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ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439-4801

SHAKEDOWN ANALYSIS OF FUSION REACTOR FIRST WALL

by

Saurin Majumdar
Fusion Power Program/Technology Development Division

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SHAKEDOWN ANALYSIS OF FUSION REACTOR FIRST WALL

Saurin Majumdar

ABSTRACT

Shakedown analyses of a typical fusion reactor first wall including coolant channels subjected to cyclic thermal / steady primary and cyclic primary / steady thermal stresses are carried out. The stresses are assumed to be predominantly of the bending type. The first cycle of loading/unloading is analyzed using elastic-plastic beam bending theory. The general problem of shakedown is solved using the shakedown theorem of perfect plasticity.

I. INTRODUCTION

During normal operations of tokamak fusion reactors, such as the International Thermonuclear Experimental Reactor (ITER), high cyclic thermal stresses are induced in the first wall by the pulsed plasma. In addition, the first wall is subjected to a steady primary stress due to coolant pressure and high cyclic transient dynamic stresses during plasma disruptions. The interaction of these stresses can potentially lead to accumulation of incremental plastic strain or ratcheting of the first wall.

Design rules for protecting a structure against ratcheting have been developed in the fission reactor codes such as the ASME Boiler and Pressure Vessels Code [1] or the French RCC-MR [2]. The ratcheting rules in these codes are derived either analytically using the Bree diagram [3] approach or experimentally. However, most of these analyses or experiments have been conducted for axisymmetric structures under axisymmetric loadings. On the other hand, the first wall of ITER may be subjected to primarily a bending type loading in which case these rules may not be directly applicable. A Bree diagram for bending loading of a first wall with a linear temperature distribution through the thickness was presented in [4].

In some designs, such as the Next European Torus (NET), the first wall has coolant holes drilled inside, as shown in Fig. 1. In such cases, the temperature distribution in the first wall is non-linear, and high stresses can develop even if the first wall is free to bend. Ratcheting tests on geometries similar to Fig. 1 have been conducted recently by Wolters et al. [5] using cyclic thermal and static mechanical loads. Finite-element calculations were conducted to confirm the experimental results [4] and recently, Wolters and Majumdar [6] developed a three-bar model for simulating the ratcheting behavior of the tests reported in [5].

Fig. 1 Typical cross-section of a first wall with a coolant channel subjected to a surface heat flux. The temperature field is idealized as bilinear in the y direction and uniform in the x direction.

The purpose of the present paper is to analytically determine the shakedown behavior of a first wall with a geometry and an idealized bilinear temperature distribution through the thickness as shown in Fig. 1. The material is assumed to be elastic-perfectly-plastic without strain hardening. The term shakedown is used here to denote load combinations under which the structure can absorb the cyclic load elastically without any cyclic plastic strain or steady increase of plastic strain. Bree diagrams are developed which may be used in the design of the first wall. Load combinations lying outside the Bree envelop may cause either cyclic plasticity or ratcheting, and are not considered here. Attention is primarily focused on situations where a cyclic thermal stress is superimposed on a steady mechanical (load-controlled) stress.

II. ANALYSIS

We consider a first wall cross-section of width unity and depth, $2d$, shown in Fig. 1, subjected to a steady primary bending moment M about x -axis (positive if it causes compressive z -stress at the plasma edge) and a pulsing surface heat flux. The applied axial (z -direction) load is assumed to be zero. During plasma-off, the temperature distribution is

uniform at T_0 , which for our present purposes is set equal to zero. The temperature distribution through the thickness during plasma-on is idealized as bilinear, as shown in Fig. 1, and is given by

$$T(y) = T \frac{\left(\frac{y}{d} - \beta_{th}\right)}{(1 - \beta_{th})} \quad \text{for } y > \beta_{th}d \quad (1a)$$

$$T(y) = 0 \quad \text{for } y \leq \beta_{th}d \quad (1b)$$

where $\beta_{th} \geq 0$ and the maximum temperature fluctuates between 0 and T . Using the Bernoulli beam theory, the stress at y is given by

$$\sigma(y) = E(\epsilon - \kappa y) + E(\epsilon_{th} - \kappa_{th}y) - E[\epsilon_p(y) + \alpha T(y)] \quad (2)$$

where E is the Young's modulus, α is the coefficient of thermal expansion, $\epsilon_p(y)$ is the plastic strain at y , ϵ is the axial strain and κ is the curvature,

$$\epsilon_{th} = \frac{\alpha}{2d} \int_{-d}^d T(y) dy = \frac{1}{4} \alpha T (1 - \beta_{th}) \quad (3a)$$

and

$$\kappa_{th} = -\frac{\alpha}{I} \int_{-d}^d T(y) y dy = -\frac{\alpha T}{4d} (1 - \beta_{th})(2 + \beta_{th}) \quad (3b)$$

