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LONG-WAVELENGTH MICROINSTABILITIES IN TOROIDAL PLASMAS

BY

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ABSTRACT

Realistic kinetic toroidal eigenmode calculations have been carried out to support a proper assessment of the influence of long-wavelength microturbulence on transport in tokamak plasmas. In order to efficiently evaluate large-scale kinetic behavior extending over many rational surfaces, significant improvements have been made to a toroidal finite element code used to analyze the fully two-dimensional (r, θ) mode structures of trapped-ion and toroidal ion temperature gradient (ITG) instabilities. It is found that even at very long wavelengths, these eigenmodes exhibit a strong ballooning character with the associated radial structure relatively insensitive to ion Landau damping at the rational surfaces. In contrast to the long-accepted picture that the radial extent of trapped-ion instabilities is characterized by the ion-gyroradius-scale associated with strong localization between adjacent rational surfaces, present results demonstrate that under realistic conditions, the actual scale is governed by the large-scale variations in the equilibrium gradients. Applications to recent measurements of fluctuation properties in TFTR L-mode plasmas indicate that the theoretical trends appear consistent with spectral characteristics as well as rough heuristic estimates of the transport level. Benchmarking calculations in support of the development of a three-dimensional toroidal gyrokinetic code indicate reasonable agreement with respect to both the properties of the eigenfunctions and the magnitude of the eigenvalues during the linear phase of the simulations of toroidal ITG instabilities.

I. INTRODUCTION

Low frequency drift-type microinstabilities driven by ion temperature gradient and/or trapped-particle dynamics have continued to be strongly supported candidates accounting for the anomalous transport observed in tokamak plasmas.¹ In assessing the proposition that these instabilities are dominant in the core (“confinement”) region, a key question concerns the role of the long wavelength modes (*i.e.*, large spatial-scale fluctuations extending over many ion gyroradii). This is a very important and relevant issue in that microwave scattering measurements have generally indicated that the largest fluctuation amplitudes occur at the longest measurable wavelengths.² The recent implementation of new diagnostics (beam-emission spectroscopy³ and microwave reflectometry⁴) capable of measuring much longer wavelength fluctuations in major devices such as TFTR has greatly heightened interest in the development of appropriate theoretical models of such phenomena. Moreover, empirical results from nondimensional transport scaling studies⁵ indicate that Bohm-like trends characteristic of long wavelength modes are more in evidence than the gyro-Bohm scaling associated with shorter wavelength microinstabilities.

Motivated by the considerations noted above, kinetic toroidal eigenmode calculations have been carried out to support the development of realistic nonlinear models capable of determining the influence of long wavelength microturbulence on tokamak transport. Fully two-dimensional (2-D) (r, θ) mode structures for trapped-ion and toroidal ion temperature gradient (ITG) instabilities have been calculated using an improved version of a toroidal finite-element code with the unique capability for evaluating large-scale kinetic behavior extending over many rational surfaces.⁶ Important new information obtainable from these studies which can be useful to the formulation of realistic nonlinear theories include a proper description of the actual radial extent of long-wavelength toroidal microinstabilities as well as the identification of their spectral range of instability. In particular, it is found that even at very long wavelengths, toroidal microinstabilities generally exhibit a strong ballooning

character with the associated radial structure relatively insensitive to ion Landau damping at the rational surfaces. The radial extent of the modes is primarily governed by the spatial variations in the equilibrium gradients. These features are in marked contrast with the long-accepted picture predicted by one-dimensional radial eigenmode calculations⁷ and supported by earlier idealized (very weak magnetic shear) 2-D calculations⁶ that trapped-ion eigenmodes are strongly localized between rational surfaces. In the present studies it is also demonstrated that except at the very longest wavelengths, a proper one-dimensional (1-D) ballooning representation analysis⁸ can capture most of the essential features of these instabilities. This is quite significant in that employed together, our 2-D and 1-D codes can now provide a unique capability for systematically calculating the linear properties of electrostatic toroidal microinstabilities over the entire spectral range of interest. Additional motivation for the present eigenmode studies is provided by the need to produce self-consistent benchmarks for three-dimensional (3-D) toroidal gyrokinetic⁹ and gyrofluid¹⁰ codes presently being developed to investigate the nonlinear properties of electrostatic microinstabilities. Comparisons with results from the linear phase of recent toroidal gyrokinetic particle code simulations of ITG modes⁹ indicate reasonable agreement with respect to both the eigenvalues and the characteristic features of the eigenfunctions.

The remainder of this paper is organized as follows. In Sec. II the main features of the two-dimensional eigenmode calculations are presented along with results illustrating the radial characteristics of trapped-ion instabilities. The 1-D ballooning representation analysis, which can provide the proper extension of the 2-D calculations into the moderate wavelength regime, is also described here. Applications of these studies to realistic TFTR tokamak conditions and to the benchmarking of recent toroidal gyrokinetic simulation results are presented in Sec. III. Finally, in Sec. IV the principal findings are summarized, and the implications of the results along with some possible future studies are discussed.

