Shock Compression and Expansion in Central Collisions

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SHOCK COMPRESSION AND EXPANSION IN CENTRAL COLLISIONS

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ABSTRACT

Physics of central symmetric reactions of heavy nuclei, in the beam energy range from few tens of MeV to a couple of GeV per nucleon, is discussed. Within transport simulations, it is shown that shock fronts perpendicular to the beam axis form in the head-on reactions. The fronts propagate into projectile and target and they separate hot compressed matter from normal matter. With an increase of the impact parameter, the angle of inclination of fronts relative to the beam axis decreases, and in-between the fronts a weak tangential discontinuity develops. Hot matter exposed to the vacuum in directions perpendicular to the shock motion (and parallel to fronts), starts to expand sideways, early within reactions. Expansion in the direction of shock motion follows, after the shocks traverse nuclei, but due to the delay does not acquire same strength. Expansion affects angular distributions and mean-energy components, and further shapes of spectra and mean energies of different particles emitted into any one direction, and also particle yields. Both the anisotropy in the expansion and a collective motion associated with the weak discontinuity, affect the magnitude of sideward flow within reaction plane. Differences in mean particle energy components in and out of the reaction plane in semicentral collisions, depend sensitively on the relative magnitude of shock speed in normal matter and speed of sound in hot matter. The missing energy, considered in the past in association with low measured pion-multiplicity in central reactions, may be identified with the energy of collective expansion.

1. Introduction

The goal of this talk is gaining a qualitative understanding of the physics of central heavy-ion reactions. Attention will focus on reactions within the range of beam energies from few tens of MeV to a couple of GeV per nucleon. When discussing general features, the collective behavior of nuclear matter will be addressed in hydrodynamic terms, exploiting e.g. the concept of a shock. The shock is a discontinuity propagating in a medium, in local parameters such as density, temperature, normal component of

velocity, and, importantly, entropy per particle. By collective motion of nuclear matter one generally understands a motion characterized by strong correlations between particle positions and momenta, of a dynamic origin.

Discussion will rely on transport-model reaction-simulations, analytic considerations, and data. Rather than going directly from formulating the assumptions of a microscopic model to comparing of calculated observables with data, we shall stop midway and examine qualitative features of the dynamics in simulations. Then we shall look how such features might be reflected in available data. The outline is as follows. After a brief description of the transport model, head-on reactions are examined first and then the more involved semicentral reactions. Next follow investigations of global and then single-particle observables, discussion of the sensitivity of results to interactions and, finally, conclusions.

2. Transport Model

Simulations are carried out within a transport model with explicit nucleon, deuteron, $A = 3$ cluster, pion, and delta and N^* degrees of freedom^{1,2,3}. In this model, quite typical for the energy regime, the phase-space occupation functions f_X of stable particles satisfy equations of the Boltzmann form

$$\frac{\partial f_X}{\partial t} + \frac{\partial E_X}{\partial \mathbf{p}_X} \cdot \frac{\partial f_X}{\partial \mathbf{r}} - \frac{\partial E_X}{\partial \mathbf{r}} \cdot \frac{\partial f_X}{\partial \mathbf{p}_X} = \mathcal{K}_X^<(1 \mp f_X) - \mathcal{K}_X^>f_X. \quad (1)$$

In (1) E_X is the single-particle energy, and $\mathcal{K}_X^<$ and $\mathcal{K}_X^>$ are, respectively, the production and absorption rates of the particle X . Squares of matrix elements within the rates are parametrized using free cross sections^{1,2}. Pions within the model are produced in two steps. First a Δ or N^* resonance is produced in an NN collision, and then the resonance decays into a nucleon and pion. Clusters are produced in few-body processes that are inverse to cluster break-up. The single-particle energies E_X are parametrized in terms of an optical potential, with a local part handled as a scalar potential, and a weaker nonlocal part added on to the particle energies in the system c.m.,

$$E_X = (p^2 + m^2)^2 + q_X \Phi + A_X U_1^C, \quad m = m_0 + A_X U^C + t_{3X} U^T, \quad (2)$$

where q_X is particle charge, Φ – Coulomb potential, A_X – particle mass number, m_0 – mass in free space, and t_{3X} – isospin component. The dependence of the potentials U on scalar densities is chosen such as in nonrelativistic calculations,