In above, I is the moment of inertia $= 2/3 d^3$. The requirement of zero axial load is satisfied by integrating Eq. 2 and using Eq 3a,

$$0 = \frac{1}{2d} \int_{-d}^d \sigma(y) dy = \epsilon - \frac{1}{2d} \int_{-d}^d \epsilon_p(y) dy \quad (4a)$$

Similarly, the requirement that the applied bending moment equal M gives

$$M = -\int_{-d}^d y \sigma(y) dy = EI \kappa + E \int_{-d}^d y \epsilon_p(y) dy \quad (4b)$$

II.A Elastic Thermal Stresses

Results from here on are presented in a nondimensional form with $M, T, \epsilon, \kappa, \sigma, \epsilon_p$, and y replacing $M/M_y, T/T_y, E\epsilon/\sigma_y, \kappa/\kappa_y, \sigma/\sigma_y, E\epsilon_p/\sigma_y$, and y/d , respectively, with σ_y as the yield stress,

$$M_y = \frac{\sigma_y I}{d},$$

$$T_y = \frac{4\sigma_y}{E\alpha(1+\beta_{th})^2},$$

and

$$\kappa_y = \frac{Ed}{\sigma_y}.$$

I is the moment of inertia. T_y is the temperature to cause first yielding by thermal stress and M_y is the bending moment to cause first yielding by primary bending stress.

Using Eqs 1-4, the elastic thermal stress distribution is as follows:

$$\text{Maximum stress at the plasma edge} = -T \text{ at } y=1 \quad (5a)$$

$$\text{stress at coolant edge} = T(1 - \beta_{th}) \text{ at } y = \beta_{th} \quad (5b)$$

$$\text{Stress at edge away from plasma} = -T \frac{(1 - \beta_{th})}{(1 + \beta_{th})} \text{ at } y=-1 \quad (5c)$$

For $\beta_{th} \geq 0$, yielding due to thermal stress always starts at the plasma edge when $T=1$.

II.B Elastic-Plastic Analysis

All elastic-plastic analyses were conducted on the basis of an elastic-perfectly plastic constitutive equation without strain hardening.

II.B.1 Initial Yield

The elastic regime for a primary bending moment satisfying $M \geq -\beta_{th}$ can be expressed as

$$M+T=1 \quad (6a)$$

When $M < -\beta_{th}$, the elastic regime is bounded by Eq. 5c which corresponds to yielding on the surface $y=-1$.

$$-M + \frac{1 - \beta_{th}}{1 + \beta_{th}} T = 1 \quad (6b)$$

II.B.2 Loading Beyond Initial Yield ($M \geq -\beta_{th}$)

Let compressive yielding occur from $y=\beta$ to $y=1$ ($\beta < 1$). Then the plastic strain distribution in the section is given by

$$\epsilon_p(y) = -(y-\beta) \left[\kappa + T \frac{(2-\beta_{th})}{(1-\beta_{th})} \right] \quad \text{for } 1 > y > \beta \quad (7a)$$

and

$$\epsilon_p(y) = 0 \quad \text{for } \beta > y > -1 \quad (7b)$$

The condition of zero axial load (Eq. 4a) with Eqs 7a-b gives

$$\epsilon = -\frac{(1-\beta)^2}{4} \left[\kappa + T \frac{(2-\beta_{th})}{(1-\beta_{th})} \right] \quad (8)$$

Similarly, Eq. 4b together with Eqs 7a-b can be used to solve for κ ,

$$\kappa = \frac{4M}{(1+\beta)^2(2-\beta)} + T \frac{(2-\beta_{th})(1-\beta)^2(2+\beta)}{(1-\beta_{th})(1+\beta)^2(2-\beta)} \quad (9)$$

Using the condition $\sigma = -1$ at $y=\beta$ gives (using Eqs 1a-b, 2, and 3a-b)

$$\epsilon = -1 + \kappa\beta - T \frac{(1-\beta_{th})}{(1+\beta_{th})^2} \left[1 + \beta(2+\beta_{th}) - \frac{4(\beta-\beta_{th})}{(1-\beta_{th})^2} \right] \quad (10)$$

Combining Eqs 8 and 10 to solve for κ gives

$$\kappa = \frac{M}{(1+\beta)^2} + \frac{(2+\beta)}{(1+\beta)^2} + T \frac{(2+\beta)}{(1-\beta_{th})} [1 - 2\beta + \beta\beta_{th}] \quad (11)$$

Eqs 9 and 11 can be solved to give

$$M + T \frac{\beta - \beta_{th}}{1 - \beta_{th}} = 2 - \beta \quad (12)$$

For a given M and T , Eq. 12 can be solved for β provided yielding does not occur at $y=\beta_{th}$.

