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TITLE: IMPURITIES AND CONDUCTIVITY IN A D-WAVE SUPERCONDUCTOR

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Impurities and Conductivity in a D-wave Superconductor*

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Abstract

Impurity scattering in the unitary limit produces low energy quasiparticles with anisotropic spectrum in a two-dimensional d-wave superconductor. We describe a new *quasi-one-dimensional* limit of the quasiparticle scattering, which might occur in a superconductor with short coherence length and with *finite* impurity potential range. The dc conductivity in a d-wave superconductor is predicted to be proportional to the normal state scattering rate and is impurity-*dependent*. We show that *quasi-one-dimensional* regime might occur in high-T_c superconductors with Zn impurities at low temperatures $T \lesssim 10$ K.

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In this short note I will address *the role of a strongly scattering impurities with finite range on the dc conductivity* in a short coherence length superconductor. This is report on the work, done in collaboration with A. Rosengren and B. Altshuler [1].

It is well known that scalar impurities are pair breakers in d-wave and any other nontrivial pairing state superconductor [2–4]. They produce a finite lifetime of the quasiparticles in the nodes of the gap, a finite density of states at low energy, and a *finite* low frequency conductivity at low temperatures, ignoring localization effects. For the special case of a 2D superconductor with a d-wave gap, a straightforward calculation yields the surprising result that dc conductivity $\sigma(\omega \to 0)$ is a "universal" number [4], *independent* of the lifetime of quasiparticle (but dependent on the anisotropy ratio of the velocities of the quasiparticle in the node of the gap) [5]. However, recent experiments on microwave absorption in YBCO crystals with Zn impurities [6] show a *linear* temperature dependence of the conductivity for pure samples, evolving to the quadratic behavior for higher impurity concentration, and low-temperature conductivity, inversely proportional to the impurity concentration.

i) We find a new *quasi-one-dimensional regime* for dc conductivity in superconductors with a short coherence length ξ, comparable to the range of impurity potential λ. The quasiparticle contribution to dc conductivity is governed by self-energy $\Sigma(\omega \to 0) = -i\gamma$ and by the phase space available for low-energy quasiparticles. The quasiparticle dispersion is strongly anisotropic in the vicinity of the nodes in a 2D d-wave superconductor: $E_k = \sqrt{v_1^2 k_1^2 + v_F^2 k_3^2}$ and $v_1/v_F \sim \Delta_0/\epsilon_F$. Here we linearized spectrum in the vicinity of the nodal point close to $(\frac{\pi}{2}, \frac{\pi}{2})$, so that k_1 is the momentum along the Fermi surface and k_3 is perpendicular. We find that the overall contribution to the conductivity depends on the ratio of the energy of the quasiparticle to the scattering rate $v_1\lambda^{-1}/\gamma$, λ is the range of the impurity potential, and we get at $T = 0$:

$$\sigma(\omega \to 0) = \frac{e^2}{2\pi\hbar} \frac{2}{\pi^2} \frac{v_F}{v_1} \left(1 + \left(\frac{\gamma}{2v_1\lambda^{-1}}\right)^2\right)^{-1/2}. \quad (1)$$

For $v_1\lambda^{-1}/\gamma \ll 1$ quasiparticle dynamics is essentially *quasi-one-dimensional* and conductivity *depends* on the impurity concentration $\sigma_{Q1D} \sim n_{imp}^{-1}$. Our model predicts that the dc conductivity at low temperature should be ***proportional to the scattering rate in the normal state***. This limit might occur in high-T_c superconductors, for which we estimate $\lambda/a \sim 1-3$ and $\Delta_0/\epsilon_F \sim 10^{-1}$. In the limit $\lambda \to 0$ Eq. (1) gives the "universal" dc conductivity $\sigma_{2D} \sim v_F/v_1$, found in [4].

To explain this effective *change of dimensionality* we note that transverse momentum is limited by $k_1 < 2/\lambda$ and quasiparticle dispersion on such a small scale is irrelevant, compared to γ. The condition for this to occur is precisely $v_1\lambda^{-1}/\gamma \ll 1$. The transverse (along the Fermi surface) scattering does not contribute effectively to the conductivity; we call this case a *quasi-one-dimensional* limit. The existence of this limit is the result of the *finite* impurity range λ [7]. In the opposite limit $v_1\lambda^{-1}/\gamma \gg 1$, which holds for "zero" impurity range, we recover standart unitary scattering results [8–10].

