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V.E. Oberacker, J.C. Wells, A.S. Umar and M.R. Strayer

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PERTURBATIVE AND NONPERTURBATIVE EM LEPTON PAIR PRODUCTION IN RELA- TIVISTIC HEAVY-ION COLLISIONS

Volker E. Oberacker^{1,3}, Jack C. Wells^{1,2,3}, A. Sait Umar^{1,3}, and
Michael R. Strayer^{1,2}

¹Center for Computationally Intensive Physics, Oak Ridge Na-
tional Laboratory, Oak Ridge, Tennessee 37831, USA;

²Physics Division, Oak Ridge National Laboratory, Oak Ridge,
Tennessee 37831, USA;

³Department of Physics & Astronomy, Vanderbilt University,
Nashville, Tennessee 37235, USA

INTRODUCTION

In this talk, I will focus on *electromagnetic* dilepton production from the QED-vacuum in relativistic heavy-ion collisions. Heavy ions in relativistic motion generate strong time-dependent EM fields with large Fourier components which give rise to sizable pair production. There are several motivations for our studies: Lepton pair production by *hadronic (Drell-Yan)* processes has been widely discussed as a possible signature of the quark-gluon plasma formation¹. The dominant background will come from electromagnetic sources and could even mask the signals from the plasma phase².

Electromagnetically produced lepton pairs also impose severe constraints on the design of relativistic heavy-ion colliders such as RHIC and LHC. In addition to the *free pair production* discussed above, *pair-production with capture* of the negatively charged lepton into a bound state is also possible, as illustrated in Figure 1. This change of the charge state of the ions is the leading mechanism for beam loss of relativistic colliders. Accurate predictions of the cross section for this process are important because the cross section increases with energy.

A few historical remarks: Free electron-pair production in the collisions of cosmic rays with nuclei was first described in 1937 by Racah³, based on the “equivalent pho-

ton method" developed by Weizsäcker and Williams⁴ and by Landau and Lifshitz⁵. In 1971, Brodsky et al.⁶ applied this technique to pair creation in particle accelerators.

Figure 1. Free electron-pair production and pair production with capture of the electron into a bound state.

One calculates classically the photon flux generated by the particles in motion and folds it with the photoproduction cross section for pairs. In 1977, Soff⁷ utilized the Weizsäcker-Williams method for the first time to describe electron-positron production in heavy-ion collisions. For a detailed discussion we refer to the review article by Bertulani and Baur⁸.

PERTURBATIVE LEPTON-PAIR PRODUCTION

The equivalent photon method relies on several approximations: it assumes that the photons are on-shell and have zero transverse momenta. This approximation is only valid in the ultra-relativistic limit. Furthermore, the corresponding S-matrix elements are singular and must be regularized with both high- and low-frequency cutoffs. A better approach is to use Feynman perturbation theory and to evaluate the lowest-order two-photon-exchange diagram for e^+e^- pair production which is depicted in Figure 2.

Figure 2. Lowest-order Feynman diagram for free e^+e^- pair production.

The two-photon-exchange diagram can be reduced to an eight-dimensional integral; this integral has been computed exactly by Bottcher and Strayer⁹ with a Monte-Carlo technique. The resulting total cross section for electron-pair production is shown in Figure 3 for $^{197}\text{Au} + ^{197}\text{Au}$ as a function of the collider energy (in units of GeV/amu). The cross section is displayed in units of $\sigma_0 = (Z\alpha)^4 \lambda_e^2 = 165$ b; at $E_c = 10$ GeV/amu (corresponding to CERN-SPS energy) the total cross section for free e^+e^- pair production is about 3 kilobarn, and at 100 GeV/amu (i.e. RHIC energy) it is about 30 kilobarn. Figure 3 also gives a comparison between the exact Monte-Carlo evaluation of the lowest-order Feynman diagram and various calculations based on the Weizsäcker-Williams method. It is striking that Racah's result is very close to the exact calculation whereas the result of Bertulani and Baur underestimates the cross section at lower energies.

Figure 3. Free electron-pair production cross section vs. collider energy.

Figure 4. Positron momentum spectrum for $^{16}\text{S} + ^{79}\text{Au}$ at $E_c = 10$ GeV/amu.

In Figure 4 we compare the prediction of the two-photon perturbation theory with data¹⁰ measured at the CERN-SPS for $^{16}\text{S} + ^{79}\text{Au}$; the equivalent collider energy is $E_c = 10 \text{ GeV/amu}$. Up to positron momenta of about 7 MeV/c, theory and experiment are in excellent agreement; at higher positron momenta, it appears that the lowest-order perturbation theory overestimates the measured differential cross section. The total measured free electron-pair production cross section is (85 ± 22) barn, whereas the Feynman-Monte Carlo calculation yields 98 barn.

