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MODELLING ATMOSPHERIC TURBULENCE EFFECTS ON GROUND-BASED TELESCOPE SYSTEMS

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Abstract. Questions still exist concerning the appropriate model for turbulence-induced phase fluctuations seen in ground-based telescopes. Bester et al. used a particular observable (slope of the Allan variance) with an infrared interferometer in an attempt to distinguish models. We have calculated that observable for Kolmogorov and "random walk" models with a variety of outer scales and altitude-dependent turbulence and wind velocity. We have found that clear distinction between models requires good data on the vertical distribution of wind and turbulence. Furthermore, measurements at time separations of order 60 s are necessary to distinguish the "random walk" model from the Kolmogorov model.

Key words: atmospheric turbulence, outer scale, Kolmogorov spectrum.

1. Introduction

Other lectures in this volume, particularly those of Fried and Roddier, have covered the effects of atmospheric refractive-index fluctuations upon the signal received by an astronomical instrument on the ground. Those lectures emphasized results for a Kolmogorov spectrum of turbulence with a large outer scale, and with relatively simple models of the vertical structure of turbulence strength and wind velocity. The measurements upon which the model is based have typically been done at temporal and spatial scales of relevance to small ground-based telescopes in the visible. An important question remains concerning the sensitivity of the quality of the images from a large-diameter ground-based telescope to the extrapolation of the turbulence model into regions of temporal and spatial separations that have not been measured.

For astronomical purposes, the goal of creating atmospheric models is to make it possible to calculate the behavior of a large-diameter ground-based telescope with an adaptive-optics system; in particular, to calculate that behavior from measurements made with much smaller systems. A measurement of r_0 alone will not allow such an extrapolation. The observable we calculate here is complicated, and will be defined later, but its purpose is to provide a quantity that can be usefully extrapolated to large telescopes. This observable obtains information about larger spatial scales by using long-time correlations and the "frozen wind" (Taylor) hypothesis.

In the present paper, we treat two turbulence spectra: the Kolmogorov model

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and a "random walk" model suggested by Bester et al.(1992) to explain their observations with an infrared interferometer. We also couple both turbulence models with complex vertical structure in turbulence strength and wind velocity.

Bester et al. suggest that measurements taken with the Infrared Stellar Interferometer on Mt. Wilson show behavior which does not correspond to a Kolmogorov spectrum with uniform wind velocity(Bester et al., 1992). They proposed a model based on refractive index fluctuations with physical sizes distributed according to random walk statistics. This model was intended as a counter-example to the usual models. Bester et al. provide references for other experiments which may have shown non-Kolmogorov behavior. The "random walk" model essentially results in a power-law spectrum but with a different power-law index than Kolmogorov.

The power-law index of the spectrum of measured phase variations due to fluctuations in the refractive index of the atmosphere is critical to the designers and users of adaptive optical systems. The system designed to a spectrum which underestimates the relative strength of high frequency phase fluctuations relative to the strength of those at low frequencies may have a larger residual wavefront phase error than expected after compensation. The situation is similar for systems which are designed to compensate at one wavelength, but primarily observe at another, since the predicted compensation error might greatly differ from the actual error.

The outer scale of turbulence describes the scale at which the power-law ceases to be relevant. That is, it is the largest spatial scale at which energy enters the "turbulent cascade". At wavenumbers smaller than the inverse of the outer scale the power spectrum of phase fluctuations should show a plateau, unless other non-turbulent processes induce temperature fluctuations in the atmosphere (and hence refractive-index fluctuations). The value of the outer scale is a much debated item, with most measurements running from a few meters to 100 meters, and sometimes much larger(Bester et al., 1992; Nightengale and Buscher, 1991; Coulman and Vernin, 1991; Colavita et al., 1987). It appears to be variable not only with site, but in time, with widely different values estimated at the same site at different times.

The significance of a small outer scale to adaptive optical systems will depend on just how small it is. Even an outer scale of 20 meters is small compared to the separation of elements in large interferometric arrays such as the VLT and the Keck I plus Keck II system, so that signals from system subelements would be completely decorrelated.

2. Model Descriptions

Let the power spectrum of refractive-index fluctuations in the atmosphere be designated by $\Phi(\kappa)$, where κ is a three-dimensional spatial wavevector. For the Kolmogorov model Φ is given by

$$\Phi(\kappa) = 0.033C_n^2 \left[\left(\frac{1.071}{L_0} \right)^2 + \kappa^2 \right]^{-11/6},$$

where C_n^2 is the strength of fluctuations and L_0 is the outer scale of turbulence. The numerator 1.071 in the outer-scale term was chosen to enable comparison to some

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other published results. In the literature, the value of the numerator is often chosen to be either 1 or 2π , so comparison of outer scale values requires keeping the specific form of the numerator in mind.

