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ANGLE INTERFEROMETER CROSS AXIS ERRORS

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Summary: Angle interferometers are commonly used to measure surface plate flatness. An error can exist when the centerline of the double corner cube mirror assembly is not square to the surface plate and the guide bar for the mirror sled is curved. Typical errors can be one to two microns per meter.

A similar error can exist in the calibration of rotary tables when the centerline of the double corner cube mirror assembly is not square to the axes of rotation of the angle calibrator and the calibrator axis is not parallel to the rotary table axis. Commercial double corner cube assemblies typically have non-parallelism errors of ten milli-radians between their centerlines and their sides and similar values for non-squareness between their centerlines and end surfaces. The authors have developed a simple method for measuring these errors and correcting them by remachining the reference surfaces.

Keywords: Metrology, Angular Measurements, Calibration

Measurement of Surface Plate Flatness

Introduction

Before the invention of the angle laser interferometer, surface plate flatness was generally measured with autocollimators and mirror sleds. Rolt, (Ref. 1) The need to adjust the body of the autocollimator in roll so the cross hairs are parallel and perpendicular to the surface plate was well known and incorporated into most surface plate flatness measurement procedures. The test for correct adjustment was (is) to manually rotate the mirror sled or autocollimator body through a small angle in yaw while observing the cross hair image. If the adjustment is correct, the return image will not appear to rise or fall with respect to the horizontal crosshair. It only takes a few minutes to make this roll adjustment using the adjusting screws on the autocollimator support stand. When the adjustment is correct, rotations of the mirror sled in yaw caused by curvature of the guiding straightedge have a negligible effect.

An autocollimator has an advantage over a laser interferometer because it can be read in the second (orthogonal) axis by estimation. An angle interferometer (Figs. 1 and 2) can only be read about one axis. This limitation complicates the problem of achieving the correct alignment.

Fig. 1. The laser angular interferometer is used to measure surface plate flatness.

Fig. 2. Definitions of "Yaw" motion and "Roll Angle"

A few years ago, we began to think about the relationship between the cross hairs of an autocollimator and the "measurement plane" of an angle interferometer. When we performed the functional test of rotating the double corner cube mirror sled in yaw by hand through a small angle we observed large changes in the interferometer reading. At first we thought the "measurement plane" of the interferometer was determined by the plane of the two outgoing laser beams and that this

plane needed to be square with the surface plate in the same sense that the measurement plane of an autocollimator (reading in pitch) needs to be square with the surface plate.

We were surprised when Dr. Ken Wayne, of Hewlett-Packard, pointed out that this is definitely not the case. The measurement plane of an angle interferometer is the plane that bisects the plane of the two outgoing beams and the two return beams. The measurement plane is independent of the angle of the interferometer and depends only on the angle of the double corner cube reflector. The explanation for this surprise is that any change of an incoming beam with respect to a corner cube results in the same change of the outgoing beam in the opposite direction.

We then realized (1) that the double corner cube assembly has no provision for adjustment, (2) that the operating instructions for angle interferometers make no mention of a functional test and (3) that the centerlines (see definition, Fig. 3) of commercially available double corner cube assemblies are not square with their feet.

Analysis of the Surface Plate Measurement Error

The double corner cube and sled assembly used to measure surface plates is illustrated in Fig. 3. The centerline is not square to the base of the housing creating a non-squareness angle ω of the centerline to the plane of the mirror feet.

Fig. 3. Analysis of errors for surface plate measurement using the angle interferometer.

In the Front View,

$$F = L \sin \omega \quad (1)$$

and in the Top View,

$$D = F \sin \phi \quad (2)$$

The angular error in the reading, in the presence of ω and ϕ , is defined as:

$$E = \arcsin(D/L) \quad (3)$$

$$E = \arcsin[(L \sin \omega)(\sin \phi)/L] \quad (4)$$

For small angles, $\sin \omega \equiv \omega$, $\sin \phi \equiv \phi$, $\sin E \equiv E$. The angular error reduces to:

$$E = \omega \phi \quad (5)$$

where ω and ϕ are expressed in radians

The error in interferometer reading is the product of the "roll angle" (non-squareness of centerline to the surface plate) and the "yaw angle" of the sled caused by a curved straightedge.

