FORCE RECONSTRUCTION USING THE SUM OF WEIGHTED ACCELERATIONS TECHNIQUE - MAX-FLAT PROCEDURE

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2001/1/204
OCT 1

ABSTRACT

Force reconstruction is a procedure in which the externally applied force is inferred from measured structural response rather than directly measured. In a recently developed technique, the response acceleration time-histories are multiplied by scalar weights and summed to produce the reconstructed force. This reconstruction is called the Sum of Weighted Accelerations Technique (SWAT). One step in the application of this technique is the calculation of the appropriate scalar weights. In this paper a new method of estimating the weights, using measured frequency response function data, is developed and contrasted with the traditional SWAT method of inverting the mode-shape matrix. The technique uses frequency response function data, but is not based on deconvolution.

INTRODUCTION

Force reconstruction is a process in which response signals from a dynamic event are used to infer what the applied force must have been to produce these responses. Force reconstruction is of interest when the input force cannot be directly measured, while response signals are easily obtained using transducers such as accelerometers or strain gages. In many structural evaluation tests, the dynamic response is not sufficient information; one may really need a description of the input force.

Any structure that is designed to impact barriers or targets may require a description of the impact forces so that interior components can be designed to withstand the shock. These structures may also have impact absorbers designed to reduce the shock on impact. To properly design an impact absorber, a description of the applied force is critical. Such structures include nuclear shipping casks with impact absorbers for accident environments [1], earth penetrators, lay-down weapons which must withstand the impact for delayed firing [2], automotive structures designed for collision impact, torpedos which must fuse and operate after impact, or any structure in which the input force or operational load is an

important design consideration. This would include payload or satellite loads during rocket launches [3].

An application that will be discussed as part of this paper is the impact into a rigid barrier of a weapon system with an energy-absorbing nose. The nose had been designed to absorb the energy of impact and to mitigate the shock to the interior components. To evaluate the crush capability of the nose, impact tests were performed, and the measured force-displacement curve was compared to the design objective. Figure 1 shows the weapon mass-mockup with the energy absorbing nose hung from its launch rail and fixture. The mass-mockup was designed to have the same mass, center-of-mass, and moment-of-inertia as the real weapon. Both axial and diagonal impact tests of the nose were planned so the mock-up needed to possess the correct rigid-body inertia as well as mass. Figure 2 shows the plastic deformation of the nose after impacting a concrete barrier at 141 f/s resulting in peak loads exceeding 500,000 lbs.

Figure 1
Weapon Mass-Mockup with Energy Absorbing Nose
Suspended from Launch Rail

This work was supported by the U. S. Department of Energy under contract No. DE-AC04-94AL85000.

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Techniques for force reconstruction have been developed over a number of years, primarily using the frequency-response-function (FRF) matrix which relates the response quantities directly to the input forces. However, the FRF matrix must be inverted to solve for the forces, and this inversion causes difficulties which limit the applicability of these techniques [4]. There have been many contributors to force reconstruction, and [4] and [5] present excellent reviews of the theory and difficulties involved.

Figure 2
Plastically Deformed Nose after Impact at 141 f/s

In the Sum of Weighted Accelerations Technique (SWAT), a different approach is developed for force reconstruction which significantly reduces the difficulties described in [4]. Instead of inverting a FRF matrix to reconstruct the applied force, the force is computed simply from a weighted sum of time domain acceleration signals. Summing data in the time domain has an inherent numerical insensitivity that makes this technique very appealing. The only difficulty with this technique is calculating the scalar weights for multiplying the acceleration signals.

In the traditional SWAT approach, one needs to know the mode-shapes of the structure with free boundary conditions, and then invert the mode-shape matrix [6]. Depending upon the locations at which the mode-shapes are measured and the accuracy of the measured mode-shapes, this matrix can be easily inverted to produce estimates of the weights. In the new approach described in this paper, frequency response function data rather than mode-shapes are used to calculate the weights. A matrix inversion is still required, but one avoids the requirement to

estimate the mode-shapes and the inaccuracies that may be generated in that process. The theory used to develop this new technique is described in a later section of the paper. This new technique uses FRF data, but is not a deconvolution technique; in that it does not invert a FRF matrix as a function of frequency. This is explained further in the Max-Flat Theory section.

