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COHERENT COMPTON X-RAY SOURCES*

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Abstract.

Coherent X-Rays can be produced by scattering of laser light by a beam of relativistic electrons, provided that the electron beam is density modulated or the scattering is done at, or near 90° . Since the coherent scattering is proportional to N^2 , where N is the number of electrons, and the incoherent scattering is proportional to N , also a modest degree of coherence can substantially increase the X-Ray yield. The theory of laser-electron beam scattering is reviewed and compared with the emission of radiation by an electron beam in an undulator. Examples of the practical implementation of an intense source of coherent X-Rays are discussed.

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Introduction.

X-Rays in the KeV range are used in Research and Industry as a diagnostic tool. Conventionally, they are produced by Röntgen tubes and their energy spectrum is characteristic of the metal of the anti cathode used. In recent years X-Rays are being produced by synchrotron radiation. The radiation spectrum from bending magnets in synchrotrons is a continuum, then desired energies, up to a few KeV, can be obtained by slicing the desired X-Ray energy out of this continuum. Higher energies are obtained by insertion devices, undulators and wigglers, consisting of arrays of magnetic dipoles with field alternately directed in opposite direction. An undulator produces a line spectrum, with a photon energy of the order of 25 KeV and multiples thereof, for the highest energy machines in existence (Argonne APS and Europe's ESRF). Wigglers produce a spectrum with an intermediate shape between bending magnets and undulators, and allow, in their super conducting implementation, to generate X-Rays of several KeV energy also in synchrotrons of smaller size (Brookhaven NSLS).

Another method to produce X-Rays is by (Compton) scattering of laser light by a beam of relativistic electrons. In a collision between a laser photon and an electron, a fraction of the energy of the electron is given to the photon, whose energy is in turn up shifted from the visible to the X-Rays. The spectrum of X-Ray energies is a continuum with a strong correlation between energy and angle, so a given X-Ray energy can be selected by an appropriate collimator. The cross section of Compton scattering of light by electrons is small, however, with present day high power lasers and high brightness electron beams, the yield of the process is interesting for many applications.

Some applications, as X-Ray microscopy or X-Ray holography require spatial and temporal coherent radiation. In a very narrow cone of radiation it is in general always possible to find some degree of coherence in the X-Rays. However, the aperture angle of the required collimator for synchrotron radiation from bending magnets or super conducting wigglers is very small and only allows a very low flux. The situation is better for undulators, since the interference that takes place in the successive periods of the device squeezes the radiation in a narrower cone, and indeed undulator radiation is being used for low intensity coherent X-Ray research. Ordinary laser-electron beam scattering only produce a marginal coherence, unless the electron beam is density modulated or the scattering is performed at large angles. In the first case, the electron beam acts as a moving diffraction grid for the primary radiation, in the second case the interaction length is so short that the laser photons are all actually reflected in a small region and the scattered radiation preserves the coherence properties of the laser.

The flux of incoherent X-Ray produced by an electron beam in a static magnetic field, as in synchrotron radiation, or by scattering by an electron beam of laser light is proportional to the

number of the electrons contained in a volume of interaction, N , The flux of coherent X-Rays is proportional to the square of this number, N^2 . Then, coherence can greatly enhance the yield of the process, and is worth pursuing even for applications, like X-Ray imaging, where the coherence *per se* is not interesting. In this case the term "collective" scattering, rather than "coherent" should be used.

Before proceeding further, it is in order to briefly discuss the definition itself of coherence. In a broad sense, radiation has spatial coherence if we can find in space a stationary set of surfaces over which the phase assumes a constant value, and has temporal coherent if it is essentially monochromatic in a given location, being the frequency spectrum the Fourier transform of the time spectrum of the radiation field. Spatial and temporal coherence can be measured from the correlation between photon counts in two locations separated by some distance or by photons counts in the same position at two distinct times. If the X-Rays are generated by free electrons, a measure of the correlation as a function of their number yields an equivalent set of definitions of coherence that implies that coherent radiation is proportional to N^2 and incoherent radiation to N . This criterion was used in an early work on synchrotron radiation, where it was shown that the degree of coherence in the continuum from the red to the UV decreases, since the number of photons per wavelength decreases along the spectrum (1).