II. ANALYSIS

Long-wavelength electrostatic microinstabilities are generally characterized by perpendicular wavelengths much larger than the ion gyroradius ($k_{\perp}\rho_i \ll 1$) and frequencies falling below or on the order of the average transit and bounce frequencies ($\omega \lesssim \omega_{ti}, \omega_{bi}$) of the ions. In toroidal systems this is called the trapped-ion regime, and the associated spatial-gradient-driven modes can be destabilized by a variety of mechanisms including non-resonant and resonant unfavorable precession drift dynamics and trapped-electron dissipative effects.^{11–13} A key factor in assessing their possible impact on transport involves the realistic determination of the actual radial extent of these instabilities. Because of the inherent difficulty of evaluating large-scale kinetic behavior extending over multiple rational surfaces, earlier calculations focused on the one-dimensional radial model with the crucial assumption that the perturbed potential is essentially constant along the field lines; i.e., no poloidal variation in the mode structure. The most comprehensive of these studies were those of Gladd and Ross⁷ whose results indicated that eigenmodes in the trapped-ion regime are localized between rational surfaces by strong ion Landau damping. Subsequently, Marchand, *et al.*⁶ carried out the first fully two-dimensional (poloidal and radial) eigenmode analysis of these instabilities and concluded that in low shear plasmas (*i.e.*, well-separated rational surfaces), the full 2-D results are in qualitative agreement with those from the 1-D radial studies. Guided by these considerations, most of the models constructed to analyze the nonlinear dynamics of long-wavelength perturbations¹⁴ have invoked the approximation that they maximize halfway between adjacent rational surfaces; *i.e.*, the radial extent of the mode is governed by the spacing between rational surfaces, $\Delta \simeq 1/nq'$, with n being the toroidal mode number and q' (the shear parameter) being the radial derivative of the safety factor. In contrast to this long-accepted picture, results from the present studies will demonstrate that under more realistic conditions, the radial extent of the mode is primarily governed by the spatial variations in the equilibrium gradients.

Standard procedures were followed in obtaining the eigenmode equation governing long-wavelength electrostatic perturbations in a toroidal system.⁶ After generating the perturbed density responses including all resonant contributions, the basic integro-differential equation is produced from the quasineutrality condition. The integral equation character comes from the poloidal structure associated with the trapped-particle dynamics, and the differential equation nature is due to the assumption that $k_r \rho_{bi} < 1$ is satisfied with k_r and ρ_{bi} being respectively the characteristic radial wave number and ion banana width. Specifically, the perturbed electrostatic potential is expressed in the form:

$$\Phi(r, \theta, \zeta, t) = \sum_m \Phi_m(r) \exp[i(n\zeta - m\theta - \omega t)] \quad (1)$$

with the radial excursions of only the trapped ions taken into account in solving for $\Phi_m(r)$. As shown in detail in Ref. 6, the final set of coupled ordinary differential equations is obtained by operating with $\int d\theta \exp(ip\theta)/2\pi$ to produce:

$$\sum_m \left\{ A_{pm}(r) \frac{\partial^2}{\partial r^2} \Phi_m + B_{pm}(r) \frac{\partial}{\partial r} \Phi_m + C_{pm}(r) \Phi_m(r) \right\} = 0, \quad (2)$$

with the matrix elements given by:

$$\begin{aligned} A_{pm}(r) = & \frac{T_e}{T_i} \frac{\epsilon^{3/2} \rho_{\theta i}^2}{2\pi^{9/2}} \int_{1-\epsilon}^{1+\epsilon} \frac{d\Lambda}{L_b^3} \int_0^\infty dY Y^{3/2} e^{-Y} \\ & \times \sum_n \frac{\omega - \omega_{*Ti}}{\omega - \omega_{Di}^{(0)} - n\omega_{bi} + i\nu_i} \sum_{n_1, n_2 \text{ odd}} I_{n_1} I_{n_2} \\ & \times [J_{n+n_1+n_2, p} J_{n, m} - 2J_{n+n_1, p} J_{n-n_2, m} + J_{n, p} J_{n-n_1-n_2, m}], \end{aligned} \quad (3)$$

$$\begin{aligned} B_{pm}(r) = & \frac{T_e}{T_i} \frac{\epsilon \rho_{\theta i}}{\pi^{7/2}} \int_{1-\epsilon}^{1+\epsilon} \frac{d\Lambda}{L_b^2} \int_0^\infty dY Y e^{-Y} \\ & \times \sum_n \frac{\omega - \omega_{*Ti}}{\omega - \omega_{Di}^{(0)} - n\omega_{bi} + i\nu_i} \\ & \times \sum_{n_1 \text{ odd}} I_{n_1} [J_{n, p} J_{n-n_1, m} - J_{n+n_1, p} J_{n, m}], \end{aligned} \quad (4)$$