There is a significant difference between SWAT based reconstruction techniques and classical force reconstruction, in that with SWAT, one can only calculate the sums of all externally applied forces and moments about the center-of-mass. The spatial distribution of the force is integrated over the structure. For many applications this is not a drawback because one often knows where the force has been applied, but not the magnitude or frequency content. This loss of spatial information also creates the strength of the SWAT technique, because by not requiring such detailed information, the technique is much more robust and insensitive to measurement noise.

This concept was first proposed by Priddy and others in [7,8,9] and extended in [10,11]. [6] contains a development of the theory for computing the weights from the mode-shape matrix. The SWAT procedure actually develops a modal filter to extract the rigid-body modes of a structure's response using a similar concept to that described by Shelley and others in their applications of modal filters [12,13] for control purposes.

SWAT PROCEDURE

The SWAT procedure, using mode-shapes to determine the scalar weights, is developed in [6] and only the key results will be summarized here. If a set of acceleration measurements, $a_i(t)$, have been made on a structure undergoing an external excitation, the generalized SWAT degree-of-freedom (dof) $s(t)$ is defined as

$$s(t) = \sum_i w_i a_i(t) = \underline{w}^T \underline{a}(t) \quad (1)$$

where w_i are scalar weights. The SWAT dof can be related to the modal coordinates as follows,

$$s(t) = \underline{w}^T [\Psi] \underline{\ddot{p}}(t) \quad (2)$$

where $[\Psi]$ is the matrix of mode-shapes with free boundary conditions and $p(t)$ are the modal coordinates. Now the weights can be chosen such that $s(t)$ is a modal coordinate, simply by requiring

$$\underline{w}^T [\Psi] = \underline{\delta}_i^T \quad (3)$$

where δ_i represents a vector of zeros except for component i which is equal to one. Substituting (3) into (2), one finds that the SWAT dof $s(t)$ is equal to the acceleration of the i th modal coordinate.

The rigid body modal coordinates are related to a sum of the externally applied forces [6] by

$$m_r \ddot{p}_r(t) = \psi_r^T f_r(t) \quad (4)$$

where m_r is the modal mass and f_r is the external force.

If (3) is generalized so that we have a set of weight vectors or a weight matrix $[w]$ then solving for the weight vectors is equivalent to inverting the mode-shape matrix.

$$[w]^T [\Psi] = I \text{ or } [\Psi]^T [w] = I \quad (5)$$

From a force reconstruction viewpoint, the rigid-body modes are of particular interest because they alone are sufficient to define the sum of the external forces with the use of equation (4). For example, if (4) is specialized for the rigid-body translation mode, ψ_1 in the X-direction, the right-hand side of (4) just sums all the applied forces in the X-direction. Then solving for the first weight vector and using these weights for the SWAT dof results in

$$s(t) = \ddot{p}_1(t) = \psi_1^T f(t) / m_1 \quad (6)$$

So with this set of weights, $s(t)$ is the modal coordinate for the first rigid-body mode, and consequently is the sum of the externally applied force in the direction of the first translation mode, divided by the modal mass.

APPLICATION OF SWAT TO WEAPON MASS-MOCKUP

To apply SWAT to a structure, the modes of vibration must be measured or calculated. For this weapon system the crushable nose was removed for a modal test, and only the elastic aft-body was measured for its modes of vibration. The force reconstructed using these mode-shapes will then be the applied force on the aft-body at the interface between the nose and the elastic aft-body.

