Nearly time-optimal feedback control of a magnetically levitated
photolithography positioning system

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ABSTRACT

This paper focuses on the development of an approximate time-optimal feedback strategy for conducting rest-to-rest maneuvers of a magnetically levitated table. Classical switching curves are modified to account for the complexities of magnetic actuation as well as the coupling of the rigid body modes through the control. A smooth blend of time-optimal and proportional-derivative controls is realized near the destination point to correct for inaccuracies produced by the approximate time-optimal strategy. Detailed computer simulations of the system indicate that this hybrid control strategy provides a significant reduction in settling time as compared to proportional-derivative control alone.

1. INTRODUCTION

Recent developments in adaptive structures technology have focused on the use of embedded sensors and actuators for active shape control. Among the various concepts in actuation, shape memory alloys¹, PZT actuators², and polyelectrolyte gels³ have received a great deal of attention. Although these materials are well suited for damping a structure's flexible body modes, they provide little control authority over rigid body motion. Consequently, applications that require rigid body maneuvers must employ alternative sources of actuation. Electromagnetic actuators offer an interesting solution by minimizing friction and providing precision control authority⁴. Recently, this method of actuation was employed to improve the process of integrated circuit (IC) manufacturing⁵. Based on advanced photolithography techniques, IC manufacturing requires precision placement of a platen containing silicon wafers. With magnetic levitation actuators, accuracy of better than 10 nm can be achieved for typical rest-to-rest maneuvers of the platen. As with other actuation devices, the use of time-optimal control strategies would improve the speed of the manufacturing process. However, the inherent nonlinearities of magnetic levitation devices preclude generating traditional time-optimal switching surfaces. In this paper, simple modifications are imposed to enable the use of modal switch curves for conducting nearly time-optimal rest-to-rest maneuvers. These modifications account for the modal coupling through the control as well as the nonlinearities associated with magnetic levitation.

The motion of the platen containing the ICs is governed by a two stage hierarchical position control scheme. The first (coarse) stage involves controlling large scale two dimensional translations via motor driven screws. The second (fine) stage involves controlling six degree of freedom rigid body motion via magnetic-levitation actuators. This paper focuses on the optimization of the fine stage control system. The interaction of the dynamics and controls of the fine stage positioning system are completely described by a previously developed computer simulation. This simulation has been shown to accurately model the behavior of the actual hardware⁶. A brief description of the system is given in the following section. In section 3, the inherent nonlinearities of the magnetic levitation actuators are addressed and an approximate time-optimal control strategy is developed. The control strategy is demonstrated on the computer simulation in section 4 and the performance is compared to the performance obtained using the original proportional-derivative (PD) controller. Although significant improvement is realized for relatively large maneuvers, hardware constraints limit the effective application of the TOC strategy for small maneuvers. Some concluding remarks are given in section 5.

2. SYSTEM DESCRIPTION

A complete description of the magnetically levitated fine stage is given in Reference 6. However, the details required for controller design are restated here for clarity. A schematic diagram of the fine stage platen is shown in Figure 1. The platen floats inside the coarse stage frame which contains supporting actuation hardware. Translational motions along the X and Y axes are limited to $\pm 200\mu\text{m}$ before the platen edges contact the coarse stage frame. Made of solid aluminum, the platen is essentially rigid and is adequately modeled as six uncoupled rigid body modes of the form

$$M\ddot{X}(t) = F(t) \quad (\text{EQ 1})$$

The mass matrix, modal displacement vector, and modal force vector are defined as

$$\begin{aligned} M &= \text{diag}[m \ m \ m \ I_x \ I_y \ I_z] \\ X(t) &= [x(t) \ y(t) \ z(t) \ \theta_x(t) \ \theta_y(t) \ \theta_z(t)]^T \\ F(t) &= [F_x(t) \ F_y(t) \ F_z(t) \ T_x(t) \ T_y(t) \ T_z(t)]^T \end{aligned} \quad (\text{EQ 2a,b,c})$$

in which m is the platen mass, I_x, I_y, I_z are the principal mass moments of inertia, $x(t), y(t), z(t), \theta_x(t), \theta_y(t), \theta_z(t)$ are the modal displacements and rotations, and $F_x(t), F_y(t), F_z(t), T_x(t), T_y(t), T_z(t)$ are the modal control forces and torques. Permanent magnets are located below the platen to offset the effects of gravity and the effects of cross-axis coupling have been shown to be insignificant⁶. Although the gyroscopic coupling terms are included in the computer simulation, they are not considered in the controller design. Furthermore, no damping terms are included in equation 1 since the magnetic levitation minimizes frictional losses.

