Hydrodynamic Instabilities and Coherent Structures^c

Progress Report

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During the project year under consideration the following progress has been made:

1. Stimulated by a reprint describing experimental results of J. Gollub and his associates on films flowing down inclined planes (see the attached preprint by Liu et al.), we (with Dr. K. Indireskumar, a postdoctoral researcher who has been supported by this grant) derived a 3-dimensional dissipative-dispersive evolution equation for the film thickness:

$$\eta_t + \eta\eta_z + \eta_{zz} - \kappa\eta_{yy} + \nabla^4\eta + \lambda\nabla^2\eta_z = 0, \quad (1)$$

where $\nabla^2 = (\partial_z)^2 + (\partial_y)^2$.

The rescaled variables and coefficients here are as follows: Let $U := gh_0^2 \sin\phi / (2\nu)$ (where ϕ is the angle of inclination, h_0 is the average film thickness, and other notations are standard), $W := \sigma / (2\mu U)$ (where σ is the surface tension), $\tilde{W} := 2W/3$; $c := (2/3)\cot\phi$, $\delta := 8R/15 - c$. Then $\kappa = c/\delta$ and $\lambda = 2/\sqrt{\tilde{W}\delta}$

The new units used in the above equation in terms of the original (based on h_0 and U) units are as follows: $L = \sqrt{\tilde{W}}/\sqrt{\delta}$ for lengths (i.g. for the spanwise coordinate y and the streamwise one, z), $T = \tilde{W}/\delta^2$ for time, and $N = \tilde{W}/(4L^3)$ for the interface deviation η .

The equation is valid if $L \ll 1$ and $R/L \ll 1$, where R is the Reynolds number based on h_0 and U .

A (pseudospectral) computer code has been created to solve the above equation with periodic boundary conditions. The spatial intervals were chosen to be sufficiently extended both in the streamwise and spanwise directions, so that the results would be insensitive to the specific choice of the boundary conditions. We simulated the above evolution equation with the values of parameters and initial conditions corresponding to the laboratory experiments (in particular, $\lambda \ll 1$, i.e. dispersion was not important in those experiments). Preliminary results showed some

stages of evolution which were similar to those observed in the experiments:

- (i) Herringbone patterns emerging via a period doubling due to the resonant nonlinear interaction of the fundamental-frequency (streamwise) mode of forcing (at the flow entrance) with the (initially negligible) oblique streamwise-subharmonic modes, at higher frequencies (see Figs. 1-8 attached);
- (ii) Synchronous (i.e. with no period doubling) transverse modulations of initially almost two-dimensional waves, at intermediate frequencies of forcing (see Figs. 9-12); and
- (iii) Solitary waves at small frequencies (i.e., at small-wavenumber initial conditions in our simulations; see Figs. 13-15).

This broad agreement with the experiments is somewhat unexpected since the validity conditions of our theory--which are obtained simultaneously with the derivation of the evolution equation in our multiparameter perturbation approach, as we have described before on more than one occasion--require that the modified Reynolds number be much less than 1, whereas the experimental value is just of order of magnitude 1, but not less than 1. This observation shows that our equation, which is of a modified Kuramoto-Sivashinsky (KS) type, does qualitatively capture the important features of those nonlinear interaction phenomena. The reason for this success may be the fact that the deviation amplitudes observed in experiments were significantly smaller than the average thickness of the film, and the small-amplitude long waves are well approximated by the KS-type equations.

We have communicated these results (in the form of contour plots, modal energy evolution plots, and 3-D surface plots, some of which are attached to this report) to Dr. J. Gollub; subsequently, he informed us that he found them interesting and stimulating for his experimental work.

Thus, we have made a significant progress in the direction of one of the broad objectives of the project (see II-4 of the *Summary of Proposed Research in our research proposal of 1993*).

2. In accordance with another objective of the project (see II-3 of the *Summary*), we have derived a dissipative-dispersive evolution equation for 3-D waves on films flowing down vertical cylinders,

$$\eta_t + \eta\eta_z + \eta_{zz} + \alpha^{-2}\eta_{\theta\theta} + \nabla^4\eta + \varepsilon\nabla^2\eta_z = 0. \quad (2)$$