and

$$\begin{aligned}
C_{pm}(r) = & \frac{\sqrt{\epsilon}}{\pi^{5/2}} \int_{1-\epsilon}^{1+\epsilon} \frac{d\Lambda}{L_b} \int_0^\infty dY \sqrt{Y} e^{-Y} \frac{\omega - \omega_{*Te}}{\omega - \omega_{De}^{(0)} + i\nu_e} J_{0,p} J_{0,m} \\
& + \frac{T_e}{T_i} \frac{\sqrt{\epsilon}}{\pi^{5/2}} \int_{1-\epsilon}^{1+\epsilon} \frac{d\Lambda}{L_b} \int_0^\infty dY \sqrt{Y} e^{-Y} \sum_n \\
& \quad \times \frac{\omega - \omega_{*Ti}}{\omega - \omega_{Di}^{(0)} - n\omega_{bi} + i\nu_i} J_{n,p} J_{n,m} \\
& + \frac{T_e}{T_i} \frac{\sqrt{\epsilon}}{\pi^{5/2}} \int_0^{1-\epsilon} \frac{d\Lambda}{L_t} \int_0^\infty dY \sqrt{Y} e^{-Y} \sum_n \\
& \quad \times \frac{(\omega - \omega_{*Ti})(\omega + i\nu_i)}{(\omega + i\nu_i)^2 - [(n + S^{(m)})\omega_{ti}]^2} K_{n,p,m} K_{n,m,m} \\
& - \left(1 + \frac{T_e}{T_i}\right) \delta_{pm}. \tag{5}
\end{aligned}$$

The associated definitions used here include: $S^{(m)}(r) \equiv nq(r) - m$, $v_t \equiv \sqrt{2T/m}$, $\rho_{\theta j} \equiv v_t cT/eB\theta$, $I_n \equiv \int_0^{\theta_0} d\theta \cos[n\omega_b \hat{t}]$, $J_{nm} \equiv \int_0^{\theta_0} d\theta [1 - \Lambda/h(\theta)]^{-1/2} \{\cos^2(n\pi/2) \cos(S^{(m)}\theta) \cos(n\omega_b \hat{t}) + \sin^2(n\pi/2) \sin(S^{(m)}\theta) \sin(n\omega_b \hat{t})\}$, $L_b \equiv (\sqrt{\epsilon}/\pi) \int_0^{\theta_0} d\theta [1 - \Lambda/h(\theta)]^{-1/2}$, $L_t \equiv (\sqrt{\epsilon}/\pi) \int_0^\pi d\theta [1 - \Lambda/h(\theta)]^{-1/2}$, $\omega_b \equiv \sqrt{\epsilon Y} v_t / 2R_0 q L_b$, $\omega_t \equiv \sqrt{\epsilon Y} v_t / R_0 q L_t$, $\theta_0 \equiv \arccos[(\Lambda - 1)/\epsilon]$, $Y \equiv E/T$, $h(\theta) \equiv 1 + \epsilon \cos(\theta)$, $\epsilon \equiv r/R_0$, $K_{n,p,m} \equiv \int_0^\pi d\theta [1 - \Lambda/h(\theta)]^{-1/2} \cos[S^{(p)}\theta - (n + S^{(m)})\omega_t \hat{t}]$, $\hat{t}(\theta, \sigma_{||}) \equiv (Rq/\sqrt{Y} v_t) \int_0^\theta d\theta' [1 - \Lambda/h(\theta')]^{-1/2}$ for $\sigma_{||} = +1$ and $\hat{t}(\theta, \sigma_{||}) \equiv \tau_b/2 - \hat{t}(\theta, +1)$ for $\sigma_{||} = -1$, for trapped particles, and $\hat{t}(\theta, \sigma_{||}) \equiv (Rq/\sqrt{Y} v_t) \sigma_{||} \int_0^\theta d\theta' [1 - \Lambda/h(\theta')]^{-1/2}$ for circulating particles.

With regard to the physics represented in Eq. (2), the first term involving the matrix $A_{pm}(r)$ accounts for the usual trapped-ion banana radial excursion effects with the diagonal elements ($p = m$) corresponding to the 1-D radial limit and off-diagonal ($p \neq m$) elements due to toroidal coupling. The second term involving the matrix $B_{pm}(r)$ represents additional contributions to the trapped-ion radial excursions from toroidal coupling alone and, as is evident from Eq. (4), vanishes in the 1-D radial limit ($p = m$). If the first two terms are neglected, then Eq. (2) reduces to the radially local eigenmode equation. The associated matrix $C_{pm}(r)$ is defined in Eq. (5) and accounts respectively for the responses of

the trapped electrons, the trapped ions, and the circulating ions. Note that the trapped-electron term includes resonant and non-resonant contributions from the precession drifts as well as collisional dissipation, and that the trapped-ion term includes all bounce and drift resonances. The circulating ion term of course contains the important Landau damping dynamics prominent in the vicinity of the rational surfaces.