Rhoades-Brown, Bottcher and Strayer¹¹ have also performed perturbative calculations of electron-pair production with electron capture into a bound state of the target atom. The two-photon-exchange Feynman diagram is shown in Figure 5.

Figure 5. Lowest-order Feynman diagram for e^+e^- pair production with bound state capture of the electron.

Figure 6. Total cross sections for electron-pair production with capture into the 1s state.

In Figure 6 we display the calculated cross sections for electron capture following pair creation from the QED-vacuum. The cross sections are given for symmetric $zA + zA$ collisions as a function of the collider energy (in units of GeV/amu). For the

system $Au + Au$ at 100 GeV/amu (RHIC energy) one finds a total cross section of 72 barn; this value corresponds to a 10 hour lifetime of the RHIC beams, enough to carry out meaningful experiments. However, it is doubtful that perturbation theory holds for very heavy systems at high energies: the effective coupling constant is no longer small compared to 1 (for a $Au + Au$ collision we have $Z\alpha \approx 0.6$), and the time-dependent electromagnetic fields generated by the projectile are 20,000 times stronger than the static Coulomb field of the target. First indications that perturbation theory may be inadequate to describe pair creation with capture have recently been reported by experimenters at the BEVALAC¹². In ${}_{92}U + Z A$ collisions at a fixed-target energy of 1 GeV/amu it was found that the cross section dependence on the atomic number of the target is $Z^{(2.8 \pm 0.25)}$. Lowest-order perturbation theory predicts a Z^2 dependence. It is therefore necessary to study lepton-pair production by non-perturbative methods.

NON-PERTURBATIVE LEPTON-PAIR PRODUCTION

We shall now outline our theoretical formalism^{13, 14} for nonperturbative lepton-pair production. Bottcher and Strayer have derived the field equations from a semi-classical least-action principle². In the following, we give an alternative derivation which is similar to the approach developed by Reinhardt et al.¹⁵ for nonrelativistic heavy-ion reactions. We start from the standard QED Lagrange density

$$\mathcal{L}_{\text{QED}} = \hat{\psi}^\dagger \gamma^0 [\gamma^\mu i \partial_\mu - 1] \hat{\psi} - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \hat{j}^\mu \hat{A}_\mu, \quad (1)$$

where

$$\hat{j}^\mu = \hat{j}_{\text{lept}}^\mu + \hat{j}_{\text{ext}}^\mu = e \hat{\psi}^\dagger \gamma^0 \gamma^\mu \hat{\psi} + \hat{j}_{\text{ext}}^\mu \quad (2)$$

denotes the current density operator which describes the lepton current and the conserved, external current of the moving heavy nuclei. The total radiation field $\hat{A}_\mu$ consists of three parts

$$\hat{A}_\mu = \hat{A}_\mu^{\text{lept}} + \hat{A}_\mu^{\text{ext}} + \hat{A}_\mu^{\text{free}}, \quad (3)$$

where $\hat{A}_\mu^{\text{lept}}$ is generated by the lepton current $\hat{j}_{\text{lept}}^\mu$ and $\hat{A}_\mu^{\text{ext}}$ by the external current $\hat{j}_{\text{ext}}^\mu$, respectively. The term $\hat{A}_\mu^{\text{free}}$ denotes the free radiation field. By varying the action integral

$$S_{\text{QED}} = \int d^4x \mathcal{L}_{\text{QED}} \quad (4)$$

with respect to the field operators $\hat{\psi}$ and $\hat{A}_\mu$, we obtain the Euler-Lagrange equations of motion for the quantum fields

$$[\gamma^\mu (i \partial_\mu + e \hat{A}_\mu) - 1] \hat{\psi}(x) = 0, \quad (5)$$