Here, $\nabla^2 := (\partial_z)^2 + (\partial_\theta)^2/\alpha$, $\alpha := 1 + 8b^2/5\sigma$ (where b is the dimensionless radius of the cylinder and σ the dimensionless surface tension), $\varepsilon^2 := (3b/W)^2/\alpha$ [where $W := \sigma/(2\mu U)$; $U := gh_0^2/(2\nu)$], and the new units, in terms of the (h_0, U) -based units, are: $L^2 = b^2/\alpha$; $T = b^4/(\tilde{W}\alpha^2)$; and $N = \tilde{W}/(4L^3)$, where $\tilde{W} := 2W/3$. In addition, we find the validity conditions for this equation:

$$L \gg 1 \text{ and } \tilde{W}/L^3 \ll 1.$$

We have also characterized the parametric conditions under which the dispersion term (the last term on the left-hand side, which was missing in the earlier--published by Shlang and Sivashinsky in 1982--equation for this flow) is important. The final states of the waves, the flows on the system attractor, are different for the strongly dispersive cases, $\varepsilon > 1$: in the computer simulations (with Drs. V. Kerchman and K. Indireskumar), we see *ordered patterns of 3-D pulses* (see Figs. 17-18)--instead of the *chaotic waves* (Fig. 16) which we see for the purely dissipative cases (i.e. those described by the original Shlang-Sivashinsky equation, which is our equation at $\varepsilon = 0$). The same holds for the limiting case of the infinite-radius cylinder, $1/\alpha = 0$, that is the vertical planar wall; an equation corresponding to this particular case of our equation has been simulated earlier by

Toh, Isikawa, and Kawahara (Phys. Rev. A, 1989), and we have registered an agreement of the corresponding subset of our observations with their results.

We have also established that, for the inclined films of section 1 above, the ordering effect of the dispersive (λ -) term (on the final wavy states) is similar to that for the cylinder case. The vertical plane wall is also a limiting case of the inclined plane (with the inclination angle becoming the right one, i.e. $\kappa=0$), and our results correctly reflect this limit. These results were reported in June of 1994 at the 11th Canadian Symposium on Fluid Dynamics (Edmonton, Canada). Also, more recently, we have had preliminary observations of the transient subharmonic herringbone patterns as well as synchronous transverse modulations of waves--first observed on inclined films--for the flows on cylinders with an appropriate periodic forcing. This theoretical prediction calls for an experimental verification. (As of today, we do not know of any experiments reported in the literature with which the theory can be compared.) We hope that these results of our research will stimulate someone to perform such experiments.

3. As has been proposed (see III-1 of the Summary), we obtained--and simulated numerically--highly nonlinear evolution equations describing large-amplitude waves in core-annular flows. The general large-amplitude equation for the vertical CAFF with imposed longitudinal pressure gradients was obtained via first deriving a large-radius longwave evolution equation for vertical CAFFs, similar to the case of one-fluid flow down a cylinder that we published before (A. Frenkel, Phys. Fluids A, October 1993). We have also derived the same large-amplitude equation in a different way: namely, directly from the Navier-Stokes problem, by using our multiparameter approach, which, in addition, yields the parametric conditions of the theory validity. From this general large-wave equation for vertical CAFFs, two interesting

particular cases were obtained: (i) The case of a single fluid falling down a cylinder is obtained in a limit as the core density approaches zero and the driving pressure gradient vanishes; (ii) a horizontal core-annular flow driven by a pressure gradient is obtained in the limit of matching densities. All these equations differ from each other in their advective terms only, and their numerical simulations show similar sequences of bifurcations. In particular, the phenomenon of coalescence of pulses for certain windows of the control parameter--that we first found for the case of a falling annular film to quantitatively explain (A. Frenkel, *Europhys. Letters* 1992; A. Frenkel, *On Evolution Equations for Thin Films Flowing down Solid Surfaces*, *Phys. Fluids A* 5 (October 1993) pp. 2342-7; V. Kerchman and A. Frenkel, "Interactions of Coherent Structures in a Film Flow: Simulations of a Highly Nonlinear Evolution Equation", *Theoret. and Computational Fluid Dynamics* 6 (1994) pp. 235-54) the experiments of Quere (*Europhys. Letters* 1990)--was suggested to also (qualitatively) explain the rupture of the core fluid in the experiments of Aul (PhD. thesis, Cornell, 1989) with core-annular flows in capillaries. This work by A.L. Frenkel and V.I. Kerchman, "On Large-Amplitude Waves in Core-Annular Flows", was published in the *Proceedings of the 14th IMACS World Congress, Georgia Inst. of Technology, Atlanta, GA, July 1994* (ed. W.F. Ames), vol. 2, pp. 697-700 (a reprint is attached). In our invited talk at that meeting, we also communicated our results on the dissipative-dispersive equations for small-amplitude waves, see sections 1 and 2 above (these latter results did not appear in the Proceedings paper; they will be published elsewhere).