The fully 2-D toroidal finite element code presently used to solve the set of coupled ordinary differential equations specified in Eq. (2) is basically an improved version of the original code developed by Marchand, *et al.*⁶ More efficient numerical methods were necessary in order to tractably carry out the studies for realistic tokamak parameters. The key modification involved implementing a more efficient algorithm for calculating the eigenfrequency (ω). After representing the unknown functions $\Phi_m(r)$ with a series of cubic B-spline finite elements and taking into account the boundary conditions, $\Phi_m(x_L) = \Phi_m(x_R) = 0$, Eq. (2) can be cast in the form of a set of algebraic equations given by

$$\sum_{m,k} M_{pk'}^{mk}(\omega) \gamma_{mk} = \lambda_j(\omega) \gamma_{pk'} = 0 \quad (6)$$

where $x \equiv r/a$ is the normalized radial coordinate, $x_L \geq 0$, and $x_R \leq 1$. The indices k and m label respectively each radial finite element and poloidal harmonic, and $\lambda_j(\omega)$ represent the individual eigenvalues of the M -matrix with the range of j determined by the product of the number of radial finite elements times the number of poloidal harmonics utilized. In principle, solving the determinant equation, $\det(M) \equiv D(\omega) = 0$, will yield the eigenfrequency. However, this root-finding procedure, which was employed in the original code,⁶ proves to be rather inefficient in practice because $D(\omega)$ is usually not a smooth function of ω and can often contain contributions from spurious roots. To remedy this difficulty, first note that $\det(M)$ can be expressed as $D(\omega) = \prod_j \lambda_j(\omega)$, so that $D(\omega) = 0$ if any individual eigenvalue of the M -matrix vanishes. Hence, the eigenfrequency corresponding to the particular eigenfunction of interest can be readily obtained by solving $\lambda_j(\omega) = 0$ for the j corresponding to that eigenfunction. This has indeed proven to be far more effective

numerically because λ_j has generally been found to be a far smoother function of ω than D and of course contains no contributions from spurious roots.

In the earlier work,⁶ the primary focus was placed on an appropriate formulation of the fully 2-D kinetic eigenmode equation in toroidal geometry together with the systematic benchmarking of the solutions in both the 1-D radially local as well as the 1-D radial limits. Although it was reported that for weak magnetic shear cases, the 2-D results were in qualitative agreement with the 1-D radial predictions, it was nevertheless pointed out for the first time that for stronger shear the radial structure seemed to be determined more by the equilibrium variations than by the position of the rational surfaces. Unfortunately, further studies were not carried out to ascertain the robustness and relevance of these trends or to determine the characteristic spectral range of long-wavelength modes. Basically, the combination of the use of relatively inefficient root-finding algorithms and the limited computing resources available at that time hampered the previous applications of the 2-D code to realistic tokamak conditions. As discussed in Sec. I, renewed interest in the long-wavelength structures has stimulated reexamination of this problem.

Armed with a much improved root-finding algorithm and, of course, with far superior computational resources, the present studies have generated significant new results together with a more realistic interpretation of some of the important trends noted previously. In particular, evidence from these calculations now indicate that for relevant tokamak parameters, the radial extent of long-wavelength toroidal eigenmodes is never really governed by the spacing between rational surfaces – even in the limit of weak magnetic shear. These points are illustrated in the following figures. First, Fig. 1 displays a plot of a representative 2-D trapped-ion eigenmode structure ($n = 5$) computed for generic TFTR-like parameters ($R = 248$ cm, $a = 85$ cm, $B = 5$ T, $n(0) = 6 \times 10^{13}$ cm⁻³, $T_e(0) = T_i(0) = 5$ keV) and the same model density, temperature, and q -profiles studied in Ref. 6. Here the parabolic-type q -profile chosen with $q(r = 0) = 1$ and $q(r = a) = 3.2$ accounts for the multiple rational surfaces. Note that the magnitude of the eigenfunction peaks around $\theta = 0$ (B-field minimum)