$$\partial_\mu \hat{F}^{\mu\nu}(x) = \hat{j}^\nu(x). \quad (6)$$

These coupled quantum field equations are difficult to solve without approximations; hence, we make the following simplifying assumptions: We neglect the leptonic current $\hat{j}_{\text{lept}}^\mu$ in Eq. (6) and the associated electromagnetic field $\hat{A}_\mu^{\text{lept}}$ in Eq. (5); this is justified because both of them are much smaller than the strong external heavy-ion current $\hat{j}_{\text{ext}}^\mu$ and the external field $\hat{A}_\mu^{\text{ext}}$, respectively. With this approximation, the field equations decouple. Furthermore, we treat the external field classically by solving Maxwell's equations, i.e. $\hat{A}_\mu^{\text{ext}} \rightarrow \langle \hat{A}_\mu^{\text{ext}} \rangle_{\text{class}} = A_\mu^{\text{ext}}$. We also neglect QED radiative corrections like vacuum polarization and self energy effects arising from interactions of the lepton field with the free radiation field, i.e. $\hat{A}_\mu^{\text{free}} = 0$. The problem is thus reduced to the solution of the time-dependent Dirac equation, Eq. (7), for the lepton field $\hat{\psi}$ interacting with an external, classical four-vector potential A_μ^{ext} determined independently by the classical Maxwell equations, Eq. (8), i.e.

$$[\gamma^\mu(i\partial_\mu + eA_\mu^{\text{ext}}) - 1]\hat{\psi}(x) = 0, \quad (7)$$

$$\partial_\mu F_{\text{ext}}^{\mu\nu}(x) = j_{\text{ext}}^\nu(x). \quad (8)$$

We study the electromagnetic production of lepton pairs in a reference frame where one of the nuclei, henceforth referred to as the target, is at rest. The target nucleus and the lepton interact via the static Coulomb field A_T^0 . The only time-dependent interaction $A_P^\mu(t)$ arises from the classical motion of the projectile. Thus, it is natural to recast the Dirac equation (7) into the Schrödinger form

$$[H_F + H_P(t)]\hat{\psi}(\vec{r}, t) = i\frac{\partial}{\partial t}\hat{\psi}(\vec{r}, t). \quad (9)$$

The Furry Hamiltonian

$$H_F = -i\vec{\alpha} \cdot \nabla + \beta - eA_T^0 \quad (10)$$

describes the leptons in the presence of the static Coulomb field of the target nucleus, and

$$H_P(t) = e\vec{\alpha} \cdot \vec{A}_P(t) - eA_P^0(t) \quad (11)$$

is the time-dependent interaction between the lepton field and the projectile. We expand now the lepton-field operator $\hat{\psi}(\vec{r}, t)$ in a complete, orthonormal set of single-particle basis states. In terms of the stationary eigenstates of the Furry Hamiltonian

$$H_F\chi_k(\vec{r}) = E_k\chi_k(\vec{r}) \quad (12)$$

we obtain

$$\hat{\psi}(\vec{r}, t) = \sum_k \hat{a}_k(t)\chi_k(\vec{r})\exp(-iE_k t), \quad (13)$$

where the $\hat{a}_k$'s are operator-valued expansion coefficients. The Furry eigenstates are the proper in- and out-states for asymptotic times $|t| \rightarrow \infty$, where the interaction $H_P(t)$ is zero. From the anticommutation relations for the fermion field operators $\hat{\psi}$ and $\hat{\psi}^\dagger$, one readily shows that $\hat{a}_k^\dagger$ and $\hat{a}_k$ describe the creation and annihilation of

leptons in state k in the static Coulomb field of the target nucleus. We also expand the lepton field in terms of the time-dependent basis of solutions to the full Dirac Hamiltonian $H_F + H_P(t)$

$$[H_F + H_P(t)]\phi_j(\vec{r}, t) = i\frac{\partial}{\partial t}\phi_j(\vec{r}, t). \quad (14)$$

This basis expansion results in

$$\hat{\psi}(\vec{r}, t) = \sum_j \hat{\alpha}_j \phi_j(\vec{r}, t), \quad (15)$$

where the $\hat{\alpha}_j$ are quasi-particle destruction operators. Under the influence of the interaction Hamiltonian $H_P(t)$ the single-particle states $\chi_j(\vec{r})$ evolve into the time-dependent states $\phi_j(\vec{r}, t)$ according to the time-dependent Dirac equation (14). Therefore, the index j does not refer to a set of good quantum numbers for $\phi_j(\vec{r}, t)$, but refers to the quantum numbers of the particular Furry state, $\chi_j(\vec{r})$, that satisfies the $t \rightarrow -\infty$ boundary condition

$$\lim_{t \rightarrow -\infty} \phi_j(\vec{r}, t) \rightarrow \chi_j(\vec{r}) \exp(-iE_j t). \quad (16)$$

Figure 7. The transitions leading to pair production with capture into the $1s$ state are indicated (see Equation (19)).

