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Tau Physics — A Theoretical Perspective*

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Tau physics 1994 — A theoretical perspective*

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1. Introduction

The origin of electroweak symmetry breaking and observed pattern of particle masses is an outstanding puzzle. To illustrate the problem, I have listed in Table 1 the spectrum of standard model elementary particles and their masses (or mass bounds). A fundamental understanding of that diverse spectrum is a major goal for our field.

Let me mention a few features of Table 1 which relate to tau physics. In the case of neutrinos, only upper mass bounds are given. Solar and atmospheric oscillation experiments hint towards very small, non-zero mass differences

$\Delta m^2 \simeq 10^{-5} \text{eV}^2$ and 10^{-2}eV^2 , but require further scrutiny before a definite positive signal can be claimed. Dark matter and cosmological missing mass arguments suggest the utility of a stable neutrino in the few to 40 eV range. The tau neutrino is a leading candidate for that role. A relatively heavy, unstable neutrino in the 1–30 MeV mass range could also impact cosmology and baryogenesis [1]. The new ALEPH bound [2] ($m_{\nu_\tau} < 23.8 \text{ MeV @ 95\% CL}$) and their 2 dimensional fit methodology are starting to confront that scenario.

The tau lepton mass is now known with remarkable precision [3], about 1 part in 10^5 . But why do charged leptons exhibit the mass hierarchy

$$m_e : m_\mu : m_\tau :: 1 : 206.77 : 3477.54 \quad (1)$$

The standard model Higgs mechanism accommodates but does not explain Eq. (1), or for that matter, why we have 3 generations. Perhaps such a hierarchy suggests some sort of nested loop mass generation, but what "new physics" is required? Does the "new physics" imply additional e - μ - τ differences other than their masses?

Even within the 3rd generation, can one understand why (roughly)

$$m_\tau : m_b : m_t :: 1 : 2.5 : 100? \quad (2)$$

Grand unified theories (GUTS) naturally accommodate $m_b^0/m_\tau^0 = 1$ and explain $m_b/m_\tau \simeq 2.5$ as a low energy QCD effect. But why is the top so heavy? Given its large mass, the top quark must be a key player in electroweak symmetry breaking, but what exactly is its role? In some 2 Higgs doublet models [4], one accounts for the large m_t/m_b by having $\tan \beta \equiv v_2/v_1$ (ratio of vacuum expectation values) $\simeq 60$. Such a large

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Table 1
Standard Model elementary particles and their masses

	Particle	Mass (GeV)
1st Generation	ν_e	$< 4.5 \times 10^{-9}$
	e	$0.51099906(15) \times 10^{-3}$
	u	5×10^{-3}
	d	9×10^{-3}
2nd Generation	ν_μ	$< 0.16 \times 10^{-3}$
	μ	$0.105658389(34)$
	c	1.35
	s	0.175
3rd Generation	ν_τ	$< 23.8 \times 10^{-3}$ (New)
	τ	1.77702 ± 0.00025
	t	174 ± 17
	b	4.5
Gauge Bosons	γ	0
	$W^\pm$	80.23 ± 0.18
	Z	91.190 ± 0.005
	Gluons	0
Higgs Scalar	H	$63 < m_H < 800$ (Theory)

value of $\tan \beta$ could have other interesting phenomenological consequences. Indeed, as we shall see, it could even affect tau decays.

The third generation of fermions is a gold mine. Heralded in by discovery of the tau, who could have guessed that it would be accompanied by such a heavy top and long lived b quark, or that its neutrino partner might constitute 90% of our universe's mass and determine its final fate. But what about the tau lepton itself? So far, it appears to be a normal (albeit oxymoronic) sequential heavy charged lepton with no unusual or exotic characteristics. It is, nevertheless, an extremely valuable testing ground for the standard model. In addition, as a member of the most massive generation, it may couple more strongly to and eventually unveil whatever "new physics" is responsible for mass. Indeed, tau studies still have the potential for revolutionary discoveries.

In the remainder of this talk, I describe some recent advances in tau physics and discuss their implications from a theoretical perspective. The

examples I have chosen include e - μ - τ universality, QCD studies, anomalous electroweak dipole moments, and forbidden decays. That list is by no means exhaustive. It should, however, demonstrate the breath of tau physics, describe some interesting new results, and point out the potential for future advances.

2. Electron-Muon-Tau Universality

The terminology e - μ - τ universality has traditionally been used to describe the fact that modulo mass effects, the electron, muon, and tau leptons all exhibit identical properties. In the standard model, universality is naturally realized by gauge interactions because the three generations of leptons all have the same $SU(2)_L \times U(1)_Y$ transformation properties and quantum numbers. As a consequence, their bare gauge couplings must be identical

$$g_{2_0}^e = g_{2_0}^\mu = g_{2_0}^\tau = g_{2_0}$$

$$g_{1_0}^e = g_{1_0}^\mu = g_{1_0}^\tau = g_{1_0} \quad (3)$$

In addition, they all must have the same bare electric charges and Z couplings parametrized with the same $\sin^2 \theta_W^0$. The relations in Eq. (3) are natural and will be preserved by the renormalized (measured) couplings up to finite calculable radiative corrections. In the standard model, those radiative corrections [5] will differ for the three species only because $m_e \neq m_\mu \neq m_\tau$ and can be easily accounted for in high precision universality tests. Of course, one can imagine "new physics" scenarios where the universality relations would be violated at the tree or loop level and then precision measurements could unveil their presence.