and exhibits no apparent sensitivity in the radial direction to the presence of the rational surfaces. This is more clearly displayed in Fig. 2 which shows the radial structure of the eigenfunction at $\theta = 0$ and also at $\theta = \pi$ (B-field maximum). Since ballooning (poloidal localization) effects are minimal at $\theta = \pi$, the sensitivity to the rational surfaces, as expected, is evident here. Of course, the amplitude of the eigenfunction is clearly much smaller than the dominant component localized near the B-field minimum. To illustrate that this trend holds true even for weak magnetic shear, Fig. 3 depicts the $n = 5$ mode structure for an artificially weak shear case (only two rational surfaces) with all other parameters the same as for Figs. 1 and 2. Although the eigenfunction is indeed localized between the rational surfaces, this is strictly the result of equilibrium variations and not the Landau damping dynamics. Specifically, comparison of Figs. 2 and 3 shows that the radial structure in the large amplitude region is generally unchanged. Hence, the ballooning character of toroidal eigenmodes will usually ensure that their actual radial extent is never significantly influenced by the presence of rational surfaces. Moreover, the associated growth rates exhibit no significant sensitivity to increased magnetic shear.

The results in Figs. 1 - 3 together with those from additional applied studies which will be described in Sec. III serve to illustrate the general trend that even at very long wavelengths (*i.e.*, $k_{\theta}\rho_i \ll 1$, $n < 10$), the mode structure of trapped-ion-type microinstabilities is dominantly influenced by ballooning features and not by the radial fine-structure effects associated with ion Landau damping dynamics near the rational surfaces. Physically, the poloidal localization is generated by magnetic trapping as well as by unfavorable curvature. The resultant coupling of the m -harmonics produces extended radial structures which are relatively insensitive to magnetic shear damping and are limited only by equilibrium variations. As illustrated in Fig. 4, the poloidal harmonics, which contribute to the magnitude of the radial structure, can *individually* extend over multiple rational surfaces. Hence, it is natural to expect that a one-dimensional ballooning representation analysis¹⁵ might capture the essential characteristics of these eigenmodes. Calculations were accordingly carried out

using a comprehensive kinetic ballooning code⁸ normally used to evaluate moderate to high toroidal mode number ($n > 10$) instabilities. The ballooning representation basically makes use of the fact that for large toroidal mode numbers, the distance between rational surfaces, $1/nq'(r)$, is much smaller than the radial scale of the equilibrium variations. A transformation is made from the usual periodic range, $0 \leq \theta \leq 2\pi$, to an infinite domain; *i.e.*, instead of Eq. (1), the perturbations are now expressed in the form

$$\Phi(r, \theta, \zeta, t) = \exp[-i\omega t + in\zeta - inq(r)(\theta - \theta_0)] \sum_{p=-\infty}^{\infty} \hat{\phi}(\theta - 2\pi p, r) \exp[inq(r)2\pi p], \quad (7)$$

with θ_0 being a free parameter to lowest order in the $1/n$ expansion. For the present problem, this leads to a one-dimensional (radially local) integral eigenmode equation in the extended θ -coordinate with the usual periodicity constraint replaced by the boundary condition that the solution be well behaved at infinity. Expanding the radially local eigenvalue around the growth rate maximum (at $r = r_0$ and $\theta_0 = \theta_{00}$) yields (to first order in $1/n$) a Weber-type equation with a solution corresponding to a "global" radial structure of the form $\Phi \propto \exp[-\alpha(r - r_0)^2/2]$ where $\alpha = nq'[\partial^2\omega^{(0)}/\partial r_0^2]^{1/2}[\partial^2\omega^{(0)}/\partial\theta_0^2]^{-1/2}$. The associated eigenfrequency is specified by

$$\omega = \omega^{(0)}(r_0, \theta_{00}) + \frac{1}{2nq'} \left(\frac{\partial^2\omega^{(0)}}{\partial r_0^2} \frac{\partial^2\omega^{(0)}}{\partial\theta_0^2} \right)^{1/2}, \quad (8)$$

with $\omega^{(0)}$ being the eigenvalue solution to the 1-D (leading order) problem. As will be illustrated in more detail in Sec. III, results from our fully 2-D calculations indicate that the toroidal eigenfunctions of interest in the present studies tend to maximize radially in the region where the spatial gradients driving the modes are strongest and exhibit a radial extent determined by such equilibrium-scale variations. Hence, this class of modes exhibit characteristics amenable to the ballooning formalism just described. As pointed out in recent studies,^{16,17} other types of toroidal eigenmodes, which are not so well radially localized, require a different type of analysis. Nevertheless, the more localized class of global modes

studied in the present paper still represent large-scale kinetic perturbations extending over many rational surfaces.

