TWO-PHASE POTENTIAL FLOW

Progress Report

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Abstract

Progress towards developing a mature science of *two-phase potential flow* is reported in five areas:

- Basic theorems
- Results for ellipsoids
- Properties of arrays of spheres
- Trajectories of interacting spheres
- Unit cell models.

1. Introduction

We have continued to pursue the objective of developing the basic science of two-phase potential flow. Remarkably, we have found that there is still scope for the development of fundamental theorems at a very general level, as well as specific results for well-defined systems, such as arrays of spheres.

Specific results during the past year include:

- Several basic theorems of field theory
- Basic theorems for spheres in potential flow
- A synthesis of all the results for ellipsoids given in Lamb¹
- Basic theorems for arrays of spheres in a potential flow
- Computation of the properties of non-isotropic arrays of spheres
- Computation of the trajectories of interacting spheres
- Further development of the basic unit cell models, such as the sphere-in-sphere model.

2. Results

a) *Basic Theorems.*

For any potential field for which

$$\mathbf{v} = -\nabla \phi \quad (1)$$

the *polarization*, or net dipole moment of internal sources, within any region is given by

$$\mathbf{D} = \int \phi \, d\mathbf{s} + \int \mathbf{r} \, \mathbf{v} \cdot d\mathbf{s} \quad (2)$$

moreover,

$$\int \phi \, d\mathbf{s} = -\frac{1}{2} \int \mathbf{r} \times (\mathbf{v} \times d\mathbf{s}) \quad (3)$$

When the region contains a particle volume V_2 translating at speed $\mathbf{v}_2$, the above equations lead to

$$\mathbf{D} = \int \phi \, d\mathbf{s} + \mathbf{v}_2 V_2 \quad (4)$$

and

$$\mathbf{D} = \frac{1}{2} \int \mathbf{r} \times d\Gamma \quad (5)$$

where

$$d\Gamma = (\mathbf{v}_2 - \mathbf{v}) \times d\mathbf{s} \quad (6)$$

is the circulation element corresponding to the velocity jump at the interface.

Equations (4) and (5) relate the *impulse* of the particle to its internal *flux sources* or surface *circulation sources*. From the impulse, the momentum and kinetic energy of the surrounding fluid can be computed for use in conservation laws.

b) Theorems for Spheres

It is fairly well-known that the potential at the center of a spherical source-free region is equal to the average potential on the surrounding sphere. We have shown that the *field* at the center is equal to the volumetric average of the *field* throughout the sphere, which is useful for evaluating certain averages.

Also:

- The dipole moment of the image in a sphere of radius "a" of an external source or dipole is equal to $-2 \pi a^3$ times the field at the center of the sphere created by that source. This is useful for evaluating the polarization when all external sources are known.
- The impulse at the surface of a sphere due to any internal source is equal to one-third the moment of that source about the center of the sphere.

c) Ellipsoids

The impulse due to a source of strength m_k at the location r_k anywhere in an ellipsoid is

$$\int \phi \, ds = m_k \underline{\underline{A}} \cdot r_k \quad (7)$$

where $\underline{\underline{A}}$ is a symmetric tensor of magnitude unity related to the added mass tensor $\underline{\underline{C}}$ by

$$\underline{\underline{C}} = (\underline{\underline{I}} - \underline{\underline{A}})^{-1} - \underline{\underline{I}} \quad (8)$$

This relationship synthesises all the results for ellipsoids in Lamb¹ and allows $\underline{\underline{A}}$ to be computed in the most convenient way without solving for the entire field.

d) Arrays of Spheres

We have derived a general "constitutive equation" relating the dipole density per unit total volume of the array, d , to the relative velocity, w , or drift flux, j_{21} , in the form

$$\underline{d} \cdot \left[(1 - \alpha) \underline{I} - \alpha \underline{\lambda} \right] = -\frac{3}{2} \alpha (1 - \alpha) \underline{w} = \frac{3}{2} \underline{j}_{21} \quad (9)$$

$\underline{\lambda}$ is a tensor that can be determined by making a sum of reflections in a single sphere from spheres within a spherical or cubical volume surrounding the sphere. This approach avoids earlier possible problems of conditional convergence. Convergence has been obtained with a variety of non-isotropic arrays, yielding results compatible with an earlier correlation by Luo².

e) *Non-isotropic arrays.*

Using the results in (d), the macroscopic properties of non-isotropic arrays have been evaluated. Examples are the "compressibility" and "particle pressure" which are presented in a recent paper³ where compatibility was demonstrated with the sphere-in-tube model of Cai and Wallis⁴.

f) *Trajectories of Spheres.*

Using the theorems in (b) we evaluated the impulses for independent spheres from the images of other interacting spheres. This enables the overall kinetic energy to be evaluated and hence the equation of motion of each sphere derived from Hamilton-Lagrange methods. The results show how the average motion interacts with relative (not necessarily random) motion: one of the fundamental questions in the field.

g) *Sphere-in-sphere model.*

Using Hamilton-Lagrange methods we have derived more general expressions for the motion under forces of the constituents of this simple "unit cell". This suggests possibilities for experimental confirmation and for extension of the approach to other unit cells that could be used to build up finite arrays, for example.

3. Publications

Since our previous progress report the following articles have appeared in the peer-reviewed literature:

Wallis, G.B., S.P. Harvey and R. DeFelice "Decompression Waves in Fluidized Beds," *Int. J. Multiphase Flow*, 19(5), 839-874, 1993.

Wallis, G.B., "The Concept of Polarization in Dispersed Two-Phase Potential Flow," *Nuclear Engineering and Design*, 141, 329-342, 1993.

Wallis, G.B., "The Particle Pressure of Arrays in a Potential Flow," *Nuclear Engineering and Design*, in press, 1994.

Cai, X. and G.B. Wallis, "A More General Cell Model for Added Mass in Two-Phase Flow," *Chemical Engineering Science*, 49(10), 1631-1638, 1994.

Other results in Section 2 are now being assembled into draft papers that will be submitted during the remainder of this contract period.

4. Personnel

Professor Graham B. Wallis directed the work. He was supported 10% of the academic year and 0.6 of a month off-term.

Two Ph. D. students, Oleg Dubitsky and Marlene McDonald were supported as graduate research assistants.

5. Award

In May, 1994, Professor Wallis received notification that he was the recipient of the "Fluids Engineering Award" from the American Society of Mechanical Engineers specifically for his work on two-phase potential flow.

References

1. Lamb, Sir Horace, *Hydrodynamics*, 6th Edition, Cambridge University Press, 1932.
2. Luo, C., "Inertial Coupling of Spherical Particles Dispersed in a Two-phase Potential Flow," Ph. D. Thesis, Thayer School of Engineering, Dartmouth College, 1992.
3. Wallis, G.B., "The Particle Pressure of Arrays in a Potential Flow," *Nuclear Engineering and Design*, in press, 1994.
4. Cai, X. and G.B. Wallis "Flow Around a Row of Spheres in a Circular Tube," *Physics of Fluids A*, 4(5), 904-912, 1992.