The Yukawa couplings of leptons to the standard model Higgs doublet explicitly violate universality. Indeed, those couplings must scale as $m_e^0 : m_\mu^0 : m_\tau^0$ in order to properly accommodate the known charged lepton masses. That universality violation manifests itself in two ways. It leads to different lepton masses and distinct Higgs scalar (and Goldstone boson) lepton couplings. Indeed, the Higgs decay rates to $\ell\bar{\ell}$ pairs ($\ell = e, \mu, \tau$) should scale as $m_e^2 : m_\mu^2 : m_\tau^2$, which could be considered a large (albeit predicted) universality violation. However, one does not have to wait for the Higgs scalar discovery. Instead, Yukawa couplings can be measured via longitudinal W boson couplings which result from charged Goldstone bosons of the Higgs doublet. Indeed, the longitudinal $W\ell\nu$ coupling is required to be

$$\frac{g_{2_0}^\ell}{\sqrt{2}} \frac{m_\ell^0}{m_W^0} \quad (4)$$

Testing that scalar coupling via decays involving π , K , D , B 's provides an independent check of the universality requirement in Eq. (3). I later give an example of "new physics" which would lead to a small "apparent" violation of Eq. (3) in leptonic τ decays, but respects universality in π decays, the two Higgs doublet model.

2.1. Charged Currents

In principle, one can directly test the $SU(2)_L$ coupling equality $g_{2_0}^e = g_{2_0}^\mu = g_{2_0}^\tau$ via leptonic W decays, $W \rightarrow \ell\bar{\nu}_\ell$ ($\ell = e, \mu, \tau$). Including radiative corrections and summing over all radiative

inclusive final states, $\Gamma(W \rightarrow \ell\bar{\nu}_\ell(\gamma)) \equiv \Gamma(W \rightarrow \ell\bar{\nu}_\ell) + \Gamma(W \rightarrow \ell\bar{\nu}_\ell\gamma) + \dots$, one predicts [5,6]

$$\frac{\Gamma(W \rightarrow \ell_1\bar{\nu}_{\ell_1}(\gamma))}{\Gamma(W \rightarrow \ell_2\bar{\nu}_{\ell_2}(\gamma))} = \frac{\left(1 + \frac{m_{\ell_1}^2}{2m_W^2}\right) \left(1 - \frac{m_{\ell_1}^2}{m_W^2}\right)^2}{\left(1 + \frac{m_{\ell_2}^2}{2m_W^2}\right) \left(1 - \frac{m_{\ell_2}^2}{m_W^2}\right)^2} \times \left(1 + \mathcal{O}\left(\alpha \frac{m_\ell^2}{m_W^2}\right)\right) \quad (5)$$

(Radiative corrections to radiative *exclusive* decay configurations can be quite large [5].) Unfortunately, direct W decay measurements currently verify coupling universality at only the 3-4% level and are not competitive with the tests I subsequently describe. One day, however, they should (statistically at least) approach the $\pm 0.1\%$ mark where things get very interesting.

A more precise test of $SU(2)_L$ gauge coupling universality involves comparing the leptonic tau decays $\tau \rightarrow \ell\nu\bar{\nu}(\gamma)$ ($\ell = e, \mu$) with $\mu \rightarrow e\nu\bar{\nu}(\gamma)$, where (γ) always denotes radiative inclusive. Including electroweak radiative corrections, one finds [7]

$$\Gamma(\mu \rightarrow e\nu\bar{\nu}(\gamma)) = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5} \frac{m_\mu^2}{m_W^2} - 2 \frac{m_e^2}{m_W^2}\right) \left(1 + \frac{\alpha(m_\mu)}{2\pi} \left(\frac{25}{4} - \pi^2\right)\right) \quad (6)$$

$$\Gamma(\tau \rightarrow \ell\nu\bar{\nu}(\gamma)) = \frac{G_\mu^2 m_\tau^5}{192\pi^3} f\left(\frac{m_\ell^2}{m_\tau^2}\right) \left(1 + \frac{3}{5} \frac{m_\tau^2}{m_W^2} - 2 \frac{m_\ell^2}{m_W^2}\right) \left(1 + \frac{\alpha(m_\tau)}{2\pi} \left(\frac{25}{4} - \pi^2\right)\right) \quad (7)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$\alpha(m_\mu) \simeq 1/136, \quad \alpha(m_\tau) \simeq 1/133.3$$

where terms of $\mathcal{O}(\alpha m_\ell^2/m_\tau^2)$ and $\mathcal{O}(\alpha m_\tau^2/m_W^2)$ have been neglected. I have, however, included a (negligible) leading $\mathcal{O}(m_\ell^2/m_W^2)$ tree level contribution from the scalar W component.

Most of the electroweak radiative corrections have been absorbed in the common G_μ . The only real difference between Eq. (6) and Eq. (7) is the trivial exchange $m_\mu \rightarrow m_\tau$, $m_e \rightarrow m_\ell$. From the muon lifetime

$$\tau_\mu = 2.19703(4) \times 10^{-6} s \quad (8)$$

one finds

$$G_\mu = 1.16639(1) \times 10^{-5} s \quad (9)$$

Using that value in Eq. (7) and normalizing with the tau lifetime, one predicts the leptonic branching ratios

$$B(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma)) = 0.17866 \left(\frac{m_\tau}{1777 \text{ MeV}} \right)^5 \times \left(\frac{\tau_{\text{tau}}}{2.916 \times 10^{-13} s} \right) \quad (10)$$

$$B(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau(\gamma)) = 0.17377 \left(\frac{m_\tau}{1777 \text{ MeV}} \right)^5 \times \left(\frac{\tau_{\text{tau}}}{2.916 \times 10^{-13} s} \right) \quad (11)$$

$$\frac{\Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau(\gamma))} = 1.0282 \quad (12)$$

Those predictions are essentially free of theoretical uncertainties and should, therefore, be tested with the highest possible experimental precision. A deviation would imply "new physics". Such tests have steadily progressed during the last few years [8]. The changes are illustrated in Table 2.