For the "random walk" model, we have used a form similar to the Kolmogorov spectrum for the spatial power spectrum

$$\Phi(\kappa) = R \left[\left(\frac{1.071}{L_0} \right)^2 + \kappa^2 \right]^{-3/2}.$$

Here R is a constant that sets the absolute level of the spectrum.

At this point we can calculate the behavior of phase fluctuations for the light from a star to a point on the ground. Most readers will be familiar with derivations of the spectrum or structure function of these phase fluctuations. Instead of these more familiar functions, we will use the Allan variance, a function that is used more often in interferometric systems than in imaging systems.

An interferometer with elements separated by a baseline B measures the phase differences $\delta\phi(B, t) = \phi(B, t) - \phi(0, t)$ between its elements. The Allan variance of phase difference fluctuations is

$$\sigma_A^2(\tau, B) = \left\langle \left[\frac{1}{\tau} \int_{t_1}^{t_1+\tau} \delta\phi(B, t) dt - \frac{1}{\tau} \int_{t_1}^{t_1+\tau} \delta\phi(B, t) dt \right]^2 \right\rangle,$$

where the $\langle \dots \rangle$ represents averaging over all the intervals of length τ in the total measurement time. By assuming the Taylor hypothesis of "frozen flow", we can connect time intervals τ to spatial separations ρ by $\rho = v\tau$. The Allan variance for a Kolmogorov spectrum and a specific choice of baseline B , outer scale, wind profile, and C_n^2 profile, is shown in Figure 1.

The long-time asymptote of the Allan variance in Figure 1 represents atmospheric behavior at large spatial separations: in this case greater than 4 m. Therefore the slope of this asymptote, denoted by β , is a measure of large-scale atmospheric behavior, the behavior that is of most relevance to large-diameter telescopes, or large-baseline interferometers. We will use β as the parameter of interest to be calculated for various atmospheric models. The definition of "long time" is not always so clearly defined as in Figure 1, so we have to define the time at which we measure β . Bester et al. measured β at times of about 10 s (Bester et al., 1992). We present here results for β at 10 s and at 60 s.

In order to calculate the Allan variance, and hence find β , we first express the power spectrum of temporal fluctuations in phase difference $W_{\delta\phi}(f)$ under conditions in which the velocity, the outer scale, and the atmospheric strength of fluctuations could vary. For the Kolmogorov model the approximate answer (we assume diffraction terms are negligible) is given by

$$W_{\delta\phi}(f) = 5.211k^2 \sum_{j=1}^N C_n^2(z_j) \Delta L_j v_j^{-1} \left\{ \frac{\Gamma(\frac{1}{2}) \Gamma(\frac{4}{3})}{\Gamma(\frac{11}{6})} \beta^{-8/3} \right. \\ \left. - 2 \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{11}{6})} \cos\left(\frac{2\pi f B \cos \theta_j}{v_j}\right) \left[\frac{a}{2\beta}\right]^{4/3} K_{4/3}(a\beta) \right\}.$$

Fig. 1. Allan variance of phase difference fluctuations for a pair of telescopes separated by a baseline of 4 meters. The asymptotic form is $\sigma_A^2(t) \sim t^{2/3}$ for small arguments and $\sigma_A^2(t) \sim t^{-1/3}$ for intervals where vt exceeds the baseline. For this example the spectrum is Kolmogorov with $L_0 = 10$ km. We have taken $v = 1$ m/s at all altitudes; therefore the Allan variance is independent of the C_n^2 profile.

where $\beta^2 = \left(\frac{2\pi f}{v_j}\right)^2 + \left(\frac{1.071}{L_{0j}}\right)^2$, $a = B \sin \theta_j$ and $K_{4/3}(a\beta)$ is the modified Bessel function. Note that the medium has been divided into N phase screens each with its own C_n^2 , L_0 , velocity v_j , wind angle relative to the baseline θ_j , and characteristic distance to the next screen ΔL_j . The wavenumber of the radiation is k . A comparable expression can be derived for the "random walk" model.

Given the above power spectrum, we can calculate the structure function and the Allan variance:

$$D_{\delta\phi}(\tau) = \frac{1}{\pi^2} \int_{-\infty}^{+\infty} W_{\delta\phi}(\omega) \sin^2\left(\frac{\omega\tau}{2}\right) d\omega.$$

and

$$\sigma_A^2(\tau) = \frac{1}{\pi^2} \int_{-\infty}^{+\infty} W_{\delta\phi}(\omega) \frac{\sin^4\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)^2} d\omega.$$

We have numerically evaluated $\sigma_A^2(\tau)$, and then found the slope β at $\tau = 10$ s and 60 s. We have used 20 atmospheric layers, allowing us to evaluate complex models based on real measurements of C_n^2 and wind as functions of altitude.