1. Production of X-Rays by scattering.

In the Compton scattering of a laser beam by a charged particle beam, the energy of the scattered photons $\hbar\omega_s$ and of the incoming photons $\hbar\omega_L$ for head-on collisions are related by (2)

$$\hbar\omega_s \cong \hbar\omega_L \frac{2(1+\beta)\gamma^2}{1+\gamma\xi+\gamma^2\theta_s^2}, \quad [1]$$

with

$$\xi = 2(1+\beta) \frac{\hbar\omega_L}{mc^2}, \quad [2]$$

and with θ_s the angle of the scattered photon with respect to the collision line. Equation [1] shows that the frequency of the scattered radiation is up shifted by a factor $\approx 4\gamma^2$ with respect to the frequency of the primary photon. Visible laser light scattered by electrons of a few tens or

hundreds of MeV of energy will produce X-Rays in the KeV range. For a derivation of [1] see the Appendix.

Equation [1] shows that the scattered photon energy has a maximum in the forward direction of the particle beam, decreases with the increasing scattering angle, and reaches 1/2 of its maximum for

$$\gamma\theta \approx 1. \quad [3]$$

It is also found that the angular intensity is maximum forward.

The total flux of the scattered radiation, with N_X the number of scattered photons per unit time, is related to the luminosity of the process as follows

$$N_X = \frac{N_e N_s}{\Sigma} f \sigma, \quad [4]$$

where $N_e (=N)$ and N_s are the number of electrons and primary photons contained in the volume of interaction, Σ the common transverse cross section of the particle and laser beams, f the frequency of encounters, and σ the scattering cross section, that reaches its maximum on the backscattering axis (Thomson)

$$\sigma_0|_{\max} = 2r_0^2 \quad ; \quad r_0 = \frac{\mu_0 e^2}{4\pi m} \quad [5]$$

r_0 is the classical radius of the electron.

The flux in [4] is calculated in the assumption that the scattering is incoherent, i.e. the laser photons may collide with electrons in any position in the electron beam, producing random phases of the scattered photons.

Let us analyze the conditions to achieve coherent scattering. In the impulse approximation, the total electromagnetic field produced by scattering by an ensemble of N electrons confined in a volume V and with a normalized spatial distribution $\rho(\mathbf{r})$ is given by the Fourier transform of the density

$$\mathbf{E}_T = \mathbf{E}F(\mathbf{q}), \quad F(\mathbf{q}) = N \int_V \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r} \quad [6]$$

where $\mathbf{E}$ is the average amplitude radiated by the single scatterer at the position $\mathbf{r}$ and $\mathbf{q}$ is the momentum transfer $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ and $F(\mathbf{q})$ is called the form factor of the distribution. The intensity of the radiation is proportional to $|\mathbf{E}_T|^2$.

When $\mathbf{q}$ is small enough that $\mathbf{q} \cdot \mathbf{r} \ll 1$ over the entire integration volume, the integral in [6] assumes a finite value, and therefore the intensity is N^2 larger than from a single electron. But, if the wavelength is much shorter than the linear dimension of the electron bunch, the exponent in [6] performs large oscillations, and $F(\mathbf{q})$ falls down very rapidly with increasing $\mathbf{q}$, so, only the incoherent fraction of the radiation survives.

To increase the coherent contribution at small wavelength, we will examine two possibilities: (i) modulate the electron beam intensity so that in the form factor $F(\mathbf{q})$ of Eq. [6] there will be a substantial contribution from the harmonic content of periods of the order of $|\mathbf{q}|^{-1}$, and (ii) perform the scattering at large angles (say, 90°) so the resulting linear dimension of the scattering volume will be very small.

It is important to observe that collective scattering and coherence in the scattered beam are not necessarily synonymous. If collective scattering occurs, in the electron reference frame (ERF) the intensity of the scattered beam increases with the square of the number of electrons involved. However, in the laboratory scattering angles and photon energies transform according to the Lorentz formulas and in many cases the resulting radiation loses its spatial coherence, measured by the phase correlation of photons separated at the detector by some distance of interest.

2. Scattering by a modulated electron beam.

If the electron density is longitudinally modulated (for simplicity, we assume here that the scattering is head-on), the electron beam is equivalent to a moving grating and we can expect Bragg diffraction to enhance the X-Ray intensity in some directions and produce coherence in the scattered beam (3).