The average leptonic branching ratios have been relatively stable, but the tau mass and lifetime have shifted since 1991. What had once appeared to be a $2\text{--}3\sigma$ universality violation has settled into accord with standard model expectations. The ratio

$$\frac{\Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma))^{\text{exp}}}{\Gamma(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau(\gamma))^{\text{exp}}} = 1.0265 \pm 0.0072 \quad (13)$$

tests $e\text{--}\mu$ universality while Eqs. (10) and (11) individually test τ universality. One finds

$$g_2^e : g_2^\mu : g_2^\tau :: 1 : 1.0008 \pm 0.0036 : 0.9983 \pm 0.0033 \quad (14)$$

Although the agreement with expectations is impressive and quite constraining, there is still room for a "new physics" deviation of $\mathcal{O}(\lesssim \pm 1\%)$ in

the leptonic branching ratios versus τ_{tau} comparison. If such a deviation were present, it might be uncovered by future experiments which aim for $\pm 0.1\%$ in the leptonic branching ratios and perhaps $\pm 0.2\%$ in τ_{tau} . The ratio in Eq. (13) should more easily reach $\pm 0.1\%$, which as we shall see, would nicely test the two Higgs doublet model.

Next, let me consider $e\text{--}\mu\text{--}\tau$ universality tests for the longitudinal (or scalar) W coupling. The best test is provided by comparing $\pi \rightarrow \ell \bar{\nu}_\ell(\gamma)$ and $\tau \rightarrow \pi \nu_\tau(\gamma)$ decays. The ratio

$$R_{e/\mu} \equiv \frac{\Gamma(\pi \rightarrow e \nu(\gamma))}{\Gamma(\pi \rightarrow \mu \nu(\gamma))} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} \times (1 + \text{Radiative Correction}) \quad (15)$$

with the radiative correction computed [9] to be -0.0376 ± 0.0004 leads to the prediction

$$R_{e/\mu}^{\text{theory}} = 1.2352 \pm 0.0005 \times 10^{-4} \quad (16)$$

which is to be compared with

$$R_{e/\mu}^{\text{exp}} = 1.2306 \pm 0.0037 \times 10^{-4} \quad (17)$$

Comparison implies

$$g_2^e/g_2^\mu = 0.9983 \pm 0.0015 \pm 0.0002 \quad (18)$$

which is the best current test of $e\text{--}\mu$ universality. However, as we shall see, it *does not rule out* the possibility of a small deviation in Eq. (13). One can also predict the ratio

$$R_{\tau/\mu} \equiv \frac{\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma))}{\Gamma(\pi \rightarrow \mu \nu(\gamma))} = \frac{1}{2} \frac{m_\tau^3}{m_\pi m_\mu^2} \times \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} (1 + \text{Radiative Correction}) \quad (19)$$

Decker and Finkemeier [10] recently found the radiative correction to be $+0.0016 \pm 0.0014$ which implies

$$B(\tau \rightarrow \pi \nu_\tau(\gamma))^{\text{theory}} = (0.1095 \pm 0.0002) \left(\frac{m_\tau}{1777 \text{ MeV}} \right)^3 \left(\frac{\tau_{\text{tau}}}{2.916 \times 10^{-13} s} \right) \quad (20)$$

Table 2

Experimental averages for the tau mass, lifetime, and leptonic branching ratios in 1991, 1992, and at this workshop 1994.

	1991	1992	1994 (This Workshop)
m_τ (MeV)	$1784.1^{+2.7}_{-3.6}$	1776.9 ± 0.5	1777.02 ± 0.25
τ_{tau} (sec)	$3.04 \pm 0.07 \times 10^{-13}$	$2.96 \pm 0.03 \times 10^{-13}$	$2.916 \pm 0.016 \times 10^{-13}$
$B(\tau \rightarrow e \nu \bar{\nu}(\gamma))$	0.1780 ± 0.0023	0.1776 ± 0.0015	0.17793 ± 0.00086
$B(\tau \rightarrow \mu \nu \bar{\nu}(\gamma))$	0.1743 ± 0.0024	0.1739 ± 0.0017	0.17333 ± 0.00087

That result should be compared with the current average

$$B(\tau \rightarrow \pi \nu_\tau(\gamma))^{\text{exp}} = 0.1106 \pm 0.0017 \quad (21)$$

which includes the new result $B(\tau \rightarrow \pi \nu_\tau(\gamma)) = 0.1090 \pm 0.0018$ reported by the ALEPH collaboration at this workshop. The agreement implies

$$g_2^T/g_2^\mu = 1.005 \pm 0.008 \quad (22)$$

which is not as precise as the test in Eq. (14), but starting to become interesting. Given the small theoretical uncertainty in Eq. (20), the experimental precision on $B(\tau \rightarrow \pi \nu_\tau(\gamma))$ should be pushed as far as possible. Similar remarks apply to the decay $\tau \rightarrow K \nu_\tau(\gamma)$.

An interesting comparison in the pursuit of "new physics" is the ratio

$$\frac{\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma))} = 0.6129 \pm 0.0018 \quad (\text{theory}) \quad (23)$$

which is to be compared with

$$\frac{\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma))} = 0.6211 \pm 0.0097 \quad (\text{experiment}) \quad (24)$$

If an apparent deviation from universality were to appear in the gauge or scalar couplings, that ratio would provide both a consistency check and a potential means of sorting out the "new physics".

2.2. Neutral Currents

Very precise tests of e - μ - τ universality in neutral current couplings are provided by the Z partial decay widths, $\Gamma(Z \rightarrow \ell \bar{\ell})$ and angular decay asymmetries. The partial widths are predicted to be (in lowest order)

$$\Gamma(Z \rightarrow \ell \bar{\ell}) = \frac{G_\mu m_Z^3}{6\sqrt{2}\pi} \left(1 - 4 \frac{m_\ell^2}{m_Z^2}\right)^{1/2} \times \left(v_\ell^2 \left(1 + 2 \frac{m_\ell^2}{m_Z^2}\right) + a_\ell^2 \left(1 - 4 \frac{m_\ell^2}{m_Z^2}\right)\right) \quad (25)$$

$$v_\ell = \frac{1}{2}(1 - 4\sin^2 \theta_W) \quad , \quad a_\ell = -1/2$$

Electroweak radiative corrections, for radiative inclusive rates $Z \rightarrow \ell \bar{\ell}(\gamma)$, are ℓ independent up to terms of $\mathcal{O}(\alpha m_\ell^2/m_Z^2)$. They basically replace G_μ by ρG_μ and v_ℓ by $\frac{1}{2}(1 - 4\kappa \sin^2 \theta_W(m_Z) \overline{MS})$, where ρ and κ are $1 + \mathcal{O}(\alpha)$ quantities. The ρ parameter includes a famous quadratic dependence on the top quark mass

$$\rho = 1 + \frac{3G_\mu}{2\sqrt{2}\pi} m_t^2 + \mathcal{O}(\alpha) \quad (26)$$

induced by the use of G_μ in Eq. (25) and is sensitive to weak isospin violating "new physics" effects.