$$U^C = -a \frac{\rho_s}{\rho^0} + b \left(\frac{\rho_s}{\rho^0} \right)^\nu, \quad U_1^C = -d \nabla^2 \left(\frac{\rho_s}{\rho^0} \right), \quad U^T = c \frac{\rho_s^T}{\rho^0}, \quad (3)$$

where

$$\rho_s = \sum_X A_X \rho_{sX}, \quad \rho_s^T = \sum_X t_{3X} \rho_{sX}, \quad \text{and} \quad \rho_{sX} = g_X \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m_{X0}}{E_X} f_X. \quad (4)$$

Fig. 1: Contour plots of baryon density in the reaction plane in Au + Au collisions at 400 MeV/nucleon. The displayed contour lines are for the densities $\rho/\rho_0 = 0.1, 0.5, 1, 1.5$, and 2 .

Unless otherwise specified, reported results are for a parameter set corresponding to a stiff equation of state ($K = 380$ MeV).

3. Head-on Collisions of Heavy Nuclei

The dynamics of the central high-energy reactions can be broken down into several stages. A 400 MeV/nucleon Au + Au system at $b = 0$, for which baryon-density contour-plots are shown in Fig. 1, will serve to illustrate points. Following an initial interpenetration of projectile and target densities, the NN collisions begin to thermalize matter in the overlap region making the momentum distribution there centered at zero momentum in the c.m.s. The density in the overlap region rises above normal and a disk of excited and compressed matter forms at the center of the system. More and more matter dives into the region with compressed matter that begins to expand in transverse directions. At late stages, when whole matter is excited, transverse expansion predominates. A further view at the situation in reaction at $b = 0$ is given in Fig. 2. It is seen that the rise of density from normal along the beam axis is accompanied by a decline in average velocity and by a rise in the entropy per nucleon.

Fig. 2: Baryon density (top panels), radial velocity (center panels), and entropy per baryon along (solid lines) and perpendicular (dashed lines) to the beam axis at different indicated times, in the $b = 0$ collision at a beam energy of 400 MeV/nucleon.

If a nuclear system were very large, then it would be necessarily ruled by laws of hydrodynamics. In the initial state of a head-on reaction, a discontinuity exists in the velocity field at the contact area of two nuclei, with the velocities in the nuclei directed opposite. Within hydrodynamics, such an initial discontinuity would have to break at finite times into two shock fronts travelling in opposite directions into the projectile and target. At low beam energies shock waves would be weak, not differing much from sound waves. At high energies, shock waves would be strong leaving behind high density and temperatures. Possibility of strong-shock formation in head-on reactions was suggested long ago^{4,5,6}. To assess whether interfaces between normal and compressed matter, cf. Figs. 1 and 2, may be interpreted as shock fronts, one can ask whether the state of the matter at the center is such as expected behind a front. To determine the properties of matter behind a front, one positions oneself in the frame of a front and considers the conservation of baryon, momentum, and energy fluxes. Upon eliminating velocities, one obtains a single called Rankine-Hugoniot equation⁷ relating the properties at the two sides of the shock

$$\frac{(e_1 + P_1)^2}{\rho_1^2} - \frac{e_0^2}{\rho_0^2} - P_1 \left(\frac{e_1 + P_1}{\rho_1^2} + \frac{e_0}{\rho_0^2} \right) = 0. \quad (5)$$

The parameters with subscripts 0 and 1 refer to the normal and shocked matter, respectively, and e and P are, respectively, the energy density (including mass) and pressure. The energy per nucleon in shocked matter e_1/ρ_1 should be taken equal to the c.m. energy per nucleon. Figure 3 displays baryon density, temperature, and entropy per baryon from (5), together with a maximum density and entropy per baryon at

Fig. 3: Solid lines show the baryon density (top panel), temperature (center panel), and entropy per baryon (bottom panel) expected behind a developed shock front at $b = 0$, as a function of beam energy, from Eq. (5). Filled circles indicate the maximum density (top panel) and entropy per baryon at maximum density (bottom panel) from the simulations of $b = 0$ Au + Au collisions at different beam energies. Crosses in the top panel indicate the maximum density from the simulations of 1 GeV/nucleon Au + Au collisions at $b = 3, 6, 8, 9, 10, 11$, and 12 fm, plotted against the beam energy scaled down by a factor equal nonrelativistically to $(1 - b^2/4R^2)$, cf. Eq. (11).

a maximum density from simulations. The latter appear to follow the general trend predicted by (5). The shocks are nearly completely developed⁸ in the high-energy simulations of head-on reactions of heavy nuclei.