The specific form of the 1-D integral eigenmode equation solved by the comprehensive kinetic ballooning code used in our investigations is described in detail in Ref. 8. It basically accounts for all the physical processes recognized as being important for low-frequency toroidal microinstabilities. These include finite gyroradius effects, finite banana orbital dynamics, collisions (modelled with a proper number conserving operator), and all of the relevant wave-particle resonances (transit, bounce, and precession-drift). Results from these calculations when compared to those from the 2-D code indicate that the 1-D ballooning analysis provides reasonable estimates for the eigenvalues well into the long-wavelength ($n < 10$) regime. Linear spectral properties for more realistic TFTR parameters have been obtained and will be discussed in Sec. III. Taken together, the 2-D and 1-D codes can now be used to calculate the linear properties of electrostatic toroidal microinstabilities over any spectral range of interest.

III. APPLICATIONS

The eigenmode analysis described in Sec. II has been applied to more realistic conditions characteristic of L-mode plasmas in the TFTR tokamak ($R = 245$ cm, $a = 80$ cm, $B = 4.8$ T, $n(0) = 4.6 \times 10^{13}$ cm $^{-3}$, $T_e(0) = 4.1$ keV, $T_i(0) = 3.3$ keV, and actual density, temperature, and q -profiles from shot number 49982). Representative results are displayed in Fig. 5 where the growth rate and the real frequency are plotted against the toroidal mode number. One-dimensional kinetic ballooning code results are represented by circles while the fully 2-D results are denoted by squares. Since the first order ($1/n$) corrections to the 1-D eigenvalues for the present cases are found to be small (less than 10%), only the lowest order results in $1/n$ are shown here. Good agreement between the 1-D and 2-D eigenvalues (within less than 25%) is found for $n = 4$ through $n = 10$. This supports the conclusion that the 1-D analysis provides reasonable estimates of the linear eigenvalues down to very long

wavelengths. For $n \leq 3$, these drift-type instabilities (ω scaling roughly with n) become weaker and the ballooning character no longer dominates. Hence, the fully 2-D calculations become necessary here. However, at the higher n -values, where the 2-D computation of eigenvalues become increasingly expensive, the present results demonstrate that the 1-D code can be applied with some confidence. This is computationally analogous to earlier MHD studies by Dewar, *et al*¹⁸ who showed that 1-D ballooning codes could produce results consistent with those from the 2-D PEST-code at moderate values of n .

With regard to the specific properties of the radial structure characterizing long-wavelength toroidal microinstabilities, it is useful to examine the radial wavenumber averaged over the "global" radial extent of the eigenfunction; *i.e.*,

$$\langle |k_r| \rangle \equiv \frac{\sum_m \int_{x_L}^{x_R} dx |\Phi_m^* (d\Phi_m/dr)|}{\sum_m \int_{x_L}^{x_R} dx |\Phi_m|^2} \quad (9)$$

This quantity along with the corresponding average poloidal wavenumber $\langle |k_\theta| \rangle$ are plotted as a function of n in Fig. 6 for the TFTR L-mode case of Fig. 5. Note that in the long wavelength regime ($n \leq 10$), results from the 2-D computations indicate that $\langle |k_r| \rangle$ remains nearly constant while $\langle |k_\theta| \rangle$ varies linearly with n . Hence, the familiar premise that the radial wavenumber characterizing ballooning modes will vary proportional to the poloidal wavenumber is not justified in the long-wavelength regime. In addition, it should be pointed out that the full width at half maximum of the eigenfunctions also remains nearly constant (*e.g.*, about 6.4 cm for the present case). This is found to be true both for the radial structure generated by the $1/n$ (first order) ballooning analysis as well as by the actual 2-D calculations.

Salient characteristic features of the eigenmode structures from Fig. 5 are highlighted by contour plots of the $n = 2$, $n = 5$, and $n = 10$ modes in Fig. 7. Depicted are: (i) the increased prominence of ballooning as n increases; (ii) an illustration of the "twisted eddies" nature¹⁹ of these extended radial structures; (iii) the transition from the $k_\theta < k_r$ regime to the $k_r < k_\theta$ regime with increasing n ; and (iv) the relative insensitivity of the radial extent

of the mode to changes in n . The extended radial structures calculated in the present studies appear to be significantly affected only by spatial variations in the equilibrium temperature and density gradients driving the instabilities. To illustrate this point, Fig. 8 depicts spatial variations in the ion temperature gradient parameter, $a/L_{Ti} \equiv -(a/T_i)(dT_i/dr)$, along with plots of the magnitude of the $n = 5$ trapped-ion eigenfunction. The curves labeled with (i) in Figs. 8(a) and 8(b) represent respectively the actual L_{Ti} -variations for the TFTR L-mode case of Fig. 5 and the corresponding eigenfunction for $n = 5$. Curves (ii) and (iii) show artificial truncations of L_{Ti} (*i.e.*, all temperature and density gradients are set to zero outside of the indicated radial region) and the associated influence on the radial extent of the eigenfunction. Hence, the picture here is that the global radial eigenmodes of interest tend to be radially localized around the maximum of a/L_{Ti} (with L_{Ti} being the dominant equilibrium gradient driving instability in the present case).