4. We (with Dr. G. Rudenko) have completed the numerical proof of negativity of the isotropic eddy viscosity for a sinusoidally oscillating triangular-eddy flow (as in I-1 of the *Summary of Proposed Research* in our research proposal of 1993), which was

suggested--but not quite proved--in our asymptotic theory (with G. Sivashinsky, Phys. Fluids A, 1992).

Recently, such a proof (with somewhat different methods) was also obtained in S. Gama, M. Vergassola and U. Frisch in their paper "Negative Eddy Viscosity in Isotropically Forced Two-Dimensional Flows: Linear and Nonlinear Dynamics", J. Fluid Mech., 1994. They reported that their numerical results showed only a slight difference from the asymptotic results of the Sivashinsky and Frenkel paper. However, in our computer-aided studies (with Dr. Rudenko), we also found a result that Gama et al. were unable to obtain with their methods: the very first destabilization of the flow occurs on intermediate, rather than long, lengthscales. In more detail, our work is described as follows:

We consider a forced viscous 2-dimensional flow which consists of a periodic array of equilateral triangular eddies whose velocities oscillate sinusoidally in time (at each point of space). The problem of linear stability with respect to disturbances of arbitrary--not necessarily large--lengthscales reduces to an eigenvalue problem--for an infinite system of ordinary differential equations with periodic coefficients. Its finite-order Galerkin truncations are integrated numerically. The large-time solutions yield the growth rates of the least-stable Bloch-type modes as functions of the parameters--the Reynolds number R , the basic frequency W , and the Bloch wavevector components μ and ν . The results converge rapidly as the truncation order is increased. They prove that the isotropic eddy viscosity, which characterizes stability for large scales (i.e. small μ and ν), is negative for certain domains of W and R --as was suggested by asymptotic results for this problem recently published by Sivashinsky and Frenkel (Phys. Fluids A, 1992). Thus, the conjecture of some previous authors that the eddy viscosity is necessarily positive insofar as it is isotropic (which was suggested by studies of a number of steady flows, including the $W=0$ case of the present triangular flow) is

refuted. However, we find that there are certain modes with finite μ and ν which are *more unstable* than the longwave modes. At certain values of the basic parameters W and R , the regions of instability in the plane of μ and ν form two small islands: a long-wave island and an intermediate-wave one. We point out the consequent possibility of an unusual theory for the weakly nonlinear evolution of small (but finite) disturbances. Certain conclusions for the linear-stability properties are obtained by using group-theoretical considerations based on the spatial symmetries of the basic flow--similarly to theories of crystals. Those theoretical conclusions are correlated with the numerical results. This work is being prepared for publication.

5. We (with my graduate student, Xiaojing Zhang) have developed an unusual generalization of Landau's equation to describe the nonlinear evolution of certain (steady) periodic flows slightly above the onset of the (intermediate-wavelength) instability. Spatially periodic (forced) viscous flows were studied before mainly for their longwave instabilities. However, for some 2-D flows, the critical normal mode (of the linearized theory) has an *intermediate*, rather than long, lengthscale (see section 4 above; the same occurs even for some steady flows, see e.g. M. Takaoka, J. Phys. Soc. Japan, 1989). For the Reynolds number slightly above its critical value, the instability may be expected to saturate. The final magnitude of a disturbance should be determined by a *Landau-type* theory. Landau's equation is well known for the case of *uniform* basic flows. However, our flows (e.g., the equilateral triangular eddy flow) are *periodic*, and the critical normal mode consists of (infinitely) many discrete Fourier components. We derive an *infinite system* of coupled ODE determining the evolution of all those Fourier coefficients. The coefficients of this (Landau-type, cubic) ODE system are expressed in terms of the basic flow. By solving finite truncations of the system, the structure of the saturated

disturbances are obtained for a number of concrete examples of periodic flows.

This work will be presented at the Annual Meeting of the Division of Fluid Dynamics of the American Physical Society (Atlanta, GA, November 1994), and the complete paper will be published elsewhere.

Plans for the next year

(a) We will continue the numerical studies of the 3-dimensional waves described by our dissipative-dispersive equations (II-3 in the Summary of our proposal of 1993).

(b) We will work on the theory of secondary instabilities of travelling wave solutions of those equations, as in the section II-2 of the Summary.

(c) In accordance with sections II-4 and III-5 of the proposal Summary, we (in collaboration with prof. A. Oron of Technion, Haifa, Israel) will study the evolution of a sheared and nonisothermal CAFF-type flow in a horizontal pipe.

(d) We are going to complete the work (II-5 of the Summary) of obtaining the extension of our large-radius evolution equation for falling annular films.

(e) We (with Dr. A. Oron of Technion) will conduct computer assisted studies of the Benney evolution equation for falling planar films, along the lines of II-1 of the proposal Summary.