Shakedown in one cycle ($M \geq -\beta_{th}$):

On unloading of thermal load, reversed yielding will not occur at $y=1$ provided (using Eq. 5a)

$$T \leq 2 \quad (13a)$$

Similarly, tensile yielding will not occur at $y=-1$ provided (using Eq. 5c),

$$\sigma(y=-1) + T \frac{(1-\beta_{th})}{(1+\beta_{th})} \leq 1 \quad (13b)$$

which on using Eqs. 1a-b, 2, 3a-b, 10 and 11 and simplifying gives

$$M \leq 1 - \frac{T^2}{2 + T \frac{(3-\beta_{th})}{(1-\beta_{th})}} \quad (13c)$$

Onset of tensile yielding at $y=\beta_{th}$ (monotonic loading) ($M \geq -\beta_{th}$):

To determine the onset of tensile yielding at $y=\beta_{th}$, set the stress at β_{th} equal to yield and solve for κ from Eqs. 1, 2, and 3,

$$\kappa = -T \frac{2-\beta_{th}}{1-\beta_{th}} + \frac{2}{\beta-\beta_{th}} \quad (14)$$

Eliminating κ from Eqs 9 and 14,

$$M + T \frac{(2-\beta_{th})}{(1-\beta_{th})} = \frac{(1+\beta)^2 (2-\beta)}{2(\beta-\beta_{th})} \quad (15)$$

For a given M , eliminating T from Eqs 12 and 15, onset of tensile yielding at $y=\beta_{th}$ occurs when

$$\beta = -1 + 2\sqrt{1 - \frac{1}{2}(\beta_{th} + M)} \quad (16a)$$

and T and M are given parametrically as functions of β ,

$$T = \frac{(1-\beta_{th})(1+2\beta_{th}+\beta^2)}{2(\beta-\beta_{th})} \quad (16b)$$

$$M = \frac{1}{2}[3 - 2\beta - 2\beta_{th} - \beta^2] \quad (16c)$$

Spreading of plastic zone from $y=\beta_{th}$ ($M \geq -\beta_{th}$):

For continued loading, let the tensile plastic zone spread from $y=\beta_1$ to $y=\beta_2$, where $\beta_2 > \beta_{th} > \beta_1$. The plastic strain distribution in the beam is then given by

$$\epsilon_p(y) = -(y-\beta) \left[\kappa + T \frac{(2-\beta_{th})}{(1-\beta_{th})} \right] \quad \text{for } y > \beta \quad (17a)$$

$$\epsilon_p(y) = 0 \quad \text{for } \beta > y > \beta_2 \quad (17b)$$

$$\epsilon_p(y) = (y-\beta_1) \left[-\kappa + T \frac{(1-\beta_{th})(2+\beta_{th})}{(1+\beta_{th})^2} \right] - T \frac{4(y-\beta_{th})}{(1-\beta_{th})(1+\beta_{th})^2} \quad \text{for } \beta_2 > y > \beta_{th} \quad (17c)$$

$$\epsilon_p(y) = (y-\beta_1) \left[-\kappa + T \frac{(1-\beta_{th})(2+\beta_{th})}{(1+\beta_{th})^2} \right] \quad \text{for } \beta_{th} > y > \beta_1 \quad (17d)$$

and

$$\epsilon_p(y) = 0 \quad \text{for } \beta_1 > y > -1 \quad (17e)$$

Zero plastic strain at $y=\beta_2$ requires (from Eq. 17c)

$$(\beta_2 - \beta_1) \left[-\kappa + T \frac{(1-\beta_{th})(2+\beta_{th})}{(1+\beta_{th})^2} \right] - T \frac{4(\beta_2 - \beta_{th})}{(1-\beta_{th})(1+\beta_{th})^2} = 0 \quad (17f)$$

Condition of zero axial load (Eq. 4a) with Eqs 17a-f gives

$$\epsilon = -\frac{1}{4} \left[(1-\beta)^2 - \beta_2(\beta_2 - \beta_1) \right] \left[\kappa + T \frac{(2-\beta_{th})}{(1-\beta_{th})} \right] - T \frac{\beta_{th}(\beta_{th} - \beta_1)}{(1-\beta_{th})(1+\beta_{th})^2} \quad (18)$$

Similarly, Eq. 4b together with Eqs 17a-f can be solved to give

$$\kappa = \frac{4 \left(M + T \frac{2+3\beta_{th}-\beta_{th}\beta_1^2}{(1-\beta_{th})(1+\beta_{th})^2} \right)}{\left[(1+\beta)^2(2-\beta) + \beta_2(\beta_2^2 - \beta_1^2) \right]} - T \frac{(2-\beta_{th})}{(1-\beta_{th})} \quad (19)$$

Substituting for κ from Eq. 19 into Eqs. 18 and 10, and eliminating ϵ from Eqs 18 and 10,

$$-M[(1+\beta)^2 - \beta_2(\beta_2 - \beta_1)] + \frac{T}{(1+\beta_{th})^2(1-\beta_{th})}[(1+\beta)^2\{\beta_{th}(1+\beta_1)^2 - \beta(1+2\beta_{th} + \beta_{th}\beta_1)\} + \beta_2(\beta_2 - \beta_1)\{(1+2\beta_{th})(2+\beta_1 + \beta_2) - \beta_{th}(1-\beta_1\beta_2)\}] = -[(1+\beta)^2(2-\beta) + \beta_2(\beta_2^2 - \beta_1^2)] \quad (20)$$

