steady-state regime of generation (A_- is time independent), we can write the first integral

$$\nu_b \int_{-\pi}^{\pi} (1 + q^2)^{1/2} d\theta_o - 0.5 \beta_- |A_-|^2 = \text{const}, \quad (2.32)$$

which is the law of energy conservation.

The efficiency of transformation of the beam energy into the electromagnetic wave energy is defined by the expression:

$$\eta = \frac{S(z=0) - S(z=L)}{S(z=0)}, \quad (2.33)$$

with $S(z)$ being the beam energy flux, which can be rewritten as

$$\eta = \frac{1}{2\pi} \int_{-\pi}^{\pi} [\gamma_0 - (1+q^2)^{1/2}] (\gamma_0 - 1)^{-1} d\theta_0. \quad (2.34)$$

The equations describing the motion of the driven beam electrons in the REB-excited field have the forms:

$$\frac{d\theta^{\text{acc}}}{d\xi'} = \beta_0 [1 + (q^{\text{acc}})^{-2}]^{1/2} - \beta_0 / \beta_0^{\text{acc}}, \quad (2.35)$$

$$\frac{dq^{\text{acc}}}{d\xi'} = \text{Re}[A_- \exp(-i\theta^{\text{acc}})] [1 + (q^{\text{acc}})^{-2}]^{1/2} R, \quad (2.36)$$

where $\xi' = (L-z)(k_0 - h_1)$, $q^{\text{acc}} = p_l^{\text{acc}} / mc$ is the dimensionless momentum of the driven beam electrons, $\beta_0^{\text{acc}} = v_0^{\text{acc}} / c$, $\theta^{\text{acc}} = \omega_0 t_l^{\text{acc}} + h_1(L-z)$, t_l^{acc} is the time of the driven beam electron arrival at the point with the coordinate $L-z$,

$R = \frac{I_0(h_1 r_2 / \gamma_0^{\text{acc}})}{a_1 I_0[(k_0 - h_1) r_1 / \gamma_0]}$ is the parameter defining the ratio of the fundamental space harmonic amplitude $a_0 = 1$ in the driven beam region to the first space harmonic amplitude a_1 in the driving beam region. Note that the R value is determined not only by the ratio of space harmonic amplitudes but also by the ratio of beams radii. The efficiency of the energy transfer from the high-current REB to accelerated electrons is dependent on this parameter. In writing (2.35), (2.36) we have used the new variables $z' \rightarrow L-z$, $p_l^{\text{acc}} \rightarrow -p_l^{\text{acc}}$, which are convenient for describing the process of relativistic electron acceleration.

The set of equations derived must be complemented with initial and boundary conditions for the field and electrons of the beams:

$$\begin{aligned} A_-(\tau=0, \xi) &= \tilde{A}, \quad A_-(\tau, \xi=\tilde{L})=0, \quad \tilde{L}=L(k_0 - h_1), \\ q(\xi=0) &= q_0 = \beta_0 \gamma_0, \quad q^{\text{acc}}(\xi'=0) = q_0^{\text{acc}} = \beta_0^{\text{acc}} \gamma_0^{\text{acc}}, \end{aligned} \quad (2.37)$$

$\tilde{\theta}(\xi=0)=\theta^{acc}(\xi'=0)\in[\tau-\pi, \tau+\pi]$,
 where $\tilde{A}$ is the initial electric field perturbation. Note, that the above given boundary and initial conditions for the field correspond to the BWO without wave reflection on the waveguide ends.

2.2 Result of numerical simulations

The investigation of the two-resonance scheme of acceleration in a corrugated waveguide was studied by numerical simulation of the set of Eqs. (2.29)-(2.31), (2.35), (2.36) for the following initial beam energies $\gamma_0=3$, $\gamma_0^{acc}=6$ and the initial electric field perturbation $\text{Re}\tilde{A}=5\cdot 10^{-3}$, $\text{Im}\tilde{A}=0$. The geometrical parameters of the slow wave structure were determined from the requirement of slowing down the phase velocity of the main space harmonic to $\gamma_{ph}=\gamma_0^{acc}=6$. According to [8], the correlation between the phase velocity of the zero space harmonic and the geometrical parameters of the waveguide is given by the expression:

$\gamma_{ph} = \frac{x}{\alpha^{1/2}} [(1+2\alpha)/(x^2-2)]^{1/2}$, $x = \alpha^{1/2} \pi r_0/D$. The parameter α was chosen equal $\alpha=8.5\cdot 10^{-2}$. Then from the last expression we can find $r_0/D \approx 1.6$. Taking this into account, we can find the amplitude of the first space harmonic a_1 and the group velocity of the wave as $a_1 \approx 0.28$, $\beta_{g1} \approx 0.8$ respectively [4]. The values of the resonance wavenumbers $h_1(\omega_0)$, $k_0-h_1(\omega_0)$ (Fig.6) were found from the condition of Cherenkov synchronism between the REB and the slow backward wave $h_1 \approx k_0/2$ [9]. Further on, we consider the radius of the driven REB to be fixed $r_2/r_0 \approx 1$. The calculations were carried out for different values of the waveguide length L , driving current I_b and radius r_1 .