We have used profiles of C_n^2 and v from various experiments. The profiles are shown in Figure 2, where the different experiments are labeled as follows: A set of profiles denoted by MK is from the Mauna Kea site (Olivier et al., 1993). The MKG profiles are the same as the MK profiles except that the ground-level v has been reduced from 3 to 1 m/s. The H-V profiles come from Hufnagel (1974) and Valley (1980). The PdM profiles come from measurements on the Pic du Midi (Hecquet and Klaus, 1989). The Mauna Kea and Pic du Midi profiles were measured at night

Fig. 2. Profiles of C_n^2 and wind speed for five different night-time cases: MK = Mauna Kea (Olivier et al., 1993), H-V = Hufnagel-Valley (Hufnagel, 1974; Valley, 1980), PdM = Pic du Midi (Hecquet and Klaus, 1989).

and the Hufnagel-Valley model was scaled to be representative of good seeing conditions. None of the models is precisely representative of the conditions encountered at Mt. Wilson; however, it is hoped that some verisimilitude is attained.

The values we obtain for β are dependent on the values of the angles, θ_j , between the baseline of the interferometer and the wind velocity, which may be a function of altitude. For the MK, MKG, and H-V profiles we have taken all θ_j to be 45° . For the PdM profiles we have assumed that the interferometer baseline were oriented North-South, and we have taken the measured profile of wind velocity (Hecquet and Klaus, 1989).

3. Results

We summarize our results in Figure 3. The left panel shows the measurements of Bester et al. for β at 10 s. The other two panels show our results for β at two different times, as a function of outer scale L_0 . The model results at 10 s show that both Kolmogorov and "random walk" models give acceptable values, provided that the profiles and outer scale are unknown. Even if the outer scale were known, different profiles create a substantial variation in β at 10 s for both types of spectrum.

On the other hand, at 60 s the values of β are much more definitive measures of the model characteristics. If the outer scale is 15 m, then extrapolating to large

Fig. 3. Allan-variance slope β . (a) Measurements of Bester et al. (1992) with an infrared interferometer with baseline 4 m and time separations of about 10 s; (b) Model results at 10 s as a function of outer scale L_0 ; (c) Model results at 60 s. Different symbols represent different sets of profiles (see Figure 2): In descending order at $L_0 = 100$ m; hexagon = MKG; square = H-V; down-pointing triangle = PdMB; circle = MK; and up-pointing triangle = PdMa.

separations gives a plateau regardless of the power law in the small-scale region. As a result the values of β cluster around -1 for all models. If the outer scale is 1000 m, then the power-law index largely determines β , and the Kolmogorov and "random walk" models are clearly distinguished. At intermediate outer scales the behavior for the two spectra is distinguishable for most profiles. It appears that if measurements of β could be made at 60 s, along with reasonable measurements of profiles, models could be usefully distinguished.

Unfortunately, at longer time intervals the chance of seeing contamination of the frequency spectrum from non-turbulent processes increases. The apparent signature of gravity waves in the atmosphere is seen in some data of Bester et al.

We have studied the differences in the 60-s β from the different profiles and have concluded that the differences can be attributed to differences in wind speed. Specifically, increasing wind speed results in a decrease in β , and vice-versa. In complicated profiles, one must consider the weighted wind speed. For example, if the wind near the ground decreased while the wind at altitude increased, the effect will depend on the relative sizes of C_n^2 at the two altitudes.

4. Conclusions

We have calculated the slope of the Allan variance for the fluctuations in the difference in phase between two elements of an interferometer. We used model atmospheres with realistic values of wind, refractive-index variations, and outer scale. We did so for the standard Kolmogorov model of turbulence and for the "random walk" model suggested by Bester et al. How well one can distinguish between spectral models seems to depend upon being able to measure the Allan variance for time intervals on the order of one minute. We do find that a Kolmogorov model combined with some experimental profiles will produce values of β at time intervals of 10 seconds which are comparable with those measured by Bester et al. Measurements at 60 s would distinguish models more clearly, and would place the effective spatial scales being measured closer to those of interest in large-diameter telescopes and interferometers. These measurements would be made significantly more useful for adaptive-optics-system design, test, and evaluation if more simultaneous measurements of relevant atmospheric quantities were made.

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