Let us now turn attention to the expansion. The hot matter in-between the shocks is exposed in the transverse directions to vacuum and, as pressure of hot matter is finite and that of vacuum is zero, the matter begins to expand collectively into transverse directions. The features of this process may be understood at a *qualitative* level in terms of the self-similar^{9,10} cylindrically-symmetric hydrodynamic expansion of an ideal gas. In the self-similar expansion, the velocity is proportional to a distance from the symmetry axis, $v = \mathcal{F}(t)r$, which roughly appears to hold in the direction of 90° in the Au + Au collision at 400 MeV/nucleon, see Fig. 2. Under an assumption of an isentropic flow, the solution to nonrelativistic hydrodynamic equations for the

Fig. 4: Time dependence of the baryon density at $r = 0$ (top panel) and the participant transverse collective energy per nucleon (bottom panel) in the $b = 0$ Au + Au reaction at 400 MeV/nucleon.

density becomes

$$\rho(r, t) = \rho(0, 0) \frac{R^2(0)}{R^2(t)} \left(1 - \frac{r^2}{R^2(t)} \right)^{1/(\gamma-1)}. \quad (6)$$

The radius R of the density distribution satisfies

$$\ddot{R}(t) = \frac{2c_s^2(0, 0)}{\gamma R(0)} \left(\frac{R(0)}{R(t)} \right)^{2\gamma-1}, \quad (7)$$

with c_s - the speed of sound and γ - the heat ratio.

According to (7) the radius should initially increase with a nearly constant acceleration. This is due to the fact that pressure does not change to lowest order with time. At later times, when R is large, the acceleration (7) tends to zero, i.e. R increases linearly with time. This corresponds to a situation where nearly all heat has been converted into energy of collective motion. The coefficient $\mathcal{F}$ in the two regimes should, respectively, rise linearly with t and fall off as an inverse of t and this type of rise and subsequent fall-off is seen for the slope of velocity in the transverse direction dv/dr in Fig. 2. The drop in acceleration of the matter radius should occur after $t \sim R/c_s$, i.e. the time it takes for a signal to propagate to the center of the system. As radius of gold nucleus is $R \simeq 7$ fm and speed of sound for an ideal gas at 400 MeV/nucleon is $c_s \simeq 0.4c$, this characteristic time for the development of expansion is $R/c_s \sim 20$ fm/c. Figure 4 shows, in particular, the evolution of the participant collective transverse energy per nucleon, calculated from the perpendicular components

of a local hydrodynamic velocity $\mathbf{v}$ in the vicinity of last collisions,

$$E_{coll}^{\perp}/A = \sum_X \int d\mathbf{r} d\mathbf{p} f_X m_X (v^{\perp})^2 \bigg/ \sum_X A_X \int d\mathbf{r} d\mathbf{p} f_X. \quad (8)$$

It is seen that the energy rises, indeed, over a period of the order of 20 fm/c.

After shocks reach the vacuum after a time $\sim (2R/v_0)(\rho_1 - \rho_0)/\rho_1 \sim 16$ fm/c, where $v_0 \approx 0.42c$ is initial velocity in the c.m. and ρ_1 is the density in the shocked region, an expansion *along* the beam axis sets in. However, as the expansion in transverse directions is already in progress and decompresses matter, the expansion in longitudinal direction does not acquire same strength.

While definite features of reaction dynamics appear to be consistent with a hydrodynamic behavior of the matter, there are also important differences. For example, in a system continuing to expanding hydrodynamically temperatures would have dropped to zero and *all* kinetic energy would get converted into collective energy of expansion^{9,10}. For that collisions would need to continue down to very low densities. The transverse collective energy in Fig. 4 freezes out at about 40 MeV/nucleon. With about 10 MeV/nucleon of longitudinal collective energy, one finds that for nucleons the net collective energy constitutes about 50% of the kinetic energy in the c.m. for a 400 MeV/nucleon reaction.

