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ALGORITHM FOR DUAL-ENERGY RADIOGRAPHIC ANALYSIS

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INTRODUCTION

The use of two or more radiographs of an object taken with different x-ray spectral characteristics to infer quantitative values of material density or Z number has been of interest to both the medical and industrial worlds for some time. One method uses monoenergetic isotopic sources with well defined energies in conjunction with standard step wedges and solving the resulting simultaneous equations. Besides the problem of finding isotopic sources with the appropriate energies, you have to have *a priori* knowledge of the materials in the object.

This paper describes an algorithm that does not impose any limitations on the energy spectrum of the sources nor require any knowledge of the object. The algorithm does require that the different radiographs have perfect spatial registration (within a pixel width) and assumes that the transmitted x-ray intensity spectra is the same (within a multiplicative constant) over the image plane. This paper is just a start in developing multi-energy techniques; objects with three or more materials have not been investigated and it is not clear just how this algorithm should be generalized to the multi-energy case.

THEORY

The algorithm is based on the work of Alvarez¹ and further developed by Stanley and LePage.² The model of photon transmission through the object is simply Beer's law without any consideration to x-ray scattering and spectral hardening. X-ray scattering, in itself, is handled automatically but the amount of scattering or beam hardening is assumed constant over the image plane. With normal radiographic geometries in which the detector is next to the object, the algorithm yields its best results when the object has a uniform radiographic thickness and the intent is to detect small changes in density or Z number.

Starting from the equation for the photon intensity transmitted through an object derived in Reference (1), an equation giving the pixel value in the final image is produced in terms of two other images.

$$Q_{ij} = C_1 L_{ij}^1 + C_2 L_{ij}^2, \quad (1)$$

where $i, j = 1, 2, \dots, N$. N equals the width and length of the image (square for simplicity) in pixels, C_1 and C_2 are scalar and constants that depend only on the x-ray energy spectra, L^1 and L^2 , are images whose pixel values depend only on the material properties of the object (density, thickness and Z number). Superscripts are NOT powers.

The image pixel value Q_{ij} is defined as:

$$P_{ij} = \left| \ln \left(\frac{I_{ij}}{I_o} \right) \right|$$

$$\bar{P} = \frac{1}{N} \sum_{ij} P_{ij}$$

$$Q_{ij} = \frac{1}{\sqrt{N}} (P_{ij} - \bar{P}), \quad (2)$$

where I_{ij} is the value of the (i, j) pixel in the original radiographic image. The Q 's form a linear vector space.

A dot product or metric in this vector space can be defined as

$$Q^1 \bullet Q^2 = K = \sum_{ij} Q_{ij}^1 \bullet Q_{ij}^2,$$

where K is a scalar and the sum is over the pixel indexes. (A point of interest: the dot product, K , is the variance or covariance of the two images.)

We haven't discussed the L images yet except to say they are only functions of the material. It seems fairly obvious that they are of interest only if they are basis images of the vector space. If they are basis images, we can get a solution to the following set of equations and calculate a pair of L 's that are orthonormal.

Assume that the L images are orthonormal and calculate the dot product of two images of the same object that are identical except taken at different energies.

$$Q^1 = C_1^1 L^1 + C_1^2 L^2 \text{ and}$$

$$Q^2 = C_2^1 L^1 + C_2^2 L^2$$

$$Q^1 \bullet Q^1 = (C_1^1)^2 + (C_1^2)^2 = K_1$$

$$Q^2 \bullet Q^2 = (C_2^1)^2 + (C_2^2)^2 = K_2 \quad (3)$$

$$Q^1 \bullet Q^2 = C_1^1 \bullet C_2^1 + C_1^2 \bullet C_2^2 = K_3,$$

where the K 's are all scalar values. Unfortunately, we also have four unknowns with only three equations. This is caused by the fact that we have not put any restrictions on the L 's except that they be orthonormal. There are an infinite number of coordinate systems all connected by a similarity transform that meets these conditions. If we arbitrarily set $C_1^1 = 0.0$, the Q^1 image lies on the L^2 axis and we have fixed the coordinate system. The C 's are now given by

$$C_1^1 = 0.0$$

$$C_2^1 = \pm \sqrt{K_1}$$

$$C_2^2 = \pm \frac{K_3}{C_2^1}$$

$$C_1^2 = \pm \sqrt{K_2 - \left(\frac{K_3}{C_2^1} \right)^2}. \quad (4)$$

These values of C are the coordinates of the two experimental images in our vector space. The only thing left to do is calculate the basis images, L^1 and L^2 , using Equation (5).

$$L_{ij}^2 = \frac{C_{ij}^1}{C_2^1}$$

$$L_{ij}^1 = \frac{Q_{ij}^2 - Q_{ij}^1 \left(\frac{C_2^2}{C_2^1} \right)}{C_2^1}. \quad (5)$$

Equations (3), (4), and (5) calculate the coordinates of the experimental images and the pixel values of the basis images in terms of the variance/covariance values of the Q images (K 's). Note that two images are subtracted pixel by pixel in Equation (5) and two scalar values are subtracted in equation (4). This means that errors are introduced in the calculations if the images are not registered exactly and if the contrast of the two images are nearly the same. Both of these errors are illustrated in the results that follow. In fact, if the contrast of the two images is exactly the same, the two images are made with the same energy spectra and there is no solution to the problem.

At this time, no particular significance can be attached to the basis images. We could have set any one of the coordinates equal to any scalar value and arrived at another solution for the C 's and L 's. A vector space has been constructed that is spanned by two basis images. The space includes radiographic images that can be produced by real x-ray sources as well as an infinite number of images (virtual) that cannot be generated. In fact, one of the basis images is real while the other is virtual. What can these virtual images tell us about the object?