Real straightedges used in industry to guide the mirror sled might easily have straightness errors of 100 micro-meters. If the non-straightness of the straightedge is a portion of a circle, it is possible to estimate the change in angle by the following small angle equation:

$$S = (1/8) T\theta$$

where:

S = non-straightness in micro-meters

T = length of the straightedge in meters

θ = the change in angle in micro-radians

Example #1:

For a straightedge having a uniformly curved non-straightness of 100 micro-meters and a length of one meter, the change in angle is:

$$\begin{aligned} \theta &= 8 S/T \\ &= 8 (100 \text{ micro-meters}) / 1 \text{ meter} \\ &= 800 \text{ micro-radians} = 0.8 \text{ milli-radians} \end{aligned}$$

If the corner cube is non-square by 10 milli-radians, the error in interferometer reading is:

$$\begin{aligned} E &= (\text{the angle of roll})(\text{the angle of yaw}) \\ E &= (10 \text{ milli-radians})(0.8 \text{ milli-radians}) = 8 \text{ micro-radians} \end{aligned}$$

The corresponding error in flatness measurement for a one meter long element of a surface plate is:

$$\begin{aligned} S &= (1/8) T\theta \\ &= 1/8 (1 \text{ meter})(8 \text{ micro-radians}) \\ &= 1 \text{ micro-meter in 1 meter} \end{aligned}$$

This error may not appear to be very serious at first glance, but it represents 40% of the working tolerance of a A-A grade granite surface plate meeting the requirements of American Standard GGG-P-463c (ref. 4).

For recommendations on controlling surface plate cross axis measurement errors, please see the **Conclusions**.

Measurement of Angular Displacement

Introduction

A similar problem occurs when using an angle interferometer and angle calibrator to evaluate angular displacement systems such as rotary tables. (see Fig. 4) Before the invention of the angle interferometer, autocollimators were used to read the error differences between polygons, serrated face calibrators, and rotary tables. The procedures specifying these methods called for tests to demonstrate that the axis of the polygon or calibrator was parallel to the rotary table axis and that the cross hairs of the autocollimator were parallel and square with the rotary table axis.

These tests were relatively easy to perform because an autocollimator can be read in two axes. The test for polygon parallelism consisted of rotating the polygon and rotary table through 360 degrees while reading the reflection of each of the mirrors of the polygon in pitch.

Fig. 4. Typical setup for calibrating a rotary table in the metrology laboratory using an angle interferometer.

The test for autocollimator squareness consisted of rotating the rotary table and polygon through small angles and checking to be sure there was a negligible pitch reading (rise or fall of the cross hair with respect to the fiducial).

The procedure for testing parallelism of the axis of serrated face calibrators is to observe the pitch angle of the flat mirror mounted on the calibrator as the rotary table is rotated through 360 degrees with the anti-rotation bar of the calibrator (or its equivalent) in place. Autocollimator squareness to the calibrator can be demonstrated by checking for pitch (rise and fall of the cross hair) as the calibrator is rotated through small angles in yaw.

An angle interferometer can be setup with the double corner cube centerline parallel to the axes to read axes parallelism error. There is no equivalent test, however, to functionally demonstrate that the double corner cube centerline is square to the axis of the calibrator since the angle interferometer is a single axis device. This error must be indirectly controlled.

Why Has This Problem Been Overlooked For So Long?

Rotary tables are traditionally calibrated in Metrology Laboratories which have good working conditions (Fig. 4). Rotary tables usually have table top surfaces that are flat and square to their axis of rotation. Angle calibrators such as polygons and serrated face calibrators usually have their bottom surfaces flat and square to their axis of rotation. When the calibrator is placed on top of the rotary table the two axes of rotation automatically become parallel within a very small angle, and the error becomes negligible.

If, however, we are calibrating the angular displacement measuring systems found in modern machine tools, we may find that a flat and square surface to mount the calibration device may not exist, or it may not exist without dismantling the machine. Working conditions may not be ideal. Working tolerances, on the other hand, can be tenths of arc-seconds.

Some examples are the turret on a fully enclosed, slant bed, turret lathe, the tilt axis of a five-axis milling machine, or the rotary table on a grinder that has a grinding spindle mounted to its top surface. A similar problem can arise when calibrating a rotary encoder. A bare shaft may be the only surface available for attachment of the calibrator.

In these situations it becomes necessary to build and install suitable bracketry to connect the calibrator to the axis to be calibrated. A question then arises as to the accuracy of the bracketry, or how well it needs to be adjusted if it is adjustable. The following analysis provides the information that is necessary to answer these questions.

Analysis Of The Angular Displacement Error

Figure 5 shows a typical setup using the double corner cube, a calibrator, and a rotary table. The anti-rotation device is in place to keep the double corner cube roughly aligned with the laser during rotation of the rotary table and calibrator. The analysis assumes that the axes of rotation are perfect, the calibrator is perfect, the interferometer is perfect, but there is an imperfection in the alignment of these components.