Subtracting the equation (using Eq. 2) for compressive yielding at $y=\beta$ from that for tensile yielding at $y=\beta_1$ and solving for κ ,

$$\kappa = \frac{2}{(\beta - \beta_1)} - \frac{T}{(1 - \beta_{th})(1 + \beta_{th})^2} \left[\frac{4(\beta_1 - \beta_{th})}{(\beta - \beta_1)} + (1 + \beta_{th})^2(2 - \beta_{th}) \right] \quad (21)$$

Eliminating κ from Eqs 19 and 21,

$$-M(\beta - \beta_1) + \frac{T}{(1 + \beta_{th})^2(1 - \beta_{th})}[(\beta_{th} - \beta_1)\{(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)\} - (2 + 3\beta_{th} - \beta_{th}\beta_1^2)(\beta - \beta_1)] = -\frac{1}{2}[(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)] \quad (22)$$

Similarly, subtracting the equation (using Eq. 2) for compressive yielding at $y=\beta$ from that for tensile yielding at $y=\beta_2$ and solving for κ ,

$$\kappa = \frac{2}{(\beta - \beta_2)} - T \frac{(2 - \beta_{th})}{(1 - \beta_{th})} \quad (23)$$

Eliminating κ from Eqs 19 and 23,

$$M(\beta - \beta_2) + T \frac{(2 + 3\beta_{th} - \beta_{th}\beta_1^2)}{(1 - \beta_{th})(1 + \beta_{th})^2} = \frac{1}{2}[(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)] \quad (24)$$

Solving for T by eliminating M from Eqs 20 and 22 and the same from Eqs 20 and 24, equating the two expressions for T , and simplifying,

$$\beta^2 = \beta_2^2 + \frac{(\beta_2 - \beta_{th})}{(\beta_{th} - \beta_1)}(1 + \beta_1)^2 \quad (25)$$

For a given β and β_1 , β_2 can be solved for from Eq. 25. Eliminating κ from Eqs 21 and 23, T can be solved for in terms of β , β_1 , and β_2 .

$$T = \frac{(1 - \beta_{th})(\beta_2 - \beta_1)(1 + \beta_{th})^2}{2(\beta_{th} - \beta_1)(\beta - \beta_2)} \quad (26)$$

M , κ and ϵ can be solved from Eqs. 24, 19, and 18, respectively and the stresses at various locations can be obtained from Eq. 2.

Shakedown in one cycle ($M \geq -\beta_{th}$):

On unloading of thermal load, reversed yielding will not occur at $y=1$ provided (using Eq. 5a)

$$T \leq 2. \quad (13a)$$

As before, tensile yielding will not occur at $y=-1$ provided (using Eq. 5c),

$$\sigma(y=-1) + T \frac{(1-\beta_{th})}{(1+\beta_{th})} \leq 1 \quad (13b)$$

However, unlike before, a closed form solution for shakedown in one cycle was not derived; it was obtained numerically.

II.B.3 Loading Beyond Initial Yield ($M \leq -\beta_{th}$):

Let compressive yielding occur from $y=-\beta$ to $y=-1$ ($0 < \beta < 1$). Then the plastic strain distribution in the section is given by

$$\epsilon_p(y) = -(y+\beta) \left[\kappa - T \frac{(1-\beta_{th})(2+\beta_{th})}{(1+\beta_{th})^2} \right] \quad \text{for } -1 < y < -\beta \quad (27a)$$

and

$$\epsilon_p(y) = 0 \quad \text{for } -\beta < y < 1 \quad (27b)$$

The condition of zero axial load (Eq. 4a) with Eqs 27a-b gives

$$\epsilon = \frac{(1-\beta)^2}{4} \left[\kappa - T \frac{(1-\beta_{th})(2+\beta_{th})}{(1+\beta_{th})^2} \right] \quad (28)$$

Similarly, Eq. 4b together with Eqs 27a-b can be used to solve for κ ,

$$\kappa = \frac{4M}{(1+\beta)^2(2-\beta)} - T \frac{(2+\beta_{th})(1-\beta_{th})(1-\beta)^2(2+\beta)}{(1+\beta_{th})^2(1+\beta)^2(2-\beta)} \quad (29)$$

Using the condition $\sigma = -1$ at $y = -\beta$ gives (using Eqs 1a-b, 2, and 3a-b)

$$\epsilon = -1 - \kappa\beta - T \frac{(1-\beta_{th})}{(1+\beta_{th})^2} [1 - \beta(2+\beta_{th})] \quad (30)$$

Combining Eqs 28 and 30 to solve for κ gives

$$\kappa = \frac{M}{(1+\beta)^2} - \frac{(2+\beta)}{(1+\beta)^2} - T \frac{(2+\beta)(1-\beta_{th})}{(1+\beta_{th})^2} [1-2\beta-\beta\beta_{th}] \quad (31)$$

Eqs 29 and 31 can be solved to give

$$-M+T \frac{(1-\beta_{th})(\beta+\beta_{th})}{(1+\beta_{th})^2} = 2-\beta \quad (32)$$

For a given M and T , Eq. 32 can be solved for β provided yielding does not occur at $y=\beta_{th}$.

