FINAL REPORT

COMPARATIVE STUDY OF THE VORTICITY FIELD IN TURBULENT FLOWS:
THEORY, EXPERIMENTS, COMPUTATIONS

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The Goal of the project was to understand the role of topology vortex lines in general and the helicity invariant (inviscid) in particular for turbulent dynamics. The project consisted of three main ingredients: theoretical, numerical and experimental.

The achievements and failures of the above are separately reported below:

1. Theoretical Aspect - Introduction

This was motivated by certain conjectures and findings made independently and simultaneously by E. Levich, A. Tsinober and H.K. Moffatt. In particular it was shown back in 1983 that fluctuations of helicity Invariant, defined by their integral variance (so-called I-invariant) if treated in the approximation of weak coupling (usually alluded to as quasi-Gaussian approximation) lead to the phenomenon of inverse cascade in 3-D turbulence. The mechanism of such 3-D inverse cascade had insignificant similarity to the mechanism of inverse cascade in 2-D turbulence. This result was skeptically meet by the turbulence community, since at least in laboratory experiments on homogeneous quasi-isotropic turbulence such inverse cascade had not been observed. It should be remarked however that in geophysical turbulence the inverse cascade has been always suspected though not loudly voiced. In all frankness for a long time we were in a state of confusion. On one hand our theoretical deduction seemed quite rigorous since the only assumption made was that of quasi-Gaussian

statistics applied to the fourth order two point correlation function routinely used in closure and models of turbulence. On the other hand direct simulation homogeneous isotropic turbulence did not reveal signs of the inverse cascade.

2. Direct Numerical Analysis - Introduction

The direct numerical simulation carried out for the purpose of clarifying our conjectures were plagued at the beginning by severe methodological difficulties. The point is that helicity is a global quantity, which contrary to energy does not have a local meaning. Furthermore helicity variance is a statistical ensemble averaged global quantity. On the other hand is customary to investigate numerical turbulent quantities locally, at a point, or to study statistical correlation functions, but not the global integral quantities. The lack of necessity to consider global statistical quantities is motivated by reference to ergodic hypothesis. This latter states that averaging over a large enough fluid volume is quite equivalent to statistical ensemble averaging.

As a rule the ergodic hypothesis works well in this context. However there are quantities for which this does apply at all. Such is helicity, a pseudo-scalar, taking both positive and negative values. For isotropic homogeneous turbulence the ensemble average would be definition zero.

The volume average, however, would never be zero, except in the limit $V \rightarrow \infty$, in particular for a finite box turbulence considered in numerical simulations, or for any bounded flow the volume average would always differ from the ensemble average. The same in a worse way is true for this helicity variance, the I-invariant.

All the above made it an acutely difficult task to find a numerical proof of our basic theoretical claim. Namely that the global statistical quantity I-invariant affects the local dynamics of turbulence. It is in this context that the controversy was generated with regard to the issue, and opinions were expressed by some that there is no influence of helicity fluctuations (or helicity itself) on the dynamics of turbulence. These opinions were based primarily on a wrong premise. Namely, helicity a global quantity was confused with the helicity density, a local quantity.

The helicity density is not Galilean invariant quantity and therefore cannot have a direct physical meaning. Thus the opinion that the helicity density does not influence turbulence dynamics was certainly true, but had nothing to do with the validity of our theoretical claim. Nevertheless the point is that only in the past phase of our research we understood how to relate the above mentioned global quantity with local properties of turbulence.

3. Theory - Results

The paradox with non-dissipation of I-invariant and subsequent implication with regard to 3-D inverse cascade was resolved in a definitive way as follows. It was suggested that the only approximation used by us (quasi-Gaussian factorization of the fourth order two point correlation function) is not valid at all. That is instead the condition for the rate of dissipation of the I-invariant such that it does not inhibit the normal energy cascade should give us information on the correct statistics of the velocity field derivatives realized in small scale turbulence. In other words given the assumption of scaling the condition for the necessary rate of dissipation of the I-invariant gives us the intermittency exponents for certain fourth order two point velocity correlation functions (including that for the rate of energy dissipation).