EXPERIMENT

The first step was to demonstrate that Equations (3), (4), and (5) produced reasonable results under ideal conditions. Ideal conditions meant that

- the two images were generated with a large energy difference,
- image registration was perfect, and
- the images did not contain any noise.

Accordingly, a step wedge of iron and aluminum with two thickness each was constructed (Fig. 1). This step wedge was radiographed on a standard x-ray machine at energies that varied from 100 pkV to 275 pkV using Kodak type M film. Lead screens were used at all energies and an extra image was made at 100 pkV without screens.

The optical densities under each step were measured and computer-generated images were made using these experimental optical densities. This guaranteed spatial registration between all the images. The computer images were noise free. Figure 2 illustrates the two images that were chosen as the raw data.

All the calculations for this experiment were carried out on a PC using the Mathematica package. We, in effect, were working with small images (two pixels by two pixels) and data storage was not a problem. Figure 3 illustrates the calculated basis images and Fig. 4 a plot of the vector space with the experimental images shown.

Using Mathematica, we were able to calculate any linear combination of the basis images and display the resulting image. Figure 5 shows images that maximized the iron and aluminum images, respectively. The coordinates of these images can be calculated by imposing conditions on the resulting images (zero aluminum and iron contrast, respectively).

The maximum contrast for iron is not located at a point but lies within a small circle located at the plotted point of Image 3. For instance, the aluminum contrast could be made zero under one thickness or the other of iron but not both simultaneously. These images of minimum aluminum contrast all fell within a small circle centered at the plotted point. The probable cause of this is the slightly different x-ray spectra under the various material thicknesses caused by different degrees of beam hardening. The same result was noted within the vicinity of Image 4.

A few empirical observations can now be made.

- The real images seem to lie along a straight line.
- Images that lie along a radial line through the origin seem to be generated with the same energy spectrum but with different contrasts.
- Images 3 and 4 are orthogonal to within experimental error.

2 mm Fe 10 mm Al	4 mm Fe 10 mm Al
2 mm Fe 20 mm Al	4 mm Fe 20 mm Al

Figure 1. Configuration of iron/aluminum step wedge.

Figure 2. Images chosen as raw data.

Figure 3. Calculated basis images.

Figure 4. Vector space showing experimental and calculated images.

Figure 5. Images that maximized iron and aluminum.

This last observation implies that, if properly scaled, Images 3 and 4 form another basis set and span the vector space. This makes sense if we observe that if we have an object that consists of two materials, we get the maximum radiographic information when one image contains only information about one material and the other image the other. Further speculation along these lines is at the end of the paper.

At this point we decided to make life difficult and try to break the system. An experiment was designed with all the parameters as far away as possible from optimum.

- The step wedge consisted of plastic and aluminum (Fig. 6).
- The detectors were two storage phosphors scanned in a Molecular Dynamics scanner (176-micron pixels).
- The x-ray source was a 150 pkV x-ray machine.
- Energy differentiation was caused by differential absorption in the plastic backing of the storage phosphors (Fig. 7).

The object consisted of two materials with reasonably close atomic numbers. We also had to take the two scanned images and register them. Noise from both the scanner and fluors were present in both images. The energy spectral difference between the two images was so small that it could not be detected by the eye when both images were displayed on the monitor. The storage phosphors also have a contrast that is equal to 1.0. In contrast, type M film at a density of 2.0 has a contrast well above 2.0. This meant that we could not expect any help

13 mm PI 4.2 mm Al	13 mm PI .25 mm Al
.76 mm PI 4.2 mm Al	.76 mm PI .25 mm Al

Figure 6. Configuration of plastic/aluminum step wedge.

Figure 7. Cut-away view of storage phosphor with backing.

from the fluors in amplifying any small contrast difference between the two images.

Since we were working with large, real images, the arithmetic was carried out on a Sun Sparc10 workstation. Figure 8 shows the actual image from one of the fluors. For all intents and purposes, the other image is identical and is not shown. Figures 9 and 10 show the basis images and Figs. 11 and 12 the results when aluminum and plastic are maximized, respectively.

The most important result from this last experiment was that numerical instabilities did not prevent a solution; we got results. While spatial registration errors are visible in all the calculated images, this can be eliminated with a better registration program. A good measure of the x-ray energy difference the two fluors saw is not available but it is certainly small and

Figure 8. An actual image.

Figure 9. Basis image L^2 .

Figure 10. Basis image L^1 .

Figure 11. Image with aluminum maximized.

Figure 12. Image with plastic maximized.

the calculation was still able to come up with a solution. Again, the two images that maximize the contrast of the two materials are orthogonal and can be used to form a basis set for their vector space.

CONCLUSIONS

We have demonstrated that a two-dimensional vector space of radiographic images can be generated from two radiographs at two different energies of an object consisting of two materials. The term "material" refers to an element, compound, or mixture that has a fixed and defined set of x-ray cross sections over a spatial length less than the image pixel size. Numerical errors result if the two images are not perfectly registered. The numerical procedures become unstable as the x-ray energy difference becomes small. We have not determined how small this must be before it becomes a problem.

The assumption that the x-ray beam quality is constant over the images can only be closely approximated when the object is relatively uniform and the purpose of the inspection is to detect small perturbations (voids, corrosion, or lack of bonding material) in the object. Even with this constraint, we obtained relatively good results when the object consisted of large thickness changes. Extrapolating these results to objects with three or more materials is risky. However, it is probably safe to say that as the number of materials increases, the number of different x-ray images required to define the vector space will increase nonlinearly and the numerical problems will increase proportionately.

REFERENCES

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