Shakedown in one cycle ($M \leq -\beta_{th}$):

On unloading of thermal load, tensile yielding will not occur at $y=1$ provided (using Eq. 5a),

$$\sigma(y=1)+T \leq 1 \quad (33a)$$

which on using Eqs. 1a-b, 2, 3a-b, 30 and 31 and simplifying gives

$$M > -1 + \frac{\left[T \frac{(1-\beta_{th})}{(1+\beta_{th})} \right]^2}{\left[2 + T \frac{(3+\beta_{th})(1-\beta_{th})}{(1+\beta_{th})^2} \right]} \quad (33b)$$

Onset of tensile yielding at $y=\beta_{th}$ (monotonic loading) ($M \leq -\beta_{th}$):

To determine the onset of tensile yielding at $y=\beta_{th}$, set the stress at β_{th} equal to yield and solve for κ from Eqs. 1, 2, and 3,

$$\kappa = T \frac{(1-\beta_{th})(2+\beta_{th})}{(1+\beta_{th})^2} - \frac{2}{\beta+\beta_{th}} \quad (34)$$

Eliminating κ from Eqs 29 and 34,

$$-M+T \frac{(1-\beta_{th})(2-\beta_{th})}{(1+\beta_{th})^2} = \frac{(1+\beta)^2(2-\beta)}{2(\beta+\beta_{th})} \quad (35)$$

For a given M , eliminating T from Eqs 32 and 35, onset of tensile yielding at $y=\beta_{th}$ occurs when

$$\beta = -1 + 2\sqrt{1 + \frac{1}{2}(\beta_{th} + M)} \quad (36a)$$

and T and M are given parametrically as functions of β ,

$$T = \frac{(1 + \beta_{th})^2 (1 - 2\beta_{th} + \beta^2)}{2(1 - \beta_{th})(\beta + \beta_{th})} \quad (36b)$$

$$M = -\frac{1}{2} [3 - 2\beta + 2\beta_{th} - \beta^2] \quad (36c)$$

Spreading of plastic zone from $y=\beta_{th}$ ($M \leq -\beta_{th}$):

For continued loading, let the tensile plastic zone spread from $y=\beta_1$ to $y=\beta_2$ where $\beta_2 > \beta_{th} > \beta_1$. The plastic strain distribution in the beam is then given by

$$\epsilon_p(y) = -(y + \beta) \left[\kappa - T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \right] \quad \text{for } -1 < y < -\beta \quad (37a)$$

$$\epsilon_p(y) = 0 \quad \text{for } -\beta < y < \beta_1 \quad (37b)$$

$$\epsilon_p(y) = (y - \beta_1) \left[-\kappa + T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \right] \quad \text{for } \beta_1 < y < \beta_{th} \quad (37c)$$

$$\epsilon_p(y) = (y - \beta_1) \left[-\kappa + T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \right] - T \frac{4(y - \beta_{th})}{(1 - \beta_{th})(1 + \beta_{th})^2} \quad \text{for } \beta_{th} < y < \beta_2 \quad (37d)$$

and

$$\epsilon_p(y) = 0 \quad \text{for } \beta_2 < y < 1 \quad (37e)$$

Zero plastic strain at $y=\beta_2$ requires (from Eq. 37d)

$$(\beta_2 - \beta_1) \left[-\kappa + T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \right] - T \frac{4(\beta_2 - \beta_{th})}{(1 - \beta_{th})(1 + \beta_{th})^2} = 0 \quad (37f)$$

Condition of zero axial load (Eq. 4a) with Eqs 37a-f gives

$$\epsilon = \frac{1}{4} \left[(1 - \beta)^2 + \beta_2 (\beta_2 - \beta_1) \right] \left[\kappa - T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \right] + T \frac{(\beta_2 - \beta_{th})(\beta_2 + \beta_{th} - \beta_1)}{(1 - \beta_{th})(1 + \beta_{th})^2} \quad (38)$$

Similarly, Eq. 4b together with Eqs 37a-f can be solved to give

$$\kappa = -\frac{4 \left(M - T \frac{\{2 - 3\beta_{th} + \beta_{th}\beta_1^2 + \beta_2(\beta_2^2 - \beta_1^2)\}}{(1 - \beta_{th})(1 + \beta_{th})^2} \right)}{[(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)]} + T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \quad (39)$$