Thus we have come to a resolution of the problem in that a requirement of normal rate of dissipation for the I-invariant, and consequently for the existence of normal energy cascade is a well defined intermittency of turbulence. It should be pointed out that empirically the existence of intermittence is well known.

Our achievement, however, is in that we relate intermittency unambiguously with the topological constraints generic for the NS equation and furthermore show that contrary to what has been often suggested intermittency in turbulence is not alien to the Kolmogorov concept of energy cascade but on the contrary it is a concomitant and a necessary condition for the existence of unimpeded energy cascade. Had it not been for intermittency the Kolmogorov turbulence would have not been possible at all.

On a more delicate level we identified the source of intermittency in physical space and tied it up with correlation and well defined coherence of certain helicity related phases in Fourier space. This latter allowed us to resolve the problem of identification of the role of global topological quantities numerically in the framework of one turbulence realization.

4. Numerical Results

On a level of one realization say the Big Box turbulence the theorem translates into a very simple prediction. Namely that independently of initial state (we consider for simplicity a decaying numerical turbulence) with regard to the helicity spectrum $H(k)$ /shell average as is usual in numerical simulations for the late stage of decay and for high enough values of wave numbers it should be $H(K) = \pm A E(k)$, where $E(k)$ is the shell averaged energy spectrum and $A = \text{const}$. Thus surprisingly for a single typical realization of turbulence the reflexional symmetry must be spontaneously violated as a result of natural turbulence dynamics (by this we mean that for all values of k in a certain range $H(k)$ should be of the same sign, either positive or negative, while the absence of correlations would have meant that the sign of $H(k)$ for various k is random!). For the ensemble of

realization, of course it stays $\langle H(k) \rangle = 0$ in isotropic turbulence. However the variance of $I(k)$ becomes much larger for high k by comparison with what this would have been had it not been for the correlations in each realization. The above conclusions were decisively reaffirmed if direct numerical simulations as well as on the basis of analysis of NASA AMES numerical data base (128 x 128 x 128 resolution).

5. Conclusions

1. Topology of vortex lines, and helicity fluctuations in particular, are responsible for intermittency phenomenon in turbulence (as a sufficient condition):
2. Intermittency of turbulence does not contravene the Kolomogorov vision of turbulence but on the contrary is necessary for the normal energy cascade to develop;
3. Intermittency can be related to certain strong phase correlations and coherence between the velocity field harmonics in Fourier space.

6. Implications

The prime implication of the above is the realization of two things:

- a) Dynamical coherence of small scale turbulence as imposed by topological properties of the vorticity field;
- b) Fragility of turbulence in the sense that if the phase coherence is disturbed by extraneous forces (say random impulses at relatively small scales) than the normal energy cascade may be interrupted and subsequently energy would start to accumulate at large scales.

The above implications may have important consequences for the problems related to organization of large coherent vortices (say in meteorological aspect) (A. Tur and E. Levich) and for the problem of turbulence drag reduction (R. Handler, E. Levich and L. Sirovich). These however are out of scope of the research carried out by us in the framework of the DOE Grant.

7. A comprehensive exposition of the above results and conclusions will be published as a part of the paper in Physica A to appear shortly (1991). This however was written after the expiration of the DOE Grant and contains results which have no direct relation our DOE sponsored research.

8. The Experimental Research

This was centered around a formidable technical problem of measuring the vorticity field with sufficient accuracy. The work was interrupted, and although certain encouraging results were obtained no robust confirmation of our theory has been possible. In order to relate the theory and experiment the local accuracy in measuring of vorticity should be sufficient to make conclusions with regard to global, integral quantities. Presently adequate for this task technologies are not known to us.

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