The Z decay asymmetries (forward-backward, τ polarization, left-right...) measure the combination

$$A_\ell = \frac{2v_\ell a_\ell}{v_\ell^2 + a_\ell^2} \quad (27)$$

Forward-backward asymmetries determine

$$A_{FB}^\ell = \frac{3}{4} A_e A_\ell \quad (28)$$

while tau polarization studies at LEP separate A_e and A_τ . (The SLAC A_{LR} measurement determines A_e .) From current fits to data (see A. Kounine's talk [11])

$$v_e = -0.0370 \pm 0.0021 \quad a_e = -0.50093 \pm 0.00064$$

$$v_\mu = -0.0308 \pm 0.0051 \quad a_\mu = -0.50164 \pm 0.00096(29)$$

$$v_\tau = -0.0386 \pm 0.0023 \quad a_\tau = -0.5026 \pm 0.0010$$

One sees that universality is even better tested than in the charged current sector, at least for the axial couplings. That is extremely important, since one might expect a modification of the axial coupling if leptons exhibited composite structure. Note, that precision knowledge of m_t in Eq. (26) and inclusion of the other radiative corrections, allow one to confirm neutral-charged current universality at the few tenths of a percent. In the future, similar precision should be reached by high statistics measurements of $\Gamma(W \rightarrow \ell \bar{\nu}_\ell(\gamma))/\Gamma(Z \rightarrow \ell \bar{\ell}(\gamma))$ at hadron colliders.

2.3. "New Physics" Constraints

The good agreement between theory and experiment in the lepton universality tests, allows one to test "new physics" appendages to the standard model. Models which were introduced to accommodate what appeared at one time to be a possible several percent deviation from charged current universality, are now well constrained by the newer more precise measurements. Nevertheless, there is still room for a deviation of $\mathcal{O}(\lesssim \pm 1\%)$ in the leptonic tau decay branching ratios from standard model predictions. Indeed, even the e - μ universality tests could show some deviation as higher precision is attained. Could realistic "new physics" scenarios actually lead to small observable deviations from e - μ - τ universality in tau decays? If so, what other implications might follow? To turn that question around, how quantitatively constraining are present-day tests and how much improvement can be expected?

In addressing those issues, I will consider two realistic scenarios: heavy neutrino mixing [12] and a 2 Higgs doublet model.

2.3.1. Heavy Neutrino Mixing

Consider the possibility of a fourth generation lepton pair (ν_L, L). (A similar discussion could be given for singlet heavy neutrinos.) If m_L and $m_{\nu_L} \gtrsim 45.5$ GeV, such a pair would not affect Z decays. Nevertheless, mixing with the third generation would modify charged current inter-

actions. Parametrizing the ν_τ - ν_L mixing by $\sin \theta$, one finds from Eq. (14),

$$\sin \theta \lesssim 0.10 \quad (30)$$

Future $\pm 0.1\%$ measurements of the tau leptonic branching ratios and τ_{tau} could improve that bound by a factor of 3. What value of $\sin \theta$ is reasonable to expect? Taken in analogy with the quark sector, one might anticipate small mixing (roughly)

$$\sin \theta \simeq \tan \theta \simeq \sqrt{\frac{m_\tau}{m_L}} \quad (31)$$

In such a scenario, the bound in Eq. (30) implies $m_L \gtrsim 178$ GeV. Such a heavy lepton would have, so far, escaped detection. A factor of 3 improvement in Eq. (30) would probe up to $m_L \simeq 1.8$ TeV, under the mixing assumption in Eq. (31). We see that mixing at the level $\sin \theta \simeq 0.05 \sim 0.10$ is in fact still quite plausible. Such mixing should also show up in the $B(\tau \rightarrow \pi \nu_\tau(\gamma))$ prediction, but not in ratios such as Eqs. (12) or (23) where the mixing effect cancels. Heavy neutrino mixing is certainly viable, and provides good motivation for continued refinements in the leptonic tau branching ratios and tau lifetime measurements.

2.3.2. Two Higgs Doublet Model

The two Higgs double scenario is a simple extension of the standard model. It is required in supersymmetric models, but could occur independent of supersymmetry. Parameterizing the Higgs sector by

$$\tan \beta = v_2/v_1 \quad (32)$$

and the physical Higgs masses

$$m_h, m_H, m_A, m_{H^\pm} \quad (33)$$

one can examine potential effects on e - μ - τ universality tests.

At the tree level, the leptonic branching ratio predictions in Eqs. (10) and (11) are reduced (roughly) [13] by a factor $\left(1 - \frac{2m_L^2}{m_{H^\pm}^2} \tan^2 \beta\right)$.

The good agreement between theory and experiment for $\Gamma(\tau \rightarrow e \nu \bar{\nu}(\gamma))/\Gamma(\tau \rightarrow \mu \nu \bar{\nu}(\gamma))$ then implies

$$m_{H^\pm} \geq 2 \tan \beta \text{ GeV} \quad (34)$$

That constraint is (roughly) equivalent to the current bound from $B \rightarrow \tau \nu X$ decays [14]. Loop induced decays $b \rightarrow s \gamma$ give the more constraining bound [15] $m_{H^\pm} > 260 \text{ GeV}$ for $\tan \beta \gtrsim 1$; however, other loop effects (particularly in SUSY models) could cancel the charged Higgs contribution and negate that bound. Future improvements in $\Gamma(\tau \rightarrow e \nu \bar{\nu}(\gamma))/\Gamma(\tau \rightarrow \mu \nu \bar{\nu}(\gamma))$ to the ± 0.001 level would probe $m_{H^\pm} \simeq 5 \tan \beta \text{ GeV}$. For $\tan \beta \simeq 60$, that becomes an interesting domain.