Substituting for κ from Eq. 39 in Eqs 30 and 38, and eliminating ϵ from Eqs 30 and 38,

$$\begin{aligned} & -M[(1 + \beta)^2 + \beta_2(\beta_2 - \beta_1)] + \frac{T}{(1 + \beta_{th})^2(1 - \beta_{th})} \left[\{(1 + \beta)^2 + \beta_2(\beta_2 - \beta_1)\} \{2 - 3\beta_{th} + \beta_{th}\beta_1^2 \right. \\ & \left. + \beta_2(\beta_2^2 - \beta_1^2)\} - \{\beta_2(\beta_2 - \beta_1) + 1 - 2\beta_{th} + \beta_{th}\beta_1\} \{(1 + \beta)^2(2 - \beta_1) + \beta_2(\beta_2^2 - \beta_1^2)\} \right] \\ & = [(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)] \end{aligned} \quad (40)$$

Subtracting the equation (using Eq. 2) for compressive yielding at $y = \beta$ from that for tensile yielding at $y = \beta_1$ and solving for κ ,

$$\kappa = -\frac{2}{(\beta + \beta_1)} + T \frac{(1 - \beta_{th})(2 + \beta_{th})}{(1 + \beta_{th})^2} \quad (41)$$

Eliminating κ from Eqs 39 and 41,

$$\begin{aligned} & -M(\beta + \beta_1) + \frac{T}{(1 + \beta_{th})^2(1 - \beta_{th})} [2 - 3\beta_{th} + \beta_{th}\beta_1^2 + \beta_2(\beta_2^2 - \beta_1^2)] \\ & = \frac{1}{2} [(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)] \end{aligned} \quad (42)$$

Similarly, subtracting the equation (using Eq. 2) for compressive yielding at $y = \beta$ from that for tensile yielding at $y = \beta_2$ and solving for κ ,

$$\kappa = -\frac{2}{(\beta + \beta_2)} + \frac{T}{(1 - \beta_{th})(1 + \beta_{th})^2} \left[(1 - \beta_{th})^2(2 + \beta_{th}) - \frac{4(\beta_2 - \beta_{th})}{(\beta + \beta_2)} \right] \quad (43)$$

Eliminating κ from Eqs 39 and 43,

$$\begin{aligned} & M(\beta + \beta_2) - \frac{T}{(1 - \beta_{th})(1 + \beta_{th})^2} \left[2 - 3\beta_{th} + \beta_{th}\beta_1^2 - (\beta_2 - \beta_{th}) \{(1 + \beta)^2(2 - \beta) \right. \\ & \left. + \beta_2(\beta_2^2 - \beta_1^2)\} \right] = -\frac{1}{2} [(1 + \beta)^2(2 - \beta) + \beta_2(\beta_2^2 - \beta_1^2)] \end{aligned} \quad (44)$$

Solving for T by eliminating M from Eqs 40 and 42 and the same from Eqs 40 and 44, equating the two expressions for T, and simplifying,

$$\beta^2 = \beta_1^2 + \frac{(\beta_{th} - \beta_1)}{(\beta_2 - \beta_{th})}(1 - \beta_2)^2 \quad (45)$$

For a given β and β_1 , β_2 can be solved for from Eq. 45. Eliminating κ from Eqs 41 and 43, T can be solved for in terms of β , β_1 , and β_2 .

$$T = \frac{(1 - \beta_{th})(\beta_2 - \beta_1)(1 + \beta_{th})^2}{2(\beta_2 - \beta_{th})(\beta + \beta_1)} \quad (46)$$

M, κ and ϵ can be solved from Eqs. 44, 39, and 38, respectively and the stresses at various locations can be obtained from Eq. 2.

Shakedown in one cycle ($M \leq -\beta_{th}$):

As before, tensile yielding will not occur at $y=1$ provided (using Eq. 5a),

$$\sigma(y=1) + T \leq 1 \quad (33a)$$

However, unlike before, a closed form solution for shakedown in one cycle was not derived; it was obtained numerically.

It can be shown that for $M \leq -\beta_{th}$ and $T=2$, there will be compressive yielding at $y=1$, and, as before, to prevent reversed tensile yielding at $y=1$ on unloading of thermal load,

$$T \leq 2 \quad (13a)$$

Calculations of shakedown beyond one cycle of loading and unloading becomes rather complicated algebraically. However, the shakedown theorem of Melan [7] can be used to determine the shakedown regime without carrying out such a detailed calculation.

II.C Application of Shakedown Theorem

Shakedown theorem of perfect plasticity (zero strain hardening) states that if a state of residual stress field satisfying equilibrium and yield condition can be found such that when superimposed on the cyclic elastic stress field the resultant stress field also satisfies yield condition everywhere, then the structure will ultimately shakedown. Unfortunately the theorem does not give any indication as to how much ratcheting strain is accumulated prior to shakedown. However, in most cases the approach to shakedown is rapid and the final ratcheting strain is approached exponentially with cycle.