One direct effect of the primarily transverse expansion in the $b = 0$ reaction simulations, is a peaking of the distributions of emitted particles at 90° in the polar angle.^{2,3} Aside from distributions, the mean components of kinetic energy

$$E_{kin} = E - m = \sqrt{m^2 + p^2} - m = \frac{(p^{\perp})^2}{m + E} + \frac{(p^{\parallel})^2}{m + E} = E^{\perp} + E^{\parallel}, \quad (9)$$

are affected, with mean transverse energy per degree of freedom exceeding the longitudinal energy, in reflection of the anisotropy in the collective energy components. For example, for protons from the 400 MeV/nucleon $b = 0$ Au + Au reaction the mean transverse and longitudinal energies are $\langle E^{\perp} \rangle \simeq 86$ MeV and $\langle E^{\parallel} \rangle \simeq 28$ MeV, respectively, while for deuterons $\langle E^{\perp} \rangle \simeq 127$ MeV and $\langle E^{\parallel} \rangle \simeq 36$ MeV. As impact parameter increases within simulations, the region with compressed matter becomes inclined at a decreasing angle with respect to the beam axis, see Fig. 1. In consequence, for parameters in excess of 3 fm, with an azimuthal-angle averaging, the polar-angle distributions become peaked at 0° and 180° . No unambiguous evidence for a 90° polar-angle peaking has been as yet seen in very central reactions, but I shall argue that same physics as at $b = 0$ is reflected in proper observables from measurements corresponding to semicentral reactions.

4. Semicentral Reactions

In the initial state, an angle α of inclination of the plane of discontinuity in velocity, relative to the beam axis, is given, in the nonrelativistic approximation, by

$$\cos \alpha \approx \frac{b}{2R}. \quad (10)$$

Fig. 5: The initial discontinuity between the projectile and target velocities breaks at a finite b into two shock fronts propagating into the projectile P and target T (thick solid lines), and a weak tangential discontinuity in-between (dotted line). Shock-front position *within* projectile at a later time is indicated with a dashed line.

While at $b = 0$ velocities are normal to the plane of discontinuity, the velocities have finite tangential components relative to that plane at finite b . Such components are continuous across shocks which detach from the initial plane of discontinuity and, with tangential components being directed opposite for matter from projectile and target, at the center of the system a so-called tangential discontinuity develops, see the sketch in Fig. 5. Density, pressure and entropy are continuous across that discontinuity. Only the tangential velocity component changes.

As only the normal velocity component drops to zero across shocks, the shocks are weaker at a finite b than in a head-on reaction. With the normal component being equal to $v_0 \sin \alpha$, the effective c.m. kinetic energy for a shock at a finite b becomes

$$\frac{E_{lab}}{4} \cdot \sin^2 \alpha = \frac{E_{lab}}{4} (1 - \cos^2 \alpha) = \frac{E_{lab}}{4} \left(1 - \frac{b^2}{4R^2}\right), \quad (11)$$

rather than $E_{lab}/4$ in the nonrelativistic approximation. In consequence the density behind a shock at a finite b should be lower than at $b = 0$ and it should, in fact, coincide with the density in a $b = 0$ reaction at beam energy reduced by a factor $(1 - b^2/4R^2)$. That is tested in the upper panel of Fig. 3. In the 1 GeV/nucleon reaction the density appears to follow the expectation up to very high impact parameters.

While a shock width generally does not change with time, when conditions at the two sides of a shock wave stay the same, the width of a weak discontinuity increases with time. That this is the case, may be seen by adapting the Navier-Stokes equations to the conditions in the vicinity of a weak discontinuity and observing that the equations reduce there to a diffusion equation for a tangential velocity component. The diffusion coefficient is equal to the kinetic viscosity coefficient $\nu = \eta/(m_N \rho)$, where η is viscosity. With an initial condition for the tangential velocity in the ideal-fluid limit in the form $v_t = v_t^0 \epsilon(r)$, the solution to the diffusion equation becomes

Fig. 6: Baryon density ρ (top panel), normal and tangential velocity-components v_n and v_t to the discontinuities in nuclear-matter (center panel), coinciding with radial and polar velocity-components, and entropy per baryon (bottom panel) as a function of the distance from the center of a 400 MeV/nucleon Au + Au system at $b = 7$ fm, along the normal to the discontinuities. At the given time $t = 13.5$ fm/c the discontinuities (at their centers) are inclined at an angle $\alpha \sim 52^\circ$ relative to the beam axis.