The modal displacements are extracted via a geometric transformation of the signals from six capacitive displacement sensors. The sensor measurements are processed in a digital signal processor with a sampling rate of 2500 Hz. Modal velocity information is obtained using the three point backward difference formula⁷

$$\dot{X}(t) \approx \frac{1}{2T} [X(t-2T) - 4X(t-T) + 3X(t)] \quad (\text{EQ 3})$$

in which T is the sampling interval. The modal displacements and velocities form the basis of the original PD control algorithm which yields a 6x1 vector of desired modal forces.

The purpose of the fine stage control system is to regulate the out-of-plane motions while conducting x-y translational motions in the range $\pm 100\mu\text{m}$. The stage provides positioning accuracies better than 10 nm and is activated after the coarse stage directs gross x-y translations via motor driven ball screws. Sixteen magnetic levitation actuators whose outputs are proportional to the square of the supply currents and inversely proportional to the square of the actuator gaps supply the forces for controlling fine stage motion. Since each magnetic levitation device is capable of producing only unidirectional pulling forces, the sixteen devices are grouped into eight bidirectional actuator pairs. The actuator forces are related to the modal forces via the geometric transformation

$$F(t) = DF_a(t) \quad (\text{EQ 4})$$

The transformation matrix and the actuator force vector are defined as

$$D = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & d_3 & d_3 & -c & c & c & -c \\ -d_3 & -d_3 & 0 & 0 & c & -c & c & -c \\ d_4 & -d_4 & d_4 & -d_4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_a(t) = [F_{a1}(t) \ F_{a2}(t) \ F_{a3}(t) \ F_{a4}(t) \ F_{a5}(t) \ F_{a6}(t) \ F_{a7}(t) \ F_{a8}(t)]^T \quad (\text{EQ 5a,b})$$

in which $c = 0.5(d_1 + d_2)$, d_1, d_2, d_3, d_4 represent actuator moment arms, and $F_{ai}(t)$, $i=1-8$ represent the forces supplied by the actuator pairs. The system parameters are summarized in Table 1.

Table 1: Fine Stage Parameters

Platen Mass (m)	0.78 slugs
X-Axis Mass Moment of Inertia (I_x)	8.4 slugs-in ²
Y-Axis Mass Moment of Inertia (I_y)	8.4 slugs-in ²
Z-Axis Mass Moment of Inertia (I_z)	15.6 slugs-in ²
Moment Arm d_1	5.145 in
Moment Arm d_2	2.712 in
Moment Arm d_3	0.658 in
Moment Arm d_4	4.75 in

Each actuator pair is nominally capable of generating +/- 15 pounds of force when the platen is centered in the coarse stage frame. However, this peak actuator force can vary from 10 to 25 pounds depending on the platen location. The desired forces from the actuator pairs are based on the desired modal forces from the control algorithm and are computed as a minimum norm solution to equation 4. The actuator gaps and desired actuator forces specify the required supply current for each actuator pair. Since an explicit expression for the required supply current is difficult to obtain due to the actuator nonlinearities, a two dimensional table is used to designate the proper currents for large forces, while a curve fit is used for small forces. The resulting actuator forces are transformed to yield the actual modal forces used to drive the table motion. Total time for input to output computation is 200 microseconds.

3. TIME-OPTIMAL CONTROL STRATEGY

As previously mentioned, the fine stage positioning system is equipped with a functioning critically damped PD controller. Without loss of generality, we assume that the origin of the state space is the target point for all rest-to-rest maneuvers in this paper. Accordingly, the PD controller computes desired modal control forces according to⁶

$$F(t) = -2\omega M\dot{X}(t) - \omega^2 MX(t) \quad (\text{EQ 6})$$

in which ω is the controller bandwidth. For nominal translations, the bandwidth is limited to approximately 50 hz due to actuator saturation which can produce an underdamped response. The relatively slow response of this controller prompted this investigation into the applicability of classical time-optimal control strategies.