In the extreme case, assume that the beam is made by a linear array of N charge blobs at equal intervals a . In this case, the density distribution and the form factor of Eq. [6] can be written as

$$\rho(z) = \sum_{n=1}^N \delta(z - na), \quad F(q_z) = \sum_{n=1}^N e^{inaq_z} \quad [7]$$

and, if the Bragg scattering condition $q_z = 2\pi m/a$, with m an integer, applies, we get from [6] and [7] the result of complete coherence

$$|\mathbf{E}_T|^2 = N^2 |\mathbf{E}|^2 \quad [8]$$

In practice, the electron beam can be modulated by passing the beam through an undulator together with a laser beam of frequency equal to the fundamental or to one of the odd harmonics of the spontaneous undulator radiation. This produces an *energy* modulation of the electron beam and, after a drift space, the energy modulation produces in turn a modulation of the longitudinal charge, as schematically shown in Fig. 1.

For this case, we will only quote the relevant results: details can be found in [3]. The basic strategy is to perform the calculations in the ERF, where in particular the integral of the form factor simplifies considerably, and then transform the results to the LAB.

First, it can be shown that the modulation of an originally gaussian density produces a series of harmonics, given in the ERF by

$$\rho(x, y, z) = \rho_0(x, y, z) \sum_{n=0}^{\infty} a_n(\eta) \cos(n\chi_b z) \quad [9]$$

with

$$\rho_0(x, y, z) = \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} e^{-\frac{1}{2} \left[\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2} \right]} \quad [10]$$

and harmonic coefficients

$$a_n(\eta) = 2J_n(n\eta) e^{-\frac{1}{2} \left(\frac{n}{n_c} \right)^2}, \quad a_0 = 1 \quad [11]$$

where J are Bessel functions, of the harmonic number n and of a quantity $\eta = \eta(z, ct)$ that depends on the modulator geometry. The coefficients [11] rapidly decrease with increasing n . n_c is a critical number that depends on the emittance of the beam and on the tolerance of the magnetic field of the undulator.

The frequencies of the harmonics of Eq. [9] in the ERF are obtained by Lorentz transformation of the frequency of the modulating laser in the LAB, of wavelength λ_b . The result is

$$\chi_b = \frac{2\pi}{\lambda_b \beta \gamma} \quad [12]$$

The next step, using the expression of Eq. [9] for the modulated density, is to calculate the form factor in a region of length Δz around the drift distance where the bunching is a maximum

$$F(\mathbf{q}) = N \sum_{n=0}^{\infty} \iint_{\Delta x} dx dy \int dz \rho_0(x, y, z) a_n \cos(n\chi_s z) e^{i\mathbf{q} \cdot \mathbf{r}} \quad [13]$$

One can perform the integrals in the transverse coordinates, x, y , and in the longitudinal, z , separately. The radial integral vanishes everywhere, except in two narrow cones, forward and backward, respectively, since the momentum transfer $\mathbf{q}$ in the exponent of the integrand of [13] is inversely proportional to the scattering laser wavelength λ_s , and the radial components of $\mathbf{r}$ at least extend to the electron beam transverse dimensions, i.e. $\sigma_x, \sigma_y / \lambda_s \gg 1$.

The z -integral will be evaluated in these two narrow cones. The result is that only for the back scattered radiation this integral is not negligible, and that the contribution to the form factor of the n -th harmonic, with the length of interaction set $=\sigma_z$ for simplicity, is proportional to N . A final result for the number of coherent photons in the backward cone of angular aperture $\Delta\Omega$ is

$$N_x^{coh} = \sqrt{\pi} \left[\frac{N a_n}{\sqrt{2\pi} \gamma} \right]^2 I_0(\pi) \frac{\sigma_z}{\beta c} \Delta\Omega \quad [14]$$

where $I_0(\mathbf{q})$ is the intensity scattered by one electron and the quantity in $[\]$ represents the number of electrons in the n mode. The cone solid angle, from the radial integration is found to be

$$\Delta\Omega = \frac{1}{4\pi\sigma_x\sigma_y} \left[\frac{\lambda_s}{(1+\beta)\gamma_n} \right]^2 \quad [15]$$

Let us compare the number of coherent photons, proportional to N^2 , given by [14] with the number of incoherent photons, given by [4], in the same solid angle, that we will rewrite in a different way

$$N_x^{inc} = 2 \frac{N}{\gamma} I_0(\pi) \frac{\sigma_z}{\beta c} \Delta\Omega \quad [16]$$

The coherent to incoherent ratio, or coherent fraction, is proportional to the square of the coefficient of the scattering harmonic

$$\mathfrak{R} = \frac{1}{4\sqrt{\pi}} \frac{N a_n^2}{\gamma_n} \quad [17]$$

3. Comparison with undulator radiation.

In synchrotron radiation sources, low intensity coherent X-Rays for experiments are obtained by selecting a very narrow cone of undulator radiation on the axis. Comparison with Compton scattering is useful to better understand the basic physical process. First, we will show that undulator radiation can be described as a scattering process of an equivalent electromagnetic wave by an electron beam.