For the tests of the impact forces on this weapon, it was desired to reconstruct the impact forces with

frequency content up to 2000 Hz. Thirteen modes were measured up to a frequency of 3600 Hz. These included both axial and lateral bending modes. Since the weapon would impact the target in both the axial and lay-down (diagonal impact) configurations, both axial and lateral forces were required. However, since it was anticipated that there would not be any forces applied across the body, bending modes in that direction were not measured. The modes and their modal frequencies are listed below with a brief description of their shapes. The first five modes were fairly uncoupled, but as the frequency increased both axial and bending deformations are coupled in the mode-shapes.

Table 1: Modal Frequencies and Shape Description

Frequency (Hz)	Shape Description
81.8	1st Bending Y
284.5	2nd Bending Y
663.6	3rd Bending Y
708.2	1st Axial
1182.6	4th Bending Y
1688.5	5th Bending Y and Flange
1753.5	5th Bending Y and Flange
1919.5	6th Bending Y, 2nd Axial, & Flange
2162.4	2nd Axial, 6th Bending Y, & Flange
2469.0	Flange
2837.3	7th Bending Y
3296.3	3rd Axial
3579.2	8th Bending Y

For the lay-down impact tests both axial and lateral forces would be applied to the body simultaneously, consequently the two reconstruction algorithms for axial and lateral forces needed to be insensitive to the occurrence of off-axis input force. If we think of the structure as a bi-axial force transducer, the cross-axis sensitivity of this force transducer needs to be small and determined beforehand as part of the "calibration" procedure.

For the reconstruction, fifteen accelerometers were used with six aligned in the axial direction and nine in the lateral direction. Equation (5) of the previous section was used to compute the weight vector for both axial input and lateral input. The thirteen elastic modes were supplemented by three rigid-body modes (axial translation, lateral translation, and rotation in the axial-lateral plane) for a total of sixteen modes with fifteen unknown weights. After experimenting with various combinations, the four modes with the highest modal frequencies were deleted, leaving fifteen unknown weights with twelve constraint equations. This set of equations was solved using the pseudo-inverse to obtain the weights as a minimum norm solution to this underdetermined system of equations.

Laboratory tests were performed to calibrate this "force transducer" and to verify the adequacy of this

technique. Some of the results of this testing will be discussed below. Figure 3 shows a typical time history from one of the accelerometers for a lateral impact from one of the calibration tests. One can clearly see the highly vibratory nature of the acceleration. Using SWAT to sum the acceleration time-histories, the acceleration of the lateral rigid-body modal coordinate is calculated and multiplied by the total mass to obtain the reconstructed force. This force is overlaid on the actual measured force in Figure 4. The reconstructed force is highlighted with small circles. The two time-histories are virtually indistinguishable with the reconstructed force exhibiting a slight ripple after the impact. Impact transient forces were used for the "calibration" as they simulated the planned-use environment. Broad-band random input could have been used as well, and similar results have been observed for other applications [3].

Figure 3
Time-History of an Accelerometer Signal Resulting from a Lateral Impact

To evaluate the cross-axis sensitivity, axial force input was used to excite the structure while applying SWAT with the lateral force reconstruction weights. Figure 5 again shows a comparison of the measured force overlaid with the reconstructed force. The reconstructed force is highlighted with small circles added to the solid line. As desired, the reconstructed force is very small compared with the actual measurement, indicating that the cross-axis sensitivity of this "transducer" is small.

Figure 4
Overlay of the Lateral Reconstructed Force with the Measured Lateral Force

To further evaluate the quality of the reconstruction process, the reconstructed force and the measured force are transformed into the frequency domain. In a frequency domain comparison, difficulties may be revealed more clearly at certain frequencies. Figure 6 shows a frequency domain overlay of the reconstructed and measured forces of Figure 4. The two curves are virtually identical at frequencies below 1000 Hz, with some deviation showing up at the higher frequencies. As expected, the particular frequencies where the deviation is the greatest are the modal frequencies.