In the case of $\tau \rightarrow \pi \nu_\tau(\gamma)$ and $\pi \rightarrow \mu \bar{\nu}_\mu(\gamma)$, both decay rates are reduced by the same $(1 - \tan^2 \beta m_\pi^2/m_{H^\pm}^2)^2$ factor (roughly for large $\tan \beta$) [14]. That factor cancels in the ratio; so, the predictions given in Eq. (19) and Eq. (20) are unaffected by $H^\pm$ at the tree level. Similarly, the $\Gamma(\pi \rightarrow e \bar{\nu}_e(\gamma))/\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu(\gamma))$ is unaffected. Hence, one has an interesting scenario in which $\Gamma(\tau \rightarrow e \nu \bar{\nu}(\gamma))/\Gamma(\tau \rightarrow \mu \nu \bar{\nu}(\gamma))$ can show a small deviation from universality while $\Gamma(\pi \rightarrow e \bar{\nu}_e(\gamma))/\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu(\gamma))$ remains unaffected. Similarly, the ratio $\Gamma(W \rightarrow e \nu(\gamma))/\Gamma(W \rightarrow \mu \nu(\gamma))$ would be unaffected.

Loop corrections can also impact e - μ - τ universality tests. In the two Higgs doublet model, such corrections reduce $B(\tau \rightarrow \ell \nu \bar{\nu}(\gamma))$ for both $\ell = e$ and μ . The reduction can be $\mathcal{O}(0.5\%)$ and thus potentially observable [13,16].

3. Tau as a QCD Laboratory

Hadronic tau decays probe $q^2 \lesssim m_\tau^2$. From a QCD perspective, that is an interesting domain both for perturbative and non-perturbative studies. The inclusive hadronic decay rate is very precisely measured and has the potential [17] to provide our best determination of the QCD gauge coupling α_s . Of course, all non-perturbative [18] effects must first be understood and properly accounted for. Hadronic spectral functions in both the vector and axial-vector channel's can also be cleanly studied [19]. Already, properties of the ρ and A mesons are best measured in tau decays. Excitations such as the $\pi'(1300)$ and $\rho'(1450)$ can also be disentangled. The spectral functions allow us to test various sum rules and thus explore

the underlying dynamics of QCD. The lessons learned will be important if we hope one day to replace the Higgs mechanism by dynamical electroweak symmetry breaking.

3.1. $\alpha_s(m_Z)_{\overline{MS}}$

The ratio

$$R_\tau \equiv \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma))} \quad (35)$$

can potentially provide our most precise determination of α_s . On the experimental side, that ratio is extremely well determined from either leptonic branching ratio

$$\begin{aligned} (1.9726 + R_\tau)^{-1} &= B(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma)) \\ &= 1.0282 B(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau(\gamma)) \end{aligned} \quad (36)$$

or from measurements of the tau lifetime and mass

$$\begin{aligned} R_\tau &= 5.5970 \left(\frac{2.916 \times 10^{-13} \text{ s}}{\tau_{\text{tau}}} \right) \left(\frac{1777 \text{ MeV}}{m_\tau} \right)^5 \\ &\quad - 1.9726 \end{aligned} \quad (37)$$

Using the current averages in Table 2 and $m_\tau = 1777.02 \pm 0.25 \text{ MeV}$, one finds

$$\begin{aligned} \tau_{\text{tau}} &= 2.916 \pm 0.016 \times 10^{-13} \text{ s} \rightarrow R_\tau \\ &= 3.624 \pm 0.31 \\ B(\tau \rightarrow e \bar{\nu}_e \nu_\tau(\gamma)) &= 0.17793 \pm 0.00086 \rightarrow R_\tau \\ &= 3.648 \pm 0.27 \\ B(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau(\gamma)) &= 0.17333 \pm 0.00087 \rightarrow R_\tau \\ &= 3.639 \pm 0.28 \end{aligned} \quad (38)$$

Those methods are consistent and collectively imply

$$R_\tau^{\text{ave}} = 3.638 \pm 0.016 \quad (39)$$

The error in R_τ^{ave} is, as we shall see, completely negligible in the extraction of α_s .

On the theoretical side, R_τ is an infrared safe quantity and thus perturbatively computable, modulo non-perturbative corrections which we later discuss. The perturbative prediction for R_τ can be written as [18]

$$R_\tau = 3(1 + \delta) \left\{ 1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.2023 \left(\frac{\alpha_s(m_\tau)}{\pi} \right)^2 + 26.366 \left(\frac{\alpha_s(m_\tau)}{\pi} \right)^3 + (K_4 + 78) \left(\frac{\alpha_s(m_\tau)}{\pi} \right)^4 + \dots \right\} \quad (40)$$

where δ parameterizes electroweak and non-perturbative QCD corrections, including quark mass effects. The value of K_4 has not been fully computed, but it is thought to be large and positive (e.g. Kataev and Starshenko [20] estimate $K_4 \simeq 27.5$). The extraction of α_s from Eq. (40) depends on δ and K_4 . In the case of δ , electroweak radiative corrections [7] give a +2% contribution while quark mass and non-perturbative condensate effects [18] give a negative contribution of similar magnitude. They approximately cancel and one finds

$$\delta = 0 \pm \Delta \quad (41)$$

where Δ is primarily the non-perturbative uncertainty. How large an error estimate one assigns to Δ significantly affects the uncertainty in α_s . Estimates for Δ range from about 1 to several percent. The effect of ultraviolet renormalons [21] is particularly difficult to assess (see G. Altarelli's talk [22]). If their contribution were better understood, R_τ would give our most precise determination of α_s . For now, I leave Δ arbitrary.