II.C.1 $M \geq 0$

For relatively small primary bending moments, using the same non-dimensionalizing scheme as in section II.A, the residual stress field is shown in Fig. 2a. The stress field satisfies equilibrium, yield condition and results in an axial load = 0 and a bending moment $M = \beta$. Note that there are no yielded zones although three points in the section reach yield stress. The maximum value of the thermal stress is obtained by superimposing the two stress fields and requiring that

$$\begin{aligned} -T+1 &\geq -1 \\ \text{i.e., } T &\leq 2 \end{aligned} \tag{47}$$

The maximum value of $\beta = \beta_2 = \frac{1}{2 - \beta_{th}}$ beyond which the yield condition is violated.

The bending moment at this stage is $M_1 = \beta_2$. The total stress field due to thermal and primary bending stresses corresponding to $\beta = \beta_2$ is shown in Fig. 2b. Note that the beam has a compressive yield zone extending from $y = 1$ to $y = \beta_2$, but no tensile yield zone.

For $M > M_1$, a residual stress field as shown in Fig. 3a is assumed. There is a compressive yield zone extending from $y = \beta_1$ to $y = \beta_2$ and a tensile yield zone extending from $y = -1$ to $y = -1 + (\beta_2 - \beta_1)$. The maximum tensile value of the total stress, shown in Fig. 3b, reaches yield when

$$\begin{aligned} \beta_2 - \beta_1 &= 1 - \beta_3 \\ \text{where } \beta_3 &= \frac{1}{2 + \beta_{th}} \end{aligned}$$

Fig. 2 (a) Distributions of elastically calculated thermal stress (dotted lines) and typical residual stresses (solid lines) corresponding to a steady positive primary bending moment without any yielding through the thickness of the first wall. (b) Distribution of combined residual and thermal stress during plasma-on condition with a compressive yield zone but no tensile yielded zone.

The tensile yield zone of the residual stress field in Fig. 3a cannot extend beyond $y = -\beta_3$ without violating the yield condition for the total stress. The value of the primary bending moment at this stage is

$$M_2 = \beta_2 + \frac{(\beta_2 + \beta_3)(1 - \beta_3)}{2} \quad (48)$$

and the maximum permissible temperature is still $T=2$.

(a)

(b)

Fig. 3 (a) Distributions of elastically calculated thermal stress (dotted lines) and residual stress (solid lines) corresponding to a steady positive primary bending moment with tensile and compressive yielding through the thickness of the first wall. (b) Distributions of combined residual and thermal stress during plasma-on condition resulting in a compressive yielded zone and onset of tensile yielding.

For $M > M_2$, the maximum tensile residual stress at the plasma edge varies between 1 and X ($-1 \leq X < 1$) and the distribution of the tensile residual stress field from $y = -1$ to $y = -\beta_1$, shown in Fig. 4a, is such that the total stress is at yield from $y = -\beta_3$ to $y = -\beta_1$, as shown in Fig 4b. The unknown quantities for the problem are stress X and distance β_1 which are determined by solving simultaneously the requirements that during plasma-on, the yield condition is satisfied at $y = 1$ and that the net axial load over the whole cross-section is zero, which lead to the following equations, respectively:

$$X = 1 - T \quad (49a)$$

$$\frac{T(1 - \beta_{th})}{(\beta_3 + \beta_{th})}(\beta_3 - \beta_1)^2 + 4\beta_1 - T(1 - \beta_2) = 0 \quad (49b)$$

and the corresponding bending moment M is

$$M = \frac{3}{2} + \frac{1}{2}\beta_1(2\beta_3 + \beta_1) - \frac{1}{4}T(1 - \beta_2)(2 + 2\beta_1 + \beta_2 + \beta_3) \quad (49c)$$

For a given T ($0 \leq T \leq 2$), Eqs 49a-c can be solved for M . The allowable temperature T reduces to zero when $X=1$ and, from Eq. 49b, when $\beta_1=0$, which when substituted in Eq. 49c gives the maximum bending moment $M_{\max}$ as

$$M_{\max} = \frac{3}{2} \quad (50)$$

independent of β_{th} .

II.C.2 $M \leq 0$:

In general, the maximum stress at the plasma edge varies between X ($-1 \leq X \leq 1$) and 1, and the distribution of the compressive stress field from $y=-1$ to $y=-\beta_3$, shown in Fig. 5b, is such that the residual stress (obtained by subtracting the elastic thermal stress from the total stress) is at compressive yield from $y = -\beta_3$ to $y = -\beta_1$, as shown in Fig 5a. The unknown quantities for the problem are stress X and distance β_1 which are determined by solving simultaneously the requirements that during plasma-on, the yield condition is satisfied at $y=1$ and that the net axial load over the whole cross-section is zero. Noting the similarity between the residual stress field of Fig. 4a and the total stress field of Fig 5b, the following equations can be directly obtained from Eqs 49a-c :

$$X = 1 - T \quad (51a)$$

$$\frac{T(1 - \beta_{th})}{(\beta_3 + \beta_{th})}(\beta_3 - \beta_1)^2 + 4\beta_1 - T(1 - \beta_2) = 0 \quad (51b)$$

and the bending moment M is

$$M = -\frac{3}{2} - \frac{1}{2}\beta_1(2\beta_3 + \beta_1) + \frac{1}{4}T(1 - \beta_2)(2 + 2\beta_1 + \beta_2 + \beta_3) \quad (51c)$$