Figure 5
Overlay of the Reconstructed Lateral Force with a Measured Axial Force Input

To directly compare the reconstructed force with the measured force in the frequency domain, the Fourier transform of the reconstructed force is divided by that of the measured force. This produces an estimate of the FRF between the SWAT calculated force and the input force. Ideally, the FRF would be flat, equal to 1.0, and purely real for the entire frequency band. In reality, there will be deviations particularly at the higher frequencies beyond the frequency of the modes which were included in the SWAT calculation. Figure 7 shows this FRF for the forces discussed in this section. One can see that the FRF is very nearly flat at the low frequencies, but has some significant deviations at particular frequencies above 1000 Hz.

Figure 6
Overlay of the Reconstructed and Measured Force
Displayed in the Frequency Domain

For comparison purposes, Figure 8 shows an FRF for one of the accelerometers due to this input force. One can see the highly modal behavior of this FRF as compared to the FRF of the SWAT dof. The SWAT FRF is essentially that of a rigid mass responding to the input force. This SWAT FRF provides the motivation for the new technique described in this paper. It would appear, if one could choose the scalar weights to maximize the flatness of the SWAT FRF, then perhaps one could obtain a better determination of the weights and also avoid calculating the mode-shapes of the structure. A technique to compute the weights using this SWAT FRF is described in the next section and has been named the Max-Flat procedure.

Figure 7
FRF of the SWAT Reconstructed Force Due to an
Input Force

MAX-FLAT THEORY

To develop the Max-Flat procedure for calculating the scalar weights used in SWAT, we start with equation (4) which is repeated below.

$$m_r \ddot{p}_r(t) = \psi_r^T f(t) \quad (7)$$

This relates a rigid-body modal coordinate $p_r(t)$ with a sum of the external applied forces, either directly applied or through the boundary conditions.

Figure 8
Typical FRF of an Accelerometer Response Due to
An Input Force

For the special case in which $f(t)$ has only one nonzero component, say component i , then (7) becomes

$$m_r \ddot{p}_r(t) = \psi_{ri} f_i(t) \quad (8)$$

Now $\ddot{p}_r(t)$ and $f_i(t)$ differ by only a constant in the time domain, and consequently their frequency domain representations will also be related by the same constant. That is, the FRF between the

response $\ddot{p}_r$ and the input force f_i is the constant ψ_{ri} / m_r . Or, if we let $H_{ri}(\omega)$ be the FRF between the rigid-body modal coordinate acceleration and the force at component i , then

$$H_{ri}(\omega) = \frac{F\{\ddot{p}_r(t)\}}{F\{f_i(t)\}} = \psi_{ri} / m_r \quad (9)$$

where $F\{\cdot\}$ indicates the Fourier transform operation. Equation (9) can be interpreted as the mass lines of a FRF when testing a free structure. If ψ_r is a translation rigid-body mode and the force component i is parallel to that translation, then the right-hand side of (9) is the reciprocal of the mass of the structure. On the other hand, if the force component i is orthogonal to that translation, ψ_{ri} will be zero, and the FRF between the modal coordinate acceleration and that input force will be zero. Similar interpretations can be made for rotational rigid-body coordinates.

Now recall the definition of the SWAT dof, equation (1).

$$s(t) = w^T a(t) \quad (10)$$

If we let $H_{ji}(\omega)$ represent the FRF between a_j and f_i then the FRF of the SWAT dof $H_{si}(\omega)$ will be

$$H_{si}(\omega) = \sum_j w_j H_{ji}(\omega) \quad (11)$$

Now to determine the requirements on the weights

w_j we require that $s(t) = \ddot{p}_r(t)$ or equivalently we require $H_{si}(\omega) = H_{ri}(\omega)$. Consequently,

$$\sum_j w_j H_{ji}(\omega) = \psi_{ri} / m_r \quad (12)$$

This equation is required for a force at any component i for any particular rigid-body coordinate r . For each rigid-body coordinate, a different weight vector will result. Thus equation (12) represents the constraint equations which determine the weights, and there is one such equation for each value of the frequency, ω . To illustrate this equation, suppose