ii) Here we will explain the assumptions we made to calculate conductivity. We assume that impurities are strong scatterers with s-wave phase shift $\delta_0(\mathbf{q}) \simeq \pi/2$, for $|\mathbf{q}| < \lambda^{-1}$, where $\mathbf{q}$ is wavevector, counted from the Fermi wavevector. This assumption is well supported by experiments on cuprates with Zn imputrities. The origin of strong potential impurity scattering in high-T_c superconductors is the highly correlated antiferromagnetic nature of the normal state. The second assumption is about the finite range λ of the impurity potential, which palys role of the momentum cut off in the momentum dependence of phase shift $\delta_0(\mathbf{q})$. It is as well motivated by the fact that high-T_c superconductors have a substantial antiferromagnetic coherence length $\xi_{AFM} \sim 3a$ at the transition temperature. A scalar impurity will produce distortions in magnetic correlations on the range of the ξ_{AFM}. On the other hand superconducting coherence length $\xi \sim 20$ Å is comparable to this scale and thus, the range of the potential is finite on the scale relevant for superconductivity. This point should be contrasted to the case of heavy-fermion superconductors, where the coherence length is $\sim 10^3$ Å, and therefore, any potential impurity will have its range substantially shorter than the coherence length. We retain this cut off finite and on the order of few lattice constants ($\lambda \sim 2a$). This implies that impurities still are well screened and s-wave scattering is dominant.

iii) To calculate the quasiparticle conductivity we use lowest order bubble diagram with self-consistent Green functions with *no vertex corrections*, see for example [4]. For the dc conductivity we get [11]:

$$\sigma(\omega \to 0) = \frac{e^2}{\hbar}\,\frac{4v_F^2}{\pi^2}\sum_{k}{}' \int d\epsilon\, (-\partial_\epsilon n(\epsilon))(|G''(\boldsymbol{k},\, \epsilon)|^2 + |F''(\boldsymbol{k},\, \epsilon)|^2), \qquad (2)$$

where, linearizing quasiparticle spectrum in the vicinity of nodes, $G''(\boldsymbol{k},\, \omega = 0) = \gamma/(\gamma^2 + (v_1 k_1)^2 + (v_F k_3)^2)$, $F''(\boldsymbol{k},\, \omega = 0) = 0$. The momentum integral in Eq. (2) is cut off at $|\mathbf{k}| \leq 2/\lambda$ and it yields the final formula Eq. (1) for $T = 0$ with $O(T^2)$ corrections.

For the particular case of strong disorder $v_1\lambda^{-1}/\gamma \ll 1$, considered in [1], relation between scattering rate in the superconducting state $\gamma = i\Sigma(\omega \to 0)$ and scattering rate in the normal state $\Gamma = n_{imp}/\pi N_0$ is $\gamma = \pi/8\ p_F\lambda\ \Gamma$. Note that the scattering rate at low temperatures is *linearly* proportional to Γ, as opposed to the $\Gamma^{\frac{1}{2}}$ dependence in the standart unitary scattering case for $v_1\lambda^{-1}/\gamma \gg 1$. The assumption $v_1\lambda^{-1}/\gamma \ll 1$ is consistent at $\lambda \sim 2a$ for $\Gamma \geq 20K$. This estimate shows that the *quasi-one-dimensional* regime of quasiparticle scattering should occur in not too clean samples at $T < \gamma$. In this limit scattering rate in the superconducting state is of the same order as the normal state scattering rate $\gamma \sim 2\Gamma \sim 40$ K for $p_F\lambda \sim 6$ and is similar to the scattering rate in the *2D* limit: $\gamma/\tilde{\gamma} \sim \sqrt{\Gamma/\Delta_0}\ p_F\lambda \simeq 1$. The finite density of states $N(\omega \to 0)/N_0 = \Gamma/\Delta_0 \sim n_{imp}$, *linear* in impurity concentration, is generated as well.

Using the above estimates we find the conductivity in *quasi-one-dimensional* regime

$$\sigma(\omega \to 0) = \frac{e^2}{\pi\hbar}\,\frac{16}{\pi^3}\,\frac{\hbar}{m\lambda^2\Gamma}. \qquad (3)$$

It is smaller than the normal state conductivity $\sigma(\omega \to 0)_{\text{normal}} = (e^2/\pi\hbar)\ (\epsilon_F/\Gamma)$ due to small factor $\hbar/(m\lambda^2\epsilon_F) \lesssim 1$ with $\lambda > a$. Conductivity is also impurity-*dependent*, $\sigma_{Q1D} \sim \Gamma^{-1} \sim n_{imp}^{-1}$. This model predicts that the dc σ at low temperatures should be *inversly proportional to the scattering rate in the normal state and to the impurity concentration.* We emphasize that *both* a higher value of the conductivity in the superconducting state *as well as* strong impurity dependence at low temperatures are observed experimentally in microwave absorption of YBCO [6]. It should be pointed out that we are interested in only elastic scattering and strong inelastic contribution to the scattering rate above T_c is not considered in this model.

iv) One can be almost certain that for dirty enough superconductors the *quasi-one-dimensional* regime will occur, since impurities will lead eventually to a very high scattering rate. The question remains about the competing phenomena, such as the localization of quasiparticles, which might occur earlier than *quasi-one-dimensional* regime. It is interesting to apply this model to the Zn impurities in the YBCO. The applicability of the results presented here to the different high-T_c materials depends on the ratio of scattering rate to the relevant quasiparticle energy. For the same impurity concentration different cuprates may be in different regimes, depending on ξ_{AFM}, Δ_0/ϵ_F, and Γ.

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