3.1 Time-optimal control considerations

Time-optimal control of low-order completely controllable systems is of the classical bang-bang form and can be readily obtained by minimizing the associated Hamiltonian and subsequently generating state based switching curves⁸. For high-order systems, numerical techniques exist for finding open-loop solutions for a given maneuver. However, several intricacies of this real system preclude the direct application of classical techniques. For example, significant delays associated with the finite sampling rate, processing operations, and actuator dynamics severely impair the control system's ability to instantly switch between full positive and full negative forces. Furthermore, the peak actuator forces are not constant but instead are nonlinear functions of the actuator gaps. And although each individual mode is completely controllable with respect to the modal control forces, the coupling of the modes through the actuator forces renders the overall system singular in the control⁹. This guarantees the presence of singular arcs in the controller signum function. Therefore, the time-optimal control is not completely specified by the simple necessary condition of optimality. Higher order necessary conditions can be considered to derive an optimal control strategy for singular systems, but the presence of nonlinear actuator dynamics coupled with the need for a simplified feedback strategy makes this approach inappropriate for this application.

The approximate technique developed in this paper is based on designing separate time-optimal control laws for each of the system's modes assuming a constant peak modal force level. Because each of these simplified modal time-optimal control problems are normal, classical bang-bang switching curves are developed to govern the modal control forces. Afterward, some modifications to the basic switching curves are required to account for the significant processing delays and actuator dynamics. Since each of the six modes in equation 1 have a similar form, the following development will concentrate on translations along the X-axis. Application of the final switching strategy to the remaining modes follows directly.

3.2 Classical rigid body time-optimal control

Assuming a constant peak modal force along the X axis, the classical time-optimal switch curve is given as a function of velocity as⁸

$$\bar{x} = \frac{-m}{2\bar{F}_x} \dot{x} |\dot{x}| \quad (\text{EQ 7})$$

where $\bar{x}$ is the modal switch curve displacement, $\dot{x}$ is the velocity, and $\bar{F}_x$ is the nominally available modal control force which is obtained by summing the nominal contributions from each of the pertinent actuators. The optimal feedback switching strategy is summarized as

$$\begin{aligned} x \neq \bar{x} &\rightarrow F_x = \bar{F}_x \text{sgn}(\bar{x} - x) \\ x = \bar{x} &\rightarrow F_x = -\bar{F}_x \text{sgn}(\dot{x}) \end{aligned} \quad (\text{EQ 8a,b})$$

According to the above control law, the modal control force assumes a maximum level (+ or -) until the modal trajectory intersects the switch curve. At that instant, the control executes a single sign switch and follows the switch curve to the origin.

3.3 Conservative control force estimates

Since the modes are coupled through the control actuators, complications to the above strategy can arise due to actuator sharing. For example, if the X translational and Z rotational modes demand large control forces, then actuator F_{a1} will likely saturate. As a result, the modal control forces required to satisfy the switching strategies are not attainable. During the early phase of a rest-to-rest maneuver, actuator saturation is not harmful since no precise path is required to transfer the mode to the switch curve¹⁰. However, in the final phase of control, precise actuation is required for the mode to follow the switch curve to the origin. Any decrease in the modal force will cause the mode to overshoot its target and produce oscillatory motion. To prevent this, the switch curve must be designed based on a conservative estimate of the maximum modal control force that can be produced under usual circumstances. The modified switch curve becomes

$$\bar{x} = \frac{-m}{2\alpha\bar{F}_x} \dot{x} |\dot{x}| \quad (\text{EQ 9})$$

where α is the scaling factor used to adjust the peak modal force. With α assigned a value between 0 and 1 depending on the number of modes participating in the maneuver, the optimal switching strategy becomes

$$\begin{aligned} x \neq \bar{x} &\rightarrow F_x = \hat{F}_x \text{sgn}(\bar{x} - x) \\ x = \bar{x} &\rightarrow F_x = -\alpha\bar{F}_x \text{sgn}(\dot{x}) \end{aligned} \quad (\text{EQ 10a,b})$$

where $\hat{F}_x$ represents all force possible. In this modified control scheme, the modal control forces assume whatever force is available until the switch curve is reached. The control then switches sign and assumes the conservative control force level, with the modal trajectory following the path traced by the switch curve. Although some loss of performance results from conservatively estimating the available control force on the switch curve, the losses related to actuator saturation are generally greater.