$v_t(r, t) = v_t^0 \text{erf}(r/2\sqrt{\nu t})$, where r is distance along a line perpendicular to the discontinuity, i.e. the width increases as $t^{1/2}$. If a nuclear system were very large, then the two shock waves and a weak discontinuity would well separate from each other, as the distance between shocks increases as t . In actual simulations, though, the tails of discontinuities overlap, cf. Fig. 6. In this figure, the center of the shock is at a position $r \sim 3.3$ fm, where density rises to the mean of that in normal matter and at the center, entropy per nucleon rises to a half of the maximum value, and magnitude of the normal velocity component drops to a half of the asymptotic value. The center of the tangential discontinuity in the figure is, on the other hand, at $r = 0$, where the tangential velocity component vanishes. The width is $\delta \sim 1.7$ fm which agrees well with $\delta = \sqrt{\eta t / m_N \rho}$ expected from the simple formula for the space and time behavior of tangential velocity.

5. Global Observables

Just as at $b = 0$, an expansion develops at a finite b in-between the shock waves, in effect of an exposure of the hot matter to vacuum. At $b < 5$ fm, in the directions

Fig. 7: Ratio of the out-of-reaction-plane mean energy-component to the lower in-plane eigenvalue of the kinetic energy tensor (crosses), as a function of the normalized participant proton multiplicity in Au + Au reactions at 400 MeV/nucleon, from measurements of Ref. 11, compared to the ratio of eigenvalues (stars) from a calculated tensor with elements modified on account of fluctuations of the estimated reaction plane about the true plane.

perpendicular to the reaction plane, variation of the baryon density, velocity and entropy with the distance from center, and with time, is difficult to distinguish from variation of these quantities in the directions perpendicular to the beam axis at $b = 0$, cf. Fig. 2. Anisotropy in the expansion, with regard to the direction perpendicular to the reaction plane and the direction of shock motion, results from the delay in the start of expansion in the second of the directions, as at $b = 0$. The anisotropy (squeeze-out) may be quantified with a ratio of the eigenvalue of the kinetic energy tensor associated with the direction out of the reaction plane, and the smaller of the eigenvalues associated with directions within the plane. As $b \rightarrow 0$, this ratio tends to $\langle E^\perp \rangle / 2 \langle E^\parallel \rangle$. The calculated ratio, corrected for fluctuations of the reaction plane, is compared to data¹¹ in Fig. 7, and a qualitative agreement is observed.

With regard to the directions within the reaction plane, both the tangential motion behind the shock waves for finite impact parameters, and anisotropy in expansion, contribute to the sideward deflection quantified usually in terms of the slope of mean momentum within the reaction plane at midrapidity,

$$F = \frac{d\langle p^x/m \rangle}{dy}. \quad (12)$$

Measured and calculated slopes in 400 MeV/nucleon Au + Au reaction are shown in Fig. 8. If the motion associated with the weak discontinuity were not contributing to the sideward deflection, then the slope would have been limited from above³ by

Fig. 8: Left panel shows the flow parameter (12) in Au + Au reactions at 250 and 400 MeV/nucleons, from the measurements of Ref. 12, as a function of the reduced participant proton multiplicity. Right panel shows the flow parameter from the simulations of Au + Au reactions at 400 MeV/nucleon, as a function of impact parameter.

$\chi/(2\sqrt{1+\chi})$ where $\chi = (\langle E^\perp \rangle/2 - \langle E^\parallel \rangle)/\langle E^\parallel \rangle$ and the energy components are from a $b = 0$ reaction. For protons from 400 MeV/nucleon reactions this limit is equal to 0.22, while slopes in Fig. 8 reach twice as high values.