The *Pitch Angle* $A(\theta)$ (Fig. 5—SIDE VIEW) is produced by the angle α . See the definitions in Fig. 5.

Fig. 5. The top of the calibrator and the corner cube are prevented from rotating while the bottom of the calibrator and the top of the rotary table rotate.

$A(\theta)$ is the projection of angle α in the pitch plane and is expressed as:

$$A(\theta) = \arctan(\tan \alpha \cos \theta)$$

for small angles, $\tan \alpha \cong \alpha$ and $\tan A(\theta) \cong A(\theta)$ this reduces to:

$$A(\theta) = \alpha \cos \theta \quad (6)$$

The *Roll Angle* $B(\theta)$ (Fig. 5—FRONT VIEW), however, is produced by the sum of the angle ω and the projection of angle α in the roll plane and is expressed as:

$$B(\theta) = \omega + \arctan(\tan \alpha \sin \theta)$$

As before this reduces to:

$$B(\theta) = \omega + \alpha \sin \theta \quad (7)$$

F is defined as the difference in height of the corner cubes:

$$F(\theta) = L \sin B(\theta) \quad (8)$$

and D is the difference in laser path lengths due to the *Roll* and *Pitch Angle* contributions:

$$D(\theta) = F(\theta) \sin A(\theta) \quad (9)$$

Combining (7) and (8) yields:

$$F(\theta) = L \sin(\omega + \alpha \sin \theta) \quad (10)$$

and combining (6), (9), and (10) yields:

$$D(\theta) = L \sin(\omega + \alpha \sin \theta) \sin(\alpha \cos \theta) \quad (11)$$

The angular error E is the angle measured from the initial position and is expressed as:

$$E(\theta) = \arcsin[D(\theta)/L] \quad (12)$$

For small angles, $\sin E(\theta) \cong E(\theta)$,

$$E(\theta) = D(\theta)/L = \sin(\omega + \alpha \sin \theta) \sin(\alpha \cos \theta) \quad (13)$$

The above expression can be further simplified to yield:

$$\begin{aligned} E(\theta) &= (\omega + \alpha \sin \theta)(\alpha \cos \theta) \\ &\text{or} \\ E(\theta) &= (\alpha^2 \sin \theta \cos \theta) + (\omega \alpha \cos \theta) \end{aligned} \quad (14)$$

$E(\theta)$ is the error from the initial position as a function of θ . For one full revolution, the total error will be the maximum error from any point to any other point. Total Error = $2[E_{\max}]$

Typical Magnitude of the Error

The error can easily amount to four arc-secs on machine tools where special bracketry is required. The four second value assumes that the non-parallelism of axes angle α is one milli-radian, and that a commercially available, unmodified, double corner cube having a 10 milli-radian parallelism error, (ω), is mounted to the face of the calibrator. Substituting in equation (14):

$$E_{\max} = 2[(1 \text{ milli-radian})^2(.707)(.707) + (10 \text{ milli-radians})(1 \text{ milli-radian})(1)]$$

$$= 21 \text{ micro-radians} = 4.3 \text{ arc-secs}$$

Some Error Remains For Perfect Corner Cubes

Equation (14) tells us that non-parallel axes will cause an error even if the corner cube centerline is perfectly square with the calibrator axis.

In the case of $\omega = \text{zero}$ equation(14) becomes:

$$E(\theta) = \alpha^2 \sin \theta \cos \theta$$

For $\theta = 45 \text{ degs.}$ (maximum error), and $\alpha = 1 \text{ milli-radians}$

$$\begin{aligned} E_{\text{max}} &= (1 \text{ milli-radian})^2 (.707)(.707) \\ &= 0.5 \text{ micro-radians} \\ &= 0.1 \text{ arc-sec.} \end{aligned}$$

$$\begin{aligned} \text{Total Error} &= 2 (E_{\text{max}}) \quad (\text{for 1 full revolution}) \\ &= 0.2 \text{ arc-secs.} \end{aligned}$$

Measuring The Centerline Parallelism Error

Fig. 6. Set up for determination of double corner cube centerline parallelism.

Measurement of the non-parallelism of the corner cube centerline to its housing can be done using conventional machine shop tools. These tools consist of an angle plate, a surface plate, a C-clamp, a straightedge, a set of angle gage blocks and the angle laser interferometer itself. Figures 6 and 7 show these components.

Figure 8 defines the angles B , W , and E . The centerline non-parallelism is B , and W is the angle plate non-squareness. E is the interferometer reading.