At an angle θ with the axis, the photon energy in the fundamental line of the spectrum is given by (4)

$$\hbar\omega_1 = \hbar\omega_0 \frac{2\gamma^2}{1 + \frac{1}{2}k^2 + \frac{1}{2}\gamma^2\theta^2}, \quad \omega_0 = \frac{2\pi}{\lambda_0}, \quad k = \gamma\theta_{\max} \quad [18]$$

where λ_0 is the period of the undulator and k a deflection parameter, proportional to the maximum angle of deflection of the electron trajectory in the undulator. Note that Eqs. [18] and [1] are very similar. The spectral width and the angular spread of the fundamental line are, respectively

$$\frac{\delta\omega}{\omega} \approx \frac{1}{N_0}, \quad \delta\theta = \frac{1}{\gamma} \sqrt{\frac{1 + \frac{1}{2}k^2}{2N_0}}, \quad [19]$$

where N_0 is the number of periods in the undulator. The intensity of the radiation in the fundamental, expressed in number of photons emitted per second and per unit horizontal and vertical angle, can be expressed as

$$\frac{d^2n}{d\theta d\phi} = \alpha N_0 \gamma^2 \frac{I}{e} F(k), \quad \alpha = \frac{1}{137} = \frac{\mu_0 e^2 c}{4\pi \hbar} \quad [20]$$

I is the particle beam current, α the fine structure constant and $F(k)$ a certain combination of Bessel functions.

In the ERF -the reference frame of the moving electron- the undulator is seen as an incoming e.m. wave traveling at the speed βc . In this frame, by relativistic transformation, a transverse electric field appears, in addition to the transverse magnetic field of the LAB -the laboratory frame of reference-

$$\text{LAB:} \begin{cases} \mathbf{E} = 0 \\ B_x = B_z = 0 \\ B_y = B_0 \cos \omega_0 t \end{cases} \quad \text{ERF:} \begin{cases} E_x = -\gamma \beta c B_0 \cos \omega_0 t \\ E_y = E_z = 0 \end{cases} \quad \begin{cases} B_x = 0 \\ B_y = \gamma B_0 \cos \omega_0 t \\ B_z = 0 \end{cases} \quad [21]$$

Then, the emission of undulator radiation can be described as a scattering process of an equivalent wave by the particle. Now, the phase is an invariant, then it must be the same whether calculated in the LAB or in the ERF (primed quantities)

$$\omega(t - z/c) = \omega'(t' - z'/c), \quad [22]$$

and, since time, space and frequency transform from LAB to ERF according to

$$t' = \gamma_z \left(t - \frac{\beta_z z}{c} \right), \quad z' = \gamma_z (z - \beta_z c t), \quad \omega' = \gamma_z \omega_0, \quad [23]$$

combining [22] and [23], one obtains -on axis- Eq. [18], i.e.

$$\omega = \omega_0 \frac{1 + \beta_z}{1 - \beta_z^2} \cong \omega_0 \frac{2\gamma^2}{1 + \frac{1}{2}k^2}, \quad \left[\beta_z^2 = 1 - \frac{1}{\gamma^2} - \beta_x^2, \quad < \beta_x > \cong \frac{1}{2} \frac{k}{\gamma} \right] \quad [24]$$

The number of equivalent photons in one period of the undulator is obtained by dividing the magnetic energy contained in a volume of length λ_0 and cross section Σ by the energy of an undulator photon

$$N_u = \frac{1}{4} \frac{\epsilon_0 c^2 B_0^2}{\hbar \omega_0} \lambda_0 \Sigma, \quad [25]$$

where B_0 is the peak magnetic field in the undulator. The number of electrons passing through the same region of space per unit time is

$$N_e = \frac{I}{e \beta c} \lambda_0, \quad [26]$$

with I the beam current. The frequency of collisions in λ_0 can be expressed as

$$f = 2 \frac{\beta c}{\lambda_0}, \quad [27]$$

and the Thomson cross section [6], recalling the definition of the fine structure constant in [20] is

$$\sigma = 2 \left(\alpha \frac{\hbar}{mc} \right) \left(\frac{\mu_0 e^2}{4\pi m} \right). \quad [28]$$