3.4 Customizing for actuator dynamics

Before applying the switching algorithm to the fine stage positioning system, other modifications to the control strategy must be made. To account for the time required by the controller to switch between peak positive and peak negative control forces, an onset switch curve is needed to begin the control transition prior to reaching the primary switch curve. Assuming that the control assumes a half cosine profile in switching between peak positive and peak negative force, the onset switch curve can be expressed as¹¹

$$\bar{x}_o = \frac{-m}{2\alpha\bar{F}_x} \dot{x} |\dot{x}| - T_t \dot{x} - \frac{2T_t^2 \alpha \bar{F}_x}{m\pi^2} \text{sgn}(\dot{x}) \quad (\text{EQ 11})$$

where $\bar{x}_o$ is the switch onset displacement and T_t is the time required for transition from peak to peak. The transition time must be selected such that the modal trajectory closely coincides with the primary switch curve when the control transition is complete. As shown in Figure 2, the region between the primary and the onset switch curves is denoted S . Also shown is a region of PD control denoted R . The control in R completes the transition to the origin and accommodates for the finite time required for the actuator force to make the transition from peak force to zero. Instead of defining a second transition time, the PD control is used because of its ability to correct for any discrepancies in the trajectory relative to the switch curve which result from modelling inaccuracies and an imprecise sampling of the phase plane. The left and right bounds are determined by equating the conservative peak force with the PD control law to obtain

$$\pm \alpha \bar{F}_x = -2m\omega \dot{x} - m\omega^2 x \quad (\text{EQ 12})$$

The upper and lower bounds of the region are based on the point of intersection of the left and right bounds with the primary switch curve as shown in Figure 2. Setting x in equation 12 equal to $\bar{x}$ in equation 9 and solving for the 4 points of intersection, the upper and lower bounds of R are given by the points closest to the origin as

$$\dot{x} = \pm (2 - \sqrt{2}) \frac{\alpha \bar{F}_x}{m\omega} \quad (\text{EQ 13})$$

Defining the bounds of R as in equation 13 ensures a smooth transition from time-optimal to PD control. Combining all of the modifications, the approximate time-optimal feedback strategy can be summarized as

$$\begin{aligned} x \in R &\rightarrow F_x = -2m\omega \dot{x} - m\omega^2 x \\ x \in S &\rightarrow F_x = -\alpha \bar{F}_x \text{sgn}(\dot{x}) \\ x \notin R \text{ or } S &\rightarrow F_x = \hat{F}_x \text{sgn}(\bar{x}_o - x) \end{aligned} \quad (\text{EQ 14a,b,c})$$

The fundamental difference between the approximate time-optimal control strategy described in equation 13 and the original PD control strategy described in equation 6 is that actuator saturation no longer limits the controller bandwidth. Instead, saturated (or nearly saturated) actuator forces are used to drive the system to a region of PD control. Although the size of the region is dictated by controller bandwidth constraints, the bandwidth is limited only by the speed of the control hardware.

4. TIME-OPTIMAL CONTROL SIMULATION

To demonstrate the approximate time-optimal control strategy developed in the previous section, we consider a rest-to-rest translation along the X axis from -150 μm to 0 μm . The nominal peak force is set to 30 lbs since two actuators drive motion in the X direction as shown in equation 5. For this example, actuator saturation is avoided by assigning the remaining five modes zero initial conditions. The control scaling factor α is accordingly set equal to unity. Based on previous experimentation, the transition time from peak positive to peak negative control force that defines the onset switch curve is set equal to 0.0023 seconds. Also, it was determined that for small maneuvers the PD bandwidth can be increased to 100 Hz before actuation delays begin to adversely affect the system response. In this example, the controller bandwidth used to define R is set equal to 85 Hz.

