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**NUMERICAL STUDY OF THE FLOW OF
GRANULAR MATERIALS DOWN AN INCLINED
PLANE USING A MODEL BASED ON A KINETIC
THEORY APPROACH**

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SUMMARY

In the previous report the governing equations for the flow of granular materials down an inclined plane, modeled by the constitutive theory proposed by Boyle and Massoudi (1990) were derived. We end up with two coupled ordinary differential equations. These equations are solved numerically subjected to the appropriate boundary conditions. The effect of various non-dimensional parameters on the volume fraction and velocity are presented in the form of graphs in this report.

INTRODUCTION

Kinetic theory approaches in the formulation of rapid flows of granular materials have attracted considerable attention in recent years. There are several models which have been suggested to describe the rapid flow of granular materials which are derived using a statistical approach. Boyle and Massoudi (1989) have written comprehensive survey article of the constitutive equations and the laws governing the flow of granular materials. A major difference between the continuum theories discussed previously and the theories based on a statistical approach is the concept of "granular temperature" which is introduced in the latter approach. Granular temperature describes the fluctuating velocity of the flow of granular solids and in this sense is similar to the temperature in a gas due to the fluctuating motion of the gas molecules. However, it is not clear how this quantity can be measured. Several models have been proposed by various investigators using ideas of kinetic theory [cf. Ogawa et al. (1980), Jenkins and Savage (1983), Lun et al. (1984), Jenkins and Richman (1985)].

GOVERNING EQUATIONS

Boyle and Massoudi (1990) proposed a model which can exhibit normal-stress effects by including the effects of the gradient of the volume distribution function, which is the noticeable difference between the model proposed by Lun, Savage and co-workers (1984).

The granular stress tensor $\mathbf{T}$ is the sum of the two terms, reflecting that momentum can be transported by the uninterrupted streaming of granules $\mathbf{T}_k$ and by the essentially instantaneous transport from one center to another during a collision $\mathbf{T}_c$. The granular stress tensor $\mathbf{T}$ follows from that of Lun et al. (1984), but there is an additional contribution to $\mathbf{T}$ that is given by

$$\mathbf{T}_m = \frac{4}{5} \eta g_0 \rho \theta v \frac{1}{(1 - v V_0)^2} (2 \mathbf{M} + \pi \mathbf{M} \mathbf{1}) \quad (1)$$

The stress tensor for a rapidly sheared granular material is found by adding the above individual contributions, which is given by

$$\begin{aligned}
\mathbf{T} = & \left\{ \rho \theta (1 + 4 \eta v g_0) + \left(\frac{2 \mu}{3 \eta (2 - \eta) g_0} (1 + \frac{8}{5} \eta v g_0) \right. \right. \\
& \left. \left. \{ 1 + \frac{8}{5} \eta (3 \eta - 2) v g_0 \} - \frac{3}{5} \mu_b \eta \right) \nabla \cdot \mathbf{u} \right. \\
& + \frac{4}{5} \eta v g_0 \rho \theta \frac{V_0^{*2} \bar{\sigma}^2}{(1 - v V_0^*)^2} \pi \mathbf{M} \left. \right\} \mathbf{1} \\
& - \left\{ \frac{2 \mu}{\eta (2 - \eta) g_0} (1 + \frac{8}{5} \eta v g_0) [1 + \frac{8}{5} \eta (3 \eta - 2) v g_0] + \frac{6}{5} \mu_b \eta \right\} \mathbf{D} \\
& + \frac{8}{5} \eta g_0 \rho \theta v \frac{V_0^{*2} \bar{\sigma}^2}{(1 - v V_0^*)^2} \mathbf{M}
\end{aligned} \tag{2}$$

$$\text{where, } \mu_b = \frac{256 \mu v^2 g_0}{5 \pi}$$

$$\mu = \frac{5 m}{16 \bar{\sigma}^2}$$

$$V_0^* = \frac{6 V_0}{\pi \bar{\sigma}^3}$$

$$\mathbf{M} = \nabla \mathbf{v} \otimes \nabla \mathbf{v}$$

$$\varepsilon = \frac{1}{2} (1 + e)$$

$$g_0 = \frac{V_0^*}{4(1 - v V_0^*)}$$

V_0 represents the volume per neighboring particle, and $\mathbf{D}$ is the stretching tensor. Also ρ is the bulk density, v is the volume fraction, θ is the granular temperature, e is the coefficient of restitution, $\bar{\sigma}$ is the diameter of the spherical particles and m is the mass.