In comparing theoretical trends with experimental observations, it is interesting to note that results from recent beam-emission spectroscopy (BES) measurements of long wavelength fluctuation properties in TFTR plasmas³ have indicated a peaking of the spectrum near $k_\theta \rho_i = 0.2$. This was found to occur in L-mode discharges with conditions comparable to those used in computing the spectral response shown on Fig. 5. The fact that the fastest growing modes here were calculated to occur for $k_\theta \rho_i$ between 0.2 and 0.5 with structures extending across many rational surfaces is consistent with trends observed in these measurements. In addition, as shown on Fig. 9, using the familiar prescription, $D \simeq \gamma/\langle k_r^2 \rangle$, leads to peak values of the diffusivity falling between 1 and 5 m²/s in the long wavelength regime. These heuristic estimates of D also appear to be consistent with the experimental results. Specifically, it was reported^{2,3} that if the radial correlation lengths and times measured in the confinement zone were used in a random walk estimate of the diffusivity, the BES results compared reasonably well with those from corresponding power balance (PB) analysis (*i.e.*, $D_{\text{BES}} = 5$ to 8 while $D_{\text{PB}} = 1.4$ to 8). With respect to possible future tests of theoretical trends, it would be very interesting to examine whether the ballooning feature prominent in

the linear eigenmode structures persists to any degree in the measured nonlinear structures. This could be most efficiently investigated using microwave reflectometry⁴ to measure the fluctuation properties at the B-field maximum ($\theta = \pi$) and compare to those observed at the B-field minimum ($\theta = 0$).

As indicated by the preceding discussion, applications of the present studies to realistic tokamak scenarios have led to some interesting comparisons with experimentally observed trends. Although the conclusions support the picture that long-wavelength toroidal microinstabilities are potentially important in the confinement zone, it should be remembered that realistic nonlinear models capable of determining their actual influence on tokamak transport still need to be developed. In order to gain better insight into the nonlinear behavior of unstable electrostatic fluctuations in long mean-free-path plasmas, a fully toroidal 3-D gyrokinetic particle code⁹ has recently been developed to study ion temperature gradient (ITG) driven instabilities. Since self-consistent benchmarks for code-validation purposes are required, another useful application of our toroidal eigenmode calculations has involved comparisons with results from the linear phase of these gyrokinetic simulations. By suppressing the perturbed density response of the non-adiabatic electrons (*i.e.*, adiabatic electron limit), the eigenmode equations (for both the 2-D and 1-D ballooning calculations) reduce to forms appropriate for analyzing toroidal ITG modes.²⁰

During the linear stage of the simulations, the dominant unstable modes exhibit ballooning-type structures qualitatively quite similar to those computed for the trapped-ion eigenmodes and displayed in Figs. 2 and 7. For these toroidal ITG modes, the ballooning in poloidal angle is due to the localization of the unfavorable curvature around the B-field minimum. As in the trapped-ion eigenmode cases discussed earlier, these modes also exhibit a global-type radial localization around the ion temperature gradient maximum with little sensitivity to ion Landau damping at the rational surfaces. Presently available computational capability limits the size of the toroidal plasmas simulated to about 100 ion gyroradii (*i.e.*, roughly 10 cm in radial extent). Eigenmode calculations were carried out using the

actual parameters from the simulations and the results compared with trends obtained from the gyrokinetic code. Reasonable agreement is found not only with respect to the characteristic features just noted for the eigenfunctions but also for the magnitude of the eigenvalues (within 25%). Benchmarking studies of this type will continue to be carried out in support of further improvements to toroidal gyrokinetic codes which would involve, for example, the implementation of important effects associated with trapped-electron dynamics and collisional dissipation.

IV. SUMMARY AND DISCUSSION

An improved version of a toroidal finite-element code⁶ with the unique capability for evaluating large-scale kinetic behavior extending over many rational surfaces has been used to study the fully 2-D (r, θ) mode structures for long wavelength trapped-ion and toroidal ion-temperature-gradient instabilities. Since these eigenmodes are found to exhibit a strong ballooning character even for very low toroidal mode numbers, complementary 1-D ballooning representation calculations have been carried out to efficiently extend the analysis of the relevant spectra to short wavelengths (*i.e.*, large toroidal mode numbers). The principal findings from these studies and some possible implications are briefly summarized in the following discussion: (i) In contrast to the long-accepted picture that long-wavelength trapped-ion-type instabilities are radially localized between adjacent rational surfaces, the present results demonstrate that under realistic conditions, the radial extent of these instabilities is primarily governed by the spatial variations in the equilibrium gradients. If used with transport estimates based on mixing length arguments, this conclusion tends to support a Bohm rather than a gyro-Bohm-type scaling since the spacing between rational surfaces is typically on the order of a few ion gyroradii while the equilibrium variations are generally characterized by the plasma scale. (ii) Employed together, our 2-D and 1-D codes can now properly evaluate the microstability properties over the entire relevant spectral range. This capability can be used to support nonlinear investigations by realistically identifying the