(a)

(b)

Fig. 4 (a) Distributions of elastically calculated thermal stress (dotted lines) and residual stress (solid lines) corresponding to a steady positive primary bending moment with tensile and compressive yielding through the thickness of the first wall. (b) Distribution of combined residual and thermal stress during plasma-on condition resulting in a compressive as well as a tensile yielded zone.

Fig. 5 (a) Distribution of residual stress corresponding to a steady negative primary bending moment. (b) Distributions of elastically calculated thermal stress (dotted lines) and combined residual and thermal stress (solid lines) during plasma-on condition.

As before, the maximum value of the negative bending moment corresponding to $T=0$ is obtained by setting $\beta_1=0$,

$$M_{\max} = -\frac{3}{2} \quad (52)$$

II.C.3 Steady Thermal and Cyclic Primary Bending Stresses:

A simple residual stress field satisfying equilibrium and yield condition, irrespective of the magnitude of the thermal loading, is the zero stress field. Therefore, the cyclic elastic stress fields corresponding to bending moments of $M = \pm 1$ together with the residual

stress field will not violate the yield condition. Thus the shakedown regime for this type of loading is

$$-1 < M < 1 \quad (53)$$

III. RESULTS

A plot of the shakedown diagram for a steady primary bending moment and cyclic thermal loading, regardless of how many cycles of load-unload cycles are required to achieve shakedown, is shown in Fig. 6. Although the axes are given in terms of bending moment and temperature, they could equally well be denoted by primary bending stress and thermal stress, respectively. Note that in contrast to the elastic regions, the shakedown regions are symmetrical about $M=0$. Both the elastic and shakedown regimes increase with increasing β_{th} , which is the distance of the coolant channel from the neutral axis (see Fig. 1).

Fig. 6 Elastic and shakedown regimes for a steady primary bending moment and cyclic thermal loading.

A disadvantage of a shakedown diagram like Fig. 6 is that we do not have any estimate of the amount of ratcheting strain accumulated prior to shakedown is achieved. A more conservative approach is to have a diagram that shows loading combinations for which shakedown is achieved in one load-unload cycle. Such plots (denoted by S_1) are shown for $\beta_{th} = 0.5, 0.25$, and 0 in Figs. 7, 8, and 9, respectively. However, note that in all cases loading can be increased significantly beyond the S_1 region without having steady ratcheting.

Fig. 7 Elastic and shakedown regimes for a steady primary bending moment and cyclic thermal loading for cases with $\beta_{th}=0.5$. S_1 denotes load combinations for which shakedown is achieved in one load-unload cycle. S_2 denotes load combinations for which shakedown is achieved in more than one load-unload cycles.

Fig. 8 Elastic and shakedown regimes for a steady primary bending moment and cyclic thermal loading for cases with $\beta_{th}=0.25$. S_1 denotes load combinations for which shakedown is achieved in one load-unload cycle. S_2 denotes load combinations for which shakedown is achieved in more than one load-unload cycles.

Fig. 9 Elastic and shakedown regimes for a steady primary bending moment and cyclic thermal loading for cases with $\beta_{th}=0$. S_1 denotes load combinations for which shakedown is achieved in one load-unload cycle. S_2 denotes load combinations for which shakedown is achieved in more than one load-unload cycles.

Finally, the shakedown diagram for steady thermal and cyclic primary bending loading is shown in Fig. 10. Note that although there is no limit to the thermal loading as long as the bending moment $M \leq M_y$, the amount of ratcheting strain prior to shakedown has not been determined.

Fig. 10 Elastic and shakedown regimes for a steady thermal loading and cyclic primary bending moment.

IV. CONCLUSIONS

Shakedown analyses of a cooled first wall were carried out for cases where the stresses are predominantly bending. Shakedown diagrams were constructed delineating zones representing elastic cycling, shakedown in one cycle, and general shakedown. These diagrams may be useful for the design of fusion reactor first wall.

Although the diagrams for eventual shakedown (S_2) are symmetrical about $M=0$, those for shakedown in one load-unload cycle (S_1) are generally not. Zones of combinations of loading for which shakedown occurs after one cycle of load-unload are significantly smaller than those leading to eventual shakedown. However, in the latter cases, an undetermined amount of ratcheting strain will accumulate prior to shakedown.

In the case of steady thermal and cyclic primary bending stress, shakedown will occur irrespective of the magnitude of the thermal stress provided the primary bending moment is less than the moment to cause first yielding of the section.

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