6. Pion and Baryon Single-Particle Observables

An important observable for particles not present in the initial state such as pions, which can be affected by collective motion, is multiplicity. It is well known that in central reactions pion multiplicities fall by nearly a factor of two below expectations from the fireball model assuming a global equilibrium. It has been suggested in the past that some of the energy in reactions is being used up to compress matter and is not available for pion production.¹³

Variation of the sum of pion and delta number in a central 800 MeV/nucleon La + La reaction is shown in Fig. 9. This sum would be equal to the final pion number if interactions stopped at any one time instant. The sum maximizes at the time $t \sim 13.5$ fm/c when shocks reach the vacuum, and the sum decreases as system expands. The decrease contrasts with naive expectations; one would expect the sum to rise continuously tending towards some equilibrium value until the freeze-out. The number of pions and deltas expected under conditions of global equilibrium, given a representative baryon density and total kinetic energy per nucleon of 1133 MeV including mass at $t \sim 13.5$ fm/c, is indicated in the figure by the upper of two

Fig. 9: Time dependence of the number of pions and deltas normalized using the baryon number, in the $b = 0$ La+La reaction at 800 MeV/nucleon, from a standard calculation (solid line) and from a calculation with an interchange of particle positions (short-dashed line). The two horizontal long-dashed lines show the equilibrium number of pions and deltas for a baryon density $\rho = 2.25 \rho_0$ and an energy per baryon of 1133 MeV and 1103 MeV, respectively. Dots show the number obtained by integrating over space the local equilibrium values of pion and delta density in a standard calculation.

horizontal long-dashed lines. The actual number turns down when away from that expectation.

The time variation of the sum of pion and delta numbers becomes understandable once the collective motion is taken into account. At the time when shocks reach the vacuum, the energy of collective expansion reaches already ~ 30 MeV/nucleon. When this energy is subtracted from the total kinetic energy to yield an energy per nucleon of 1103 MeV available locally, then an equilibrium number of pions and deltas is obtained indicated by the lower of two horizontal long-dashed lines in Fig. 9. The actual number of pions and deltas comes close to that value and then turns down because collective energy increases further, less energy becomes available locally, and system reabsorbs deltas.

The effect of collective expansion on pion multiplicity is finally illustrated with Fig. 10 which shows data, results from standard calculations, and results from calculations with correlation between positions and momenta, characterizing collective motion, destroyed during evolution. The extinguishing of collective motion is carried out by interchanging particle positions with no change to particle momenta, when interactions are still frequent. In Fig. 10, it is apparent that the results of standard

Fig. 10: Ratio of the mean pion multiplicity to the number of participant nucleons in central symmetric reactions, as a function of laboratory energy. The solid and dashed lines represent, respectively, the results of standard calculations and of calculations with particle positions interchanged, for the La+La $b = 0$ reactions. The circles represent data of Ref. 13.

calculations, with the collective motion developed, agree reasonably with data. The extinguishing of the motion increases the multiplicity by nearly a factor of two in a dramatic disagreement with the data.

Left panel of Fig. 11 shows momentum distributions of different particles emitted from central La + La reactions at 800 MeV/nucleon, in the vicinity of 90° in the c.m. For a large thermalized system, one would expect exponential distributions with same slope corresponding to the temperature inverse. Simulations, though, yield distributions that are increasingly concave down as particle mass increases. The high energy slopes decrease with the increasing mass, corresponding to apparent temperatures of 70, ~ 110 , ~ 145 , and ~ 175 MeV, for pions, protons, deuterons, and helions, respectively.

The specific features of calculated distributions are primarily associated with the collective motion. This becomes apparent when examining in turn the distributions in right panel of Fig. 11, from calculations with the correlations characteristic for collective motion destroyed. The latter distributions turn out to be nearly exponential with same slopes. The collective energy developed in the La + La reaction within the standard simulation, as calculated from dynamics similarly to (8), is of the order of 75 MeV/nucleon. For radial collective expansion with a single collective velocity, this corresponds to $v \sim 0.4c$. The remaining energy for protons corresponds to a