Figure 9 defines E_1 and E_2 as the change in interferometer reading when the angle plate/corner cube assembly is rotated through the yaw angle

R (Fig. 6) in setup #1 and setup #2. This procedure relies upon the angle plate being repeatably rotated through angle R .

Setup #1 shows an apex on one end of the corner cube which is located downward. This apex does not exist on real corner cubes. It is shown to illustrate the "upside down" reversal of the corner cube as the angle

plate is reversed. It is, of course, necessary to remove the C-clamp to permit the reversal of the corner cube with respect to the angle plate. Some care is necessary to be sure that the corner cube housing surface contacts the same area on the angle plate. Extraordinary care is not necessary, since we only need to measure the angle "B" to a tolerance of a few tenths of a milli-radian.

Fig.7. Set up used to determine centerline parallelism.

Fig. 8. Definitions for angles W, B, and E.

Figure 9 also shows that we have a sign reversal on the effect of non-squareness of the angle plate for setup #1 and setup #2.

Figure 10 shows that the centerline non-parallelism "B" is the arithmetic average of the interferometer readings for setup #1 and setup #2 divided by the yaw angle R.

Fig. 9. Two set-ups are necessary to determine the angle B.

Fig. 10. Angle B is determined with this equation.

Correcting The Centerline Parallelism Error

Once the angle B is measured it is not too difficult to set up a sine plate or sine vise on a surface grinder, adjust it to the angle B, clamp the corner cube in place and regrind the housing. Figure 11 is a photograph of the setup. The variation in the width of the chamfer gives an idea of the magnitude of the original angle B which was 9.2

milli-radians. The original chamfer of 1 mm has been reduced to nearly zero at one end.

After one side has been ground parallel with the centerline, it is relatively easy to grind the opposite side parallel with the first.

Fig. 11. The centerline parallelism is corrected by grinding the side of the housing.

Correcting The Centerline Squareness Error

For surface plate measurements it is important that the corner cube centerline be square to the end of its housing. Regrinding of the end can also be done on the surface grinder by clamping the reground side of the housing to an accurate angle plate.

If the end of the housing is square to the centerline, the only remaining error is the possibility of non-parallelism of the top of the mirror sled to its feet. Regrinding of this surface is also relatively easy to do if measurements show that it is necessary.

Conclusions:

1. Angle interferometer cross axis errors for surface plates must be controlled by controlling the arithmetic product of:
 - (a) the curvature of guide bars, and
 - (b) the squareness of the corner cube centerline to the feet of its mirror support sled.

Equation 5 (see also Fig. 3) provides the necessary guidance to determine the tolerances for guide bar curvature and centerline squareness relative to the flatness accuracy required. Functional tests to demonstrate these tolerances should be written into future procedures.

2. Angle interferometer cross axis errors for angular displacement systems must be controlled by controlling the arithmetic product of:

- (a) the parallelism error of the axes of rotation, and
- (b) the squareness error of the corner cube centerline to the calibrator axis.

The squareness error of the corner cube centerline to the calibrator axis must be controlled by controlling the arithmetic product of:

- (i) the parallelism error of the corner cube centerline to its housing, and
- (ii) the face runout of the housing with respect to the calibrator axis.

Figure 5 and equation 14 provide the necessary guidance to determine the tolerances for axes parallelism and centerline squareness relative to the measurement accuracy required. Functional tests to demonstrate these tolerances should be written into future procedures.

The tests illustrated in Fig. 12 are examples of such tests. Figure 12a shows the test for axes parallelism. The anti-rotation bar is in place and the indicator is reading the rise and fall of one point on the housing as the rotary table is rotated through 360°.

Figure 12b shows the test for centerline squareness to the calibrator axis. The anti-rotation bar is removed, and the difference in indicator readings on the housing at any two points 180° apart is read as the calibrator is rotated.

If the calibrator design requires the table top to lift to permit rotation, the indicator may be subject to overtravel. This problem can be avoided by turning the rotary table sufficiently to allow the indicator to clear the corner cube housing when the calibrator top is lifted and turned. The rotary table is returned to the same angular position for the second reading.

This procedure may lead to suggestions that the corner cube be squared to the rotary table instead of the calibrator. This is undesirable, since the angle ω in equation 14 becomes equal to α instead of zero. The total error then becomes 3 times as large or 0.6 arc-secs for $\alpha =$ one milli-radian.

a. Top of Rotary Table and Bottom of Calibrator Rotates

b. Rotary Table is Stationary and Top of Calibrator Rotates

Fig. 12. a) Test for axes parallelism b) Test for centerline squareness to calibrator.

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