We first consider the case, when $r_1/r_0=0.5$ ($G=0.51$, $R=3.6$). The space-charge limiting current I_{sc1} for a thin hollow REB with this radius is equal to $I_{sc1}=17\text{kA}(\gamma^{2/3}-1)^{3/2}/[2\ln(r_0/r_1)] \approx 14\text{ kA}$. The waveguide length is chosen to be $\tilde{L}=44$ from the condition for the onset of a steady-state harmonic of generation, which is close to the optimum in efficiency $\gamma_0=3$ [10]. For these parameters the oscillations get excited at $\nu_b=6\cdot 10^{-4}$ ($I_b \approx 5\text{kA}$) and the steady-state harmonic of generation sets in. Qualitatively, the excitation of stationary RF fields by a high-current REB can be described as

follows (Fig.7). At the linear stage there is an exponential growth of the output-signal amplitude. At the same time the maximum amplitude value shifts from the output to the input of the waveguide at a velocity close to the group one. Then, after a typical amplitude peak in the vicinity of $\xi=0$ there occur damped oscillations about stationary state. The process is completed by a steady-state field distribution along the interaction space. Note, that the efficiency of the stationary regime of generation (2.34) is about $\eta \approx 17\%$.

As current I_b approaches to the value corresponding to the limiting vacuum current I_{sol} , the electric field intensity increases (Fig.8). In this case, in the range of v_b values considered the steady-state one-frequency mode of generation remains stable. The stability of stationary states of the electrodynamic system at $\tilde{L}=44$ is due to the fact that the RF space charge field of the high-current REB prevents the nonlinear rearrangement of beam electrons, which is associated with arising nonstationary self-modulation regimes of generation. A more detailed discussion of this problem can be found in [11].

Let us now consider the process of charged particle acceleration by a monochromatic electromagnetic back-ward wave. Figs.9,10 show the phase planes of the driving beam particles q^{acc} , φ^{acc} , where $\varphi^{acc} = \varphi - \vartheta^{acc}$, φ is the phase of the complex amplitude of the back-ward wave $A_- = |A_-| \exp(i\varphi)$; and the dependences of electron momenta q^{acc} on the initial phase ϑ_0^{acc} of electrons. It is seen from these figures, that already at the beginning of the interaction space the bunches of accelerated and decelerated particles are formed. The accelerated electrons make up approximately 20% of the total amount of beam particles. For the parameters considered the maximum energy of the accelerated electrons is about three times as large as its initial value is about three times as large as its initial value w_0^{acc} . The relative energy scattering of the accelerated particle is no more than 10%. As the numerical analysis shows, the phase difference for the particles belonging to the accelerated bunch $(\Delta\varphi^{acc} = \varphi^{acc}(\xi'=\tilde{L}) - \varphi^{acc}(\xi'=0))$ is approximately equal to π and throughout the length of the interaction space ϑ^{acc} weakly changes its initial value ϑ_0^{acc} . Therefore, the electric field, which acts on the accelerated electrons, changes its polarity only near $\xi'=\tilde{L}$. So, the energy gain of the particles of the driven REB has mono-

tonous character. This is illustrated by Fig.11, which shows the dependence of the electron momentum on the longitudinal coordinate ξ' of one of the particles belonging to the accelerated bunch.

The decrease of the driving REB radius $r_1/r_0=0.3$ ($G=0.9$, $R=4.1$, $I_{sol}=8$ kA) does not essentially change the process of generation of RF electromagnetic fields. At the same time the increase of the transformation parameter R allows one to reach a greater acceleration rate in comparison with the case of $r_1/r_0=0.5$. Fig.12 shows the maximum energy gain of the accelerated particles ΔW^{acc} versus the driving beam current I_b . It is evident that in a wide interval of the driving beam currents an effective charged particle acceleration takes place. Here we should note that at $r_1/r_0 \rightarrow 0$ along with the growing transformation coefficient R , the coupling between the driving beam and the resonance backward electromagnetic wave becomes weaker (ν_b decreases). Besides, the space-charge limiting beam current value in the waveguide I_{sol} also decreases. So, further reduction of the driving beam radius ($r_1/r_0 < 0.3$) seems unreasonable.

Now we present the estimates of the maximum longitudinal component of the electric field and the acceleration rate which can take place in the two-resonance acceleration scheme under consideration for the following parameters: $r_1/r_0=0.5$; $\nu_b=1.6 \cdot 10^{-3}$ ($I_b \approx 13$ kA), $D=1.2$ cm. For these parameters the maximum electric field intensity in the region of driven REB (Fig.8) is:

$|E_{-}|_{max} \approx 0.14 mc^2 (k_0 - h_1) R / a_1 e I_0 (\alpha r_1) \approx 190$ Mv/m. The length of the slow-wave structure for $D=1.2$ cm is equal, accordingly, to 17 cm. As it follows Fig.8, at this length the acceleration rate is about $T \approx 35$ Mev/m. It should be stressed that significant difference between $|E_{-}|_{max}$ and the acceleration rate T is due to an inhomogeneous distribution of the longitudinal component of the electric field along the slow-wave structure (Fig.8), and to the variation of electric field phase ϕ during acceleration of charged particles.