Fig. 11: Momentum distribution of protons (circles), deuterons (triangles), helions (inverted triangles), negative (squares) and neutral (diamonds) pions from central 800 MeV/nucleon La + La reaction, in the vicinity of 90° in the c.m. Left panel shows the results of calculations at $b = 1$ fm (filled symbols) and data of Ref. 14 (open symbols). Right panel shows the results of calculations with particle positions interchanged during evolution. Solid lines in the left panel indicate results of the instantaneous freeze-out model, for protons and neutral pions. Long- and short-dashed lines for neutral pions indicate a contribution of free pions at freeze-out and a contribution from Δ decays, respectively. Straight parallel lines in the right panel serve to guide the eye.

temperature $T \simeq 55$ MeV. Spectra expected for pions and protons,

$$\frac{d^3 N_p}{dp^3} = \frac{2V}{(2\pi)^3} \exp\left(\frac{\mu_p - \gamma E_p}{T}\right) \sinh\left(\frac{\gamma p v}{T}\right) \bigg/ \left(\frac{\gamma p v}{T}\right), \quad (13)$$

expected under the assumption of an instantaneous freeze-out in the case of radial expansion, for the above parameters, are indicated in the left panel of Fig. 11 with solid lines. These spectra come close to those from simulations. The slopes of the spectra from simulations decrease with particle mass, because the sensitivity to the collective motion increases with mass. For pions sensitivity to collective motion is enhanced due to relativistic effects.

By fitting measured spectra of intermediate mass fragments with formulas of the type (13) or by using an increase of the mean energy with mass, nonrelativistically

$$\langle E_{A+1} \rangle - \langle E_A \rangle \simeq m_N \langle v^2 \rangle / 2 = E_N^{coll}, \quad (14)$$

collective energy values have been recently determined in a number of central systems^{15,16,17}.

The findings are generally close to the values obtained from dynamics in present calculations. Thus within preliminary analysis in Ref. 16, energy values of 18, 31, and 52 MeV/nucleon were deduced in central Au + Au reactions at beam energies of 150, 250, and 400 MeV/nucleon, while simulations yield values of 21, 35, and 55 MeV/nucleon at the respective beam energies.

7. Dependence on Interactions

As is apparent from the discussion of reaction dynamics, the separation into collective and thermal energy occurs at freeze-out. Correspondingly, the magnitude of collective energy is rather insensitive to the nuclear equation of state; in $b = 0$ Au + Au reaction at 400 MeV/nucleon same collective energy is obtained, within 1 MeV/nucleon, for the nuclear compressibility equal to 380 MeV and to 210 MeV. On the other hand, the energy is sensitive to the effective NN cross sections. Change of the cross sections by 30% leads to a 15% change in the collective energy.

Anisotropy in the collective expansion, whether in the central or semicentral collisions, arises from the delay of collective expansion in the direction of shock motion, relative to the expansion in the directions parallel to shock fronts. This anisotropy is thus expected to depend on the ratio of the delay time $t_{sh} \sim (2R - W)/v_0^{sh}$, where W is shock width and v_0^{sh} is shock speed relative to intact matter, to the time characterizing the development of an expansion $t_{exp} \sim R/c_s$, where c_s is speed of sound in hot matter,

$$\frac{t_{sh}}{t_{exp}} = \frac{(2 - W/R) c_s}{v_0^{sh}}. \quad (15)$$

With the speeds in (15) depending on nuclear compressibility, the anisotropy should then be sensitive to the nuclear equation of state. E.g. in the Au + Au semicentral 400 MeV/nucleon reaction simulations, a change of K from 380 MeV to 210 MeV leads to 40% reduction in the anisotropy quantified with a deviation of the ratio in Fig. 7 from 1, confirming this expectation.

8. Conclusions

Within transport simulations of central energetic reactions of heavy nuclei, formation of shock fronts separating the cold from hot nuclear matter is observed. In the semicentral reactions a weak tangential discontinuity develops in-between these fronts. The hot compressed matter exposed to the vacuum in directions parallel to shock fronts begins to expand collectively into these directions. The expansion affects particle angular distributions and mean energy components and further shapes of spectra and mean energies of particles emitted into any one direction. Data which support the scenario include those on squeeze-out, sideward flow, pion yields, and on spectra and mean energies of different particles.

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