ψ_r is the rigid-body translation mode-shape in the x-direction of a structure moving in the x-y plane. Thus $\psi_r = (1, 0, 1, 0, 1, 0, \dots)$. For force applied at node 1 in the x-direction (the first component), $\psi_{r1} = 1.0$, then the right-hand side of (12) would be $1/m$ as the modal mass would be the total mass m . On the contrary, if the force is applied at node 1 in the y-direction (the second component), $\psi_{r2} = 0.0$, then the right-hand side of (12) would be zero.

Consequently, depending upon the number of equations desired or the number of FRF's measured for various input forces f_i , the equations represented by (12) can be stacked together to form a highly over-constrained set of equations. These equations hold for each discrete value of ω , so depending on the discrete frequency resolution, this can be a vastly over-constrained system of equations. Further, since the FRF's, H , are complex quantities, there are real and imaginary parts to each equation. The imaginary parts are all equal to zero as the right-hand side of (12) and the weights are purely real. These over-constrained equations can be solved easily using the least squares solution and employing the pseudo-inverse of the matrix of FRF data. In the next section an actual solution of these equations will be illustrated, and the particular data included in the matrix will be discussed.

This method of determining the SWAT weights would be equivalent to inverting the mode-shape matrix to find the weights as in (5), if the modes could be perfectly computed without error and if the FRF data had no noise contamination. As mentioned earlier, this Max-Flat method uses FRF data but is not a deconvolution technique. Ordinarily, in deconvolution, the FRF matrix is inverted at each frequency line with the accompanying difficulties involved in the inversion near the resonant frequencies. The Max-Flat technique stacks up the FRF data into one large matrix with each new frequency representing an equation in the matrix which is used to determine the weights.

APPLICATION OF MAX-FLAT TO WEAPON MASS-MOCKUP

The Max-Flat theory which uses FRF data to determine the scalar weights used in the SWAT force reconstruction was developed in the previous section. To clarify the assembly of the set of equations that are used to define the weights, define a new matrix A, such that

$$A_{kj}^i = H_{ji}(\omega_k) \quad (13)$$

where ω_k is a discrete frequency value, so that a row of A are the values of H at each frequency increment, and the columns of A represent the dof's instrumented with accelerometers. Matrix A is also supplemented with additional blocks of equations, using FRF data for forces at other locations (component i), which are aligned and orthogonal to the desired reconstruction direction. FRF data from orthogonal input force is important to insure that the cross-axis sensitivity is constrained to be small. Assembling the matrix A as described produces the set of equations shown below.

$$\begin{bmatrix} A_{kj}^1 \\ \vdots \\ A_{kj}^2 \\ \vdots \end{bmatrix} w = \frac{1}{m_r} \begin{bmatrix} \Psi_{r1} \\ \Psi_{r1} \\ \Psi_{r2} \\ \vdots \end{bmatrix} \quad (14)$$

Ordinarily, this equation will be highly over-constrained and can be solved in a least squares sense. Recall that each equation has a real and an imaginary part which doubles the total number of equations, with the weights and the right-hand side purely real.

In practice, to obtain best values for the weights, it has been found that not all of the available equations should be used to define the weights, and that some equations at particular frequencies should be emphasized more than others in the least squares solution. First, any location where force input is anticipated should be included in the A matrix. Frequently, if FRF data is not included for a particular input location, then Max-Flat will not produce weights which will reconstruct force input at that location.

Secondly, the imaginary part of the FRF data is used with scaling factors applied to the equations at resonant frequencies. In this example those

equations were scaled by a factors of ten to give them more authority in the least squares process. Recall that the components in right-hand side of (14) will be zero for each equation involving the imaginary part of the FRF data.