Finally, the cone of aperture of the radiation in the fundamental frequency, from [19] is given by

$$\Delta\theta\Delta\phi \cong \frac{1}{2\gamma^2 N_0}, \quad [29]$$

Combine [18] through [29] into the expression for the photon flux of Eq. [6], and find

$$\frac{n_1}{\Delta\theta\Delta\phi} \cong \alpha N_0 \gamma^2 \frac{I}{e} k^2, \quad [30]$$

i.e: [20], since -from (4) and the expansions for the Bessel functions for small values of k - it is $F(k) \cong k^2$. Note that the expression [30] implies that the incoherent flux in the fundamental is due to the scattering from a region of length λ_0 in the undulator.

How about coherence from an undulator? Undulator radiation is not intrinsically coherent (5), however experiments of X-Ray microscopy and Holography have been performed using undulators. Spatial coherence has been obtained by imaging a pinhole illuminated by the undulator at large distance from the subject, in a way to remain well within the diffraction limit

$$\sigma \times \theta << \lambda_x \quad [31]$$

with σ the radius of the pinhole and θ the angle of collection (Fig. 2), temporal coherence by the use of a monochromator, since the spectral line width is given in [19] no matter how small the pinhole is. Coherence is mainly limited by the finite emittance of the electron beam(6). The coherent flux is generally very small.

4. Coherent scattering at 90°.

In the scattering, collective effects also arise if the electron bunch length is much shorter than the incoming radiation wavelength (3, 7), so all the electrons could radiate in phase (reflection). For optical or shorter wavelengths, this is difficult to achieve in head-on collisions, since typical electron pulses cannot be much shorter than a few millimeters. A possibility is that electron and

photon beams cross each other at a large angle (8). This considerably shortens the interaction region and can lead to collective effects and to some degree of coherence of the Compton scattered photon beam.

In this case, a fraction of photons is continually scattered in phase in the forward direction of the electrons by a thin slice of the electron beam (a true reflection). However, the scattering done by successive slices on successive photons does not add in phase, since there is no phase relation between the scattering by successive slices. So, the contribution by successive slices add incoherently, until the electron bunch is gone.

Let us recall Eq. [6] for the form factor of the scattering and explicitly discuss the case of laser electron Compton scattering at 90° in the laboratory. Assume to have the usual parallel beam pulse of N electrons traveling at a speed βc along the z axis with gaussian density distribution described by Eq. [10]. The laser beam of wavelength λ_S is directed along x and is focused right on the electron trajectory, with an elliptical spot size of half axes η, ξ along the z and y axis respectively. The momentum/position uncertainty principle limits the laser beam minimum dimensions to $\lambda/2\pi$.

Like in Sec. 2, we will limit ourselves to quote the relevant mathematical results, referring for details to (8). Also in this case the calculations have been performed in the ERF, where everything is easier.

First, it can be shown that at the waist location ($x_L = 0$) the *laser* photon density in the ERF can be written as

$$\rho_S(x, y, z, \ell, m) = \frac{dN_S}{dx dy dz d\ell dm} = \gamma(1 - \beta m) \rho_{S,L} \quad [32]$$

with $\rho_{S,L}$ the corresponding density in the LAB, a function of the three spatial coordinates and of the angles of the incoming laser beam (LAB) θ_L, ϕ_L as defined in Fig. 3

$$\ell = \sin \theta_L \cos \phi_L, \quad m = \sin \theta_L \sin \phi_L \quad [33]$$

After the relativistic transformation, for each value of ℓ, m in the LAB, the photon density [32] in the ERF system appears concentrated in a cylindrical volume almost perpendicular to the y - z plane and sweeping across the electron pulse at the speed βc . Since the Lorentz transformation compression factor γ squeezes the cylinder along the z direction, the laser light collides only against a tiny slice of the electron bunch. The shape of this slice defines the volume of the electron pulse over which the integration of the form factor must be performed. For each specified value of ℓ, m , the form factor becomes

$$F(\mathbf{q}, \ell, m) = N \iiint_{-\infty, +\infty} \rho_e(x, y, z) e^{-\Gamma} e^{i\mathbf{q} \cdot \mathbf{r}} dx dy dz \quad [34]$$

with Γ a certain function of the position and angles.