Rewriting R_τ at an arbitrary scale $\mu < m_\tau$ (actually less than m_c such that $N_F = 3$ quark flavors applies)

$$R_\tau = 3(1 \pm \Delta) \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} + [5.2023 - 2.25\ell n \left(\frac{m_\tau^2}{\mu^2} \right)] \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 + \left[26.366 - 27.41\ell n \left(\frac{m_\tau^2}{\mu^2} \right) + 5.0625\ell n^2 \left(\frac{m_\tau^2}{\mu^2} \right) \right] \left(\frac{\alpha_s(\mu)}{\pi} \right)^3 + \left[K_4 + 78 - 229.65\ell n \left(\frac{m_\tau^2}{\mu^2} \right) + 101.5\ell n^2 \left(\frac{m_\tau^2}{\mu^2} \right) \right. \right.$$

$$\left. - 11.39\ell n^3 \left(\frac{m_\tau^2}{\mu^2} \right) \right] \left(\frac{\alpha_s(\mu)}{\pi} \right)^4 + \dots \right\} \quad (42)$$

Of course $\mu \frac{d}{d\mu} R_\tau = 0$; however, truncation of the perturbative expansion introduces some μ dependence. Arguments as to what value of μ should be used are hotly debated. I will consider two cases: 1) $\mu = m_c \simeq 1.35$ GeV, the largest scale where $N_F = 3$ applies in the $\overline{MS}$ scheme and 2) $\mu_0 \simeq 805$ MeV, a relatively small scale where the coupling is large but the extracted value of $\Lambda_{\overline{MS}}$ is relatively flat or stable (optimized).

For the first case

$$R_\tau = 3(1 \pm \Delta) \left\{ 1 + \frac{\alpha_s(m_c)}{\pi} + 3.9645 \left(\frac{\alpha_s(m_c)}{\pi} \right)^2 + 12.822 \left(\frac{\alpha_s(m_c)}{\pi} \right)^3 + (K_4 - 19.5) \left(\frac{\alpha_s(m_c)}{\pi} \right)^4 + \dots \right\} \quad (43)$$

while for the second

$$R_\tau = 3(1 \pm \Delta) \left\{ 1 + \frac{\alpha_s(\mu_0)}{\pi} + 1.6398 \left(\frac{\alpha_s(\mu_0)}{\pi} \right)^2 - 4.3418 \left(\frac{\alpha_s(\mu_0)}{\pi} \right)^3 + (K_4 - 76.4) \left(\frac{\alpha_s(\mu_0)}{\pi} \right)^4 + \dots \right\} \quad (44)$$

Employing R_τ from Eq. (39) and assuming small Δ , one finds

$$\frac{\alpha_s(m_c)}{\pi} = 0.1253 - 0.00009(K_4 - 19.5) \pm 0.002 \mp 0.46\Delta \quad \text{case 1} \quad (45)$$

$$\frac{\alpha_s(\mu_0)}{\pi} = 0.1842 - 0.001(K_4 - 76.4) \pm 0.004 \mp \Delta \quad \text{case 2} \quad (46)$$

where the K_4 term has been treated as a perturbation. Note that case 2 is more sensitive to the uncertainty in K_4 and requires a large positive K_4 if it is to be at all stable. The values in Eqs. (45) and (46) translate to

$$\alpha_s(m_Z)_{\overline{MS}} = 0.1200 - 0.000025(K_4 - 19.5)$$

$$\pm 0.0005 \mp 0.1\Delta \quad (\text{case 1}) \quad (47)$$

$$\alpha_s(m_Z)_{\overline{MS}} = 0.1175 - 0.0001(K_4 - 76.4)$$

$$\pm 0.0005 \mp 0.1\Delta \quad (\text{case 2}) \quad (48)$$

The two cases agree for $K_4 \simeq 62$. I will assume that value but allow a fairly generous ± 0.002 truncation (or K_4 uncertainty) error. In that way, one finds

$$\alpha_s(m_Z)_{\overline{MS}} = 0.1190 \pm 0.0005 \pm 0.0020 \mp 0.1\Delta \quad (49)$$

If $\Delta \lesssim 0.01$, as originally expected, Eq. (49) represents the current best determination of $\alpha_s(m_Z)$. It is, therefore, imperative that the effect of ultra-violet renormalon corrections be scrutinized and their impact be assessed [23].

To illustrate the importance of precisely determining α_s (and the other gauge couplings), I give the "average" supersymmetry mass scale prediction of SUSY SU(5)

$$m_{\text{SUSY}} \simeq 100m_Z \exp \left[\frac{3\pi}{4} \left(\frac{1 - 5\sin^2 \theta_W(m_Z)_{\overline{MS}}}{\alpha(m_Z)_{\overline{MS}}} + \frac{7}{3\alpha_s(m_Z)_{\overline{MS}}} \right) \right] \quad (50)$$

Assuming

$\alpha(m_Z)_{\overline{MS}} = 1/127.9$, $\sin^2 \theta_W(m_Z)_{\overline{MS}} \simeq 0.232$, and $\alpha_s(m_Z)_{\overline{MS}} \simeq 0.119$ leads to $m_{\text{SUSY}} \simeq 1$ TeV which thrills SUSY enthusiasts. However, the Δ shift in Eq. (49) would multiply that value by $\exp(\pm 40\Delta)$ which is considerable if Δ is relatively large. (A downward shift in $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ can also significantly impact m_{SUSY} .)