Our aim is to treat pair production by completely ionized heavy atoms. In this case, the QED vacuum state in the external Coulomb field of the target nucleus has the structure

$$|\Phi_0\rangle = \prod_{i < F} (\hat{a}_i^\dagger) |0\rangle, \quad (17)$$

where $i < F$ denotes states below the Fermi energy $E_F = -m_0c^2$. The time-evolved QED vacuum state is given by the time-evolution operator $\hat{U}(t, t_0)$

$$|\Phi_0(t)\rangle = \hat{U}(t, t_0 \rightarrow -\infty)|\Phi_0\rangle \quad (18)$$

By making the assumption that $|\Phi_0(t)\rangle$ is at all times a single Slater determinant, one can derive expressions for the inclusive number of leptons and antileptons created after the collision. The total number of leptons captured into the $1s$ state following pair creation from the QED vacuum is given by projecting the time-evolved single-particle state $\phi_{1s}(t \rightarrow \infty)$ onto the complete set of Dirac continuum states $\chi_k^{(-)}$ with negative energy ($k < F$), i.e.

$$N_{capt}^{(1s)}(t \rightarrow \infty) = \sum_{k < F} |\langle \chi_k^{(-)} | \phi_{1s}(t \rightarrow \infty) \rangle|^2. \quad (19)$$

To compute probabilities for free lepton-pair production is computationally much more involved because one has to time-propagate a large number of negative-energy states and project these onto the Furry continuum states with positive energy.

SOLUTION OF THE TIME-DEPENDENT DIRAC EQUATION

There are essentially two different approaches to the numerical solution of the time-dependent Dirac equation (14): lattice calculations^{2, 13, 14, 16} and the coupled-channels formalism^{17, 18}. We utilize here a 3-dimensional Cartesian lattice and the Basis-Spline collocation method. Details will be discussed in the presentation by Jack Wells. The four Dirac spinor components $\psi^{(p)}(x, y, z, \tau)$, $p = (1, 2, 3, 4)$ are expanded in terms of a product of Basis-Spline functions $B_i^M(x)$ which are piecewise-continuous polynomials of order $(M - 1)$. The B-Splines are generalizations of the "finite elements" that are widely used in computational physics. The Dirac spinor components $\psi^{(p)}$ are represented on a rectangular cartesian lattice $(x_\alpha, y_\beta, z_\gamma)$ i.e. the lattice representation of the spinor wave function $\psi^{(p)}$ is a vector ψ with $N = 4 \cdot N_x \cdot N_y \cdot N_z$ complex components. The original partial differential equation (14) is transformed into a matrix equation of the form

$$\mathbf{H}\psi(\tau) = i \frac{\partial \psi}{\partial \tau}, \quad (20)$$

where we use boldface type for matrices and vectors. It is impossible to store $\mathbf{H}$ in computer memory, since this would require the storage of N^2 complex double-precision numbers. Hence, we must resort to iterative methods for the solution of the matrix equation which do not require the storage of $\mathbf{H}$.

We solve the time-dependent Dirac equation in two steps: first, we consider the static Coulomb problem at time $\tau \rightarrow -\infty$, i.e. the lepton bound to a heavy nucleus. The stationary Dirac equation for the ground-state spinor is given by

$$\mathbf{H}_0\psi_{gs} = E_{gs}\psi_{gs} \quad (21)$$

The static problem is solved by an iterative procedure such as the Lanczos algorithm¹⁴ or the damped relaxation method¹⁹.

We solve the time-dependent equation (20) by a Taylor-expansion of the propagator. For an infinitesimal time step $\Delta\tau$ we find

$$\psi(\tau + \Delta\tau) = U(\tau + \Delta\tau, \tau) \psi(\tau) \approx \left(1 + \sum_{n=1}^N \frac{(-i\Delta\tau H)^n}{n!}\right) \psi(\tau). \quad (22)$$

We have thus reduced the original problem to a series of (matrix) $\times$ (vector) operations which can be executed with high efficiency on vector or parallel supercomputers without explicitly storing the matrix in memory.

RESULTS

In Figure 8 we show the probability of muon-pair production with capture into the K-shell as a function of time. The system studied is $^{197}\text{Au} + ^{197}\text{Au}$ at $E_c = 100$ GeV/amu for the grazing impact parameter $b = 8.72\lambda_\mu = 16.3\text{fm}$. We notice that the probability increases very steeply as the two nuclei approach each other; after the classical turning point has been passed $P(t)$ reaches a constant value of about 10^{-2} .

Figure 8. Muon-pair production with capture for a $Au + Au$ collision at RHIC energy.

Further results will be presented by Jack Wells at this conference, in particular the impact parameter dependence of the capture cross section.

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