Third, it was found, for this application, that the real part of the FRF data did not contribute to the quality of the solution and could be deleted, if three additional constraint equations were added. These constraint equations were the mode-shapes of the rigid-body modes for the axial, lateral, and rotational directions. These three equations, in essence, contain all the information in the FRF real parts that is not already contained in the imaginary parts. So if $\underline{\Psi}_r$ represents a rigid-body mode, we add to the A matrix the equations

$$\underline{\Psi}_r^T w = \Psi_{ri} / m_r \quad (15)$$

for each of the rigid-body modes. The right-hand side of (15) will be one over the modal mass in the direction of force reconstruction, and zero in the other directions. If this was a three dimensional problem, then a total for six rigid-body mode-shape equations would be added. These equations were also scaled by a factor of ten in the least squares solution.

The Max-Flat procedure was applied to the weapon mass-mockup to obtain weights for the reconstruction and evaluated in a calibration procedure in the lab before full field testing. Figures 9 and 10 show a reconstructed force for an axial impact, using the Max-Flat weights. Figure 9 is the overlay in the time domain of the reconstructed force with the measured force; they are virtually identical. Figure 10 shows the computed SWAT FRF as defined in (11). The FRF is fairly flat for frequencies below 1000 Hz, but less so at the higher frequencies, although it appears to be quite adequate as the time domain data in Figure 10 reveals almost a perfect reconstruction.

For the lateral force input, Figures 11 and 12 display the same results as the above figures. The overlay of the force time histories is shown in Figure 11. Again, the reconstruction is very good, although the peak force was missed slightly by the reconstructed force. There is more ripple after the applied force has ended. The SWAT FRF is shown in Figure 12 for the lateral input. Here the FRF is very flat below 1000 Hz, but has significant difficulty maintaining flatness at particular frequencies above 1000 Hz, although again this does not significantly affect the time domain reconstruction. Perhaps if those equations were weighted more heavily in the least squares solution, the flatness of the FRF could be improved.

Figure 9
Overlay of Max-Flat Reconstructed Force with
Measure Force for Axial Input

Figure 10
SWAT FRF Using Max-Flat Weights for Axial Input

Figure 11
Overlay of Max-Flat Reconstructed Force with
Measured Force for Lateral Input

Figure 12
SWAT FRF Using Max-Flat Weights for Lateral Input

CONCLUSIONS

As a force reconstruction technique, SWAT (Sum of Weighted Accelerations Technique) performs very well. It avoids the ill-conditioning problems that have affected other techniques by demanding less information. SWAT produces the sums of all applied forces and moments about the center-of-mass, but does not provide the spacial distribution of the applied force.

SWAT sums acceleration time histories to obtain the acceleration of the rigid-body modal coordinates, which then provides the sum of applied forces with the appropriate modal mass scaling. SWAT does not perform frequency domain filtering, but uses the concept of modal filtering to eliminate response due to the elastic modes.

The conventional SWAT procedure inverts the mode-shape matrix to obtain the scalar weights. The new Max-Flat procedure uses data from the FRF's to assemble a set of over-determined equations to calculate the weights. Max-Flat attempts to create a FRF between the reconstructed force and the measured force which is flat and purely real. Both procedures perform very well under ideal conditions. The Max-Flat procedure does not require a determination of the mode-shapes but works directly with the FRF's, consequently avoiding possible errors resulting from mode-shape estimation. Max-Flat will also be sensitive to modes out of the primary frequency band as they affect the FRF within the frequency band. Lastly, Max-Flat is a more interactive procedure in solving for the weights because the scaling of certain equations can be changed to give them more authority in the least squares solution, and the results can be immediately visualized in the resulting SWAT FRF.

In assembling the set of over-determined equations for Max-Flat, the real parts of the complex equations were not used, but replaced by additional equations representing the rigid-body mode shapes. These added equations were scaled by a factor of ten. All of the imaginary parts of the equations were used in the matrix assembly, but certain equations around the resonant frequencies were also scaled by ten.

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