The x mode phase plane trajectory and the calculated switch curves are shown in Figure 3. Initially, a full positive force drives the trajectory toward the onset switch curve. At the first sample point beyond this curve, a control switch is initiated and the trajectory begins its downward descent toward the origin. Although the transition to full negative control places the trajectory very near the primary switch curve, the trajectory slightly overshoots the curve and the origin. The control in R compensates for this discrepancy, eventually driving the system to the origin. As shown in Figure 4, a comparison of the displacement profiles for this control and the original 50 Hz bandwidth PD control demonstrates a 50% decrease in settling time is achieved using the time-optimal feedback strategy. This improvement results from the superior exploitation of the available control authority as shown in Figure 5. The desired and actual control forces for both controls illustrate that the time-optimal control provides a larger impulse during the early phase of the maneuver. Furthermore, this figure demonstrates a smooth blend of the desired controls as the trajectory enters R at approximately 0.01 seconds.

To illustrate the effects of actuator sharing, a combined translation and rotation rest-to-rest maneuver is considered. An initial θ_z of -3000 $\mu\text{radians}$ enhances the initial conditions of the previous example. Considering equation 5, the nominal torque available for controlling this mode is 285 inch-pounds. To avoid actuator saturation in

the final phases of the control, a scaling factor α of 0.6 is used to design the switch curves for both the x and θ_z modes. The x and θ_z phase plane trajectories are shown in Figures 6 and 7, respectively. While the x trajectory drastically undershoots the primary switch curve and the origin, the θ_z trajectory almost exactly follows its primary switch curve. As shown in Figures 8 and 9, these trajectories produce 20% and 30% decreases in the settling time as compared to the original PD control. The undershoot of the x trajectory is caused by the 15 pound saturation of the control force as shown in Figure 10. Since this is well below the 30 pound nominal peak force, the transition to the negative control force requires only about 0.0015 seconds and is completed well before the trajectory encounters the primary switch curve. This undershoot causes an abrupt transition to the PD region at 0.013 seconds. Conversely, the close correlation of the θ_z trajectory and its switch curve produces a smooth transition to R as shown in Figure 11.

In the previous examples, the size of the region R was designated by bandwidth constraints of the controller hardware. The controller bandwidth was increased from the initial 50 Hz to 85 Hz by removing the stigma of actuator saturation during the early phases of the control. Note that for small maneuvers, that is initial conditions inside R , the time-optimal control strategy provides no improvement since the bandwidth of the original controller could be increased without saturating the actuators. Further improvements over the original control can be realized by improving the response time of the actuation hardware. Such improvements will permit further increases in the PD bandwidth and reduce the size of R .

5. CONCLUSIONS

This paper described an approximation to time-optimal control of a nonlinear photolithography positioning system. The major elements of the system dynamics and control system were discussed, highlighting the difficulties associated with developing an exact time-optimal feedback control strategy. As an alternative approach, the system was decomposed into normal modal problems and an approximate time-optimal strategy was developed. The control was demonstrated on a detailed computer simulation of the system and the performance was compared to the original PD controller. In the cases considered, the time-optimal strategy offered significant reductions in settling time as compared to the PD control. However, performance enhancements can only be realized for relatively large maneuvers due to the limitations on the response time of the electromagnetic actuators. Application of the time-optimal strategy for smaller maneuvers will require hardware modifications to increase the speed of the processing and actuation subsystems.

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Figure 1. - Fine Stage Platen with Magnetic Levitation Actuators

Figure 2. - Time-Optimal Control Regions

Figure 3. - X Phase Plane Trajectory for a Commanded X Translation.

Figure 4. - Comparison of Displacements for a Commanded X Translation.

Figure 5. - Comparison of Control Forces for Commanded X Translation.

Figure 6. - X Phase Plane Trajectory for a Commanded X Translation and Z Rotation.

Figure 7. - Theta-Z Phase Plane Trajectory Commanded X Translation and Z Rotation.

Figure 8. - Comparison of Displacements for a Commanded X Translation and Z Rotation.

Figure 9. - Comparison of Angles for a Commanded X Translation and Z Rotation.

Figure 10. - Comparison of Control Forces for a Commanded X Translation and Z Rotation.

Figure 11. - Comparison of Control Torques for a Commanded X Translation and Z Rotation.

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