The above given results were obtained at a fixed $\tilde{L}$ value, when a steady state regime of RF field generation was attained for the self-oscillation system. With an increasing waveguide length, the steady-state regime becomes unstable and changes into a self-modulation one [11]. The structure length value at which this regime sets in, depends on the initial REB energy and current. For the self-modulation regime of generation the field distribution along

the waveguide appears to be a sequence of amplitude $|A|$ peaks and dips of different height and shape. Numerical calculations have shown that this field distribution leads to a nonmonotone energy gain by driven beam electrons. As a result, the acceleration rate falls down to $T < 35$ Mev/m. So, in the two-resonance acceleration scheme considered the most effective acceleration regime is accomplished when a steady-state regime of generation is reached in the system.

Basic results. Conclusions

In conclusion let us formulate the basic results obtained in this work.

In section I of the work considers the acceleration of electrons by the high-current REB in the plasma waveguide.

1.1 The excitation of accelerating fields by the modulated high-current REB in the plasma waveguide has been treated by numerical means. The preliminary modulation of the beam is shown to shorten the distance to the first maximum of the accelerating field amplitude. The RF space charge field is established to increase the field amplitude in the region of the driven beam over all the accelerating gap ($0 \leq \xi \leq 1$).

1.2 A process of accelerating the electrons of the driven beam is described. The estimates of the maximum strength of the accelerating field and the acceleration rate for electrons are given. The preliminary modulation of the driving REB is established to enhance the acceleration rate of driven beam electrons.

1.3 The effect of plasma inhomogeneity on the process of plasma wave excitation has been considered. Admissible gradients of plasma inhomogeneity are found, at which there is no breakdown of plasma wave excitation and acceleration of driven beam electrons.

In section II we have considered the two-beam scheme of charged particle acceleration in a vacuum corrugated waveguide.

2.1 In the framework of the space-time model the nonlinear set of equations has been derived to describe the process of electromagnetic wave generation by a high-current REB. The set of equations has been analyzed by numerical simulation.

2.2 The RF field excitation by a high-current REB has been analysed for different beam current and electrodynamic structure length

values.

2.3 The process of accelerated particle bunch formation in the backward electromagnetic wave field of a corrugated waveguide has been investigated. It is found that with the onset of a steady-state regime of generation in the self-oscillation system the energy gain by the accelerated particle is of monotone character. The amount of electrons trapped into the the acceleration regime makes up about 20% of the total amount of the driven REB particles.

2.4 Estimates are given for the peak intensity of the accelerating field and the rate of electron acceleration.

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Figure Caption

Fig.1 Amplitude $|A(\xi)|$ (a) and phase $\text{Arg } A(\xi)$ (b) versus the longitudinal coordinate ξ . 1 corresponds to the dependences obtained without the RF space charge. 2 corresponds to the dependences obtained with taking into account the RF space charge.

Fig.2 $|\tilde{A}(\xi)|$ (a) and $\text{Arg } \tilde{A}(\xi)$ (b) versus the longitudinal coordinate ξ .

Fig.3 Electron momenta q^{aoo} versus the initial phases φ_0^{aoo} .

a) $\xi=24$, b) $\xi=48$, c) $\xi=72$, d) $\xi=96$.

Fig.4 Momentum of the accelerated particle versus the longitudinal coordinate ξ . $\varphi_0^{\text{aoo}}=\pi$.

Fig.5 A sketch of a periodic waveguide with accelerating and accelerated beams.

Fig.6 Qualitative dispersion picture of the first three radial modes of the corrugated wall waveguide.

Fig.7 Space-time evolution of the backward wave amplitude $|A_-(\tau, \xi)|$. $\nu_b=6 \cdot 10^{-4}$.

Fig.8 Space distribution of the electric field in a corrugated waveguide. 1.- $\nu_b=6 \cdot 10^{-4}$; 2.- $\nu_b=1.2 \cdot 10^{-3}$; 3.- $\nu_b=1.6 \cdot 10^{-3}$.

Fig.9 Phase plane of the driven beam for $\nu_b=1.6 \cdot 10^{-3}$ and different ξ' . a) $\xi'=11$; b) $\xi'=22$; c) $\xi'=33$; d) $\xi'=44$.

Fig.10 Dependences of electron momenta q^{aoo} on the initial phase φ_0^{aoo} . a) $\xi'=11$; b) $\xi'=22$; c) $\xi'=33$; d) $\xi'=44$.

Fig.11 Electron momentum q^{aoo} vs the longitudinal space coordinate.

Fig.12 The maximum energy gain of the accelerated particles vs the driving REB current I_b . 1.- $r_1/r_0=0.5$; 2.- $r_1/r_0=0.3$.

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Fig. 12

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