The amplitude of the form factor of [34] represents the number of electrons contained in the volume of interaction. These electrons scatter in phase because they have been selected in a region of linear dimensions along the z axis of the same order of the photon wavelength. Therefore, depending upon the number of electrons that can be packed in one wavelength, the scattered intensity can be significantly enhanced.

For a small solid angle around the direction $\theta_L = 0$

$$\Delta\Omega = 4\pi \sin^2 \frac{1}{2} \Delta\theta_L \quad [35]$$

the resulting form factor is

$$F(\mathbf{q}, \theta_L \equiv 0) \equiv N \frac{\xi\eta}{\sqrt{(\xi^2 + \sigma_y^2)(\eta^2 + \gamma^2 \sigma_z^2)}} e^{-\frac{1}{2}\left(\frac{\beta c t}{\sigma_x}\right)^2} e^{-\frac{1}{4}\left(\frac{q_x^2}{u^2} + \frac{q_y^2}{v^2} + \frac{q_z^2}{f^2}\right)} \quad [36]$$

with u , v , and f some amplitude of the momentum transfer components. This expression is strongly peaked at the values

$$q_x = q_y = 0, \quad q_z = -2\beta\gamma k_L \quad [37]$$

and vanishes everywhere else. So, it describes a photon pulse of length $\sigma_x/\sqrt{2}\beta$ that propagates in the ERF along the direction specified by

$$\cos\theta = \frac{1}{\gamma}, \quad \phi = \frac{\pi}{2} \quad [38]$$

Equations [37] shows that in the scattering the x - and y - components of the momentum remain unchanged while the z - component changes sign. This means that the direction where the coherence builds up is obtained by reflecting the direction of the incoming photon momentum on a plane perpendicular to the z -axis. The angle θ in the ERF of [38] sensibly transforms to $\theta_L \equiv 0$ in the LAB.

Finally, the coherent fraction within the solid angle [35] is

$$\Re(\theta_L \equiv 0) = \frac{N}{4\pi} \frac{(\lambda_s/\gamma^2)}{\sigma_{s,L}} e^{-4\beta^2}, \quad \left[\text{if } \xi = \eta = \sigma_y = \frac{\lambda_L}{2\pi} \right] \quad [39]$$

i.e. directly proportional to the number of electrons contained in a wavelength of the scattered radiation.

5. Practical implementation.

Let us schematically describe possible setups to implement a case of Compton scattering of a laser beam by a modulated electron beam and of 90° scattering. Also, let us attempt a numerical estimate of the expected rates and of the coherence properties of X-Rays for the two cases. As a comparison, we can estimate the corresponding quantities for an undulator.

5.1. Scattering by a modulated beam.

A possible setup is shown in Fig. 4. The electron beam is passed through an undulator, together with the beam of a modulating laser. The combined effect of the undulator and the laser produces a modulation of the electron energy. After a drift space, the energy modulation produces a density modulation. A second laser is focused at the location of maximum modulation, and the resulting X-Ray beam is collected in the direction of propagation of the electrons (actually, the electron beam is deflected away).

The modulating and scattering lasers can in principle be the same laser. An optical cavity encompassing the undulator can be used to increase the photon flux. An electron recirculator can be used to increase the electron beam intensity.

Possible numerical values are shown in Table I.

5.2. Scattering at 90°

Figure 5 shows the geometry of a possible setup. A laser in a confocal cavity, in order to increase the photon flux, collides at 90° with an electron beam. The X-Rays are emitted in the general direction of propagation of the electrons. Numerical estimates are given in Table II.

5.3. Coherent radiation from an undulator.

Some coherence is obtained in an X-Ray beam from an undulator provided that the photons are collected within an angle with the axis that satisfies the diffraction condition of Eq. [3.13], and Fig. 2. Numerical values are given in Table III.

Conclusions.

We claim that coherent effects arise in Compton scattering by a modulated electron beam or at large angles (in particular at 90°). The first option implies a very good quality electron beam, as obtained in state of the art recirculated Van de Graaf electrostatic accelerators. The second option requires that the laser beam is strongly focused on the electron trajectory and the electron density is sufficiently high. These conditions can be achieved with present day lasers and electron high gradient, low emittance, linacs. In this case we have shown that the coherent beam of X-Rays is contained in a narrow cone, with an intensity several hundred times higher than the corresponding incoherent yield. This coherent radiation is directed in the LAB practically along the electron beam and can be easily selected with a suitable collimator.