damped (stable) versus driven (unstable) regions in the spectrum of interest. (iii) Comparisons with recent results from BES measurements of long-wavelength fluctuation properties in TFTR L-mode plasmas indicate that the theoretical trends appear to be consistent with both the spectral characteristics (peaking near $k_\theta \rho_i = 0.2$) and heuristic estimates of the transport level (D between 1 and 8 m²/s). This should encourage further applied studies of this type as well as possibly motivate the use of diagnostics such as microwave reflectometry to investigate the ballooning features predicted for these toroidal microinstabilities. (iv) Comparisons with results from the linear phase of recent 3-D toroidal gyrokinetic simulations of ITG modes have yielded reasonable agreement with respect to the magnitude of the eigenvalues (within 25%) as well as with the characteristic features of the eigenfunctions. This has served to illustrate how systematic eigenmode studies can effectively complement the development of nonlinear codes by providing valuable self-consistent benchmarks in the linear regime. It is also interesting to note that nonlinear results⁹ from the gyrokinetic simulations studied here have indicated that significant remnants of the ballooning character as well as the extended radial feature observed in the linear phase can persist.

With regard to ongoing and future studies of long-wavelength toroidal microinstabilities, one obvious area of interest concerns their sensitivity to sheared toroidal flow effects. For example, radially sheared toroidal flows generated by neutral-beam heating might well influence the extended radial structures emphasized in the current investigations. In order to properly address issues of this type, the gyrokinetic equation including sheared equilibrium flows has been derived and applied to the present problem. Details of the analysis will be presented in a future publication. Before leaving this topic, it is appropriate to comment that the strong stabilizing influence of “poloidal” velocity shear on ITG modes in sheared slab calculations^{21,22} may not readily extend to the toroidal case. As illustrated by the ion Landau damping issue, the importance of dynamics strongly localized at an individual rational surface can be readily mitigated by the strong ballooning effects associated with toroidal microinstabilities. In addition, the heuristic argument that stability naturally ensues when

the frequency shift produced by velocity shear exceeds the magnitude of the eigenfrequency can also be quite misleading. Even for sheared slab cases, computed results from Ref. 22 clearly show that when this heuristic criterion is met, the growth rate can remain unchanged while the eigenfunction simply adjusts its shape to accomodate the velocity shear modification. Finally, recent analysis²³ of single-particle ion orbit properties indicates that within the context of the simplest local theory, toroidal plasma rotation can increase both the fraction of trapped particles and their precession drift velocity and thereby tend to further destabilize the trapped-ion modes.

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FIG. 1. Fully 2-D (r, θ) trapped-ion eigenmode structure ($n=5$) illustrates large scale kinetic behavior extending over many rational surfaces for generic TFTR-like parameters (i.e., $R = 248$ cm, $a = 85$ cm, $B = 5$ T, $n(0) = 6 \times 10^{13} \text{ cm}^{-3}$, $T_e(0) = T_i(0) = 5$ keV, and parabolic-type q -profile with $q(0) = 1$ and $q(a) = 3.2$).

FIG. 2. Comparison of the small amplitude radial structure at $\theta = \pi$ (no ballooning) with the large amplitude radial structure at $\theta = 0$ (maximum ballooning) for the $n=5$ trapped-ion eigenmode.

FIG. 3. Artificially weak shear limit (only two rational surfaces) calculated for the $n = 5$ trapped-ion eigenmode of Figs. 1 and 2.

FIG. 4. Contributions of the individual poloidal (m) harmonics to the magnitude of the radial structure of the $n = 5$ trapped-ion eigenmode of Fig. 1.

FIG. 5. Representative linear spectrum (growth rate (a) and real frequency (b) vs. $k_\theta \rho_i \propto$ toroidal mode number) for realistic TFTR L-mode discharge parameters

FIG. 6. Average radial wavenumber $\langle k_r \rangle$ as a function of toroidal mode number n .

FIG. 7. Contour plots of the $n=2$, 5, and 10 eigenmode structures from Fig. 5.

FIG. 8. Example of the dependence of the radial extent of the 2-D eigenfunction at $\theta = 0$ (b) on the spatial variation of one of the equilibrium gradients driving instability (a) (All radial temperature and density gradients are set to zero outside the indicated radial extents i, ii, and iii.)

FIG. 9. Heuristic estimate of the diffusivity, $D = \gamma / \langle k_r^2 \rangle$, for the representative L-mode spectrum of Fig. 5.

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