3.2. Sum Rules

The $J = 1$ vector and axial-vector spectral functions $\rho_V(s)$ and $\rho_A(s)$ can be obtained from $\tau \rightarrow \nu_\tau + n\pi$ data using [19]

$$\frac{d\Gamma(\tau \rightarrow \nu_\tau + n\pi)}{ds} = \frac{G_\mu^2 |V_{ud}|^2}{32\pi^2 m_\tau^3} (1.02)(m_\tau^2 + 2s) \times (m_\tau^2 - s)^2 \begin{cases} 4\pi\rho_V(s) & n = 2, 4, \dots \\ 4\pi\rho_A(s) & n = 3, 5, \dots \end{cases} \quad (51)$$

Using ρ_V and ρ_A , one can test the Weinberg sum rules [24]

$$\begin{aligned} \frac{f_\pi^2}{2} &= \int ds (\rho_V(s) - \rho_A(s)) \\ 0 &= \int ds s (\rho_V(s) - \rho_A(s)) \end{aligned} \quad (52)$$

or, even better, the more convergent Das-Mathur-Okubo PCAC sum rule [25,26]

$$L_{10} = \frac{-S_{\text{QCD}}}{16\pi} = -\frac{1}{4} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} [\rho_V(s) - \rho_A(s)] \quad (53)$$

where L_{10} is one of the coefficients of the chiral lagrangian. Radiative π decays give [26] $L_{10} = -6.4 \pm 0.3 \times 10^{-3}$ while tau decays (roughly) give $L_{10} \simeq -6 \times 10^{-3}$. That value of L_{10} corresponds to $S_{\text{QCD}} \simeq +0.3$ which is used to draw the inference [27,28]

$$S = +0.1 N_{\text{TC}} N_D \quad (54)$$

for the Peskin-Takeuchi [27] technicolor loop parameter ($N_{\text{TC}} = \text{No. of technicolors}$ $N_D = \text{No. of doublets}$).

One can also relate Eq. (53) to the pion electromagnetic polarizability [28,29]

$$\alpha_E = \frac{\alpha}{3m_\pi} \langle r_\pi^2 \rangle - \frac{2\alpha}{m_\pi f_\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} [\rho_V(s) - \rho_A(s)] \quad (55)$$

where $\langle r_\pi^2 \rangle \simeq 0.44 fm^2$ is the pion charge radius. Employing $f_\pi \simeq 130.7$ MeV and $L_{10} \simeq -6 \times 10^{-3}$ implies $\alpha_E = 3.9 \times 10^{-4} fm^3$ which is somewhat above the prediction $\alpha_E = 2.8 \times 10^{-4} fm^3$ from chiral perturbation theory. It would be interesting to carry out a new more precise determination of $\rho_V(s) - \rho_A(s)$ from tau decays (and above $s = m_\tau^2$ from $e^+e^- \rightarrow \text{hadrons}$) for the purpose of refining α_E .

A similar discussion can be given for $J = 1$, $S = 1$ tau decay spectral functions involving kaons. However, SU(3) breaking effects such as the strange quark mass are potentially more important in such analyses.

4. Exotic Tau Properties and “Forbidden” Decays

So far, the tau has proven to be a “normal” sequential lepton. However, as we scrutinize it more closely, it may reveal some surprises. Here, I briefly comment on anomalous electroweak dipole moments and neutrinoless tau-number violating decays. Constraints on baryon number violating tau decays are also discussed.

4.1. Anomalous Electroweak Dipole Moments

Whatever mechanism is responsible for mass generation, should also induce anomalous electroweak dipole moments (as well as chromodipole moments for quarks). Indeed, the chiral changing structure of masses and dipole moments are similar; so, dipole moments may provide important insight regarding the origin of mass.

In a recent study [30], I found the very general result that a fermion of mass, m_f , generated by physics at a scale $M \gg m_f$, will have an induced anomalous dipole moment

$$\delta a_f \sim \frac{m_f^2}{M^2} \quad (56)$$

That result holds for γ and Z moment couplings, a_f^γ and a_f^Z , as well as transition W dipole couplings a_f^W .

Dynamical symmetry breaking, such as extended technicolor, is particularly interesting because the M in Eq. (56) can scale inversely with m_f (the largest fermion masses may be induced by the smallest M , e.g. $m_f \sim m^2/M$ in walking technicolor with $m \simeq 300$ GeV). If that is the case, the top quark’s electroweak and chromo moments should prove to be the most interesting. Here, however, I limit my discussion to the tau.

From the general discussion given above, one might expect effects of “new physics” on electroweak dipole moments to scale as

$$\delta a_e : \delta a_\mu : \delta a_\tau :: m_e^n : m_\mu^n : m_\tau^n \quad (57)$$

$n = 2, 3, \text{ or } 4$

with $n = 4$ in walking extended technicolor scenarios. Similarly, in the case of electric dipole moments, one expects

$$d_e : d_\mu : d_\tau :: m_e^{n-1} : m_\mu^{n-1} : m_\tau^{n-1} \quad (58)$$

which is lower by one power because anomalous magnetic moments $\equiv \frac{e}{2m_\ell} \delta a_\ell$. Of course, T violating d_ℓ moments can have a complicated dependence on the underlying physics of CP violation. I, nevertheless, use Eqs. (57) and (58) to guide my discussion.

At present, one has the bounds on “new physics” effects [31]

$$\begin{aligned} |\delta a_e| &\lesssim 5 \times 10^{-11}, |\delta a_\mu| \lesssim 10^{-8}, |\delta a_\tau| \lesssim 6 \times 10^{-3} \\ |d_e| &< 10^{-26} e\text{-cm}, |d_\mu| \lesssim 7 \times 10^{-19} e\text{-cm}, \\ |d_\tau| &\lesssim 2 \times 10^{-17} e\text{-cm} \end{aligned} \quad (59)$$

Comparing the electron and muon, one sees that allowing for the scaling in Eq. (57), δa_μ is a much better probe of “new physics” than δa_e (a BNL experiment aims to further improve δa_μ by a factor of 20). In the case of electric dipole moments, their roles are reversed. In fact, d_e is one of the most promising possibilities for uncovering a new source of CP violation.

How does the tau compare? If we take the most promising case, $n = 4$ in Eqs. (57) and (58), the tau is very competitive in the comparison. However, the effect must be large enough to be experimentally observable. Taking $M \gtrsim 100$ GeV for the scale of tau mass generation, I expect tau dipole moment searches to become interesting when they begin to probe $|\delta a_\tau| \lesssim 10^{-4}$ and $|d_\tau| \lesssim 5 \times 10^{-19} e\text{-cm}$. So, they are still more than an order of magnitude away. (Can we push those bounds by a factor of 100?) Nevertheless, one should continue the adiabatic experimental improvements in δa_τ and d_τ (particularly in Z decays). We may get lucky and discover an unanticipated large effect.