The calculations shown are analytical on some schematic cases, but it is clear that a complete picture of the radiation pattern would require a numerical study of the form factor integral, as done to calculate incoherent back scattered intensities (9).

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Appendix. Derivation of the Compton Scattering Formulas.

Equation [1] of the text is derived by a comparison of the kinematics of scattering in the LAB (non-primed quantities) and in the ERF (primed quantities). Energy and momentum

$$\left[\begin{array}{lll} & \text{energy} & \text{momentum} \\ \text{photon} & \varepsilon = \hbar\omega & \mathbf{k} = \frac{\hbar\omega}{c} \mathbf{n} \\ \text{particle} & E = \gamma mc^2 & \mathbf{p} = \gamma m \mathbf{v} \end{array} \right] \quad [40]$$

obey the conservation laws

$$\text{LAB:} \begin{cases} \mathbf{p}_0 + \mathbf{k}_0 = \mathbf{p}_1 + \mathbf{k}_1 \\ E_0 + k_0 c = E_1 + k_1 c \end{cases} \quad \text{ERF:} \begin{cases} E'_0 = mc^2 \\ \mathbf{k}'_0 = \mathbf{p}'_1 + \mathbf{k}'_1 \\ E'_0 + k'_0 c = E'_1 + k'_1 c \end{cases} \quad [41]$$

From these and with the aid of Fig. 6, the Compton scattering formula is derived in the ERF

$$k'_1 = \frac{k'_0}{1 + \frac{c k'_0}{E'_0} [1 - \cos(\theta'_1 - \theta'_0)]}, \quad [42]$$

Equation [42], using the relativistic transformations of angles from ERF to LAB

$$\tan \theta = \frac{\sin \theta'}{\gamma(\cos \theta' + \beta)}, \quad \phi = \phi', \quad [43]$$

and assuming $\theta_0 = \pi$ (back scattering), readily yields in the LAB Eq. [1] of the text.

References.

1. C. BÉNARD AND M. ROUSSEAU, *J. Opt Soc. Amer.* **64**, 1433 (1974).
2. R. H. MILBURN, *Phys. Rev. Lett.* **10**, 75 (1963).
F. R. ARUTYUNIAN AND V. A. TUMANIAN, *Phys. Lett.* **4**, 176 (1963).
L. FEDERICI, ET AL., *Nuovo Cimento* **59B**, 247 (1980).
A. M. SANDORFI ET AL., *IEEE Trans. Nucl. Sci.* **NS-30**, 3083 (1983).
3. A. LUCCIO, G. MATONE, L. MICELI AND G. GIORDANO, *Laser and Particle Beams*, **8**, 383 (1990).
4. J. KIRZ ET AL. (Eds.), "X-Ray Data Booklet" *Lawrence Berkeley Laboratory*, **PUB-490** (1985)
5. M. R. HOWELLS AND B. M. KINCAID, in "Proceedings of the NATO Advanced Study Institute: New Directions in Research with Third Generation Soft X-Ray Sources" edited by A.S. SCHLACHTER AND F. W. WUILLEUMIER, Kluwer Academic Publishers, 1993, in press.
6. J. KIRZ, *Private communication*.

7. C. PELLEGRINI, in "Proc. Conference on New Horizons in Electromagnetic Physics", Charlottesville, Virginia, 1982.
8. G. GIORDANO, A. LUCCIO, G. MATONE and L. MICELI,
Preprint BNL-49502 (1993).
9. C-M TANG, B. HAFIZI AND S. K. RIDE. *Naval Research Laboratory Report*
NRL/MR/6791--93-7328 (1993). Also: "Proc. XIV Int. Free Electron Laser Conf.",
Kobe, Japan, Aug. 24-28, 1992.