(f) We will work on the general Kolmogorov flow--periodic in both space and time, but not necessarily in the sinusoidal way (see I-3 of the Summary).

Alex Frenkel

Alex Frenkel, Principal Investigator

Figure Captions

Fig. 1. Film surface at the beginning of the evolution of an inclined planar film (described by eq. 1) further reflected in Figures 2-8: $\eta(T=0)=1.2\cos 15z/q+0.02\cos 7.5z/q\cos y/p$ +small amplitude white noise (SAWN).

Fig. 2. Contour plot corresponding to Fig. 1.

Fig. 3. Evolution of "energy" contained in modes with belonging to a same azimuthal wavenumber n .

Fig. 4. Interaction of the fundamental mode ($n=15$, $m=0$) and its oblique subharmonic.

Fig. 5. Herringbone pattern which results (at a transient time) from the mode interaction of Fig. 4 (compare with Fig. 4(a) of Liu et al. 1994).

Fig. 6. Surface plot corresponding to Fig. 5.

Fig. 7. Chaotic surface at a sufficiently large time.

Fig. 8. Contour plot corresponding to Fig. 7.

Fig. 9. Initial state of a run (to which also Figures 10-12 belong) corresponding to forcing at an intermediate frequency: $\eta(T=0) = 1.2\cos 9z/q + 0.05\cos[18(z-z_1)/q] + 0.05\cos[27(z-z_2)/q] + 0.05\{\cos[9(z-z_3)/q]+\cos[18(z-z_4)/q]+\cos[27(z-z_5)/q]\}\cos y/p$, where the phases z_j are selected randomly with a uniform probability distribution on $[-\pi, \pi]$.

Fig. 10. Plot of the modal energy.

Fig. 11. Synchronous transverse modulation of waves at transient stage of the film evolution (compare to Fig. 1(a) of Liu et al. 1994)

Fig. 12. Surface plot corresponding to fig. 11.

Fig. 13. Initial state (with a small wavenumber) for the run which leads to Figs. 14 and 15: $\eta(T=0)=1.2\cos 2z/q$ +SAWN.

Fig. 14. Solitary waves (a transient stage of evolution).

Fig. 15. Cross-section of the surface of Fig. 14 at an arbitrarily chosen value of y (compare to Fig. 2(b) of Liu et al. 1994).

Fig. 16. Chaotic waves on falling annular films at large times, which is typical of purely dissipative cases (simulations of eq. 2 with a small value of ε).

Fig. 17. Final pulses ordered by the action of the dispersive terms of eq. (2) (with a large value of ε).

Fig. 18. Contour plot corresponding to Fig. 17; the order in the arrangement of pulses is clearly evident.

$\kappa=0.56$ $\lambda=0.057$ $T=0.0$

FIG. 1

$\kappa=0.56$ $\lambda=0.057$ $T=0.0$

FIG. 2

$$\kappa=0.56 \quad \lambda=0.057$$

$$\eta(y,z)=\sum_{m,n} a_{mn} \exp(i n y / p + i m z / q)$$

$$p=8$$

$$q=16$$

$$\int \eta^2 = \sum_{m,n} |a_{mn}|^2$$

FIG. 3

$\kappa=0.56$ $\lambda=0.057$

$p=8$ $q=16$

FIG. 4

$\kappa=0.56$ $\lambda=0.057$ $T=1.5$

FIG. 5

FIG. 6

$\kappa=0.56$ $\lambda=0.057$ $T=9.0$

FIG. 7

$\kappa=0.56$ $\lambda=0.057$ $T=9.0$

FIG. 8

$\kappa=0.18$ $\lambda=0.05$ $T=0.0$

FIG. 9

$$\kappa=0.18 \quad \lambda=0.05$$

$$\eta(y,z)=\sum_{m,n} a_{mn} \exp(i n y/p + i m z/q)$$

$$p=8$$

$$q=16$$

$$\int \eta^2 = \sum_{m,n} |a_{mn}|^2$$

—	$n = 0$
⋯	$n = 1$
- -	$n = 2$

FIG. 10

$\kappa=0.18$ $\lambda=0.05$ $T=9.0$

FIG. 11

$\kappa=0.18$ $\lambda=0.05$ $T=9.0$

FIG. 12

$\kappa=0.56$ $\lambda=0.057$ $T=0$

FIG. 13

$\kappa=0.56$ $\lambda=0.057$ $T=1.2$

FIG. 14

$\kappa=0.56$ $\lambda=0.057$ $T=1.2$

FIG. 15

FIG. 16

FIG. 17

$\alpha=2.5$ $\epsilon=10.0$

FIG. 18