4.2. Neutrinoless Tau-Number Violating Decays

The CLEOII collaboration [32] recently published new bounds on “forbidden” tau number violating decays such as $\tau \rightarrow \mu\gamma$, $e\gamma$, $\mu^-\mu^+\mu^-$, $\mu^-\pi^+\pi^-$, μ^-K^* etc. Their sensitivity has reached a few $\times 10^{-6}$ bounds for those branching ratios, and one can expect some improvement

as statistics improve (until backgrounds become a problem). That progress has been impressive and hopefully will continue, but how does it compare with other lepton flavor changing tests?

Muon number violating processes give extremely restrictive bounds and they are continuing to improve (see Table 3). Those bounds constrain "new physics" and often translate into bounds on other "forbidden" decays. For example, the bounds on $\mu \rightarrow 3e$ and $\mu N \rightarrow eN$ in Table 3 imply

$$\begin{aligned} B(Z \rightarrow \bar{\mu}e) &< 10^{-12} && \text{from } B(\mu \rightarrow 3e) \\ B(Z \rightarrow \bar{\mu}e) &< 10^{-13} && \text{from } R(\mu N \rightarrow eN) \quad (60) \end{aligned}$$

Present direct Z decay bounds $B(Z \rightarrow \bar{\mu}e) < 10^{-6}$ are obviously not competitive.

In the case of tau-number violation, comparison with Table 3 is only possible in specific well defined models. In some cases, standard model forbidden tau decays might occur at an observable rate while the rare processes in Table 3 remain suppressed. So, one should have an open mind and continue to push constraints on $\tau \rightarrow \mu\gamma, e\gamma, 3\mu \dots$ as far as possible.

An interesting comparison is tau number violation in Z decays versus tau decays (just as Eq. (60) compares Z and μ). For example, if the decay $Z \rightarrow \mu\bar{\tau}$ can occur, then the Z should also mediate $\tau \rightarrow 3\mu, \mu\pi^+\pi^-, \mu e^+e^-$ etc. One finds roughly (assuming no unusual cancellation) for a $Z\mu\tau$ coupling

$$B(\tau \rightarrow \mu e^+e^-) \simeq \frac{1}{6} B(Z \rightarrow \mu\bar{\tau}) \quad (61)$$

and similar relations for $\tau \rightarrow 3\mu, \mu\pi^+\pi^-$ etc. So, searches for $Z \rightarrow \mu\bar{\tau}$ are currently at about the same level of sensitivity as searches for $\tau \rightarrow \mu X$. Both are starting to push 10^{-6} . Can we find a way to reach 10^{-7} bounds? That seems to require a high luminosity tau-charm factory or experiments at hadron collider facilities. In the latter category, one might search for "forbidden" reconstructable decays such as $\tau \rightarrow 3\mu$ using $W \rightarrow \tau\nu_\tau$ or $b \rightarrow c\tau\nu_\tau$ as copious sources of taus. Of course, in a hadron collider environment, backgrounds rather than statistics may be the limitation.

4.3. Baryon Number Violation

An amusing idea initiated by the ARGUS Collaboration [33] was to search for baryon number violating tau decays $\tau \rightarrow \bar{p}\gamma, \bar{p}\pi^0$, and $\bar{p}\eta$. (Those decays conserve $B-L$.) Of course, none were seen and branching ratio bounds in the $10^{-3} \sim 10^{-4}$ range were found. Here, I would like to point out that indirect bounds can be put on those decays using proton decay constraints ($\tau_p \lesssim 10^{32}$ yr) and known tau properties. (If $\tau \rightarrow \bar{p}\gamma$, then $p \rightarrow$ "virtual τ " $\rightarrow \nu_\tau \pi^+$ for example.) Depending on assumptions regarding the amplitude, one finds (very roughly) from proton lifetime bounds

$$B(\tau \rightarrow \bar{p} + \text{anything}) < 10^{-40} \sim 10^{-48}! \quad (62)$$

One could consider other kinematically allowed baryon number violating decays such as $\tau \rightarrow \Lambda\pi^-, \Sigma^-\pi^0 \dots$ Since those baryons couple to protons (albeit weakly), constraints on such branching ratios are comparable to or perhaps 10^8 times Eq. (62) which is, nevertheless, very restrictive. So, it appears that observable baryon number violating tau decays are effectively ruled out.

5. Conclusion

The discovery of the tau was an unexpected glorious event. It ushered in the third generation of fermions and provided us with new experimental and theoretical challenges. Presently, the ongoing tau studies at LEP and CESR are producing some of the most interesting results in particle physics. Indeed, more new results were presented at this tau workshop than I have heard for a long time. The advent of B factories and perhaps a tau-charm factory point to a bright future.

It will be interesting to keep an eye on measurements of the tau lifetime and branching ratios as they push towards $\pm 0.1\%$ precision. Anomalies in $Z \rightarrow \tau^+\tau^-$ are also possible at that level. Continuing work on hadronic tau decays should provide more accurate data on light resonances, sum rules etc. and a better understanding of the α_s determination uncertainty.

Searches for exotic tau properties and decays are starting to become interesting. If we can successfully push anomalous dipole moment measurements and "forbidden" decay searches

Table 3

Bounds on muon-number violation and expected progress by ongoing experiments

Present Bound	Future Sensitivity (Current Exps.)
$B(\mu \rightarrow 3e) < 1 \times 10^{-12}$	—
$B(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$	$\sim 10^{-13}$
$R(\mu N \rightarrow eN) < 1 \times 10^{-12}$	$\sim 10^{-13}$
$B(K_L \rightarrow \mu e) < 9 \times 10^{-11}$	$\sim 10^{-12}$
$B(K \rightarrow \pi\mu e) < 2 \times 10^{-10}$	$\sim 3 \times 10^{-12}$

another 2 orders of magnitude, we will enter an uncharted domain where surprises may await us. Perhaps the tau will unveil its purpose and help us decipher the basic laws of nature.

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