Table I. Example of Coherent Scattering by a Modulated Electron Beam

Electron Beam (Recirculated Van de Graaf)		
Electron energy	E	9.30 MeV
Current	$\langle I \rangle$	40 μ A
Energy spread ^a	σ_E	$7 \cdot 10^{-7}$
Bunch length	$\sigma_z L$	15 cm
Beam transv. dimensions	$\sigma_{x,y}$	0.15 mm
Bunching: Undulator		
Period ^b	λ_0	3.48 mm
Peak field	B_0	0.41 T
No. of periods	N_0	24
Deflection parameter	k	0.134
Bunching and Scattering: Laser (CO ₂ , pulsed, doubled)		
Laser wavelength	λ_L	5.3 μ m
Laser pulse length		1 nsec
Peak Power for Bunching	P_b	36 KW
Peak Power for Scattering	P_s	5 GW
X-Rays		
Wavelength	λ_X	4 nm
Coherence enhancement ^c	$\mathfrak{R}$	$1.41 \cdot 10^4$
Coherent photons per pulse	N_X^{coh}	$5.3 \cdot 10^6$
Coherent cone	$\Delta\Omega$	$5.66 \cdot 10^{-11}$ sterad

^a Advertised energy spread for edge-of-technology machines

^b Micro undulators. E.g. NSLS-Brookhaven

^c Equation [17]

Table II. Example of Coherent Scattering at 90°

Electron Bunch		
Number of electrons	N	$5 \cdot 10^{11}$
Electron energy	E	33 MeV ($\gamma = 64.5$)
Bunch Length (LAB)	$\sigma_z L$	3 mm (10 psec)
Emittance	ϵ	$2 \cdot 10^{-7}$ m-rad
Laser (CO ₂), pulsed		
Wavelength	λ	10.6 μm
Power, pulse length	W	1 Joule/100 psec
Repetition Rate		1 KHz
Scattering		
X-Ray Energy	$\hbar\omega$	1 KeV
Laser and beam spot size	$\xi = \eta = \sigma_x = \sigma_y = \frac{\lambda}{2\pi}$	1.68 μm
Central cone aperture	$\Delta\theta_L$	10°
No. of passes in cavity		100
Coherent Enhancement ^a	$\mathfrak{R}$	620
Coherent photons	N_X^{coh}	$5.7 \cdot 10^{10} \text{ sec}^{-1}$
X-Ray beam direction ^b	(θ, ϕ)	(89.1°, 90°)

^a Equation [39]

^b Equation [38]

Table III. Undulator radiation. Example for a Scanning Microscope

Storage ring: APS-Argonne		
Electron energy	E	7 GeV
Beam current	I	200 mA
Emittance	ϵ	74 nm-rad
Coupling	χ	0.1
Beam envelope functions	$\beta_x \beta_y$	13 & 10 m
Beam size ^a	$\sigma_x \sigma_y$	974 & 85.4 μm
Undulator		
Period	λ_0	3.3 cm
Number of periods	N_0	72
Peak field	B_0	0.75 T
Deflection parameter ^b	k	2.312
Fundamental wavelength ^c	λ_1	0.323 nm (3.84 KeV)
Natural line width	$\delta\omega/\omega$	0.0139
Scanning Microscope		
Pinhole aperture	σ	=vertical beam size
Angle of coherence ^d	λ/σ	$3.78 \cdot 10^{-6}$ rad
Solid angle	$\Delta\Omega$	$4.49 \cdot 10^{-11}$ sterad
Photon flux in $\Delta\Omega$ ^e	n_{ph}	$1.27 \cdot 10^{15} \text{ sec}^{-1}$
Coherent fraction ^f	$\mathfrak{R}$	$1.98 \cdot 10^{-4}$

$$^a \sigma_x = \sqrt{\frac{\epsilon\beta_x}{1+\chi^2}}, \quad \sigma_y = \chi \sqrt{\frac{\epsilon\beta_y}{1+\chi^2}}$$

$$^b k = \frac{e}{2\pi mc} \lambda_0 B_0$$

$$^c \lambda_1 = \lambda_0 \frac{1 + \frac{1}{2}k^2}{2\gamma^2}$$

$$^d \text{Equation [31]}$$

$$^e \text{Equation [20]}$$

$$^f \mathfrak{R} = \frac{(\lambda/\pi)^2}{\epsilon_x \epsilon_y} = \frac{1}{\pi^2} \left[\frac{\lambda(1+\chi^2)}{\chi\epsilon} \right]^2$$

Fig. 1. Klystron diagram. It shows the distance vs. time traveled by a particle beam with sinusoidal modulated kinetic energy. The bunching in density is apparent.

Fig. 2. Definition of the diffraction cone for undulator radiation.
 2σ is the diameter of the pinhole.

Fig. 3. Definition of the LAB angles for 90° scattering.

Fig. 4. Basic set up for scattering by a modulated electron beam.

Fig. 5. Basic set up for scattering at 90°.

Fig. 6. Kinematics of Compton back scattering in the LAB and in the PRF.

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