Consider the flow of granular materials modeled by the above model down an inclined plane (cf. Figure 1) due to the action of gravity [Savage (1979), Johnson and Jackson (1987), Johnson, et al. (1990), Hui, et al. (1984), Richman and Marciniec (1990),

Hutter, et al. (1986a, b)]. In this problem we consider steady one dimensional flow of incompressible granular materials (i.e. $\rho_p = \text{constant}$) down an inclined plane, where the angle of inclination is α . The governing equations of motion are the conservation of mass and momentum. We assume that the granular temperature is constant and the volume fraction, and velocity to be of the form:

$$\begin{aligned} v &= v(y) \\ u &= U(y)i \end{aligned} \quad (3)$$

For the above flow field, the conservation of mass is automatically satisfied. From the balance of linear momentum the non-dimensional equations are

$$\begin{aligned} \frac{2 D_3 v^2}{(1-vV_0^*)^3} \frac{dv}{d\bar{y}} \frac{d^2 v}{d\bar{y}^2} + \frac{D_3 v (2+v V_0^*)}{(1-vV_0^*)^4} \left\{ \frac{dv}{d\bar{y}} \right\}^3 \\ + \left\{ D_1 + \frac{D_2 v (2-vV_0^*)}{(1-vV_0^*)^2} \right\} \frac{dv}{d\bar{y}} = -v \cos \alpha \end{aligned} \quad (4)$$

$$\begin{aligned} \left\{ D_4 (1-vV_0^*) + D_5 v + \frac{D_6 v^2}{(1-vV_0^*)} \right\} \frac{d^2 \bar{U}}{d\bar{y}^2} \\ + \left\{ D_3 - D_4 V_0^* + \frac{D_6 v (2-vV_0^*)}{(1-vV_0^*)^2} \right\} \frac{dv}{d\bar{y}} \frac{d\bar{U}}{d\bar{y}} = -v \sin \alpha \end{aligned} \quad (5)$$

$$\text{with, } \bar{y} = \frac{y}{L}; \quad \bar{U} = \frac{U}{u_0} \quad (6)$$

where L is the characteristic length and u_0 is the reference velocity. And the boundary conditions become

$$\bar{U} = 0 \quad \text{at } \bar{y} = 1 \text{ (on the inclined plane)} \quad (7)$$

$$N = \int_0^1 v d\bar{y} \quad (8)$$

and,

$$\frac{d\bar{U}}{d\bar{y}} = 0 \quad (9)$$

$$v = 0 \quad \text{at } \bar{y} = 1 \text{ (at the free surface)} \quad (10)$$

$$D_1 = \frac{E_1}{L \gamma g}; \quad D_2 = \frac{E_2 V_0^*}{L \gamma g}; \quad D_3 = \frac{3 E_6 V_0^*}{L^3 \gamma g}$$

$$D_4 = \frac{6 E_3 u_0}{L^2 \gamma g V_0^*}; \quad D_5 = \frac{3 E_4 u_0}{2 L^2 \gamma g}; \quad D_6 = \frac{E_7 u_0 V_0^*}{8 L^2 \gamma g} \quad (11)$$

where,

$$E_1 = \gamma \theta, \quad E_2 = \gamma \theta \eta$$

$$E_3 = \frac{2 \mu}{3 \eta (2-\eta)}, \quad E_4 = \frac{16 \mu (3\eta-1)}{15 (2-\eta)}$$

$$E_5 = \mu \eta \left\{ \frac{128 (3\eta-2)}{75 (2-\eta)} - \frac{768}{25 \pi} \right\}$$

$$E_6 = \frac{4}{5} \gamma \theta \eta \bar{\sigma}^2 V_0^{*2}$$

$$E_7 = \mu \eta \left\{ \frac{128 (3\eta-2)}{25 (2-\eta)} + \frac{1536}{25 \pi} \right\} \quad (12)$$

Results and Discussion

The system of equations (4) and (5), with the boundary conditions (7), (8), (9) and (10) are solved numerically using a collocation code COLSYS [cf. Ascher et al. (1981)]. A parametric study for the volume fraction and velocity profiles was carried out for the base solution and the results are presented in the form of graphs.

The manner in which the volume fraction and velocity profiles change with D_2 and N is shown in Figures 2 and 3 respectively. Notice, that the volume fraction decreases from the surface of the plane to the free surface and attains a zero value for the volume fraction at the free surface due to such a prescription for the volume fraction at the free surface. Figure 4 shows the effect of D_4 on the velocity profiles. Here, the velocity decreases as D_4 increases. Finally, Figure 5 depicts how the velocity profile changes, when D_6 is changed.

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Figure 1. Flow Down An Inclined Plane

Figure 2. Effect of D_2 and N on Volume Fraction Profiles.

Figure 3. Effect of D_2 and N on Velocity Profiles.

Figure 4. Effect of D_4 on Velocity Profiles.

Figure 5. Effect of D_6 on Velocity Profiles.

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